REAL TIME ESTIMATION OF THE HEAVING AND PITCHING

MOTIONS OF A SHIP USING A KALMAN FILTER

by

MICHAEL TRIANTAFYLLOU

MICHAEL ATHANS

LABORATORY FOR INFORMATION AND DECISION SYSTEMS

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE, MASSACHUSETTS 02139

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Michael Triantafyllou   Michael Athans

Ocean Engineering Dept., Electrical Engineering and
                          Computer Science Dept.,
     and Laboratory for Information and Decision Systems
          Massachusetts Institute of Technology

ABSTRACT

Real time ship motion estimation is of
interest to naval ship operators for
vertical aircraft landing, as well as
to the offshore industry for installing
rigs and transferring equipment in
rough seas.

In the present study the estimation of
the heave and pitch motion of a ship is
considered, using Kalman filtering tech-
niques. A significant part of the study
is devoted to constructing appropriate
models for the sea and the ship.

The governing equations are obtained
from hydrodynamic considerations in the
form of linear differential equations
with frequency dependent coefficients.
In addition, non-minimum phase charac-
teristics are obtained due to the spa-
tial integration of the water wave
forces.

The resulting transfer matrix function is
irrational and non-minimum phase. The con-
ditions for a finite-dimensional approxi-
mation are considered and the impact of
the various parameters is assessed. A
numerical application is considered for a
DD-963 destroyer.

1. INTRODUCTION

The real time estimation of the ship mo-
tions, velocities and accelerations is
very important for applications in rough
seas, such as the landing of aircraft or
helicopters on ships, the transfer of
cargo, or heavy equipment at sea and off-
shore rig installation.

In a study on the vertical landing of
aircraft on destroyers, it was found that
the modeling of the ship motions plays a
very important role in any automatic land-
ing control scheme [4].

In the present study the accurate finite
dimensional modeling of the heave and
pitch motions of a ship is considered,
for application in the real time estima-
tions of the motions, velocities and
accelerations using Kalman filter tech-
niques. Only two measurements are assumed
available, the heave and pitch motion.

2. EQUATIONS OF MOTION

2a. Definitions

The rigid body motions of a ship in six
degrees of freedom are shown in Figure 1:
We define the $x_1 z_1$ plane to coincide with
the symmetry plane of the ship, with the
z_1 axis pointing vertically upwards, and
the y_1 axis to port, so as to obtain an
orthogonal right-hand system. Then the
linear motions along the x_1, y_1, z_1 axes
are surge, sway and heave respectively.
We consider small motions so that the ten-
sor of angular displacements can be re-
placed by a vector of small angular dis-
placements, which are roll, pitch, yaw
around the x_1, y_1, z_1 axes respectively.
The assumption of small wave induced mo-
tions is justified, given the large mass
of the vessel and the small wave amplitude
to length ratio. Very good predictions
have been obtained in the literature us-
ing this assumption [5].

2b. Derivation of the Equations of Motion

The complex problem of wave induced mo-
tions can be simplified within linear
theory as follows: The incident waves are
diffracted by the vessel, while waves are
radiated away from the ship as she osci-
llates. The diffraction waves can be
found by assuming the vessel motionless
within the incident waves, while the ra-
diation waves are found by oscillating the
vessel in calm water [5]. The force caus-
ed by the incident and diffraction waves
is called the exciting force and can be
separated in the part caused by the inci-
dent waves (Froude-Krylov force) and the
diffraction force. The force caused by
the radiation waves is proportional to the
motion amplitude so if the motion is

$$\underline{x}(t) = \underline{x}_0 e^{i\omega t} \qquad (1)$$

with $\underline{x}_0$ complex, then the radiation force is

$$\underline{F}_r(t) = T(\omega_0)\underline{x}_0 e^{i\omega_0 t} \qquad (2)$$

The frequency dependent constant $T(\omega_0)$ can be proved to include the hydrostatic terms, terms modeling the energy loss due to the outgoing waves (damping terms), and terms representing the inertia of the entrained fluid (added mass terms) [1]. For this reason we can rewrite (2) as

$$F_r(t) = -A(\omega_0)\ddot{x}(t) - B(\omega_0)\dot{x}(t) - Cx(t) \qquad (2a)$$

where the added mass $A(\omega)$ and damping $B(\omega)$ are frequency dependent, because the radiation waves are different at various frequencies.

The exciting force can be written as

$$F_r(t) = F_0(\omega_0)e^{i\omega_0 t} \qquad (3)$$

so by Newton's law

$$-M x_0 \omega^2_0 e^{i\omega_0 t} = \{F_0(\omega_0)+[A(\omega_0)\omega^2_0 - i\omega_0 B(\omega_0) - C]x_0\}e^{i\omega_0 t} \qquad (4)$$

By dropping $e^{i\omega_0 t}$, we can rewrite equation (4) as:

$$\{-[M+A(\omega_0)]\omega^2_0 + i\omega_0 B(\omega_0) + C\}x_0 = F_0(\omega_0) \qquad (4a)$$

Ultimately, we wish to obtain the response in a random sea, so equation (4a) must be extended for a random sea. This can be done by obtaining the inverse Fourier transfrom of (4), i.e.

$$\int_{-\infty}^{\infty} K_\alpha(t-\tau)\ddot{x}(\tau)d\tau + \int_{-\infty}^{\infty} K_v(t-\tau)\dot{x}(\tau)d\tau + C\,x(t) = \int_{-\infty}^{\infty} K_f(t-\tau)\eta(t)d\tau \qquad (5)$$

where K_α, K_v, K_f the inverse Fourier transform of $-\omega^2[M+A(\omega)]$, $i\omega B(\omega)$ and $F_0(\omega)$ respectively. The random undisturbed wave elevation is denoted by $\eta(t)$. Equation (5) is not popular with hydrodynamicists because the effort required to evaluate the kernels K_α, K_v, K_f is by far greater than to find the added mass, damping and exciting force. For this reason, equation (5) is rewritten in a hybrid form as follows:

$$-[M+A(\omega)]\ddot{x}(t) + B(\omega)\dot{x}(t) + C\,x(t) = F(\omega)\eta(t) \qquad (6)$$

This is an integro-differential equation (or differential equation with frequency dependent coefficients), whose meaning is in the sense of equation (5). It should be noted that $-\omega^2 A(\omega)$, $i\omega B(\omega)$ are caused by the same radiation waves, so they are related by the Cramers-Kröning relations of causal systems.

2c. Speed Effects

When the ship is heaving with a small angle Θ and at the same time is moving forward with speed U, then a heave velocity results, which is $\dot{x} = U\Theta$. The effect of the forward speed, therefore, is to couple the various motions by speed dependent coefficients. As it can be found in Appendix 1, there are simplified expressions for the added mass, damping and exciting force with a parametric dependence on the speed U.

2d. Frequency of Encounter

An additional effect of the ship speed is the change in the frequency of encounter. If the incident wave has a frequency ω then the frequency of encounter ω_e is, for deep water:

$$\omega_e = \omega + \frac{\omega^2}{g} U \cos\phi \qquad (7)$$

Although the exciting force changes with frequency ω_e, its value is the same as if the frequency were ω (plus any speed dependent terms).

2e. Equations of Motion

Within linear theory and using the ship symmetry, the heave and pitch motions are not coupled with the group of sway, roll, yaw motions. We can express the heave-pitch motions therefore as:

$$\left\{ \begin{bmatrix} m & 0 \\ 0 & Iy \end{bmatrix} + \begin{bmatrix} A_{33} & A_{35} \\ A_{53} & A_{55} \end{bmatrix} \right\} \underline{\ddot{x}}_v + \begin{bmatrix} B_{33} & B_{35} \\ B_{53} & B_{55} \end{bmatrix} \underline{\dot{x}}_v + \begin{bmatrix} C_{33} & C_{35} \\ C_{53} & C_{55} \end{bmatrix} \underline{x}_v = \begin{bmatrix} F_3 \\ F_5 \end{bmatrix} \eta \qquad (8)$$

Where A_{ij}, B_{ij}, C_{ij} the added mass, damping, hydrostatic coefficient matrices respectively; F_j the exciting forces; η the wave elevation;

$$\underline{x}_v = \{x_3, x_5\}^T \qquad (9)$$

The subscript $_3$ refers to heave and $_5$ to pitch (consistent with the hydrodynamic literature). The frequency and velocity dependence is not written explicitly, but is understood, as described in the previous sections.

2f. Exciting Force Approximation

The heave exciting force amplitude presents several peaks and zeros as can be seen in figure 2. Within the wave frequency range, only the first peak is important, so that a relatively simple finite dimensional approximation can be used.

As it was mentioned before, the exciting force depends on the wave frequency ω and

varies with frequency ω_e, while it is parametrically dependent on the speed and the wave angle.

In order to model the DD-963 destroyer, the M.I.T. five degrees of freedom sea-keeping program [2] was used to derive hydrodynamic results. The following model was derived to model the shape of the heave force at V = 0 and $\phi = 0$ (no speed, head seas)

$$F_3(s) = \frac{\alpha_1\,\eta}{\left[1 + 2J\,\dfrac{s}{\omega_a} + \dfrac{s^2}{\omega^2_a}\right]^2} \qquad (10)$$

Where J = 0.707 chosen for optimal response of a second order system, α_1 a constant determined from hydrodynamic data, η the wave elevation and ω_a the corner frequency.

This approximation was extended to include the effects of speed and wave angle by defining:

$$\omega_a = \sqrt{\frac{2\pi\,g}{L\cos\phi + B}} + \frac{2\pi}{L\cos\phi + B}\,U\cos\phi \qquad (11)$$

where L is the ship length, B the beam.

For long waves, the heave force and the pitching movement are 90° out of phase. This means that the transfer function between heave and pitch is a non-minimum phase one, because the amplitude is constant, while the phase is 90°. We choose to attribute the non-minimum phase to pitch. Also, the pitch angle tends to the wave slope for large wavelengths, so the pitching moment can be written as

$$F_5 = \alpha_2\,\frac{1 - s/\omega_0}{1 + s/\omega_0}\,\frac{s^2\cos\phi}{\left[1+2J\dfrac{s}{\omega_a}+\left(\dfrac{s}{\omega_a}\right)^2\right]^2}\,\eta \qquad (12)$$

Where α_2 a constant to be determined, ω_a is the same (for simplicity) as in equation (11), while ω_0 models the non-minimum phase characteristics, and was chosen to be equal to the wave spectrum modal frequency.

2g. Added Mass and Damping

By using equation (9), we can rewrite the equations of motion as

$$\begin{bmatrix} T_{33}s^2+ Ms^2+ C_{33} & T_{35} + UsT_{33} + C_{35} \\ T_{53}s^2- UsT_{33}+C_{35}s^2 & + T_{55}s^2 - U^2T_{33} +C_{55} \end{bmatrix} *$$

$$* \begin{bmatrix} X_3 \\ X_5 \end{bmatrix} = \begin{bmatrix} F_3 \\ F_5 \end{bmatrix}\,\eta \qquad (13)$$

Here we construct a simplified model where

$$T_{ij} = A_{ij} - \frac{B_{ij}}{i\,\omega} \qquad (14)$$

with A_{ij}, B_{ij} to be evaluated from the hydrodynamic data.

3. WAVE SPECTRUM MODELING

The waves in a specific location are composed of waves generated by the local wind, and waves generated by a distant storm, which are usually characterized as swell. As a result, the wave spectrum contains two peaks, one very narrow at low frequencies (swell), and one relatively wide and at higher frequencies (local storm).

The Bretschneider spectrum was chosen for the present application, which allows one to specify both the intensity (significant wave height H) and the location of the peak (modal frequency ωm). Two such spectra can be used to model a double peaked spectrum. The form of the spectrum has as follows:

$$S(\omega) = \frac{1.25}{4}\,H^2\frac{\omega^4_m}{\omega^5}\,\exp\{-1.25\,(\frac{\omega_m}{\omega})^4\} \qquad (15)$$

The spectrum was developed by Bretschneider for the North Atlantic, for unidirectional seas, with unlimited fetch, infinite depth and no swell. It was developed to satisfy asymptotic theoretical predictions and to fit North Atlantic data. It was found to fit reasonably well in any sea location. The spectrum, for ship coordinates, has as follows

$$S(\omega_e) = \left[\frac{S(\omega)}{d\omega_e/d\omega}\right]; \omega = f(\omega_e) \qquad (16)$$

where

$$\omega = f(\dot\omega_e) = \frac{-1 + \sqrt{1 + 4\omega_e\,\dfrac{U\cos\phi}{g}}}{2\,\dfrac{U}{g}\cos\phi} \qquad (17)$$

A rational approximation was found to (16) subject to (17) in the following form

$$S_a(\omega_e) = \frac{1.25}{4\omega_m}\,H^2 B(\alpha)\,\frac{\omega^4_e}{\left[1+(\dfrac{\omega_e}{\omega_0(\alpha)})^4\right]^3} \qquad (18)$$

where $B(\alpha)$, $\omega_0(\alpha)$, functions of $\alpha = \dfrac{U}{g}\,\omega_m\cos\phi$ $\qquad (19)$

The corresponding causal system has transfer function:

$$H_a(s) = \sqrt{S_0}\,\frac{s^2}{\left[1 + 2J\,\dfrac{s}{\omega_0} + (\dfrac{s}{\omega_0})^2\right]^3} \qquad (20)$$

where

$$S_0 = \frac{1.25}{4\omega_m}\,H^2\,B(\alpha),\quad J = 0.707 \qquad (21ab)$$

It should be noted that the Bretschneider spectrum $S_b(\omega)$ is related to the actual spectrum $S(\omega)$, defined as the Fourier transform of the autocorrelation, by the relation

$$S_b(\omega) = \frac{1}{\pi} S(\omega) \qquad \omega > 0 \qquad (22)$$

4. STATE SPACE MODEL

If s refers to the equations modeling the sea, f to the equations for the force and m to the ship equations, then the overall equations are written as

$$\dot{\underline{x}} = A\underline{x} + Bw$$
$$y = C\underline{x} \qquad (23ab)$$

with $A = \begin{bmatrix} A_s & \emptyset & \emptyset \\ \emptyset & A_m & B_m \cdot C_f \\ B_f C_s & \emptyset & A_f \end{bmatrix}$ $B = \begin{bmatrix} B_s \\ \emptyset \\ \emptyset \end{bmatrix}$ (24abcd)

$$C = \begin{bmatrix} \emptyset & C_m & \emptyset \end{bmatrix} \qquad x = \begin{bmatrix} \underline{x}_s & \underline{x}_m & \underline{x}_f \end{bmatrix}^T$$

and $A_s = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\omega_o^2 & -2J\omega_o & 0 & \omega_o^2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega_o^2 & -2J\omega_o & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\omega_o^2 & -2J\omega_o \end{bmatrix}$

$A_m = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ 0 & 0 & 0 & 1 \\ \beta_5 & \beta_6 & \beta_7 & \beta_8 \end{bmatrix}$

$A_f = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_a^2 & -2J\omega_a & \omega_a^2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\omega_a^2 & -2J\omega_a & 0 \\ \theta_1 & \theta_2 & \theta_3 & \theta_4 & 1 \end{bmatrix}$

(25abc)

$$B_s = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$C_s = \begin{bmatrix} \sqrt{S_0} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (26ab)$$

$$B_f = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T$$

$$C_f = \begin{bmatrix} \alpha_1 \cdot \omega^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha^2 \cos\phi \end{bmatrix} \qquad (27ab)$$

$$C_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (28ab)$$

where

$$\{D_{ij}\} = \{A_{ij}\}^{-1}$$

$$\begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ \beta_5 & \beta_6 & \beta_7 & \beta_8 \end{bmatrix} = -\{D_{ij}\} \begin{bmatrix} C_{33} & B_{33} & C_{35} & B_{35} \\ C_{53} & B_{53} & C_{55} & B_{55} \end{bmatrix}$$

(29ab)

and

$$\theta_1 = -(2J\omega_a^5 + \omega_a^6)$$

$$\theta_2 = -(\omega_a^4(4J^2-1) + 2J\omega_a^3)$$

$$\theta_3 = \omega_a^4 + 2J\omega_a^5$$

$$\theta_4 = -\omega_a^6 \qquad (30abcd)$$

KALMAN FILTER

The gyroscopes of the ship can provide very accurate measurements of angles. The linear motions require the installation of accelerometers. For simplicity, in the present application it is assumed that the heave and pitch motions are measured only.

The accuracy of the measurements can be very good. The noise is due primarily to the structural vibrations of the ship, which for the present application were found to be as much as 0.5 ft. in amplitude, and with frequency content close to 12 rad./sec. (as opposed to the wave frequencies, which are less than 1 rad./sec.)

APPLICATION

An example has been worked out for a DD-963 destroyer, whose length L, beam B and draft T are

$$L = 529 \text{ ft.}, \quad B = 55 \text{ ft.}, \quad T = 18 \text{ ft.}$$

The hydrodynamic characteristics were obtained using the M.I.T. sea-keeping program [2], and were approximated as outlined above, for speed $U = 21$ ft./sec. and wave angle $0°$

Subsequently, a Kalman filter was designed, using the M.I.T.-LIDS software [3] and for driving noise and measurement noise intensities:

$$U_1 = \pi \qquad U_2 = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.0003 \end{bmatrix}$$

corresponding to sea state 5 ($H = 10$ ft., $\omega_m = 0.72$ rad./sec.

The filter poles are within a radius of 1.3 rad./sec. A typical simulation of heave and pitch motions and their predictions is given in figures 3,4 (assuming exact knowledge of the sea parameters H and ω_m). By using the same filter for various combinations of H, ω_m (other than the design values) it was found that the performance of the filter depends primarily on the accurate estimation of the modal frequency ω_m: Figures 5 and 6.

The simulations indicate the good accuracy that can be obtained by using the present formulation. The most important parameter is the modal frequency, whose influence, although smooth, is quite critical. The at sea estimation of the modal frequency can be done by using the zero up-crossing

period $\bar{T}$. Typically

$$\omega_m \simeq 0.7 \ \frac{2\pi}{\bar{T}}$$

A scheme for on-line estimation of ω_m is currently tested by extending the Kalman filter to estimate ω_m. Finally figure 7 illustrates the influence of a second peak (swell) on the performance of the filter. It is clearly seen that the second peak must be modelled, if accurate predictions are desired.

SUMMARY

A finite dimensional approximation of the heave and pitch equations of motion of a ship, as obtained from hydrodynamic theory, has been outlined. The important features are:

    (a) the frequency dependence of the coefficients of the equation of motion

    (b) the non-minimum phase characteristics between heave and pitch, due to the spatial integration of the hydrodynamic forces

    (c) the dependence of the hydrodynamic quantities on the speed of the vessel and the wave angle.

A Kalman filter can be used to estimate the motions, velocities, etc., whose performance primarily depends on an accurate estimation of the sea spectrum modal frequency. Very good estimation is obtained when all the features, mentioned above, are included.

ACKNOWLEDGEMENTS

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(4) McMuldroch, C.G.,"VTOL Controls for Shipboard Landing," Laboratory for Information and Decision Systems, Report LIDS-TH-928, 1979, M.I.T.,Cambridge,Mass.

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APPENDIX 1

The following relations provide the hydrodynamic coefficients and forces as functions of the speed U:

$$A_{33} = A_{33}^{\circ}$$

$$B_{33} = B_{33}^{\circ}$$

$$A_{35} = A_{35}^{\circ} - \frac{U}{\omega^2} B_{33}^{\circ}$$

$$B_{35} = B_{35}^{\circ} + U A_{33}^{\circ}$$

$$A_{53} = A_{53}^{\circ} + \frac{U}{\omega^2} B_{33}^{\circ}$$

$$B_{53} = B_{53}^{\circ} - U A_{33}^{\circ}$$

$$A_{55} = A_{55}^{\circ} + \frac{U^2}{\omega^2} A_{33}^{\circ}$$

$$B_{55} = B_{55}^{\circ} + \frac{U^2}{\omega^2} B_{33}^{\circ}$$

The superscript $^{\circ}$ denotes quantities at zero speed. The $A_{11}^{\circ}, B_{11}^{\circ}$ were found using the M.I.T. Seakeeping program [2].

$$F_3 = \rho \ \eta \int (f_3 + h_3) \ d\xi$$
$$F_5 = - \rho \ \eta \int [\xi \ (f_3 + h_3) + \frac{U}{i\omega} h_3] \ d\xi$$

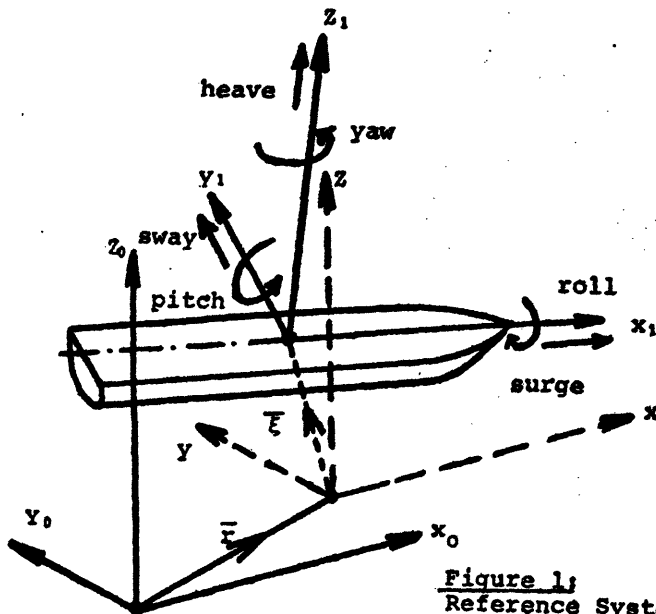where f_j is the sectional Froude-Kryloff force and h_j the sectional diffraction force.

Figure 1:
Reference Systems

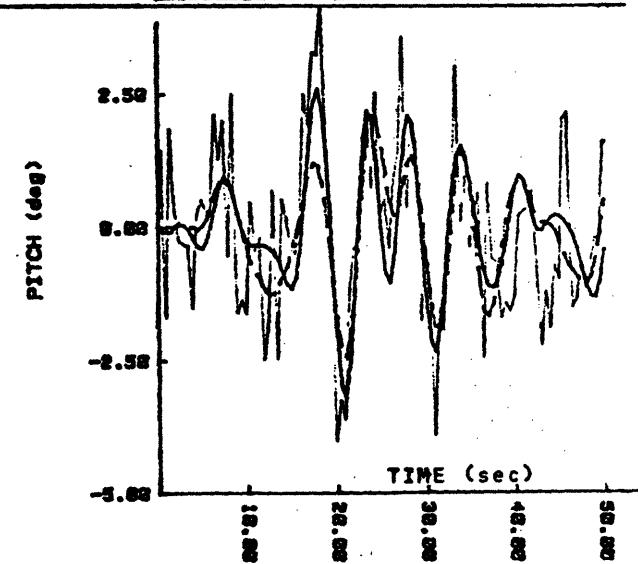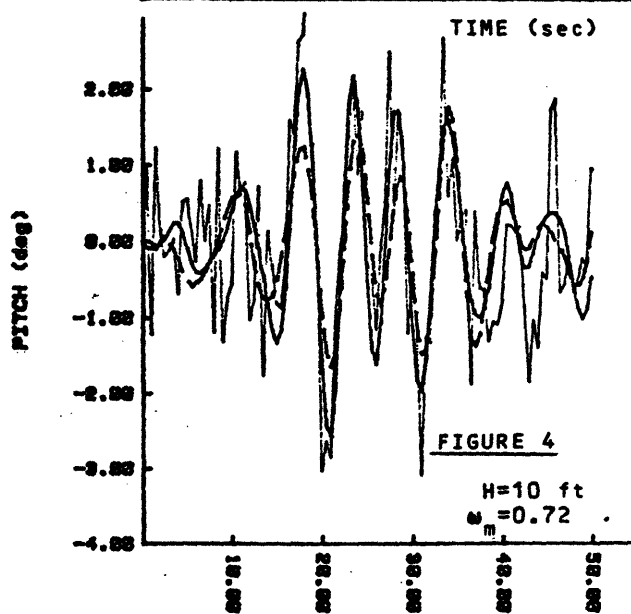FIGURE 2

Actual
Noisy measurement
Estimated

FIGURE 3

H=10 ft
$\omega_m = 0.72$

HEAVE (ft)

TIME (sec)

FIGURE 4

H=10 ft
$\omega_m = 0.72$

PITCH (deg)

FIGURE 5

H=10 ft
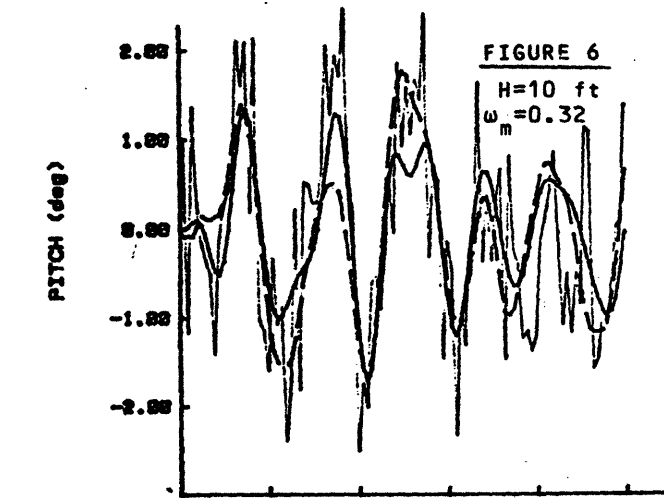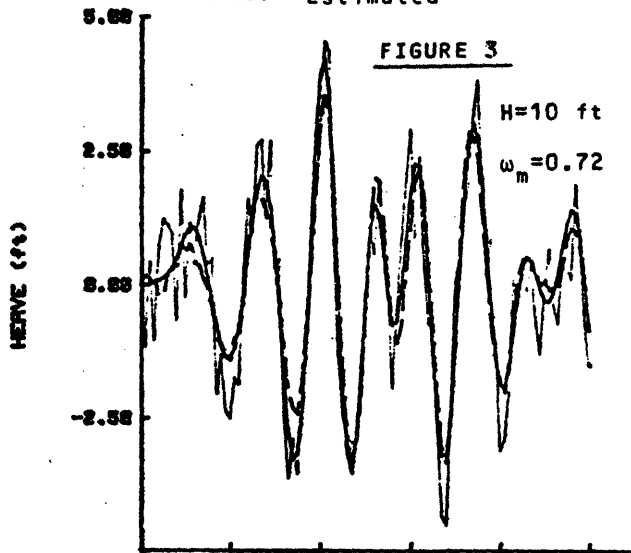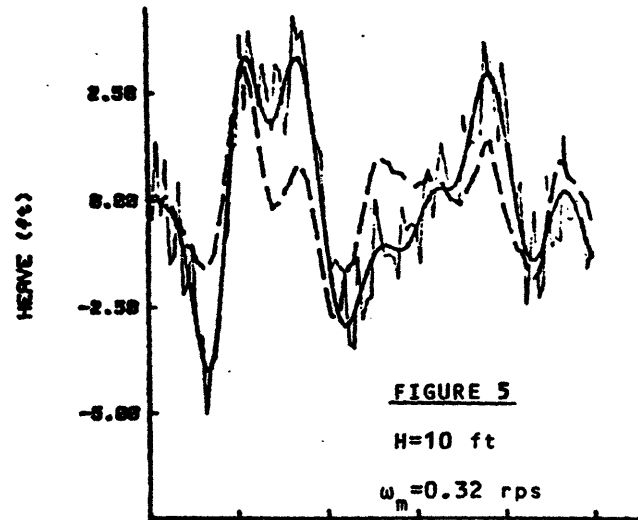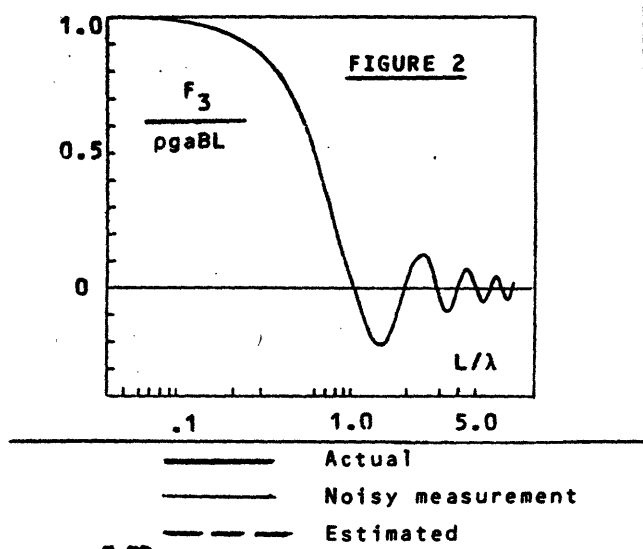$\omega_m = 0.32$ rps

HEAVE (ft)

FIGURE 6

H=10 ft
$\omega_m = 0.32$

PITCH (deg)

TIME (sec)

FIGURE 7   Double peak spectrum

PITCH (deg)