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Real Hypersurfaces in Complex Projective Space Whose Structure Jacobi Operator Is Cyclic-Ryan Parallel

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Abstract. We classify real hypersurfaces in complex projective space whose structure Jacobi operator satisfies a certain cyclic condition.

1. Introduction

Let $\mathbb{C}P^m$, $m \geq 3$, be a complex projective space endowed with the metric g of constant holomorphic sectional curvature 4. Let M be a connected real hypersurface of $\mathbb{C}P^m$ without boundary. Let J denote the complex structure of $\mathbb{C}P^m$ and N a locally defined unit normal vector field on M. Then $-JN = \xi$ is a tangent vector field to M called the structure vector field on M. We also call $\mathbb{D}$ the maximal holomorphic distribution on M, that is, the distribution on M given by all vectors orthogonal to ξ at any point of M.

The study of real hypersurfaces in nonflat complex space forms is a classical topic in Differential Geometry. The classification of homogeneous real hypersurfaces in $\mathbb{C}P^m$ was obtained by Takagi, see [14], [15], [16], and is given by the following list: A_1 : Geodesic hyperspheres. A_2 : Tubes over totally geodesic complex projective spaces. B : Tubes over complex quadrics and $\mathbb{R}P^m$. C : Tubes over the Segre embedding of $\mathbb{C}P^1$x$\mathbb{C}P^n$, where $2n + 1 = m$ and $m \geq 5$. D : Tubes over the Plucker embedding of the complex Grassmann manifold $G(2, 5)$. In this case $m = 9$. E : Tubes over the cannonical embedding of the Hermitian symmetric space $SO(10)/U(5)$. In this case $m = 15$.

Other examples of real hypersurfaces are ruled real ones, that were introduced by Kimura, [5]: Take a regular curve γ in $\mathbb{C}P^m$ with tangent vector field X. At each

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point of γ there is a unique complex projective hyperplane cutting γ so as to be orthogonal not only to X but also to JX. The union of these hyperplanes is called a ruled real hypersurface. It will be an embedded hypersurface locally although globally it will in general have self-intersections and singularities. Equivalently a ruled real hypersurface is such that $\mathbb{D}$ is integrable or $g(A\mathbb{D},\mathbb{D}) = 0$, where A denotes the shape operator of the immersion. For further examples of ruled real hypersurfaces see [7].

Except these real hypersurfaces there are very few examples of real hypersurfaces in $\mathbb{C}P^n$.

On the other hand, Jacobi fields along geodesics of a given Riemannian manifold $(\tilde{M},\tilde{g})$ satisfy a very well-known differential equation. This classical differential equation naturally inspires the so-called Jacobi operator. That is, if $\tilde{R}$ is the curvature operator of $\tilde{M}$, and X is any tangent vector field to $\tilde{M}$, the Jacobi operator (with respect to X) at $p \in M$, $\tilde{R}_X \in$End$(T_p\tilde{M})$, is defined as $(\tilde{R}_XY)(p) = (\tilde{R}(Y,X)X)(p)$ for all $Y \in T_p\tilde{M}$, being a selfadjoint endomorphism of the tangent bundle $T\tilde{M}$ of $\tilde{M}$. Clearly, each tangent vector field X to $\tilde{M}$ provides a Jacobi operator with respect to X.

The study of Riemannian manifolds by means of their Jacobi operators has been developed following several ideas. For instance, in [1], it is pointed out that (locally) symmetric spaces of rank 1 (among them complex space forms) satisfy that all the eigenvalues of $\tilde{R}_X$ have constant multiplicities and are independent of the point and the tangent vector X.

Let M be a real hypersurface in a complex projective space and let ξ be the structure vector field on M. We will call the Jacobi operator on M with respect to ξ the structure Jacobi operator on M. Then the structure Jacobi operator $R_\xi \in End(T_pM)$ is given by $(R_\xi(Y))(p) = (R(Y,\xi)\xi)(p)$ for any $Y \in T_pM$, $p \in M$, where R denotes the curvature operator of M in $\mathbb{C}P^m$. Some papers devoted to study several conditions on the structure Jacobi operator of a real hypersurface in $\mathbb{C}P^m$ are [2], [3], [4].

Recently, [9], we have proved the non-existence of real hypersurfaces in $\mathbb{C}P^m$ with parallel structure Jacobi operator. Also in [10], [11], [12], [13] we have studied distinct conditions on the structure Jacobi operator (Lie parallelism, Lie ξ-parallelism, $\mathbb{D}$-parallelism, and so on).

For any vector fields X,Y tangent to M, $R(X,Y)$ operates as a derivation on the algebra of tensor fields on M. For a tensor field F of type (r,s), $R(X,Y).F = \nabla_X\nabla_YF - \nabla_Y\nabla_XF - \nabla_{[X,Y]}F$. In the case of $F = R_\xi$, we get $(R(X,Y).R_\xi)Z = R(X,Y)(R_\xi(Z)) - R_\xi(R(X,Y)Z)$, for any X,Y,Z tangent to M.

The purpose of the present paper is to study a weaker condition than structure Jacobi operator being parallel for a real hypersurface of $\mathbb{C}P^m$. In fact we will study the condition

$$(R(X,Y).R_\xi)Z + (R(Y,Z).R_\xi)X + (R(Z,X).R_\xi)Y = 0 \tag{1.1}$$

for any X,Y,Z tangent to M. Due to the literature we propose to call them real hypersurfaces with cyclic-Ryan parallel structure Jacobi operator.

We will obtain the following

Theorem. *Let M be a real hypersurface of $\mathbb{C}P^m$, $m \geq 3$. Then M has cyclic-Ryan parallel structure Jacobi operator if and only if M is locally congruent either to a geodesic hypersphere or to a tube of radius $\pi/4$ over a complex submanifold in $\mathbb{C}P^m$.*

2. Preliminaries

Thoughout this paper, all manifolds, vector fields, etc., will be considered of class C^∞ unless otherwise stated. Let M be a connected real hypersurface in $\mathbb{C}P^m$, $m \geq 2$, without boundary. Let N be a locally defined unit normal vector field on M. Let ∇ be the Levi-Civita connection on M and (J, g) the Kaehlerian structure of $\mathbb{C}P^m$.

For any vector field X tangent to M we write $JX = \phi X + \eta(X)N$, and $-JN = \xi$. Then (ϕ, ξ, η, g) is an almost contact metric structure on M. That is, we have

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.1}$$

for any tangent vectors X, Y to M. From (2.1) we obtain

$$\phi\xi = 0, \quad \eta(X) = g(X, \xi). \tag{2.2}$$

From the parallelism of J we get

$$(\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi \tag{2.3}$$

and

$$\nabla_X \xi = \phi AX \tag{2.4}$$

for any X, Y tangent to M, where A denotes the shape operator of the immersion. As the ambient space has holomorphic sectional curvature 4, the equations of Gauss and Codazzi are given, respectively, by

$$\begin{aligned} R(X,Y)Z \;=\; & g(Y,Z)X - g(X,Z)Y + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y \\ & -2g(\phi X, Y)\phi Z + g(AY, Z)AX - g(AX, Z)AY, \end{aligned} \tag{2.5}$$

and

$$(\nabla_X A)Y - (\nabla_Y A)X = \eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi \tag{2.6}$$

for any tangent vectors X, Y, Z to M, where R is the curvature tensor of M.

In the sequel we need the following results:

Theorem 2.1 ([6]). *A real hypersurface M of $\mathbb{C}P^m$, $m \geq 3$ satisfies $R(X,Y)AZ + R(Y,Z)AX + R(Z,X)AY = 0$, for any X, Y, Z tangent to M if and only if it is* *locally congruent to a geodesic hypersphere.*

Theorem 2.1 ([9]. *There exist no real hypersurfaces M in $\mathbb{C}P^m$, $m \geq 3$, such that the shape operator is given by $A\xi = \xi + \beta U$, $AU = \beta\xi + (\beta^2 - 1)U$, $A\phi U = -\phi U$, $AX = -X$, for any tangent vector X orthogonal to Span$\{\xi, U, \phi U\}$, where U is a unit vector field in $\mathbb{D}$ and β is a nonvanishig smooth function defined on M.*

3. Proof of the theorem

Bearing in mind Bianchi identity, (1.1) is equivalent to have $R(X,Y)(R_\xi(Z)) + R(Y,Z)\ (R_\xi(X)) + R(Z,X)(R_\xi(Y)) = 0$. As $R_\xi(Z) = Z - g(Z,\xi)\xi + g(A\xi,\xi)AZ - g(AZ,\xi)A\xi$, we get $R(X,Y)(R_\xi(Z)) = R(X,Y)Z - g(Z,\xi)R(X,Y)\xi + g(A\xi,\xi)\ R(X,Y)AZ - g(AZ,\xi)\ R(X,Y)A\xi$. So our condition is equivalent to $-g(Z,\xi)\ R(X,Y)\xi - g(X,\xi)R(Y,Z)\xi - g(Y,\xi)R(Z,X)\xi + g(A\xi,\xi)[R(X,Y)AZ + R(Y,Z)AX + R(Z,X)AY] - g(AZ,\xi)R(X,Y)\ A\xi - g(AY,\xi)R(Z,X)A\xi - g(AX,\xi)R(Y,Z)A\xi = 0$. From Gauss equation we obtain

$$\begin{aligned}(3.1)\quad & -g(Z,\xi)(g(AY,\xi)AX - g(AX,\xi)AY) - g(X,\xi)(g(AZ,\xi)AY - g(AY,\xi)AZ)\\ & -g(Y,\xi)(g(AX,\xi)AZ - g(AZ,\xi)AX) + g(A\xi,\xi)(g(\phi Y,AZ)\phi X\\ & -g(\phi X,AZ)\phi Y - 2g(\phi X,Y)\phi AZ + g(\phi Z,AX)\phi Y - g(\phi Y,AX)\phi Z\\ & -2g(\phi Y,Z)\phi AX + g(\phi X,AY)\phi Z - g(\phi Z,AY)\phi X - 2g(\phi Z,X)\phi AY)\\ & -g(AZ,\xi)(g(\phi Y,A\xi)\phi X - g(\phi X,A\xi)\phi Y - 2g(\phi X,Y)\phi A\xi\\ & +g(AY,A\xi)AX - g(AX,A\xi)AY) - g(AX,\xi)(g(\phi Z,A\xi)\phi Y - g(\phi Y,A\xi)\phi Z\\ & -2g(\phi Y,Z)\phi A\xi + g(AZ,A\xi)AY - g(AY,A\xi)AZ) - g(AY,\xi)(g(\phi X,A\xi)\phi Z\\ & -g(\phi Z,A\xi)\phi X - 2g(\phi Z,X)\phi A\xi + g(AX,A\xi)AZ - g(AZ,A\xi)AX) = 0\end{aligned}$$

for any X,Y,Z tangent to M. First we suppose that M is Hopf, that is, $A\xi = \alpha\xi$, for a certain function α. Then (3.1) becomes

$$(3.2)\qquad \alpha(R(X,Y)AZ + R(Y,Z)AX + R(Z,X)AY) = 0$$

for any X,Y,Z tangent to M. Thus if $\alpha \neq 0$, $R(X,Y)AZ + R(Y,Z)AX + R(Z,X)AY = 0$. From Theorem 2.1, M must be locally congruent to a geodesic hypersphere. If $\alpha = 0$, then M is locally congruent to a tube of radius $\pi/4$ over a complex submanifold of $\mathbb{C}P^m$.

From now on we suppose that M is not Hopf. Thus locally we can write $A\xi = \alpha\xi + \beta U$, where U is a unit vector field in $\mathbb{D}$ and β a nonnull function. Introducing this expression into (3.1) we get

$$\begin{aligned}(3.3)\quad & -\beta g(Z,\xi)(g(Y,U)AX - g(X,U)AY) - \beta g(X,\xi)(g(Z,U)AY - g(Y,U)AZ)\\ & -\beta g(Y,\xi)(g(X,U)AZ - g(Z,U)AX) + \alpha(g((A\phi + \phi A)Y,Z)\phi X\\ & -2g(\phi Y,Z)\phi AX + g((A\phi + \phi A)Z,X)\phi Y - 2g(\phi Z,X)\phi AY\end{aligned}$$

$$+g((A\phi + \phi A)X, Y)\phi Z - 2g(\phi X, Y)\phi AZ) - g(AZ, \xi)$$
$$(\beta g(\phi Y, U)\phi X - \beta g(\phi X, U)\phi Y - 2\beta g(\phi X, Y)\phi U$$
$$+g(AY, A\xi)AX - g(AX, A\xi)AY) - g(AX, \xi)(\beta g(\phi Z, U)\phi Y$$
$$-\beta g(\phi Y, U)\phi Z - 2\beta g(\phi Y, Z)\phi U + g(AZ, A\xi)AY$$
$$-g(AY, A\xi)AZ) - g(AY, \xi)(\beta g(\phi X, U)\phi Z - \beta g(\phi Z, U)\phi X$$
$$-2\beta g(\phi Z, X)\phi U + g(AX, A\xi)AZ - g(AZ, A\xi)AX) = 0$$

for any X, Y, Z tangent to M. From now on we will call $\mathbb{D}_U$ the subspace of TM orthogonal to the subspace spanned by $\xi, U, \phi U$. Taking $Z = \xi$, $Y = U$, $X = \phi U$ in (3.3) we obtain $\beta g(A\phi U, U) = 0$. Thus

$$g(AU, \phi U) = 0. \tag{3.4}$$

Taking $Z = \xi$, $Y = U$, $X \in \mathbb{D}_U$ in (3.3) we have

$$g(AU, X) = 0 \tag{3.5}$$

for any $X \in \mathbb{D}_U$. From(3.4) and (3.5) we obtain $AU = \beta\xi + g(AU, U)U$. If we take $Z = U$, $Y = \phi U$, $X \in \mathbb{D}_U$ in (3.3) we get $-\alpha g(AU, U)\phi X - \alpha g(A\phi U, \phi U)\phi X + 2\alpha\phi AX - \alpha g(A\phi U, X)U + \alpha g(A\phi X, \phi U)\phi U + \beta^2\phi X = 0$. If $\alpha = 0$ this yields $\beta^2\phi X = 0$ which is impossible. Thus $\alpha \neq 0$. Taking the scalar product with ϕU,

$$g(A\phi X, \phi U) = 0 \tag{3.6}$$

for any $X \in \mathbb{D}_U$. Thus ϕU is principal and the above expression reduces to $-\alpha g(AU, U)\ \phi X - \alpha g(A\phi U, \phi U)\phi X + 2\alpha\phi AX + \beta^2\phi X = 0$, for any $X \in \mathbb{D}_U$. If we apply ϕ we obtain $\alpha g(AU, U)X + \alpha g(A\phi U, \phi U)X - 2\alpha AX - \beta^2 X = 0$ for any $X \in \mathbb{D}_U$. It follows

$$AX = ((g(AU, U) + g(A\phi U, \phi U))/2) - (\beta^2/2\alpha))X \tag{3.7}$$

for any $X \in \mathbb{D}_U$. If we take $X \in \mathbb{D}_U$, $Y = \phi X$, $Z = U$ in (3.3) and its scalar product with ϕU we get

$$\alpha(g(A\phi X, \phi X) + g(AX, X) - 2g(AU, U)) + 2\beta^2 = 0 \tag{3.8}$$

for any $X \in \mathbb{D}_U$. From (3.7) and (3.8) we obtain

$$g(AU, U) = g(A\phi U, \phi U) + (\beta^2/\alpha). \tag{3.9}$$

Taking $X \in \mathbb{D}_U$, $Y = \phi X$, $Z = \phi U$ in (3.3) and its scalar product with U it follows

$$\begin{aligned} g(A\phi U, \phi U) &= g(AX, X), \\ g(AU, U) &= g(AX, X) + (\beta^2/\alpha) \end{aligned} \tag{3.10}$$

for any $X \in \mathbb{D}_U$. If we call $A\phi U = \gamma\phi U$, then $g(AU, U) = \gamma + (\beta^2/\alpha)$.

Consider two orthonormal vector fields $X, Y \in \mathbb{D}_U$. Codazzi equation gives $(\nabla_X A)Y - (\nabla_Y A)X = -2g(\phi X, Y)\xi$. That is, $X(\gamma)Y - Y(\gamma)X + \gamma[X, Y] - A[X, Y] = -2g(\phi X, Y)\xi$. Taking the scalar product of this expression and ξ we get

$$(\gamma - \alpha)g([X, Y], \xi) - \beta g([X, Y], U) = -2g(\phi X, Y). \tag{3.11}$$

And its scalar product with U gives

$$\alpha g([X, Y], \xi) + \beta g([X, Y], U) = 0. \tag{3.12}$$

As $g([X, Y], \xi) = g(X, \nabla_Y \xi) - g(Y, \nabla_X \xi) = g(X, \phi AY) - g(Y, \phi AX) = -2\gamma g(\phi X, Y)$, from (3.11) and (3.12) we have

$$\gamma^2 = 1. \tag{3.13}$$

Now if we take $X \in \mathbb{D}_U$, $Y = U$, $Z = \xi$ in (3.3) we obtain $(1 + \gamma\alpha)\beta\gamma X = 0$. This yields

$$1 + \alpha\gamma = 0. \tag{3.14}$$

From (3.13) and (3.14) we have two possibilities: i) $\gamma = -1$, $\alpha = 1$ or ii) $\gamma = 1$, $\alpha = -1$.

From Theorem 2.2 case i) cannot occur. So we consider case ii), that is, $A\xi = -\xi + \beta U$, $AU = \beta\xi + (1 - \beta^2)U$, $A\phi U = U$, $AX = X$, for any $X \in \mathbb{D}_U$. Take $X \in \mathbb{D}_U$. Codazzi equation gives $(\nabla_X A)U - (\nabla_U A)X = 0$. This yields $X(\beta)\xi + \beta\phi X + X(1 - \beta^2)U + (1 - \beta^2)\nabla_X U - A\nabla_X U - \nabla_U X + A\nabla_U X = 0$. Taking the scalar product of this equality and U we get

$$g(\nabla_U U, X) = 2X(\beta)/\beta, \tag{3.15}$$

and the scalar product with ξ yields

$$g(\nabla_U U, X) = X(\beta)/\beta. \tag{3.16}$$

From (3.15) and (3.16) we get

$$X(\beta) = 0 \tag{3.17}$$

for any $X \in \mathbb{D}_U$. The scalar product of the above expression and X gives

$$g(\nabla_X U, X) = 0 \tag{3.18}$$

for any $X \in \mathbb{D}_U$.

If we develop $(\nabla_{X+U} A)\xi - (\nabla_\xi A)(X + U) = -\phi X - \phi U$ and take its scalar product with $X \in \mathbb{D}_U$ we obtain $\beta g(\nabla_X U, X) + \beta g(\nabla_U U, X) + \beta^2 g(\nabla_\xi U, X) = 0$. From (3.17) and (3.18) this yields

$$g(\nabla_\xi U, X) = 0 \tag{3.19}$$

for any $X \in \mathbb{D}_U$.

Developing $(\nabla_{X+\phi U}A)\xi - (\nabla_\xi A)(X+\phi U) = -\phi X + U$ and taking its scalar product with U, bearing in mind (3.17), (3.18) and (3.19) we have

$$(\phi U)(\beta) + (1-2\beta^2) - \beta^2 g(\nabla_\xi \phi U, U) = 0. \tag{3.20}$$

and taking its scalar product with ξ it follows

$$g(\nabla_\xi \phi U, U) = -4. \tag{3.21}$$

From (3.20) and (3.21) we get

$$(\phi U)(\beta) = -(2\beta^2+1). \tag{3.22}$$

If we develop $(\nabla_U A)\xi - (\nabla_\xi A)U = -\phi U$ and take its scalar product with U we obtain

$$U(\beta) = -2\beta\xi(\beta) \tag{3.23}$$

and its scalar product with ξ gives

$$\xi(\beta) = 0. \tag{3.24}$$

From (3.17), (3.22), (3.23) and (3.24) we get

$$grad(\beta) = -(2\beta^2+1)\phi U. \tag{3.25}$$

Thus $\nabla_X grad(\beta) = -4\beta X(\beta)\phi U - (2\beta^2+1)\nabla_X \phi U$ for any X tangent to M. Therefore, for any Y tangent to M we have $g(\nabla_X grad(\beta), Y) = -4\beta X(\beta) g(\phi U, Y) - (2\beta^2+1) g(\nabla_X \phi U, Y)$. Thus $g(\nabla_X grad(\beta), Y) - g(\nabla_Y grad(\beta), X) = 4\beta(Y(\beta)$ $g(\phi U, X) - X(\beta) g(\phi U, Y)) + (2\beta^2+1)(g(\nabla_Y \phi U, X) - g(\nabla_X \phi U, Y))$.

As $g(\nabla_X grad(\beta), Y) - g(\nabla_Y grad(\beta), X) = 0$, it follows

$$\begin{aligned} &4\beta(Y(\beta)g(\phi U, X) - X(\beta)g(\phi U, Y)) \\ &\quad + (2\beta^2+1)(g(\nabla_Y \phi U, X) - g(\nabla_X \phi U, Y)) = 0 \end{aligned} \tag{3.26}$$

for any X, Y tangent to M. Taking $Y = \xi$ in (3.26), for any X tangent to M we get $g(\nabla_\xi \phi U, X) = g(\nabla_X \phi U, \xi)$. Taking $X = U$ we get

$$g(\nabla_\xi \phi U, U) = \beta^2 - 1. \tag{3.27}$$

From (3.21) and (3.27) we obtain $\beta^2 = -3$, which is impossible, finishing the proof.

As a consequence we obtain

Corollary 3.1. *There exist no real hypersurfaces in $\mathbb{C}P^m$, $m \geq 3$ satisfying $R.R_\xi = 0$.*

Proof. As this condition implies that M has cyclic-Ryan parallel structure Jacobi operator, it must be Hopf. So $A\xi = \alpha\xi$. Then if we develop $R_\xi(R(X,\xi)\xi) = 0$, with $X \in \mathbb{D}$ such that $AX = \lambda X$, we get

$$\alpha^2\lambda^2 + 2\alpha\lambda + 1 = 0. \tag{3.28}$$

If $\alpha = 0$, (3.28) gives a contradiction. Thus M must be locally congruent to a geodesic hypersphere. In this case, $\alpha = 2cot(2r)$, $\lambda = cot(r)$, $r \neq \pi/4$, $0 < r < \pi/2$. Thus (3.28) does not hold and we finish the proof. □

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