

Real exchange rate persistence and the excess return puzzle: the case of Switzerland versus USA

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Abstract

The fact that nominal exchange rates tend to undergo wide swings around their long-run equilibrium values is often referred to as the PPP puzzle. Closely related to this is the excess return puzzle describing the fact that the domestic/foreign interest rate differential deviates persistently from the expected change in the nominal exchange rate. The paper shows that the puzzles can be reconciled by allowing the expectations formation to be based on imperfect information rather than on so called rational expectations. The results, using the $I(2)$ Cointegrated VAR model, suggest that self-reinforcing feedback mechanisms are the major cause to the pronounced persistence in the Swiss-US parity conditions. Excess return is explained by an uncertainty premium that is proxied by the persistent PPP gap.

Keywords: Long swings, Imperfect knowledge, $I(2)$ analysis, Self-reinforcing feedback.

JEL codes: C32, C51, F31

1 Introduction

It is a well-established fact that the ratio of domestic to foreign goods prices typically changes only slowly while the nominal exchange rate undergoes large, persistent swings. As a result, both the real and the nominal exchange rate exhibit large swings away and towards long-run benchmark values. These long swings around long-run fundamental values, are hard to reconcile with standard monetary models based on Rational Expectations (RE) and a representative agent endowed with essentially perfect knowledge. As a result, alternative approaches that rely on heterogeneous agents and imperfect knowledge have been proposed in the literature. We call these “imperfect knowledge” based models in contrast to the conventional RE representative agent models.

Common to these models is that today’s asset price depends on future prices which, in varying degree, are forecasted under imperfect knowledge and, thus, can deviate from the expected future prices as derived under RE. For example, Hommes (2005) and Hommes et al. (2005a, 2005b) develop models for a financial market populated by fundamentalists and chartists, where fundamentalists use long-term expectations based on economic fundamentals and chartists are trend-followers using short-term expectations¹. Positive feedback can arise when the latter dominate the market. Adam and Marcet (2011) propose a separation of standard RE rationality into an internal and an external component. They show that positive feedback can arise in a model where internal

¹For a detailed overview, see the handbook chapter by Hommes (2006).

rationality is maintained but external rationality is relaxed due to imperfect market knowledge. Heemeijer et al. (2009) use experiments and find that prices converge to its fundamental level under negative feedback but fail to do so under positive feedback.²

When heterogeneous agents make forecasts under imperfect knowledge, causal relationships are unlikely to remain constant in the aggregate and one would in general expect model parameters to be changing. Frydman and Goldberg (2007, 2011) develop a theoretical framework where agents' expectations are formed in the context of imperfect knowledge about the shape of underlying causal relations assumed to be subject to structural change.

Johansen et al. (2010) and Juselius (2015) estimate a near $I(2)$ CVAR model for German-US data in the post Bretton Woods/pre-EMU period and find that the persistency features of the data strongly support imperfect knowledge rather than RE based models. This paper investigates whether similar results can be found when comparing a much smaller, but financially important country, such as Switzerland with the dominant world economy, the USA. We follow a similar empirical $I(2)$ methodology as it allows us to explore the dynamics of positive and negative feedback mechanisms that have generated the long and persistent swings in the data. Since Switzerland is not a member of the euro area, the sample covers the whole Bretton Woods period up to the present date.

The paper is organized as follows: Section 2 documents the departure from basic parity conditions in the Swiss-US foreign exchange market in the data. Section 3 discusses a theoretical framework for real exchange rate persistence. Section 4 outlines the $I(2)$ CVAR model for the Swiss-US data and reports tests of various hypotheses on the long-run relations. Section 5 discusses the pulling forces in the system. Finally, Section 6 concludes.

2 Graphical analysis of the basic parity conditions

The Swiss-US real exchange rate and the interest-rate differentials have been characterized by long and persistent swings over the last 35 years.³ In the following, we present graphical evidence that the basic parity conditions like purchasing power parity (PPP), uncovered interest rate parity (UIP) and the spread between long and short-term interest rates appear nonstationary, in contrast to what standard rational expectations models would suggest.

The upper panel of Figure ?? shows that the Swiss-US nominal exchange rate and relative prices follow a similar downward-sloping trend over the very long run. In the medium run, the nominal exchange rate fluctuates with long persistent swings—sometimes lasting more than ten years—around its long-run benchmark PPP value, defined as $p_{1,t} - p_{2,t}$ with $p_{1,t}$ being the log of the Swiss CPI, $p_{2,t}$ the log of the US CPI, and $e_{12,t}$ the log of the spot Swiss franc-US dollar exchange rate. A subscript 1 stands for Switzerland (the domestic country) and a subscript 2 for the US (the foreign country). The upper panel of Figure ?? also shows that it is the nominal exchange rate that exhibits the long persistent swings, whereas the relative price level has moved

²For similar results see Hommes (2013), Frydman and Goldberg (2013) and Anufriev et al. (2013). A special issue of the Journal of Economic Methodology (2013) on Reflexivity and Economics discusses various aspects of positive and negative feedback mechanisms, see Hands (2013).

³All data for the empirical analysis are from the International Financial Statistics (IFS) data base and are quarterly averages of monthly data. The sample period runs from the first quarter of 1976 to the last quarter of 2013.

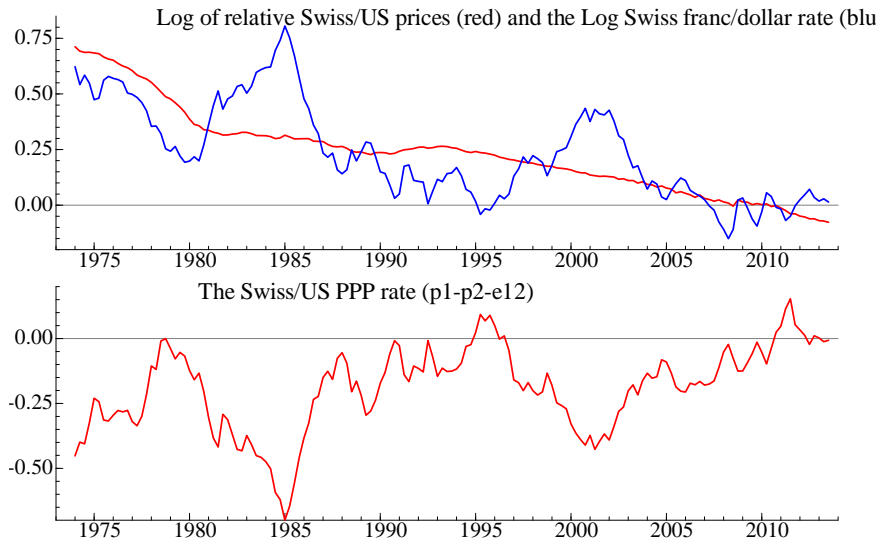


Figure 1: The log exchange rate and the log relative price between Switzerland and US (upper panel) and the deviation from PPP (lower panel)

much more smoothly over the sample period. Consequently, the real and the nominal exchange rate look very similar, which seems untenable with standard theories (Rogoff, 1996). Since the largest part of the transactions in the Swiss franc-US dollar currency market is related to financial speculation and only a small part to the trade with goods, the large fluctuations in the real exchange rate are likely to be associated with financial market participants' behavior.⁴

The prolonged movements away from PPP, lasting five to six years or longer, are likely to trigger off compensating movements in other variables associated with the real exchange rate such as long and short-term real interest rates. When inspecting the short and the long-term real interest rate for both countries we find that all four real interest rates drift off in a nonstationary manner; the US rates and the Swiss rates, respectively, behave similarly in this regard. The three-month real interest rate differential and the ten-year real interest differential between Switzerland and the US, both together with the departures from PPP, are shown in Figure ???. To calculate real interest rates, we use the actual inflation rate as a proxy for expected inflation.⁵ Both the short-term and the long-term real interest rate differential co-move quite closely with the deviation from PPP.

Figure ??? shows the US-Swiss short and long-term interest rate differential together with the inflation differential. While the interest rate differentials have narrowed considerably since the nineties and the inflation spread has become small and quite stable since 1983, the latter does not seem to co-move strongly with the former. The short and the long-term interest rate differentials differ to some extent as the short rates respond more strongly to central bank decisions whereas the long rates react more to financial market conditions. The subsequent empirical analysis will include both of them.

Based on the expectations hypothesis of the term structure the long-term interest

⁴According to the BIS triennial survey the turnover in the USD/CHF currency pair averaged 184 billion USD *per day* in April 2013. By contrast, the monthly trade volume (i.e. exports plus imports) between Switzerland and the US amounted to about 2.7 billion USD in April 2013.

⁵To make interest rates comparable with the inflation rate, which is the quarterly difference of the log price level, we divide the interest rate expressed in percent by 400.

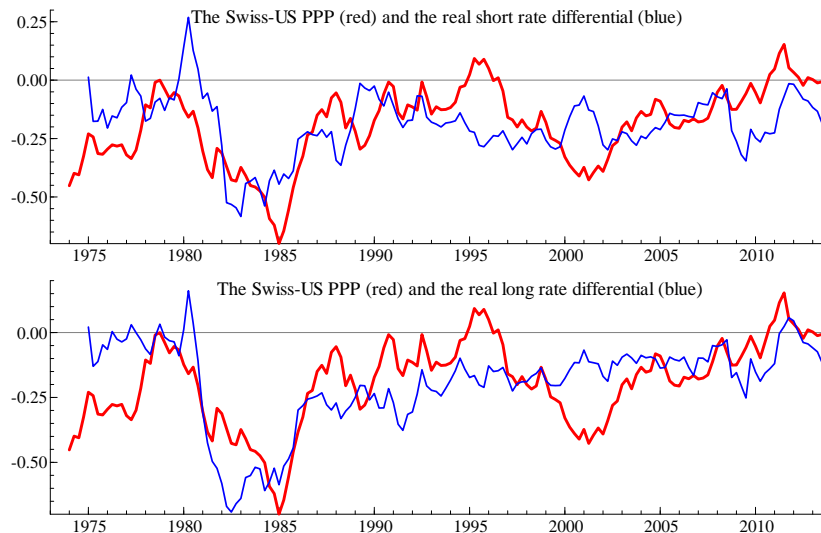


Figure 2: Deviation from PPP together with the real three-month interest rate differential (upper panel) and the real ten-year interest rate differential (lower panel). The graphs have been scaled to have the same range and mean.

rate should be a weighted average of current and expected short-term rates, implying that the term spread should be stationary (Campbell and Shiller, 1987). Empirically, however, both the Swiss and the US spread look nonstationary, or at least very persistent, suggesting that the short and the long-term rate may not share a common stochastic trend over this sample period.

3 Modelling real exchange rate persistence

Standard models generally interpret the economy's behavior through the lens of a representative agent with rational expectations, the behavior of which will secure that prices return to equilibrium when exogenous shocks have pushed them away. In such a world the economic system is always moving towards a model-specific equilibrium, albeit possibly with some sluggishness and basic parity conditions, such as PPP, UIP and the Fisher parity, would be stationary processes. The pronounced persistence characterizing these parities as shown in Figures 1 and 2 seem inconsistent with stationarity and we shall argue below that it can be reconciled with the theory of imperfect knowledge.

If purchasing power parity (PPP) prevails in the goods market, then one would expect the nominal exchange rate to roughly follow the relative prices. The PPP puzzle describes the empirical fact that the real exchange rate defined as

$$q_t = e_{12,t} - (p_{1,t} - p_{2,t})$$

often tend to move in long persistent swings and that the volatility of the nominal exchange rate is very large compared to the relative price.

If uncovered interest rate parity prevails in the foreign currency market then the interest rate differential, $(i_{1,t} - i_{2,t})$, would reflect the expected change in the exchange rate, $e_{12,t+1}^e - e_{12,t}$, where a superscript e characterizes expected values made at time t . The excess return puzzle is about the empirical fact that excess return defined as

$$exr_t = (i_{1,t} - i_{2,t}) - (e_{12,t+1}^e - e_{12,t}) \quad (1)$$

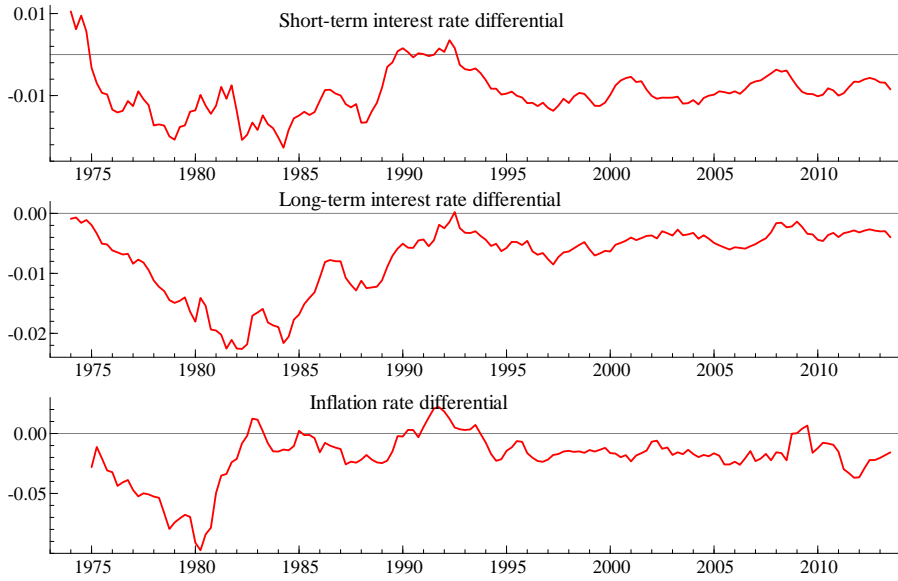


Figure 3: Three-month interest rate differential (upper panel), ten-year bond rate differential (middle panel), and the inflation rate differential (lower panel).

often behaves like a nonstationary process. Next, we shall argue that non-constant parameters due to expectation formation based on imperfect knowledge can result in near $I(2)$ -type persistence for the basic parity conditions.

Assume that the nominal exchange rate depends on the expected relative price as well as other changes in important determinants, v_t^e , for example interest rates, income, etc. and that financial actors attach different weights, B_t , to the relative price depending on how far away the nominal exchange rate is from its fundamental PPP value, i.e.:

$$e_{12,t} = A + B_t(p_1 - p_2)_{t+1}^e + v_{t+1}^e. \quad (2)$$

where the relative price is assumed to follow a random walk so that its forecast $(p_1 - p_2)_{t+1}^e = (p_1 - p_2)_t$.

To be able to associate expected with observed values we assume first that expected values of relative inflation rates cannot deviate persistently from actual values so that the forecast error $\{\Delta(p_1 - p_2)_{t+1} - \Delta(p_1 - p_2)_{t+1}^e\} \sim I(0)$, and second that $\{\Delta(p_1 - p_2)_{t+1} - \Delta(p_1 - p_2)_t\} \sim I(0)$ so that $\Delta(p_1 - p_2)_t$ is at most $I(1)$. Under these assumptions the cointegration properties are robust to using actual values instead of the (generally unknown) expected values. See Juselius (2015) for a detailed discussion. Using this, the change in the nominal exchange rate can be expressed as

$$\Delta e_{12,t} = B_t \Delta(p_1 - p_2)_t + \Delta B_t (p_1 - p_2)_t + \Delta v_{t+1}^e,$$

where Δv_{t+1}^e is assumed to be noisy $I(0)$ process. Simulations suggest that a change in ΔB_t has to be implausibly large for $\Delta B_t (p_1 - p_2)_t$ to have a noticeable effect on $\Delta e_{12,t}$. Hence, we assume similarly as Frydman and Goldberg (2007) that $|\Delta B_t (p_1 - p_2)_t| \ll |B_t \Delta(p_1 - p_2)_t|$ and that

$$\Delta e_{12,t} \simeq B_t \Delta(p_1 - p_2)_t + \Delta v_t^e. \quad (3)$$

To be able to estimate the above model with the data using the constant parameter CVAR model we need to address the problem of time-varying parameters inherent in (3).

To study the properties of this type of model, Tabor (2014, Chapter 3) simulates a simple model, $Y_t = \beta_t' X_t + \varepsilon_t$, where the coefficient vector $\beta_t = \beta_0 + \rho\beta_{t-1} + \varepsilon_{\beta,t}$ is stochastically non-trending with $0 < \rho < 1$ and where the adjustment back to $\beta_t' X_t$ is immediate. By fitting a constant parameter CVAR model to the simulated data (implying that $(\beta_t - \beta)X_t$ becomes part of the residual) the estimated gap term, $Y_t - \hat{\beta}' X_t$, is shown to be highly persistent. The closer ρ is to 1, the more persistent it is and the smaller is the adjustment coefficient α (while still highly significant). As long as $\rho < 1$, the mean value of the estimated $\hat{\beta}_t$ is approximately equal to its true value β . Thus, the pronounced persistence that often characterizes constant parameter models of asset prices may very well be the result of time-varying coefficients as a result of forecasting under imperfect knowledge.

Now assume that in (3) $B_t = b_0 + \rho B_{t-1} + \varepsilon_{\beta,t}$ and $E(B_t) = (b_0/1 - \rho) = 1$. If ρ is close to, but not one, then the Tabor (2014) results predict that $\Delta e_{12,t} - \Delta(p_{1,t} - p_{2,t})$ should be a persistent near $I(1)$ process. It follows then that $(e_{12,t} - p_{1,t} + p_{2,t})$ is a near $I(2)$ process. Thus, while the real exchange rate should behave like $I(0)$ or near $I(1)$ process in the REH-based models, it should behave like a near $I(2)$ process in imperfect knowledge based models.

As discussed in Juselius (2014), this process can be approximated by a random walk with a time-varying drift term (see also Frydman and Goldberg, 2007):

$$\Delta q_t = \zeta_t + e_t^q \quad (4)$$

where e_t^q is stationary and the drift term, ζ_t , is assumed to follow an autoregressive process:

$$\zeta_t = \rho_t \zeta_{t-1} + \varepsilon_t^\zeta.$$

The drift term, ζ_t , is related to the time-varying coefficients, $(\beta_t - \beta)\Delta q_t$, as a result of forecasting based on imperfect knowledge. The parameter $\rho_t \approx 1$ in periods when the PPP gap is moderately sized (i.e. the proportion of chartists is high) and $\rho_t \ll 1$ when the gap is large (i.e. the proportion of fundamentalists is high). Since the periods when $\rho_{j,t} \ll 1$ are likely to be short compared to the ones when $\rho_{j,t} \approx 1$, the average $\bar{\rho}_j$ is assumed to be close to 1 (Frydman and Goldberg, 2007). Hence, ζ_t can be considered a near $I(1)$ process. The noise component, ε_t^q , is likely to be large relative to the drift component, ε_t^ζ . The former is associated with volatile expectations to short-run changes in important determinants, v_t^e in (2), and the latter with small changes in $(\beta_t - \beta)\Delta q_t$ associated with smooth momentum trading along the trend. As a result the signal-to-noise ratio is likely to be small, i.e. $\sigma_{\varepsilon,\zeta}^2 \ll \sigma_{\varepsilon,q}^2$. In such a case it is often difficult to detect the tiny, but persistent, drift term (??) by econometric testing. For example, Juselius (2014) shows by simulations that the univariate Dickey-Fuller tests essentially never finds a second large root when $\bar{\rho} = 0.9$ and $\sigma_{\varepsilon,\zeta}/\sigma_{\varepsilon,q} = 0.15$ (a typical value for many foreign exchange markets) whereas the multivariate trace tests finds it in the majority of all cases.

Such persistent swings in the real exchange rate are likely to affect the speculative demand for foreign currency. To control for this feature Frydman and Goldberg (2007) add an uncertainty premium to the UIP condition and the excess return becomes:

$$exr_t^{IK} = (i_{d,t} - i_{f,t}) - \Delta e_{12,t+1}^e + up_t \quad (5)$$

where up_t stands for an uncertainty premium measuring agents' loss aversion⁶. The

⁶The assumption that agents are loss averse, rather than risk averse, builds on the prospect theory by Kahneman and Tversky (1979).

condition (5) describes an economy where financial actors require a minimum return - an uncertainty premium - to speculate in the foreign exchange market. When the asset price moves away from its long-run value, the uncertainty premium starts increasing until the price reverses back towards equilibrium. In the foreign currency markets the uncertainty premium is likely to be closely associated with the PPP gap (as well as other relevant gaps, for example the current account deficit, see Frydman and Goldberg, 2007)⁷. Using the PPP gap, the excess return can now be formulated as:

$$exr_t^{IK} = (i_{d,t} - i_{f,t}) - \Delta e_{12,t+1}^e + f(p_{d,t} - p_{f,t} - e_{12,t}) \quad (6)$$

Thus, the expected change in nominal exchange rate is not directly associated with the observed interest rate differential, but with the interest rate differential corrected for the PPP gap.

4 Specifying the CVAR model for the Swiss-US data

It is useful to formulate the $I(2)$ CVAR model in acceleration rates, changes and levels (see Juselius 2006). We here use a Maximum Likelihood parametrization first suggested by Johansen (1997) and then modified by Paruolo and Rahbek (1999):

$$\Delta^2 x_t = \alpha(\beta' x_{t-1} + \delta' \Delta x_{t-1}) + \zeta \tau' \Delta x_{t-1} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t, \quad (7)$$

where α is a $p \times r$ matrix of adjustment coefficients, β is a $(p+1) \times r$ matrix describing long-run relationships among the variables, $\delta = \psi' \bar{\tau}' \tau' \tau' \tau'$ ⁸ is a $(p+1) \times r$ matrix of coefficients determined so that $(\beta' x_{t-1} + \delta' \Delta x_{t-1}) \sim I(0)$, $\tau = [\beta, \beta_{\perp 1}]$ is a $(p+1) \times (r+d_1)$ matrix describing stationary relationships among the differenced variables with $\beta_{\perp 1}$ being the orthogonal complement of $[\beta, \beta_{\perp 2}]$, ζ is a $p \times (p-d_2)$ matrix of medium run adjustment coefficients, p is the dimension of the data vector, r is the number of multicointegration relations, d_1 is the number of cointegration relations that can only become stationary by differencing and d_2 is the number of $I(2)$ trends. The data vector is given by $x_t = [p_{1,t}, p_{2,t}, e_{12,t}, b_{1,t}, b_{2,t}, s_{1,t}, s_{2,t}]$ where p_t stands for CPI prices, $e_{12,t}$ for the nominal Swiss franc-US dollar rate, b_t for the ten-year bond rates, s_t for the three-month interest rates, a subscript 1 for Switzerland and a subscript 2 for the US. The model includes seasonal dummies for the full sample period, and three additional seasonal dummies from 2000:1 to account for a change in the seasonal pattern of the Swiss CPI. The dummy vector, D_t , includes three impulse dummies at 1980:2, 1982:4 and 2008:4 to control for very large residuals in the model due to specific events. The sample period is 1975:1-2013:3.

4.1 The choice of rank indices

The hypothesis that x_t is $I(1)$ is formulated as a reduced rank hypothesis on $\Pi = \alpha\beta'$, and that it is $I(2)$ as an additional reduced rank hypothesis, $\alpha'_{\perp} \Gamma \beta_{\perp} = \xi \eta'$, where ξ, η are $(p-r) \times d_1$ and $\alpha_{\perp}, \beta_{\perp}$ are orthogonal complements of α, β . The determination of the reduced rank indices is based on the maximum likelihood trace test procedure in Nielsen and Rahbek (2007). The results are reported in Table 1. The procedure starts with the most restricted model ($r = 0, d_1 = 0, d_2 = 7$)⁹, where d_2 is the number of $I(2)$

⁷The assumption in Hommes (2005) that the proportion of chartists relative to fundamentalists decrease as the PPP gap grows is likely to capture a similar gap effect.

⁸Using the notation of Johansen (1997).

⁹The tests for $r = 0, 1, 2$ and 3 were all rejected and the Table only reports results for $r = 4, 5, 6, 7$.

Table 1: I(2) trace tests

| Rank test statistics | | | | | | | | |
|---|-----|-----------|------------------|------------------|------------------------|-----------------|-----------------|-----------|
| $p - r$ | r | $d_2 = 5$ | $d_2 = 4$ | $d_2 = 3$ | $d_2 = 2$ | $d_2 = 1$ | $d_2 = 0$ | |
| 4 | 3 | | 213.38 [0.00] | 149.00 [0.00] | 101.08 [0.01] | 86.90 [0.01] | 84.43 [0.00] | |
| | 3 | 4 | | 97.60 [0.01] | 65.09 [0.11] | 54.14 [0.05] | 50.46 [0.01] | |
| | 2 | 5 | | | 35.17 [0.53] | 27.05 [0.30] | 23.09 [0.11] | |
| | 1 | 6 | | | | 15.46 [0.20] | 7.49 [0.31] | |
| The modulus of the seven largest characteristic roots for $r = 5$ | | | | | | | | |
| $r = 5, d_2 = 0$ | | | 1.00 | 1.00 | 0.96 | 0.88 | 0.88 | 0.69 0.66 |
| $r = 5, d_2 = 2$ | | | 1.00 | 1.00 | 1.00 | 1.00 | 0.75 | 0.69 0.69 |

trends, and continues row-wise until the first non-rejection at $(r = 4, d_1 = 1, d_2 = 2)$. However, this is a borderline case corresponding to a p-value of only 0.11, whereas the next non-rejection at $(r = 5, d_1 = 0, d_2 = 2)$ has a p-value of 0.53. In both cases the result is consistent with five relations that cointegrate from $I(2)$ to $I(1)$. In the former case there are four polynomially cointegrating relations $\beta'x_t + \delta'\Delta x_t$ plus one relation, $\beta'_{\perp 1}\Delta x_t$, which can only become stationary by differencing. In the latter case stationarity is achieved exclusively by polynomial cointegration. From a practical point of view, the difference between the two cases is small and we continue with $(r = 5, d_1 = 0, d_2 = 2)$ as it was more strongly supported by the test.

At the bottom of Table 1 the moduli of the seven largest roots are reported for the choice of $r = 5$ and $d_2 = 0, 2$. The choice of $d_2 = 2$ eliminates all large roots in the model, whereas the choice of $d_2 = 0$ (i.e. treating the variables as $I(1)$) would leave a very large root with a modulus of 0.96 and a complex pair of roots with a modulus of 0.88 in the model. The former is likely to describe the long smooth movements in relative prices and the latter the long persistent swings in nominal exchange rates around the relative price.¹⁰

Because the long swings should be bounded according to the IKE theory, a near unit root model would be more consistent with the theoretical framework. The fact that the null of a double unit root cannot be rejected based on a p-value of 0.53 does not preclude that alternative hypothesis of a near unit root can be correct. But since approximating a near unit root with a unit root allows us to exploit the different persistency profiles of the data and to study the dynamics of the error increasing/correcting behavior of the system we continue with the choice of $(r = 5, d_1 = 0, d_2 = 2)$.

4.2 Integration and cointegration properties

Juselius (2015) derived the time-series properties of the nominal exchange rate, prices and interest rates in a rational expectations versus imperfect knowledge based monetary model for exchange rate determination. She finds that the nominal exchange rate and the interest rates should be $I(1)$ in the RE case whereas near $I(2)$ in the IKE case. This difference in the order of integration is related to the uncertainty premium discussed in Section 3. Prices should be approximately $I(2)$ in both cases as they are not in general subject to speculation. The deviations from basic parities such as the PPP, and the Fisher parities should be stationary or at most near $I(1)$ under rational expectations,

¹⁰Johansen et al. (2010) obtain similar results based on US-German data for the pre-EMU period.

Table 2: Testing hypotheses of $I(1)$ versus $I(2)$

| | p_1 | p_2 | e_{12} | b_1 | b_2 | s_1 | s_2 | $\chi^2(v)$ | $p - val$ |
|--------------------|-------|-------|----------|-------|-------|-------|-------|-------------|-----------|
| \mathcal{H}_1 | 1.0 | -1.0 | - | - | - | - | - | 21.5(3) | 0.00 |
| \mathcal{H}_2 | - | - | 1.0 | - | - | - | - | 21.6(3) | 0.00 |
| \mathcal{H}_3 | 1.0 | -1.0 | -1.0 | - | - | - | - | 22.1(3) | 0.00 |
| \mathcal{H}_4 | - | - | - | 1.0 | - | - | - | 27.7(3) | 0.00 |
| \mathcal{H}_5 | - | - | - | - | 1.0 | - | - | 22.1(3) | 0.00 |
| \mathcal{H}_6 | - | - | - | 1.0 | -1.0 | - | - | 23.4(3) | 0.00 |
| \mathcal{H}_7 | - | - | - | - | - | 1.0 | - | 9.6(3) | 0.02 |
| \mathcal{H}_8 | - | - | - | - | - | - | 1.0 | 22.6(3) | 0.00 |
| \mathcal{H}_9 | - | - | - | - | - | 1.0 | -1.0 | 23.0(3) | 0.00 |
| \mathcal{H}_{10} | - | - | - | 1 | - | -1 | - | 2.5(3) | 0.48 |
| \mathcal{H}_{11} | - | - | - | - | 1 | - | -1 | 7.1(3) | 0.07 |

but near $I(2)$ under imperfect knowledge. The relative price should be $I(1)$ and the term spread $I(0)$ under RE, whereas near $I(2)$ and $I(1)$, respectively, under imperfect knowledge.

The above integration and cointegration hypotheses can be tested by the Likelihood Ratio tests described in Johansen et al. (2010). Table ?? reports the test results. The hypotheses that the relative price level, the nominal and the real exchange rate, the nominal interest rates, the short and the long-term interest differentials are $I(1)$ are all rejected¹¹ whereas the hypotheses that the Swiss and the US short-long spreads are $I(1)$ cannot be rejected with a p-value of 0.48 and 0.07 respectively. Thus, the results support imperfect knowledge based expectations.

It needs to be emphasized that the near $I(2)$ persistence of the nominal exchange rate and the interest rates is primarily associated with the shocks to the drift term, ζ_t in (4). When the shocks to the drift term are tiny compared to the shocks to the process itself, as is usually the case with the nominal exchange rate, the drift term might be hard to catch sight of because of the large short-run volatility. See, for example Figure 1, panel 1.

5 The pulling forces

The case $\{r = 5, d_1 = 0, d_2 = 2\}$ defines five stationary polynomially cointegrating relations, $\beta'_i x_t + \delta'_i \Delta x_t$, $i = 1, \dots, 5$. They can be interpreted as dynamic equilibrium relations in the following sense: When data are $I(2)$, $\beta'_i x_t$ is generally $I(1)$ and can be given an interpretation as an equilibrium error that exhibits pronounced persistence. In such a case, it is useful to interpret the coefficients α and δ as two levels of equilibrium correction: The δ adjustment describes how the growth rates, Δx_t , adjust to the long-run equilibrium errors, $\beta'_i x_t$; the α adjustment describes how the acceleration rates, $\Delta^2 x_t$, adjust to the dynamic equilibrium relations, $\beta'_i x_t + \delta'_i \Delta x_t$. This is illustrated below for the variable $x_{i,t}$:

$$\Delta^2 x_{i,t} = \dots \sum_{i=1}^r \alpha_{ij} (\delta'_i \Delta x_{t-1} + \beta'_i x_{t-2}) + \dots, j = 1, \dots, p \quad (8)$$

¹¹Note, however, that applying a near unit root correction to these tests is likely to increase the p-values to some extent. Such a correction has not been applied here as the correct size is not yet known.

where $\delta'_i = [\delta_{i1}, \dots, \delta_{ij}, \dots, \delta_{ip}]$ and β'_i is similarly defined.

The long and persistent swings away from PPP would generally imply equilibrium error increasing behavior (positive feedback) in the medium run but error correcting behavior (negative feedback) in the long run. However, in a world populated by actors with rational expectations, the CVAR would still be an appropriate empirical framework, but would give results showing pure equilibrium error correction (Juselius, 2015).

Error increasing and error correcting behavior can be empirically studied by checking the signs of β, δ , and α in the following way: If $\delta_{ij}\beta_{ij} > 0$, then $\Delta x_{i,t}$, is equilibrium error correcting to $\beta' x_t$; if $\alpha_{ij}\beta_{ij} < 0$ then the acceleration rate, $\Delta^2 x_{i,t}$, is equilibrium correcting to $(\beta' x_t + \delta' \Delta x_t)$; if $\alpha_{ij}\delta_{ij} < 0$ then the acceleration rates, $\Delta^2 x_{i,t}$, are equilibrium correcting to $\delta'_i \Delta x_{t-1}$. In all other cases the system is equilibrium error increasing. Whether a variable is equilibrium error correcting or equilibrium error increasing is vital for understanding the mechanisms behind self-reinforcing feedback in the system. We shall focus on this particular feature when discussing the results.

Since all characteristic roots were inside the unit circle¹² the system is stable, as it should be, implying that any equilibrium error increasing behavior is compensated by error correcting behavior somewhere else in the system. Thus, while variables can move away from their long-run stable equilibrium path for extended periods of time, sooner or later the equilibrating forces will set in, for example due to an increasing uncertainty premium, and pull the variable back towards equilibrium.

The finding of two stochastic near $I(2)$ trends in Section 4 suggests that neither relative prices and the nominal exchange rates nor Swiss and US prices have moved closely together over this period. We have, therefore, chosen to identify the five cointegration relations by associating each of the five variables, the *ppp* gap $p_1 - p_2 - e_{12}$, the nominal exchange rate, e_{12} , the price differential, $p_1 - p_2$, Swiss prices, p_1 , and US prices, p_2 , with suitable combinations of the four interests rates. Since all relations need a linear trend to become stationary, the latter variables should be interpreted as deviations from their long-term trends, basically capturing business cycle fluctuations in prices and exchange rates. For example, in the *ppp* relation the trend may describe a small but significant productivity differential between the two countries.

The structure in Table ?? imposes one over-identifying restriction on β , accepted with a p-value of 0.92 but, because of the complexity of the asymptotic distribution of the δ coefficients, the latter have not been subject to over-identifying restrictions. Since the $\beta' x_t$ relations are generally defining cointegration from $I(2)$ to $I(1)$, the above relations need to be combined with the growth rates to become stationary. As discussed above, the α and the δ coefficients describe the adjustment dynamics and allow us to investigate how changes, $\Delta x_{i,t}$, and acceleration rates, $\Delta^2 x_{i,t}$, respond to imbalances the system. To facilitate interpretation, statistically significant coefficients of β and α (and large coefficients of δ) are given in bold face.¹³ Error-increasing coefficients are given in italics.

The first β relation is identified by incorporating the main features of the mechanism that governs the behavior of prices, interest rates and the exchange rate as expressed by (6). Thus, $\hat{\beta}'_1 x_t \sim I(1)$ captures the main theoretical hypothesis:

$$\hat{\beta}'_1 x_t = (s_{1,t} - s_{2,t}) - \omega_1 \widehat{ppp}_t - \omega_2 (b_{1,t} - b_{2,t}). \quad (9)$$

¹²Roots inside the unit disk imply non-explosive behavior as they are calculated as eigenvalues of the characteristic polynomials (Juselius, 2006).

¹³Note that all β coefficients have very large t ratios, actually sufficiently large to be statistically significant also after having applied a near unit root correction. However, the significance of the δ coefficients is at this stage a guesswork.

Table 3: The pulling forces

| An overidentified β structure $\chi^2(1) = 0.01[0.92]$ | | | | | | | | |
|--|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|-------------------------|-------------------------|--------------------------|
| | p_1 | p_2 | e_{12} | b_1 | b_2 | s_1 | s_2 | <i>trend</i> |
| β_1 | 0.02 [19.09] | -0.02 [-19.09] | -0.02 [-19.09] | 0.63 [11.09] | -0.63 [-11.09] | -1.00 | 1.00 | -0.00 [-15.62] |
| δ_1 | 0.53 | -0.55 | -0.67 | 0.04 | -0.09 | <i>0.03</i> | <i>-0.08</i> | 0.00 |
| α_1 | <i>1.43</i> [2.67] | -0.29 [-0.54] | 8.96 [1.37] | -0.39 [-4.67] | <i>-0.54</i> [-3.78] | -0.23 [-1.16] | -0.56 [-4.07] | |
| β_2 | 0.01 [23.56] | -0.01 [-23.56] | — | -0.43 [-16.67] | 1.00 | 0.43 [16.67] | -1.00 | 0.00 [17.08] |
| δ_2 | <i>-0.19</i> | -0.11 | 0.14 | -0.01 | 0.01 | -0.01 | 0.00 | -0.00 |
| α_2 | -5.14 [-5.49] | -0.09 [-0.10] | -39.05 [-3.42] | -0.23 [-1.57] | 0.27 [1.08] | -2.34 [-6.86] | 0.18 [0.75] | |
| β_3 | — | — | 0.05 [25.23] | — | — | 1.00 | — | 0.00 [34.77] |
| δ_3 | -0.29 | 1.56 | 0.78 | -0.06 | 0.15 | <i>-0.04</i> | 0.14 | -0.07 |
| α_3 | 0.59 [3.19] | -0.38 [-2.09] | <i>3.65</i> [1.62] | <i>-0.13</i> [-4.41] | -0.21 [-4.22] | -0.07 [-1.09] | -0.21 [-4.40] | |
| β_4 | -0.02 [-82.73] | — | — | -0.54 [-6.48] | — | 1.00 | — | 0.00 [8.36] |
| δ_4 | -0.76 | 0.04 | 0.72 | -0.04 | 0.06 | <i>-0.03</i> | 0.05 | 0.08 |
| α_4 | 3.02 [6.67] | 0.17 [0.39] | 16.74 [3.03] | -0.13 [-1.89] | -0.39 [-3.22] | <i>0.51</i> [3.10] | -0.38 [-3.21] | |
| β_5 | — | -0.00 [-51.96] | — | — | -0.89 [-14.31] | — | 1.00 | 0.00 [10.19] |
| δ_5 | -0.01 | <i>0.08</i> | 0.03 | -0.00 | 0.01 | -0.00 | 0.01 | 0.01 |
| α_5 | -7.59 [-4.85] | -1.93 [-1.24] | -63.14 [-3.31] | -0.16 [-0.63] | 0.58 [1.38] | -3.01 [-5.26] | 0.14 [0.35] | |

where $\widehat{ppp}_t = (p_{1,t} - p_{2,t} - e_{12,t}) - \omega_3 t$ stands for trend-adjusted PPP gap. Juselius (2015) showed that, under imperfect knowledge, the interest rate spread corrected for the uncertainty premium should be $I(1)$. In the present case we have two gap effects, the PPP gap and the long-term bond rate gap, that can be interpreted as a combined measure of the uncertainty premium agents require to enter a trade. By combining (9) with the differenced process, $\hat{\delta}' \Delta x_t \sim I(1)$, as a proxy for $\Delta e_{12,t+1}^e$ in (6), stationarity is achieved so that:

$$\left\{ \hat{\beta}'_1 x_t + 0.55(\Delta p_{1,t} - \Delta p_{2,t}) - 0.67 \Delta e_{12,t} + 0.04 \Delta b_{1,t} + 0.03 \Delta s_{1,t} - 0.09 \Delta b_{2,t} - 0.08 \Delta s_{2,t} \right\} \sim I(0)$$

The estimates of δ_1 suggest that the inflation rate differential is an important determinant in this respect, possibly measuring short-term exchange rate expectations as postulated by (6).

In terms of adjustment behavior, all variables except the two short-term interest rates are equilibrium error correcting in the medium run. This would be consistent with monetary policy driving the relation away from equilibrium but market-determined variables adjusting back over the medium run. The nominal exchange rate, the Swiss bond rate, and the US short-term rate are equilibrium error correcting in the long run, though the coefficient on the exchange rate is not significant. That prices are equilibrium error increasing even in the long run can seem surprising, but might reflect the finding in the previous section that both price differentials were near $I(2)$. The Swiss bond rate and the Swiss short-term rate are both error increasing in the long run, though the Swiss short-term rate not very significantly so.

The second relation is a relationship between the trend-adjusted price differential and the two short-long spreads that can be interpreted as reflecting the expectations

hypothesis of the term structure and relative monetary policy in the two countries. The size of the δ coefficients shows that changes in Swiss and US prices and the nominal exchange rate also enter this relation in an economically significant way. Swiss prices and the Swiss and US short-term rates are error increasing in the medium run but error correcting in the long run. Most adjustment coefficients on the US variables, however, are not very significant implying that it is primarily a Swiss relation. When the Swiss short-long spread is above its equilibrium value, the Swiss franc depreciates in the medium run. This could be a situation in which Swiss short-term rates are relatively low, reflecting an expansive monetary policy. Capital flows out of Switzerland could lead to a depreciation of the Swiss franc in such a situation. In the long run, the Swiss franc appreciates.

The third relation shows that the trend-adjusted nominal exchange rate has been negatively related to the Swiss short rate, meaning that the Swiss franc tends to be stronger when Swiss short-term rate is high. The exchange rate is error correcting in the medium run but error increasing in the long run. When the short rate is above its equilibrium value, Swiss prices have tended to go down in the medium run but up in the long run. US prices tend to move in the opposite way. The Swiss short rate is not significantly adjusting, suggesting that it is exogenous to this relation. Instead the Swiss bond rate has decreased significantly both in the medium and the long run, probably reflecting capital inflows in situations with relatively tight Swiss monetary policy. The US rates have been increasing in the medium run which would be consistent with a capital-flows interpretation. They have been decreasing in the long run.

The fourth relation shows that Swiss trend-adjusted prices have been positively comoving with the Swiss short-term relative to the long-term interest rate. Hence, the short-term relative to the long-term rate has been higher when Swiss prices have been above their long-term trend and vice versa. This relation can be seen as a characterization of Swiss domestic monetary policy, influencing the slope of the yield curve. Swiss prices are equilibrium correcting in the medium and the long run, whereas the coefficients on the US price level are small and insignificant. The Swiss franc has appreciated when Swiss prices have been above their equilibrium value, probably reflecting relatively tight monetary policy and the resulting capital flows. Though the Swiss bond rate is equilibrium error correcting in the medium run, both Swiss interest rates are error increasing in the long run (the long rate less significantly so). The two US interest rates increase in the medium run (probably because of capital flows), but fall in the long run.

The fifth relation presents the US equivalent to the fourth relation. Trend-adjusted US prices have been positively associated with the US short-term relative to the long-term interest rate. The US short rate is not significantly adjusting and, hence, is exogenous to this relation. By contrast, the US long rate is error correcting though not very significantly so. The most obvious difference to the fourth relation is that US prices have been equilibrium error increasing both in the medium and the long run, but not very significantly so. The US dollar has appreciated when the US short-term rate has been above its equilibrium value, which would be consistent with capital flows into the US. This matches with the results of Bruno and Shin (2014) who provide evidence that monetary policy shocks in the US lead to international spillover through capital flows in the banking sector. Swiss prices and the Swiss short-term rate have decreased when the US short-term rate has been above its equilibrium value.

Altogether the results suggest that the persistent movements away from equilibrium values visible in Figure 1-5 are associated with lack of equilibrium correcting behavior in prices as well as interest rates. The US interest rates, in particular, showed a tendency for

self-reinforcing behavior in the hypothetical relation (??) pointing to the importance of the US dollar as an international reserve currency. Overall, divergent monetary policies and the resulting international spillovers seem to be responsible for the long swings in interest rate differentials and the real exchange rate.

6 Concluding discussion

Real exchange rates, real interest rates and interest rate differentials tend to exhibit a pronounced persistence which seems untenable with standard models based on RE. Here we argue that expectation formation based on imperfect knowledge and fundamental uncertainty is consistent with these long, persistent swings around long-run equilibrium values we see in the data. Such swings signal the presence of self-reinforcing feedback mechanisms somewhere in the economic system. The econometric problem is to identify the channels through which they work. For this purpose, the Cointegrated VAR for $I(2)$ data is tailor-made as it can describe equilibrium error increasing adjustment behavior in the medium run combined with error correction in the long run.

We found a number of such positive and negative feedback mechanisms in the foreign exchange market. While essentially all variables showed some evidence of error increasing behavior, the strongest and most significant error increasing behavior was associated with US interest rates, suggesting that speculative asset flows in to and out of the US play a significant role for the determination of exchange rates, interest rates and prices. In spite of the strong evidence of self-reinforcing feedback mechanisms, the system itself was still found to be stable in the sense that all characteristic roots were either on or inside the unit circle. In terms of cointegration we found that persistent movements in one parity were counteracted by similar persistent movements in another. For example, persistent movements in the short-term interest rate differential were cointegrated with persistent movements in the long-term bond differential and deviations from PPP. By interpreting the latter two as a proxy for an uncertainty premium in the foreign exchange market as proposed by Frydman and Goldberg (2007), the results provided strong empirical support for uncovered interest parity being stationary once an adjustment for uncertainty is allowed for. Thus, much of the forward premium puzzle seem to disappear when accounting for an uncertainty premium in the foreign exchange markets.

As mentioned in the introduction, the chartists/fundamentalists model represents another way of modelling positive feedback behavior. In these models agents switch endogenously between a mean-reverting fundamentalist and a trend-following chartist strategy. They have been taken to the data by applying a non-linear $AR(1)$ model with a time-varying coefficient which is supposed to capture market sentiment. In periods of strong trend-following, such a model can temporarily describe explosive behavior.¹⁴ In contrast, we use an $I(2)$ CVAR model to describe persistent swings as a result of two types of adjustment. Strong trend-following behavior appears as error-increasing behavior away from long-run fundamental values (positive feedback) and reversal to fundamental values as error-correcting behavior (negative feedback).

¹⁴For example De Jong et al. (2010) estimated such a model to exchange rates and Boswijk et al. (2007), Hommes and in't Veld (2014), and Lof (2015) to stock prices.

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