Real-time Error Control for Surgical Simulation

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Introduction

Real-time simulations are becoming increasingly common for various applications, from geometric design [1, 2] to medical simulation [3].
Two of the main factors concurrently involved in defining the *accuracy* of surgical simulations are: the modeling error and the discretization error. Most work in the area has been looking at the above sources of error as a compounded, lumped, overall error. Little or no work has been done to discriminate between modeling error (e.g. needle-tissue interaction, choice of constitutive models) and discretization error (use of approximation methods like FEM). However, it is impossible to validate the complete surgical simulation approach and, more importantly, to understand the sources of error, without evaluating both the discretization error and the modeling error.
Our objective is thus to devise a robust and fast approach to measure the discretization error via a posteriori error estimates, which are then used for local remeshing in surgical simulations. To ensure that the approach can be used in clinical practice, the method should be robust enough to deal, as realistically as possible, with the interaction of surgical tools with the organ, and fast enough for real-time simulations. The approach should also lead to an improved convergence so that an *economical* mesh is obtained at each time step. The final goal is to achieve optimal convergence and the most economical mesh, which will be studied in our future work.

Methods

We use corotational elasticity and a needle/tissue interaction model based on friction. The problem is solved using hexahedron-based finite elements within SOFA[†]. The local h-refinement strategy is based upon simple Zienkiewicz-Zhu a posteriori error estimation method. Zienkiewicz-Zhu smoothing procedure [4] is used to first recover the stress field. Then, we define the approximate error of an element Ω_e as

$$\eta_e = \sqrt{\int_{\Omega^e} (\boldsymbol{\epsilon}^h - \boldsymbol{\epsilon}^s)^T (\boldsymbol{\sigma}^h - \boldsymbol{\sigma}^s) \mathrm{d}\Omega}, \tag{1}$$

which is the energy norm of the distance between the FEM solution (denoted by h) and an improved (recovered) solution (denoted by s). We mark an element for refinement if

$$\eta_e \geq \theta \eta_M \text{ with } 0 < \theta < 1 \text{ and } \eta_M = \max_e \eta_e. \tag{2}$$

Results

We control the local and global error level in the mechanical fields (e.g. displacement or stresses) during the simulation. We show the convergence of the algorithm on academic examples, and

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demonstrate its practical usability on a percutaneous procedure involving needle insertion in a liver. For the latter case, we compare the force displacement curves obtained from the proposed adaptive algorithm with that obtained from a uniform refinement approach (see [5]).

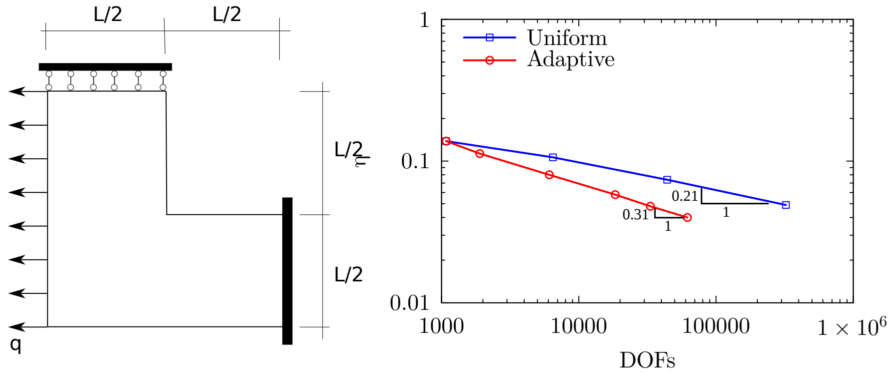

Fig. 1: Convergence of the relative error on L-shaped domain test, comparison between uniform and adaptive refinements.

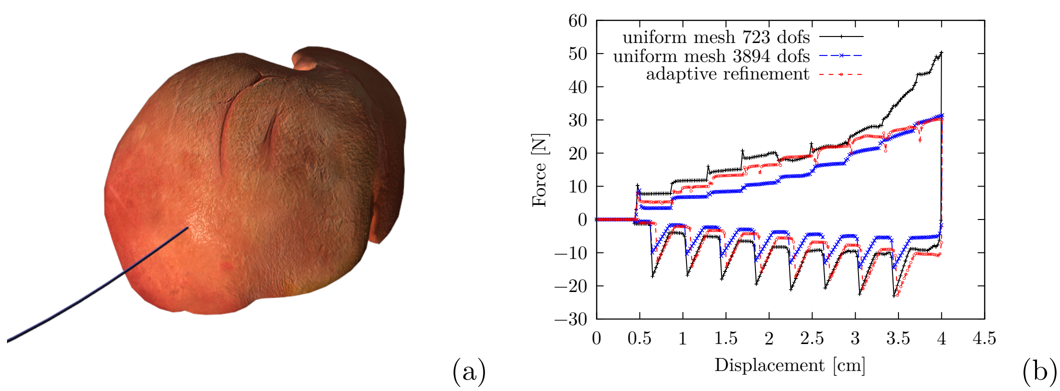

Fig. 2: Needle insertion simulation into a liver (a). Needle-liver phantom interaction force during needle insertion and pullback (b). The interaction force varies due to advancing friction and tissue cutting strength, globally increases (with positive values) during the insertion stage. At 4 cm of the needle-tip displacement, the needle is retracted and the interaction force changes the direction, varies due to retrograding friction, globally decreases and gets zero when the needle is completely pulled out. The maximum DOFs (at the end of insertion step) for adaptive refinement schemes is 1140.

References

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