# Real Earnings and Business Cycles: New Evidence 

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#### Abstract

In the time domain, the observed cyclical behavior of the real wage hides a range of economic influences that give rise to cycles of differing lengths and strengths. This may serve to produce a distorted picture of wage cyclicality. Here, we employ frequency domain methods that allow us to assess the relative contribution of cyclical frequency bands on real wage earnings. Earnings are decomposed into standard and overtime components. We also distinguish between consumption and production wages. Frequency domain analysis is carried out in relation to wages alone and to wages in relation to output and employment cycles. Our univariate analysis suggests that, in general, the dominant cycle followed by output, employment, real consumer and producer wages and their components is 5-7 years. Consistent with previous findings reported in the macro-level literature, our bi-variate results show that the various measures of the wage are generally not linked to the employment cycle. However, and in sharp contrast with previous macro-level studies we find strong procyclical links between the consumer wage and its overtime components and the output cycle, especially at the 5-7 years frequency.


Keywords: Business cycles, real wages, co-movement, spectral analysis

Observed real wages are not constant over the cycle, but neither do they exhibit consistent pro- or counter-cyclical movements. This suggests that any attempt to assign systematic real wage movements a central role in an explanation of business cycles is doomed to failure. (Lucas, 1977)

## 1 Introduction

Recent years have witnessed a marked shift in economists' views of the behavior of real wages over the business cycle. The prevailing wisdom, emanating largely from aggregate time series investigations, is that wages are at most weakly procyclical. For example, Abraham and Haltiwanger (1995) conclude, "Correcting for all of the measurement problems, estimation problems, and composition problems does not lead to a finding of systematically procyclical or countercyclical real wages." In contrast, evidence based on individual-level longitudinal surveys (e.g. Bils, 1985; Solon et al., 1994) supports the notion that wages are strongly procyclical. This paper shows that another type of data disaggregation adds significant new insights into the issue of aggregate wage cyclicality. This concerns observing wage behavior in the frequency domain. We are interested in real annual hourly earnings in U.S. manufacturing where earnings are separated into those deflated by consumer prices and those by producer prices. In addition to the insights gained by applying frequency based methods to standard measures of the wage, we also find that our approach provides even more value-added if we break down earnings into constituent parts. Thus we also provide a method of earnings decomposition into the standard hourly wage, the overtime mark-up, and the proportion of overtime workers in the total workforce. ${ }^{1}$

To appreciate the potential value-added of employing frequency methods, consider three cycles of relatively short, medium and long periods. Although by no means hard and fast or exhaustive representations, these might consist

[^1]of (respectively) a wage contract cycle, a business cycle and a product cycle. Each type may associate systematically with the real wage. The relative strength and direction of the associations may differ, however. The start of a three-year United States wage contract, for example, may coincide with wage adjustments designed to correct for unforeseen economic events at the previous negotiation time point. This process may be expected to generate a mix of pro- and countercyclical wage effects through time depending on the direction of deviations from expected outcomes. Additionally, the wage may respond positively to the business cycle. For instance, where compensation relates to marginal product, human capital investment may produce procyclical wages stemming from the fixity of the labor input. The wage may also associate positively with the product cycle. Top quality workers earning relatively high pay may be matched with new and innovative products with strong growth potential. As these products are eventually superseded by new innovations, wages may subsequently be associated with the hire of relatively poor quality and less well-remunerated workers.

Yet, all three cyclical effects will serve to condition a long time series of the real wage. This gives rise to a series of critical questions. Which, if any, is the frequency band dominating the cyclical behavior of the wage? If a given frequency dominates, what direction and strength of cyclicality does it exhibit? Pursuing such lines of enquiry leads to a more general question. Is the observed wage cyclicality in the frequency domain supportive of the general view arising from aggregate time series analysis or does it serve to modify that view? A seeming low correlation between the wage and a measure of the cycle may simply reflect the fact that the underlying time series is composed of a number of cyclical components that are of different amplitudes and timing. Separately, one or more bands may display strong evidence of a systematic cyclical relationship. Taken together, countervailing influences may serve to mask underlying patterns.

The analysis of the wage's spectral representation allows us to tackle directly these issues since it can be decomposed into cyclical components defined over multiple economic cycle frequencies. The starting point is univariate analysis. A stationary time series can be broken down into superimposed
harmonic waves of varying phases and amplitudes. To determine the length of the dominant cycle it is necessary to search the spectrum between the endpoints of the entire frequency interval and select the cycle that contributes the greatest portion to the total variance of the wage. An apparent 5 -years cycle may simply reflect the fact that the time series possesses a dominant cycle of that length. Alternatively, it may disguise the fact that there are two other underlying cycles - one longer and one shorter than 5 years - that combine to give the appearance of a prevailing 5 -years cycle. Or there may be more than two underlying cycles. Investigating the frequency domain not only allows us to identify the number of cycles a series possesses but also to determine the contribution of each cycle to explaining the total variance of the wage and whether at this frequency the explained variance is significant.

However, the main economic interest behind the study of the frequency domain lies in an extended framework. We cannot discern whether observed cycles in the wage reflect underlying economic conditions unless we relate them to other variables that are both reflective of the ups and downs of economic activity and exhibit strong associations with the wage itself. Following the existing literature on measuring wage cyclicality, we adopt output and employment for this latter role. On the wage side, we focus on average hourly wage earnings. Moreover, we develop a method of decomposing hourly earnings into the standard wage, the premium mark-up, and the proportion of overtime workers. Spectral methods are applied to each component. We find that not only does each component respond to output and employment cycles but that responses vary significantly across components. ${ }^{2}$ This allows us to gain insights into why earlier studies have observed differences in the cyclical behavior of wage rates and wage earnings.

The results of our univariate analysis suggest that, in general, the dominant cycle followed by output, employment, real consumer and producer wages and their components is 5-7 years. Consistent with previous findings reported in the macro-level literature, our bi-variate results show that

[^2]the various measures of the wage are generally not linked to the employment cycle; although we do find significant positive links between overtime components and employment. Also, and in sharp contrast with previous macro-level studies we find strong procyclical links between the consumer wage and its components and the output cycle especially at the 5-7 years frequency.

The rest of the paper is organized as follows. Sections 2 and 3 set out the econometric method and a brief explanation of our method for decomposing hourly earnings respectively. Section 4 reports our finding and Section 5 concludes. In addition, we include Appendices which provide more detail on (i) the modified Baxter-King method used for filtering the data; (ii) how we estimate the spectra; and (iii) our method for decomposing the wage.

## 2 Econometric Method

### 2.1 Univariate Measure

In empirical research on economic cycles, the predominant paradigm has been to examine auto- or cross-covariances in the time domain. ${ }^{3}$ The information on the cyclical structure contained in the autocovariance function can be transformed into frequency domain, revealing a more detailed picture. The spectrum of a process is defined as the Fourier transform of the autocovariance function $\gamma_{x}(\tau), \tau=0, \pm 1, \pm 2, \ldots$ :

$$
\begin{equation*}
f_{x}(\omega)=\frac{1}{2 \pi} \sum_{\tau=-\infty}^{\infty} \gamma_{x}(\tau) e^{-i \omega \tau} ; \omega \in[-\pi, \pi] . \tag{1}
\end{equation*}
$$

The interpretation is like that of a probability density function; $f_{x}(\omega) d \omega$ is the part of the overall variance of $X_{t}$ which is due to the component with frequencies over the interval $[\omega, \omega+d \omega]$. Spectral analysis permits a natural decomposition of the variance of a series into the contribution of cyclical components defined over frequency bands we are interested in. In terms

[^3]of real wages, these may relate to 3 - years wage contract cycles, 5-7 years business cycles and still longer cycles generated, for instance, by product and process innovations.

### 2.2 Multivariate Measures

Identifying each of the multiple wage cycles that combine to produce the observed wage time series does not in itself contain the most interesting information from an economist's viewpoint. To achieve this, we would need to establish what each of the wages represents, if anything, from an economic perspective. The common procedure is to test whether or not a cyclical indicators can explain significant degrees of the wage variation. One such indicator may be output deviations from trend, selected to capture business cycle activity. If output deviations are strongly correlated, say, with a 5-7 years wage cycle, and if that frequency range represents a dominant explanation of wage movements, we would be inclined to lean towards a business cycle explanation of wage cyclicality. Following the literature we employ output and employment deviations from trend as proxies for the business cycle and expect each to offer greater or lesser co-variations with different wage cycles.

Suppose that the peaks and troughs of an influential constituent cycle of the wage time series coincide with the respective turning points of the selected business cycle measure. Then we would conclude that the wage is both procyclical and in phase with the cycle. But the two series may be highly procyclical and out of phase. For example, in common with time series analyses, adjustment impediments associated with bargaining may lead to consistent phase lags of the wage to the cycle. Or the two series may be partly in phase and partly out of phase.

We apply frequency domain techniques to offer detailed insights into these aspects of wage cyclicality. We first consider 'explained variance' from a frequency domain perspective. This is achieved via the squared coherency measure ( $s c$ ) which assesses the degree of linear relationship between cyclical components of two series $X_{t}$ and $Y_{t}$, frequency by frequency. The $s c$ is defined
as

$$
\begin{equation*}
s c(\omega)=\frac{\left|f_{y x}(\omega)\right|^{2}}{f_{x}(\omega) f_{y}(\omega)} ; \quad 0 \leq s c(\omega) \leq 1 \tag{2}
\end{equation*}
$$

where $f_{x}(\omega)$ is the spectrum of the series $X_{t}$, and $f_{y x}(\omega)$ is the cross-spectrum for $Y_{t}$ and $X_{t} .{ }^{4}$ Using this expression, we can decompose $f_{y}(\omega)$ into an explained and an unexplained part. Integrating it over the frequency band $[-\pi, \pi]$ gives

$$
\begin{equation*}
\underbrace{\int_{-\pi}^{\pi} f_{y}(\omega) d \omega}_{\gamma_{y}(0)}=\underbrace{\int_{-\pi}^{\pi} s c(\omega) f_{y}(\omega) d \omega}_{\text {"explained" variance }}+\underbrace{\int_{-\pi}^{\pi} f_{u}(\omega) d \omega}_{\tilde{\sigma} 2} \tag{3}
\end{equation*}
$$

The first term on the right in equation (3) is the product of squared coherency between $X_{t}$ and $Y_{t}$ and the spectrum of $Y_{t}$; the second term is the spectrum of the residual. This equality holds for every frequency band $\left[\omega_{1}, \omega_{2}\right]$. Comparing the area under the spectrum of the explained component to the area under $Y$ 's spectrum in a frequency interval $\left[\omega_{1}, \omega_{2}\right]$ yields a measure of the explanatory power of $X$, analogous to an $R^{2}$ in the time domain. In contrast to sc however, $R^{2}$ is constant across all frequencies.

As pointed out by Croux et al. (2001), a measure like the squared coherency presented above is not suited for analyzing the co-movement of time series because it does not contain information about possible phase shifts between cycles in the series $X_{t}$ and $Y_{t}$. In this sense, the correlation coefficient in the time domain is more informative, since it is calculated lag by lag provides both information on the lead-lag structure and the degree of linear relationship between the two series. We can overcome this problem by also presenting the phase spectrum. This spectrum is difficult to interpret, since it is only defined $\bmod 2 \pi$, and cannot easily be summarized over a frequency band like in the case of the explained variance.

Croux et al. (2001) propose an alternative measure, the so-called dynamic

[^4]correlation $\rho(\omega)$, which measures the correlation between the "in-phase" components of the two series at a frequency $\omega$ :
\[

$$
\begin{equation*}
\rho(\omega)=\frac{c_{x y}(\omega)}{\sqrt{f_{x}(\omega) f_{y}(\omega)}} ; \quad-1 \leq \rho(\omega) \leq 1 . \tag{4}
\end{equation*}
$$

\]

Using the cross spectrum,

$$
f_{y x}(\omega)=c_{y x}(\omega)-i q_{y x}(\omega),
$$

which is the Forier transform of the covariance function of $Y_{t}$ and $X_{t},{ }^{5}$ we can write

$$
s c(\omega)=\frac{\left|f_{x y}(\omega)\right|^{2}}{f_{x}(\omega) f_{y}(\omega)}=\frac{c_{x y}(\omega)^{2}+q_{x y}(\omega)^{2}}{f_{x}(\omega) f_{y}(\omega)} .
$$

We can use this expression to further decompose equation (3):

$$
\begin{align*}
& \int_{-\pi}^{\pi} f_{x}(\omega) d \omega=\int_{-\pi}^{\pi} s c(\omega) f_{x}(\omega) d \omega+\int_{-\pi}^{\pi} f_{u}(\omega) d \omega= \\
& =\underbrace{\int_{-\pi}^{\pi} \frac{c_{x y}(\omega)^{2}}{f_{x}(\omega) f_{y}(\omega)} f_{x}(\omega) d \omega}_{\text {"explained" variance (in-phase) }}+\underbrace{\int_{-\pi}^{\pi} \frac{q_{x y}(\omega)^{2}}{f_{x}(\omega) f_{y}(\omega)} f_{x}(\omega) d \omega}_{\text {"explained" variance (out-of-phase) }}+ \\
& +\underbrace{\int_{-\pi}^{\pi} f_{u}(\omega) d \omega}_{\text {"unexplained" variance }} .
\end{align*}
$$

Thus, it is possible to decompose explained variance into the "in-phase" component and the "out-of-phase" component, adding some information on the importance of the phase shift in a frequency interval to the $R^{2}$ interpretation in equation (3) above.

We present this measure together with explained variance and phase shift in Section 4 to give a complete picture of the co-movement of real wages with the business cycle.

[^5]
## 3 Real hourly earnings decomposition and the cycle

We next deal with three measurement issues relating to real wage earnings. First, we describe how we decompose wage earnings. Second, we give $a$ priori reasons for expecting cyclical effects to vary across nominal earnings components. Third, we discuss implications of choosing either consumer or producer price deflators.

### 3.1 Nominal earnings decomposition

A critical feature of earnings decomposition concerns the distinction between the hourly standard wage rate and overtime payments. Overtime has been an important recent phenomenon in the United States. During the 1980s and 1990s, the proportion of overtime workers in manufacturing grew to 40 percent of the workforce. From the early 1990s trough to early 1997, average weekly overtime in manufacturing increased by 1.6 hours to reach 4.9 hours, the highest since the Bureau of Labor Statistics first recorded these data in 1956 (Hetrick, 2000).

There is a general problem in attempting to break down average earnings expressions, including the BLS measure used here, into their core component parts. They are typically expressed as arithmetic averages of standard and overtime parts of remuneration and their additive nature prevents simple decomposition. A way round the difficulty is to express earnings in terms of a geometric rather than arithmetic average. We show in Appendix C that we are able to define average nominal earnings, $A$, in our Bureau of Labor Statistics data as

$$
\begin{equation*}
A_{t}=E_{t}^{\lambda_{t}} W_{t}^{1-\lambda_{t}} \tag{5}
\end{equation*}
$$

where, $E$ is average hourly earnings of overtime workers, $W$ is the average standard hourly wage rate, and $\lambda$ is the proportion of the total workforce working overtime. Additionally, given Fair Labor Standard Act regulations that set maximum weekly standard hours at 40 and the minimum overtime
premium at $1.5, E$ can be expressed as

$$
\begin{equation*}
E_{t}=\mu_{t} W_{t}, \tag{6}
\end{equation*}
$$

where $\mu$ is the mark-up required to convert the average standard wage to average wage earnings. Specifically, the mark-up is given by

$$
\begin{equation*}
\mu_{t}=\left(40+1.5 V_{t}\right) /\left(40+V_{t}\right) \tag{7}
\end{equation*}
$$

where $V$ is average weekly overtime hours of overtime workers.
Substituting (6) into (5) and taking logs gives

$$
\begin{equation*}
\ln A_{t}=\ln W_{t}+\lambda_{t} \ln \mu_{t} . \tag{8}
\end{equation*}
$$

If $\lambda=0$, all workers receive the average standard wage rate and $A=W$ in (5). If $\lambda>0$, then $A>W$ due to the fact that a proportion of weekly hours for a proportion of workers are compensated at the hourly premium rate. ${ }^{6}$

The extent to which $A$ diverges from $W$ depends in (8) on (a) the size of the average premium mark-up, $\mu$ and (b) the proportion of overtime to total workers, $\lambda$. Essentially, changes in $\mu$ are solely dependent, as shown in (7), on changes in average overtime hours of overtime workers, $V$. For the great majority of workers, the maximum number of weekly hours before premium rates apply together with the level of the premium are fixed by legislation. By contrast, changes in $\lambda$ in (8) are employment driven, dependent on the proportion of workers choosing to undertake overtime. Together, these variables recognize an essential overtime breakdown underlined in the work of Trejo (1993) dealing with union effects on overtime working.

Why is it important to differentiate between $W, \mu$ and $\lambda$ in (8)? Standard and overtime components in the wage may well differ with respect to the cyclical indicator with which they most strongly co-vary as well as the degree to which they are in phase with their dominant cyclical influence. Suppose at the trough of an output or employment cycle firms hold underutilized

[^6]labor stock. The initial phase of the upturn may be dominated by labor dishoarding which involves effective hours being brought into line with paidfor hours. Planned increases in the stock of employment will depend on the anticipated time required to restore normal work intensity together with the expected degree and length of the ensuing growth period. In the later phases of the upturn, employment adjustment lags associated with search, hiring, and training may require firms to resort to temporary hours increases as an employment buffer (Nadiri and Rosen, 1973).

Overtime hours would be expected to feature prominently in this latter adjustment process. The degree to which the firm will extend overtime working will depend on the relative costs of alternative buffers, such as a greater than planned run-down of inventories (Topel, 1982). To the extent that overtime is used to offset shortfalls in planned employment growth especially at times of exceptional demand peaks - overtime cycles are likely to be relatively short and, given potential substitution, may well correlate strongly with changes in business inventories. Moreover, since premium rates apply automatically to changes in overtime hours - with no pay negotiation involved - we would expect that, to a large degree, overtime pay would be in-phase with the cycle (see the discussion in Hall and Lilien, 1979).

Overtime adjustments occur not only through hours adjustments, $\mu$, but also through employment adjustments, $\lambda$. Whether or not these two variables are close substitutes is an empirical question. Speedier responses may be achieved through changing the hours of workers already committed to overtime rather than through persuading marginal workers to move in and out of overtime completely. In this case, $\mu$ rather than $\lambda$ is more likely to represent the in-phase buffer response to high demand discussed above. Changes in $\lambda$, by contrast, may represent a longer term restructuring of work organization.

### 3.2 Real earnings and the choice of price deflator

Time series analysts have found that the choice of price deflator has a strong bearing on the observed degree of cyclical wage responsiveness (see, e.g. Abra-
ham and Haltiwanger, 1995). Generally, wages deflated by consumption prices, $C p$, are found to be more procyclical than wages deflated by production prices, $P p$. We label these, respectively, consumption and production wages.

A potentially important issue concerning the cyclical behaviors of $C p$ and $P p$ is the length of adjustment lag between actual and desired prices. For example, delayed price changes on the producers' side may involve component parts supplied to assembly plants by subcontractors that are based on fixed prices over specified contract periods. As for consumers, make-to-order companies will supply products at a future date, but at currently specified prices. It is difficult, a priori, to form expectations about the relative lag-lengths of $C p$ and $P p$ but it is important that the methodology allows us to test for systematic response differences between consumption and production wages.

## 4 Findings

Before we can analyze the cyclical structure of wages, we need to ensure that the data series are stationary. ${ }^{7}$ To this end we employ the modified BKM filter ${ }^{8}$ instead of other standard filters which have well known deficiencies. ${ }^{9}$ As discussed in Section 2.1, we can decompose a series into cyclical components, defined over multiple frequency bands. A graphical representation of this decomposition across all frequency bands is provided in Figures 1 and

[^7]2. These Figures show the auto-spectra, the share of variance attributed to cycles in employment $(N)$ and output $(Y)$, and the in-phase component of these shares. This information helps to assess both the strength and direction of the relationship between the various wages and the two cycles. For example, Figure 1 suggests that a relatively larger part of the total variance in the various consumer wages is explained by employment than in the corresponding cases for the producer wages. ${ }^{10}$ However, there does not appear to be evidence of a procyclical link between consumer and producer wages and employment since the in-phase component of the explained share is very small for both sets of wages (although slightly less so for the consumer wages). In contrast, evidence of a procyclical relationship between $\mu$ and $\lambda$ and employment is suggested by the relatively large in-phase components. More evidence of potentially procyclical links between consumer wages, $\mu, \lambda$ and output can also be found in Figure 2.

While the results of the graphical analysis are useful, it will be further revealing to focus not only on the entire frequency range but also on specific ones representing, for example, short (Kitchin), medium (Juglar) and long (Kuznetz) cycles. Moreover, in what follows we provide quantitative information regarding the degree and direction of relationship between wages and the cycles by undertaking hypothesis testing in the context of the frequency analysis. Thus, our empirical research strategy is to build out from the foundation of the univariate analysis of wave decomposition. As discussed in section 2, univariate methodology in the frequency domain allows us to establish the dominant cycle length for each of the variables under study. We are then in a position to examine the degree to which the frequency of the wage and its components coheres with each of the business cycle indicators. In both the univariate and multivariate exercises we undertake tests of the relevant hypotheses using Monte-Carlo methods which will described in more detail below.

[^8]Figure 1: Spectral Decomposition ( $N$ )


Figure 2: Spectral Decomposition $(Y)$


### 4.1 Univariate results

Let's first turn to the univariate results reported in Table 1. The variables included in the first column of Table 1 are the consumer and producer wages and their constituent parts (see equation 5) as well as the two measures of the business cycle. The next column gives the cycle length at which the explained variance is maximized across the entire spectrum for each series, e.g. $\mu$ has a cycle length of 4.18 years. The final three columns show the share of the total variance of each filtered series that is explained by the composite of waves in the respective frequency range. Taking the premium markup, $\mu$, as an example, $0.01,0.07$ and 0.59 of the total variance earnings is explained by the composite waves in the (1) $7-10$, (2) 5-7 and (3) 3-5 years frequency ranges, respectively, where only the third band is found to yield a significant share of the total variance. ${ }^{11}$

Given the above, the findings in Table 1 can be summarized as follows. First, the cycle of the premium mark-up, $\mu$, is predominantly explained within the shortest (3-5 years) range whereas the cycle for the proportion of overtime workers, $\lambda$, is within the 5-7 years range. Second, the production wage has two significant cycles that fall into the 5-7 and 7-10 years ranges, with the dominant cycle being the former range. The consumption wage contains one significant cycle in the 3-5 years range. Thus, with the exception of $\mu$, the preponderance of evidence suggests that the dominant cycle range for the consumer and producer wages and their components is $3-5$ years. Table 1 also shows that the 3-5 years range dominates for output and employment.

[^9]Table 1: Earnings (1959-1997): Share of Total Variance

|  | Cycle | $(1)$ | $(2)$ | $(3)$ |
| :--- | :--- | :--- | :--- | :---: |
| $A / C p$ | 5.94 | 0.15 | $0.46^{\star \star \star}$ | 0.31 |
| $A / P p$ | 6.73 | $0.24^{\star \star}$ | $0.42^{\star \star \star}$ | 0.24 |
| $E / C p$ | 5.42 | 0.12 | $0.44^{\star \star \star}$ | 0.32 |
| $E / P p$ | 6.74 | $0.24^{\star \star}$ | $0.42^{\star \star \star}$ | 0.23 |
| $W / C p$ | 5.82 | 0.14 | $0.43^{\star \star \star}$ | 0.33 |
| $W / P p$ | 6.74 | $0.24^{\star \star}$ | $0.41^{\star \star \star}$ | 0.25 |
| $\mu$ | 4.18 | 0.01 | 0.07 | $0.59^{\star \star \star}$ |
| $\lambda$ | 5.94 | 0.08 | $0.64^{\star \star \star}$ | 0.22 |
| $N$ | 5.94 | 0.09 | $0.43^{\star \star \star}$ | 0.36 |
| $Y$ | 5.75 | 0.08 | $0.49^{\star \star \star}$ | 0.33 |
| Notes: |  |  |  |  |

(i) $A$ : average nominal earnings, $E$ : average hourly earnings of overtime workers; $W$ : standard hourly wage; $\mu$ : premium markup; $\lambda$ : proportion of workers working overtime; $P p$ : producer price index; $C p$ : consumer price index; $Y$ : output, $N$ : employment; (ii) (1): 7-10 years (Juglar cycle), (2): 5-7 years, (3): 3-5 years (Kitchin cycle); (iii) ${ }^{* *} / \star * *$ : Share of Total Variance is significant at the $5 / 1$ per cent level.

### 4.2 Multivariate results

Analogous to the univariate approach, we require a multivariate method that allows us to achieve two objectives. First, we would like to find out at which business cycle frequency the ratio of explained to unexplained variance is at a maximum. Second, it is important to determine whether the share of variance explained by output and employment, in a specific frequency band, is significant. ${ }^{12}$ In other words, we would like to test the null hypothesis

[^10]that the real wage component(s) and the business cyclical indicators are unrelated in a specific frequency band. As in the univariate case, the data may reveal multiple cycles between any two series. The cells of Table 2 refer to the proportion of total variance in the respective frequency range for each component of the real wage explained by the variance of $Y$ and $N$ respectively. Consider $A / C_{p}$ in Table 2 using the $Y$ cycle. This case reveals that $0.74,0.67,0.90$ and 0.56 of the total variance of $A / C_{p}$ is explained by the total variance in $Y$ in, respectively, the (1) entire, (2) 7-10, (3) 5-7 and (4) 3-5 years frequency ranges.

Table 2: Real Earnings and Business Cycles (1959-1997): Explained Variance

(i) $Y$ : output, $N$ : employment, $A$ : average nominal earnings, $E$ : average hourly earnings of overtime workers; $W$ : standard hourly wage; $\mu$ : premium markup; $\lambda$ : proportion of workers working overtime; $P p$ : producer price index; $C p$ : consumer price index; (ii) Explained Variance: share of variance explained by variance of cycle measure in the respective frequency range; (iii) (1): entire range, (2): 7-10 years (Juglar cycle), (3): 5-7 years, (4): 3-5 years (Kitchin cycle); (iv) **/***: Explained Variance is significant at the $5 / 1$ per cent level.

Thus, the results in Table 2 suggest that the explained variance is significant (at the 5 per cent level or less) in 14 cases for $Y$ and 6 cases for $N$, over all measures and for all frequency ranges. In particular, with the exception of $\mu$ and $\lambda$, none of the other wage measures is significantly related to the
$s=2000$ so that we can obtain an empirical distribution of $\rho_{X Y}$ under the null conditional on the series we are examining. Note that Priestley (1981, p705-706) develops a similar test of zero coherency for the classical spectral estimate, the periodogram.
employment cycle. The lack of relationship at the $5 \%$ level also applies to the association between producer wages and the output cycle. In contrast, all measures of consumer wages significantly cohere with the output cycle at the (7-10 year) and (5-7 year) ranges with the latter being the dominate range. Moreover, $\mu$ and $\lambda$ also significantly relate to the output cycle for all cycle ranges (except the 7-10 years range for $\mu$ ). The dominate ranges in these cases are (3-5 years) for $\mu$ and (5-7 years) for $\lambda$.

We next turn to the issue of establishing whether the significant results from Table 2 are pro- or counter- or a-cyclical. This can be achieved by working out whether the share of variance attributed to cycles in employment and output in Table 2 are in-phase or out-of-phase. These calculations are reported in Table 3 and show that the explained variance from Table 2 can be decomposed into in- and out-of phase components. For ease of interpretation, each of the in- and out-of-phase components in Table 3 has been normalized by its corresponding explained variance value in Table 2. Thus an in-phase (out-of-phase) share greater than $50 \%$ indicates that a variable is procyclical (countercyclical). A-cyclical cases include all the insignificant cases from Table 2.

Given the above background and concentrating on the significant results in Table 2, we find in Table 3 that 86 per cent of the cases are procyclical (i.e. in-phase share $>50 \%$ ) for $Y$, and $67 \%$ for $N$. If we further concentrate on only the dominate cycle range in Table 2 only, we can safely conclude that all of the consumer wages $\left(A / C_{p}, E / C_{p}, W / C_{p}\right)$ and the proportion of overtime workers, $\lambda$, are procyclical related with output in the 5-7 years range (e.g. in-phase shares of $0.72,0.64,0.68$ and 0.90 respectively). We can also draw the same conclusion for the overtime markup, $\mu$, and output cycles but for the shorter 3-5 years range. When turning to employment and the wage measures in the dominant cycle ranges, there is strong evidence of procyclical links for $\lambda$ and $\mu$ in the 5-7 years and 3-5 years ranges respectively.
Table 3: Real Earnings and Business Cycles: Variance Decomposition


## 5 Conclusions

The basic premise underlying our interest in applying frequency methods to the issue of wage cyclicality was that the observed behavior of the real wage in the time domain might produce a distorted picture of wage cyclicality due its inability to capture cycles across multiple frequencies. While the consensus view, from macro-level time domain studies, does indeed appear to prevail when considering the employment cycle and standard wage measures in the frequency domain, it does not hold when our new composite wages measures and the employment cycle are considered. Moreover, and in sharp contrast to the received wisdom regarding wage cyclicality, we find that all measures of the consumer wage and its components are significantly procyclically related to the output cycle especially, in the 3-5 years range.

Given the importance attached to the "stylized facts" in the specification, calibration, and estimation of macroeconomic models, we conclude that further application of frequency methods to other relationships of interest to macroeconomists would be a fruitful and worthwhile endeavor.

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## Appendices

## A Modified Baxter-King Filter

Baxter and King (1999) construct a bandpass filter of finite order $K$ which is optimal in the sense that it is an approximate bandpass filter with trendreducing properties and symmetric weights, which ensure that there is no phase shift in the filter output. In time domain, the impact of the filter on an input series $y_{t}$ is given by the finite moving average ${ }^{13} \tilde{y}_{t}=\sum_{j=-K}^{K} a_{j} L^{j} y_{t}$. In the frequency domain, the filter is characterised by its Fourier transform $\alpha(\omega) .{ }^{14}$ To find the weights $a_{j}$, one solves the minimisation problem

$$
\begin{equation*}
\min _{a_{j}} Q=\int_{-\pi}^{\pi}|\beta(\omega)-\alpha(\omega)|^{2} d \omega \text {, s.t. } \alpha(0)=0 \tag{A1}
\end{equation*}
$$

where $|\beta(\omega)|$ is the "ideal" filter gain with cut-off frequencies $\omega_{1}$ and $\omega_{2} .{ }^{15}$ The constraint ensures that the resulting filter has trend reducing properties. ${ }^{16}$

Solving the minimisation problem leads to the following results: ${ }^{17}$

$$
\begin{align*}
a_{j} & =b_{j}+\theta ; j=0, \pm 1, \ldots, \pm K \\
b_{j} & = \begin{cases}\frac{\omega_{2}-\omega_{1}}{\pi} & \text { if } j=0 \\
\frac{1}{\pi j}\left(\sin \omega_{2} j-\sin \omega_{1} j\right) & \text { if } j= \pm 1, \pm 2, \ldots\end{cases}  \tag{A2}\\
\theta & =\frac{-\sum_{j=-K}^{K} b_{j}}{2 K+1}
\end{align*}
$$

The original Baxter-King filter has an undesireable property, which is

[^11]known as Gibb's phenomenon, due to the fact that the ideal filter, which is a discontinuous function of $\omega$, is approximated by a finite Fourier series. This approximation leads to side lobes in the gain function of the filter. (Priestley 1981, p. 561-3, Koopmans 1974, p. 187-9). While the relative contribution of some components for the overall variance of the series is exaggerated (i.e. they are multiplied by a gain greater than 1 ), other components are suppressed (i.e. they are multiplied by a gain less than 1).

An obvious solution to this problem is to increase the filter length. But since we are restricted by the limited availability of economic data, there is not much to be gained from changing the length of the filter. A more appropriate solution is to apply spectral windows. As an example, consider the so called Lanczos's $\sigma$ factors (Bloomfield 1976, p. 129-137). We replace the truncated weights of the optimal filter $b_{j}$ in equation (A2) by the modified weights $b_{j}^{\star}$, which are obtained from

$$
\begin{equation*}
b_{j}^{\star}=b_{j} \frac{\sin ((2 \pi j) /(2 K+1))}{(2 \pi j) /(2 K+1)} ;|j|=1, \ldots, K . \tag{A3}
\end{equation*}
$$

After this step, the modified filter weights of the Baxter-King filter $a_{j}^{\star}$ can be calculated as demonstrated above (Woitek, 1998). The results of the ADF test reported in Table 4 show that filtering is indeed necessary.

Table 4: Unit Root Test

| Variable | t-statistic | Variable | t-statistic | Variable | t-statistic |
| :--- | :---: | :--- | :---: | :--- | :---: |
| $A / C p$ | -2.14 | $E / C p$ | -2.39 | $W / C p$ | -2.04 |
| $A / P p$ | -2.10 | $E / P p$ | -2.26 | $W / P p$ | -2.16 |
| $\mu$ | -1.59 | $\lambda$ | -1.25 |  |  |
| $N$ | -2.80 | $Y$ | -3.68 |  |  |
| $N=33$, constant and trend in the alternative. The critical value (1\% significance |  |  |  |  |  |
| level) is -4.29 (McKinnon 1996$).$ |  |  |  |  |  |
| $A:$ average nominal earnings, $E$ : average hourly earnings of overtime workers; $W$ : |  |  |  |  |  |
| standard hourly wage; $\mu$ : premium markup; $\lambda:$ proportion of workers working over- |  |  |  |  |  |
| time; $P p:$ producer price index; $C p:$ consumer price index; $Y:$ output, $N:$ employ- |  |  |  |  |  |
| ment |  |  |  |  |  |

## B Estimation of the Spectrum

To estimate the spectra, we fit autoregressive models in the time domain, ${ }^{18}$ and calculate the spectral representation of the estimated models. This method is based on the seminal work by Burg (1967), who shows that the resulting spectrum is formally identical to a spectrum derived on the Maximum Entropy Principle. This is seen to be a more reasonable approach then the normally used periodogram estimator. The periodogram employs the assumption that all the covariances outside the sample period are zero. Given that economic time series are notoriously short, this seems to be a problematic assumption. ${ }^{19}$ Consider a univariate AR model of order $p$, with residual variance $\sigma^{2}$. The spectrum is given by

$$
\begin{equation*}
f(\omega)=\frac{1}{2 \pi} \frac{\sigma 2}{\left|1-\sum_{j=1}^{p} \alpha_{j} e^{-i \omega j}\right|^{2}} ; \omega \in[-\pi, \pi] . \tag{A4}
\end{equation*}
$$

Equation (A4) is the analogue to the univariate spectrum in equation (1). With a VAR model of order $p$, the spectral density matrix is given by

$$
\begin{equation*}
\mathbf{F}(\omega)=\frac{1}{2 \pi} \mathbf{A}(\omega)^{-1} \boldsymbol{\Sigma} \mathbf{A}(\omega)^{-\star} ; \omega \in[-\pi, \pi] . \tag{A5}
\end{equation*}
$$

$\boldsymbol{\Sigma}$ is the error variance-covariance matrix of the model, and $\boldsymbol{A}(\omega)$ is the Fourier transform of the matrix lag polynomial $\mathbf{A}(L)=\mathbf{I}-\mathbf{A}_{1} L-\cdots-\mathbf{A}_{p} L^{p} .{ }^{20}$ The diagonal elements of this matrix are the analogue to the univariate spectrum in equation (1), and the off-diagonal elements are the cross-spectra defined in footnote 7.

[^12]
## C Derivation of earnings expression (6) from BLS data

The BLS calculate the earnings rate ${ }^{21}$ (i.e. average hourly earnings), by dividing gross payrolls, $G P$, by total hours, thus

$$
\begin{equation*}
A_{t}=\frac{G P_{t}}{N_{t} H_{t}} \tag{A6}
\end{equation*}
$$

We can decompose equation (A6) by differentiating between overtime workers and workers working only standard hours. Under the FLSA, overtime is compensated at a premium rate for hours in excess of 40 per week. We assume that overtime workers are compensated for 40 weekly hours at the standard rate and then at the mandated premium rate for additional weekly hours. ${ }^{22}$

Accordingly, we may re-express equation (A6) in the form

$$
\begin{equation*}
A_{t}=\frac{N_{t} W_{t} H_{t}^{s}+N_{t}^{o} W_{t}\left(40-H_{t}^{s}\right)+N_{t}^{o} W_{t}^{o} V_{t}}{N_{t} H_{t}^{s}+N_{t}^{o}\left(40-H_{t}^{s}\right)+N_{t}^{o} V_{t}} \tag{A7}
\end{equation*}
$$

where, $N^{o} \leq N$ are the number of employees working overtime, $W$ and $W^{o}$ are the average standard and average overtime hourly wage rates, $H^{s}$ is average weekly standard hours of non-overtime workers, $V=\left(H^{o}-40\right)$ is average overtime hours of overtime workers, and $H^{o}$ is average total weekly hours worked by overtime workers. The numerator of (A7) comprises three parts. The first term allows for all $N$ workers to be paid at the standard rate, $W$ for standard hours, $H^{s}$. These latter hours are averaged over nonovertime workers and we expect $H^{s}<40$. Therefore, the second term allows for the fact that $N^{o}$ overtime workers, assumed to be working 40 standard hours, are compensated at $W$ for $\left(40-H^{s}\right)$ hours. The final term shows that overtime workers are further compensated at the overtime rate, $W^{o}$ for

[^13]overtime hours, $V$. The maximum number of standard hours and the hourly overtime premium are fixed by legislation.

There are two problems with the definition of $A$ in (A7): (i) the BLS Establishments Survey does not provide data on $N^{o}$ and $V$; (ii) the arithmetic average used to calculate (A7) is additive and accordingly cannot be algebraically decomposed into its separate parts. We deal with each of these problems in turn.

The wage rate - i.e. $W_{t}$ in (A7) - is approximated by BLS by adjusting average hourly earnings through the elimination of premium pay for overtime at a rate of time

$$
\begin{equation*}
W_{t}=\frac{G P_{t}}{N_{t}\left[\bar{H}_{t}^{s}+1.5 \times \bar{H}_{t}^{o}\right]} \tag{A8}
\end{equation*}
$$

where $\bar{H}^{s}\left(=H-\bar{H}^{o}\right)$ and $\bar{H}^{o}$ are, respectively, standard and overtime hours averaged over all workers (i.e. overtime and non-overtime workers). ${ }^{23}$ No adjustment is made for other premium payment provisions such as holiday work, late shift work, and premium overtime rates other than those at time and one-half. $W$ is calculated only for manufacturing industries because data on overtime hours are not calculated in other industries. This is the principal reason why we concentrate our attention on the manufacturing sector.

The BLS Current Population Survey does gather (unpublished) data pertaining to $N^{o} .{ }^{24}$ Strictly, $N^{o}$ defines the number of workers working in excess of 40 weekly hours and, as mentioned previously, this is how we define overtime workers, that is

$$
\begin{equation*}
\lambda_{t}=\frac{N_{t}^{o}}{N_{t}} \tag{A9}
\end{equation*}
$$

[^14]where $\lambda_{t}$ is the proportion of workers working overtime at time $t$. Then, $H^{s}$ in (A7), the number of standard hours worked by non overtime workers is given by
\[

$$
\begin{equation*}
H_{t}^{s}=\frac{\bar{H}_{t}^{s}-\lambda_{t} \times 40}{1-\lambda_{t}} \tag{A10}
\end{equation*}
$$

\]

Further, $V$ in (A7) is given by

$$
\begin{equation*}
V_{t}=\frac{\bar{H}_{t}^{o}}{\lambda_{t}} \tag{A11}
\end{equation*}
$$

The RHS of (A7) decomposes $A$ into the contributions of total and overtime workers. To differentiate explicitly between workers working overtime and those working only standard hours while retaining $A$, we re-express real earnings as a geometric instead of an arithmetic average; thus our equation (5) in the text

$$
\begin{equation*}
A_{t}^{\star}=E_{t}^{\lambda_{t}} W_{t}^{1-\lambda_{t}} \tag{A12}
\end{equation*}
$$

where, $E$ is average hourly earnings of overtime workers and $W$ and $\lambda$ are defined in (A8) and (A9), respectively. Additionally, given FLSA regulations, $E$ can be expressed in terms of (6) and (7) as in the main text, with $V$ defined in (A11).

All employment, hours and earnings data are from the BLS Establishments Survey. The proportion of employees working greater than 40 hours per week are unpublished annual figures from the BLS Current Population Survey. The index of industrial production and price data are from the Federal Reserve Board and the BLS respectively. ${ }^{25}$

[^15]
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[^1]:    ${ }^{1}$ To obtain this breakdown we employ unpublished data relating to hours of work provided by the U.S. Bureau of Labor Statistics. Despite the availability of quarterly and even monthly data for some of the other series employed in this study, we are restricted to the annual frequency in our econometric analysis since our unpublished data are only provided on an annual frequency from 1959 to 1997 (for further details, see Appendix C).

[^2]:    ${ }^{2}$ Along these lines, see the longitudinal micro study of Devereux (2001) where it is found that components of wage earnings respond very differently to cyclical movements in economic activity as measured by the change in the rate of unemployment.

[^3]:    ${ }^{3}$ Widely cited examples are Kydland and Prescott $(1982,1990)$ and Backus and Kehoe (1992). For exceptions working in the frequency domain, see Altug (1989) and A'Hearn and Woitek (2001).

[^4]:    ${ }^{4}$ The cross-spectrum is the Fourier transform of the cross-covariance function:

    $$
    f_{y x}(\omega)=\frac{1}{2 \pi} \sum_{\tau=-\infty}^{\infty} \gamma_{y x}(\tau) e^{-i \omega \tau} ; \omega \in[-\pi, \pi]
    $$

[^5]:    ${ }^{5} c_{y x}(\omega)$ is the cospectrum and $q_{y x}(\omega)$ is the quadrature spectrum.

[^6]:    ${ }^{6}$ In our data set, movements in geometric and arithmetic average wage earnings correspond almost identically, e.g. the simple correlation is 0.99 .

[^7]:    ${ }^{7}$ See Table 5 in the Appendix which shows that none of the data under consideration is stationary in levels.
    ${ }^{8}$ The modified Baxter-King Filter (Woitek, 1998) uses Lanczos' $\sigma$ factors to deal with the problem of spurious side lobes, which invariably arises with finite length filters. In contrast to the original filter proposed by Baxter and King (1999), our cut-off period is 15 years (low-pass filter), allowing us to analyse cycles as long as the Juglar cycle. Following the suggestion in Baxter and King (1999) for annual data, the filter length is set to 3 (see Appendix A for details).
    ${ }^{9}$ Recently it has been demonstrated by Cogley and Nason (1995), King and Rebelo (1993) and Harvey and Jaeger (1993), that the widely used Hodrick-Prescott filter (Hodrick and Prescott 1997) is likely to generate spurious cyclical structure at business cycle frequencies if applied to difference stationary series. Similar points can be made with respect to the Baxter-King Filter (Guay and St-Amant 2005), and to moving-average filters in general (Osborn 1995). Moreover, there is the danger of spurious correlation between Hodrick-Prescott filtered series (Harvey and Jaeger 1993).

[^8]:    ${ }^{10}$ This finding is also echoed in Figure 2.

[^9]:    ${ }^{11}$ To establish significance we follow Reiter and Woitek (1999) and simulate white-noise processes to assess whether the share of total variance in the frequency intervals of interest is significantly different from the result we would obtain if the data generating process was white noise. For example, we fit an AR model of order 5 to a white noise process, which has the same variance as the series under analysis, and repeat this 2000 times. We then use the univariate spectral measures from this experiment to derive the empirical distribution under the null hypothesis (i.e. no cyclical structure).

[^10]:    ${ }^{12}$ To determine whether the explained variance, $\rho_{X Y}$ between two series $Y$ and $X$, in the relevant frequency band $\left[\omega_{1}, \omega_{2}\right]$, is significantly different from zero we implement the following procedure. First, we fit AR models to $Y$ and $X$ and, second, we conduct a parametric bootstrap to simulate the model under the null hypothesis (i.e. no interaction between the series). This produces a simulated series $\left(Y_{t}^{S} X_{t}^{S}\right)$ that has the univariate characteristics of the underlying data, but without interaction. Third, we fit a VAR of fixed order to $\left(Y_{t}^{S} X_{t}^{S}\right)$ and calculate $\rho_{X Y}^{S}$. Fourth, these steps are then repeated for

[^11]:    ${ }^{13} L$ denotes the backshift operator $\left(L^{n} y_{t}=y_{t-n}\right)$.
    ${ }^{14}$ See e.g. Koopmans (1974), p. 165 ff.
    ${ }^{15}$ The gain of a filter measures the change in the amplitude of the input components if transformed by the filter. The ideal bandpass filter gain $|\beta(\omega)|$ takes the value 1 in the frequency interval $\left[\omega_{1}, \omega_{2}\right]$ and 0 outside this interval.
    ${ }^{16}$ To remove the component with the frequency $\omega=0$ from the series, the filter weights must sum to zero (Baxter and King, 1999).
    ${ }^{17}$ The filter is symmetric (i.e. $a_{j}=a_{-j}$ ), and therefore does not impose a phase shift on the output.

[^12]:    ${ }^{18}$ The order is determined by AIC.
    ${ }^{19}$ See the discussion in Priestley (1981, p. 432, 604-607). A recent application to economic time series is A'Hearn and Woitek (2001).
    ${ }^{20} L$ is the backshift operator; the superscript ' $\star$ ' denotes the complex conjugate transpose.

[^13]:    ${ }^{21}$ See, BLS Handbook of Methods, 1997, Ch. 2 Employment, Hours, and Earnings from the Establishment Survey.
    ${ }^{22}$ Actually, there is evidence (Trejo, 1993) that some overtime workers receive the premium before the 40 hour limit. Unfortunately, our data are such that we cannot accommodate this possibility.

[^14]:    ${ }^{23}$ Note that (A6) follows the definition given by BLS (Handbook of Methods, 1997, Ch. 2, p 22):"[Average hourly earnings excluding overtime] ... are computed by dividing the total production payroll ... by the sum of the total production worker hours and one-half of the total overtime hours, which is equivalent to the payroll divided by standard hours."
    ${ }^{24}$ Basic information regarding the Survey and the published data can be found in BLS Handbook (Chapter 1) and the February 1994 issue of the BLS publication called Employment and Earning. There is a complication with these data. If, for example, a person worked 40 hours a week at a manufacturing job and then worked another 20 hours in the same week as a clerk in a store, that person would be shown as working 60 hours that week in their manufacturing job. We note that, while dual job holding is generally an important phenomenon, it is clear from the breakdowns of rates of dual job holding provided by Paxson and Sicherman (1996) that manufacturing occupations tend to exhibit below-average rates.

[^15]:    ${ }^{25}$ The producer price index is for all commodities and the consumer price index is for all urban consumers. Note that the latter (CPI-U-X1) is a BLS retrospectively calculated estimate of how the all items index would have moved (from 1967 to 1982) had the CPI been calculated using the flow of services approach to shelter. Prior to 1967 we use the official series reported by the BLS. In December 1982 the production CPI began to use flow of services approach. Hence as in other studies of wages (see Abraham and Haltiwanger, 1995) we employ the CPI, which is most consistent with the series currently reported by the BLS. We thank the BLS for providing the unpublished 1967-82 consumer price data as well as the Current Population Survey data.

