



Real-time frequency dynamics and high-resolution spectra of a semiconductor laser with delayed feedback

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The unstable emission of semiconductor lasers due to delayed optical feedback is characterized by combined intensity and frequency dynamics. Nevertheless, real-time experimental investigations have so far been restricted to measurements of intensity dynamics only. Detailed analysis and comparison with numerical models, therefore, have suffered from limited experimental information. Here, we report the simultaneous determination of the lasers optical emission intensity and emission frequency with high temporal resolution. The frequency dynamics is made accessible using a heterodyne detection scheme, in which a beat signal between the delayed feedback laser and a reference laser is generated. Our experiment provides insight into the overall spectral drift on nanosecond timescales, the spectral distribution of the unstable pulsations and the role of the individual external cavity modes. This opens new perspectives for the analysis, understanding and functional utilization of delayed feedback semiconductor lasers.

Semiconductor lasers with delayed optical feedback have received considerable attention from different communities. Complex dynamical behavior in the emission intensity, induced even by weak optical feedback, can act as nuisance in communication systems by reducing the signal to noise ratio. At the same time, the complex dynamics can be utilized for chaos communication encryption schemes¹, random number generation² and rainbow refractometry³. From a fundamental point of view, semiconductor lasers with delayed feedback serve as excellent test-beds for delayed complex systems in general. The delayed optical feedback has a profound impact on the mode structure of the laser. In conjunction with the laser cavity, the external delay line forms a compound cavity system. As a consequence one obtains an ensemble of external cavity modes⁴. When plotted against the laser's carrier inversion, these external cavity modes form the so called mode ellipse. The dynamical processes of the feedback laser can then be interpreted as a trajectory moving along this ellipse. Apart from already available techniques to highly resolve the temporal intensity dynamics in these systems, a thorough understanding of their highly resolved spectral properties and their real-time spectral dynamics is of great significance. Therefore, it is desirable to gain access to the spectral characteristics in a quality and practicality comparable to what is the current state of the art for the emission intensity.

Due to the strong amplitude-phase coupling in semiconductor lasers, intensity and frequency dynamics are tightly linked. Both are required to obtain satisfactory insight into phase space dynamics and to understand the mechanisms involved⁵. Detection of the intensity dynamics in real-time with high bandwidth has already been possible⁶. It was used for investigations of phenomena like low frequency fluctuations and coherence collapse⁷. The equally important dynamics in frequency has however not been accessible experimentally in satisfying quality.

In numerical modeling this information is often extracted in form of the average frequency f , defined by the change in phase $\Phi(t)$ over one delay τ_D as $f = [\Phi(t) - \Phi(t - \tau_D)] / (2\pi\tau_D)$. Previous experiments on frequency dynamics of single mode lasers were limited in temporal resolution or time-trace length. In an inspiring, early experiment, single shots of optical spectra of a single longitudinal mode were recorded with 10 ns temporal resolution at different temporal positions within a low frequency event⁸. Later, spectral dynamics was reconstructed in a time averaged fashion⁹ or deduced from comparison with numerical modeling¹⁰. In multi-mode laser experiments, the total intensities of longitudinal modes of the semiconductor laser have been resolved¹¹, however providing no information on the frequency dynamics of a single longitudinal mode family.



Despite of the immense importance of the frequency dynamics for the understanding of the delayed feedback induced instabilities, the access to nanosecond-resolved frequency dynamics, along with the intensity dynamics and high-resolution time-integrated optical spectra, was lacking. Concerning the accurate measurements of the time-integrated optical emission spectra, a new generation of high resolution optical spectrum analyzers (HROSA¹²) now allows for ~ 10 MHz resolution. Such spectra capture the time averaged dynamics of the chaotic itinerancy along the external cavity modes of the mode ellipse⁵, which is a fingerprint of frequency and intensity dynamics. Here, we employ a heterodyne detection scheme, combined with a sliding Fourier transform (sFFT) analysis of the detected heterodyne timetraces, to extract the frequency dynamics with nanosecond time resolution. In addition, the heterodyne method also allows to extract time averaged spectra, which pushes the resolution limit even further compared to most existing techniques. In the case of using an ideal reference source, our scheme allows for a maximum spectral resolution of 0.625 kHz, enabling us to spectrally resolve and characterize the ~ 900 external cavity modes.

Results

Experimental setup. Heterodyne detection is a well-established and reliable technique, used in spectrum analyzers as well as in low level signal detection. We use a semiconductor laser as reference, detuned in frequency from the delayed feedback laser. Figure 1 shows a schematic representation of the setup based on fiber-optical components. The laser subject to delayed optical feedback (LI) is an Eblana fiber-pigtailed discrete mode laser without integrated optical isolator. The laser is a single mode device and even under optical feedback shows a longitudinal side mode suppression ratio of more than 40 dB. Therefore, contributions of the other longitudinal modes of the laser diode to the dynamics can be neglected. The delayed optical feedback is realized by forming a fiber loop with an optical circulator (Circ) directly attached to the fiber-pigtail of the laser. By adding a 50/50 two-by-one optical splitter (50/50), we re-inject $\sim 50\%$ of the in-coupled power back into the feedback loop. Using an optical attenuator (Att), we have further reduced the feedback strength. The final feedback strength amounts to a 15% of the emitted power being injected back into the laser, resulting in a threshold reduction of 7%. A polarization controller (Pol) is added to the feedback loop for polarization alignment between optical feedback and emission of LI. The optical signal coupled out from the feedback loop is used for signal detection. As reference laser we employ a commercial tunable laser source. The reference laser (LII) is directly connected to an optical isolator, suppressing undesired optical feedback and thereby avoiding complex dynamical

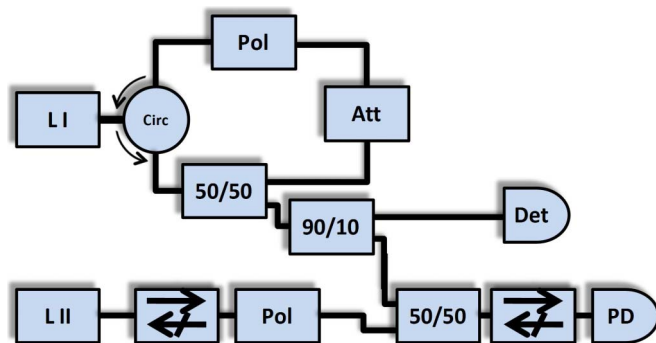


Figure 1 | Schematic of the experimental setup for heterodyne detection of the delayed feedback laser emission frequency. Laser one (LI) is subjected to delayed optical feedback. A second laser (LII) serves as a reference source, with the heterodyne signal created by the fast photo diode (PD).

behavior resulting in artifacts in the heterodyne spectrum. The optical polarization of LII is aligned to LI using another polarization controller. The heterodyne signal, created by optical interference of LI and LII on the fast photo diode (PD), is recorded with an analog bandwidth of 16 GHz. Back reflections from the PD into the optical setup are avoided by adding another optical isolator in front of the detector.

The created heterodyne signal is given by

$$I_{opt} = (E_{LI} + E_{LII})^2 = E_{LI}^2 + 2E_{LI}E_{LII} + E_{LII}^2 \quad (1)$$

where I_{opt} is the recorded optical intensity, and E_{LI} and E_{LII} are the optical fields of LI and LII impinging on the PD, respectively. By substituting $E_i = A_i(t) \cos(2\pi f_i t)$, ($i = \{LI; LII\}$), one can rewrite the interference term $I_{beat}(t) = 2E_{LI}E_{LII}$ in Eq. (1) as

$$I_{beat}(t) = \frac{A_{LI}^2(t) + A_{LII}^2(t)}{2} + A_{LI}(t)A_{LII}(t) \cos(2\pi df_{LI}(t)t); \quad (2)$$

using $df_{LI}(t) = f_{LII}(t) - f_{LI}(t)$. In the heterodyne detection scheme, only the part determined by the frequency difference $f_{LII} - f_{LI}$ is recorded, since the optical frequencies and their summations are too fast to be detected.

From equation (2) follows that the heterodyne signal is not exclusively governed by the dynamics of df_{LI} , but also by temporal fluctuations of the individual field amplitudes (A_i). Particularly the broad band optical intensity dynamics of LI and the strongly damped, but in some cases nevertheless non-negligible relaxation oscillations of LII would contribute to the frequency spectrum of the beat signal.

In order to obtain the Fourier transform of the $\cos(2\pi df_{LI}(t)t)$ term in equation (2), we simultaneously record the intensity dynamics of LI and LII. We can then extract the isolated frequency dynamics of LI via

$$S_{LI,t}(f) = \left(\left[\tilde{I}_{beat,t}(f) - \frac{\tilde{A}_{LI,t}^2(f) + \tilde{A}_{LII,t}^2(f)}{2} \right] \hat{*} \tilde{A}_{LI,t}(f) \right) \hat{*} \tilde{A}_{LII,t}(f), \quad (3)$$

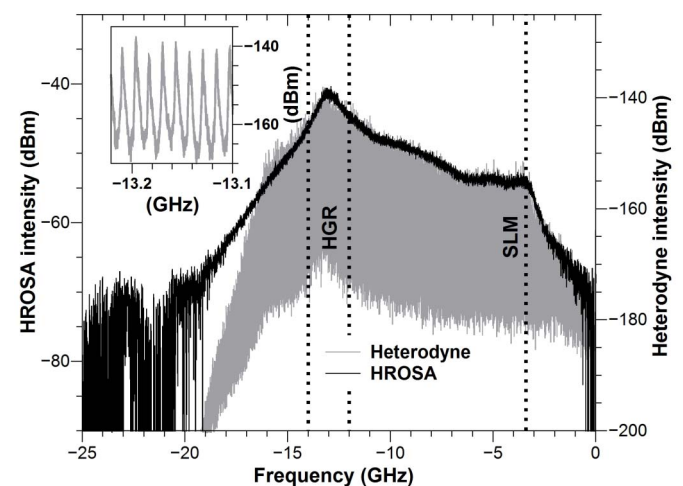


Figure 2 | Typical time averaged emission spectrum of the delayed feedback laser. The gray trace is constructed using the FFT of an entire heterodyne timetrace. The black curve is the spectrum obtained from a high resolution optical spectrum analyzer. For clarity we indicate the solitary laser mode (SLM) and the high gain region (HGR) via dotted lines. The inset presents a zoom of the heterodyne spectrum, revealing the individual cavity modes with an average width of 3 MHz.



where the symbol $\tilde{}$ depicts a Fourier transformation, symbol \otimes stands for a de-convolution and $S_{LI,t}(f)$ is the optical spectrum of LI at time t' , excluding intensity dynamics of LI.

Proof of principle. As a proof of principle, we compute the optical spectrum using the fast Fourier transform (FFT) of the entire heterodyne timetrace, length $\sim 10 \mu\text{s}$, and compare it to the optical spectrum recorded with a HROSA. The timetraces were recorded using an oscilloscope with 16 GHz analog bandwidth, 40 GSamples/s sampling rate and a memory of 400 thousand data points per time trace. For extracting the optical spectrum we include the frequency and intensity dynamics of LI, processing the heterodyne data according to Eq. (3). To gain high-resolution spectral information about the external cavity modes, we employ a narrow linewidth (100 kHz) tunable laser source as LI. The reference is tuned to a frequency which is 3.4 GHz higher than the frequency of the solitary laser mode of LI. Since this reference laser exhibits strong suppression of the relaxation oscillations (-60 dB), the reference laser intensity dynamics can be neglected in Eq. (3).

Both, HROSA and time averaged heterodyne spectra are shown in Fig. 2, with the x-axis depicting the detuning to the reference laser in GHz. The two spectra agree almost perfectly, illustrating the accuracy of our procedure for extracting optical spectra. For frequencies greater than 16 GHz, one can identify the decreasing sensitivity of the heterodyne measurement. The maximum spectral resolution is determined by the signal detection bandwidth of 16 GHz and the recording system's memory (32 million samples), resulting in a theoretical limit of 0.625 kHz in our setup. Practically, in this experiment, resolution was limited by the reference laser's linewidth (1 MHz). By using a reference with a narrower linewidth the system should be able to achieve the theoretical limit. We demonstrate this still very high resolution by clearly resolving the external cavity modes with a mode separation of $\sim 13.4 \text{ MHz}$ in the optical spectrum of our delayed feedback laser.

Real time measurement of frequency dynamics. The dynamics of the laser frequency have been obtained by applying a sFFT, where we used Eq. (3) on a data window of size $\Delta\tau$, centered at time t' . We continuously move the window across the entire heterodyne

timetrace, calculating optical snapshot spectra for each time window of length $\Delta\tau$. Due to the nature of the sFFT, temporal and spectral resolution cannot be chosen independently, since spectral resolution is given by $\Delta\nu = \frac{1}{2\Delta\tau}$. We chose $\Delta\tau = 4 \text{ ns}$, allowing for a resolution of $\Delta\nu = 125 \text{ MHz}$. Using such snapshot spectra, we define the instantaneous frequency of the laser as $\nu_{i'} = \max(df_{LI,t}(f))$.

Figure 3 shows the intensity and optical emission frequency dynamics of LI recorded in real-time, where frequency information was extracted using a five nearest neighbor averaging of the sFFT spectra. In this case LI was driven 3% above the solitary laser threshold current. Under such conditions, the optical emission of Fig. 3 a) displays low frequency fluctuations (LFF) with its fast pulsations⁶.

In Fig. 3b), one can clearly see that, when LI reaches the frequencies within the high gain region (HGR, $\sim -13 \text{ GHz}$), it is after a while rapidly re-injected into a region close to the solitary laser mode (SLM, -3.4 GHz). As HGR we denote the optical frequency range close to the maximum gain mode at the lower frequency end of the mode ellipse, where optical gain is largest. From there it again starts a smooth drift towards the HGR, on its trajectory visiting a significant number of external cavity modes between the SLM and HGR. We would like to stress that all dynamics occurs exclusively on the mode ellipse of a single longitudinal mode of the solitary laser. For our device the longitudinal mode spacing is of the order of $\sim 150 \text{ GHz}$, with the additional longitudinal modes experiencing a suppression of more than 40 dB. Intensity and frequency traces agree with the standard observations and interpretations of LFFs⁵.

A full spectrogram of the frequency dynamics is shown in Fig. 3c). At any moment the spectral width is several GHz, reflecting the bandwidth of the irregular picosecond pulsations. Accordingly, Fig. 3, illustrates that at each time the real-time optical spectra originates from contributions of several cavity modes. Please note that the time-dependent spectrograms contain significantly more information than the commonly plotted average frequency f . Altogether, the real-time extraction of the frequency dynamics, depicted in panels b) and c), allows for the first time an accurate reconstruction how the trajectory moves along the ECMs of the mode ellipse⁵.

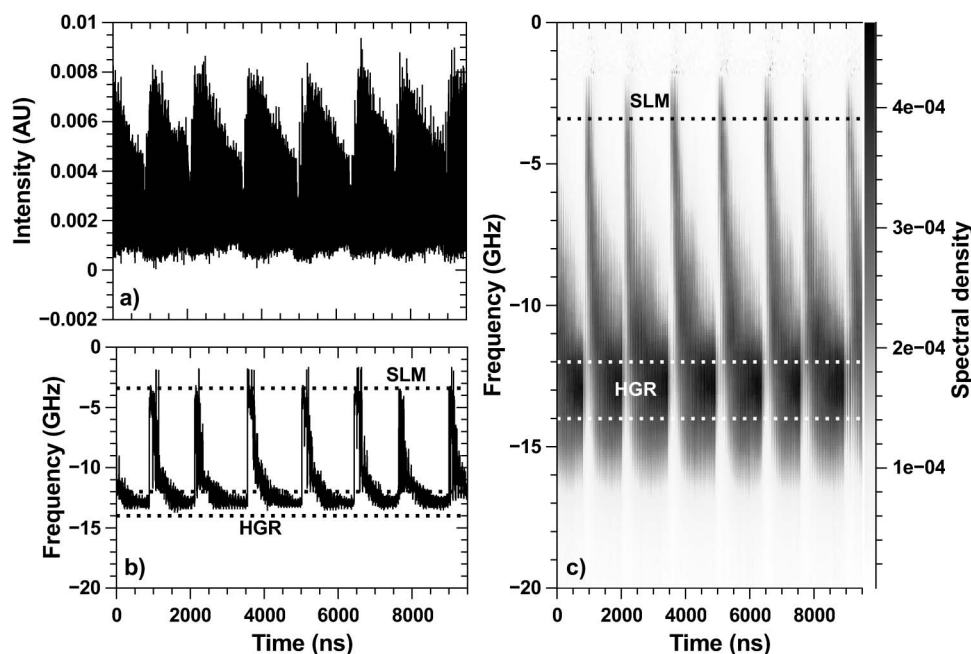


Figure 3 | Dynamics of the delayed feedback laser. Panel a) shows the intensity dynamics of LI. Panel b) shows the dynamics of the peak spectral component. Panel c) depicts the dynamics of the entire emission spectrum.



High resolution optical spectra. Even more detailed information can be extracted from applying the heterodyne technique, providing further insights into optical characteristics during the chaotic itinerancy process. In particular we are able to characterize the optical linewidth of the individual external cavity modes. For long delays (~ 100 ns), the external cavity mode spacing is reduced to the order of 10 MHz, setting the precondition for clearly resolving individual modes to ~ 100 kHz.

The widths of the individual ECM is displayed in the upper panel of Fig. 4. They were extracted from the heterodyne spectrum also displayed in Fig. 2. In addition, we applied 10 data points averaging on the optical spectrum. Under these conditions for LI, the optical emission consists of ~ 900 modes. The linewidth of the external cavity modes exhibits a characteristic dependence on their spectral position, with a minimum linewidth of 2 MHz inside the HGR of the mode ellipse. The full benefit of our approach can be appreciated by interpreting the linewidth of Fig. 4 with the aid of data presented in Fig. 3. Narrow modal widths are obtained for emission in the HGR region, in which, according to Fig. 3b), the system resides for a duration of several μ s. In contrast, the fast drift of the laser from the SLM towards the HGR results in increased modal linewidths.

Based on the real-time frequency dynamics shown in Fig. 3b), we can extract the relaxation time of the ECM versus their optical frequency, thereby linking it to their position on the mode ellipse. For this, we calculate the frequency trajectory derivative versus position and multiply it by the full width half maximum of the individual ECMs, obtaining the residence time for each ECM and normalize it to the timetrace length. Using this information, we calculate the ECM's relaxation rate, with the data displayed in the lower panel of Fig. 4. In the region between the SLM and the HGR (3 to 10 GHz), the linewidths of the ECM is dominated by their lifetime. Around the HGR the lifetime of the modes increases significantly. This effect is only partially reflected by the decreasing linewidth of the modes inside the HGR. It will be interesting to clarify in the future, in

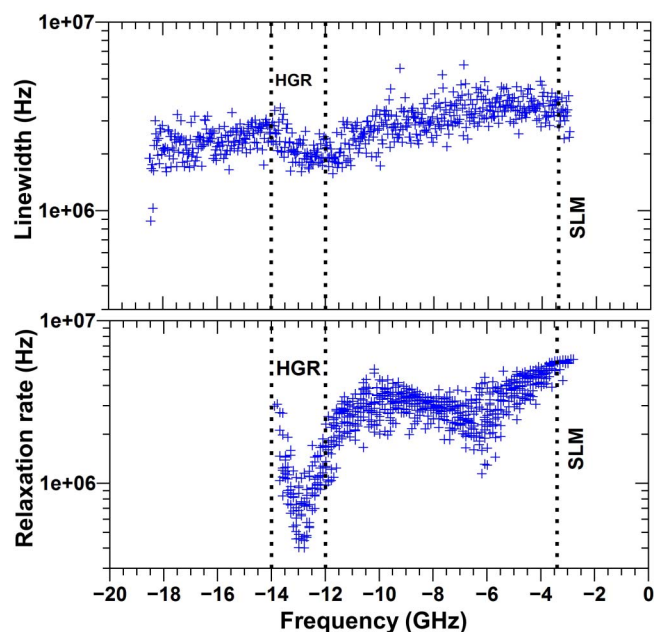


Figure 4 | Linewidth and relaxation rate of the individual external cavity modes of the feedback laser. Especially in the HGR, we observe a notable difference between relaxation rate (lower panel) and linewidth (upper panel). There, the strong decrease in relaxation rate does not result in a comparable linewidth decrease. This could point to additional de-coherence mechanisms, setting a lower limit of the modal widths. Each data point corresponds to one individually resolved external cavity mode.

how far in this region the linewidth is determined by de-coherence mechanisms, rather than relaxation.

Discussion

We demonstrate a heterodyne scheme which enables the accurate extraction of the spectral properties of delayed feedback lasers. Our experiments represent a powerful approach to gain insight into the temporal dynamics in the optical spectral domain. Employing the heterodyne scheme for time-averaged spectroscopy allows to extract optical spectra of the external cavity mode at unprecedented resolution.

We measure, for the first time, the frequency excursion of the laser along its mode ellipse, along with the intensity dynamics, in real-time. In consequence, it has become possible to capture the key dynamical features of a delayed feedback laser. This is illustrated by analyzing the nanosecond frequency drift, the pulsation spectrum at different times. In the future, time resolved spectra extracted via our experimental scheme allow a direct, more complete comparison between the dynamics of a delayed feedback semiconductor laser and theoretical models. Theoretical models, typically employed for simulating dynamics of semiconductor lasers, e.g. the Lang-Kobayashi equations, are based on dynamics of the carriers, as well as amplitude and frequency dynamics of the optical fields. Our experiment therefore enables access to a key dynamical variable of these models for the first time in an adequate quality. In addition, an in depth analysis of the intensity-phase coupling under dynamical conditions might become possible.

Apart from extracting the delay phase, typically used in simulations, we extract the entire dynamical optical spectrum. Therefore, the full spectral distribution of the frequency dynamics, involving emission from hundreds to thousands of external cavity modes, becomes accessible.

Intriguing new possibilities arise from ultra-high spectral resolution, which we have achieved using time-averaged heterodyne spectroscopy. When using our experimental scheme, one can extract the lineshape of the individual external cavity modes also for delay times of the order of ~ 100 ns. Access to the spectral features of each individual external cavity mode allows for a new quality in the comparison between experiment and theory. Combining the dynamical and time-averaged heterodyne spectra makes it possible to compare the relaxation rate of individual external cavity modes and their linewidths. This enables an analysis of the influence of de-coherence mechanisms on the optical spectrum.

Methods

Real-time timetraces of the heterodyne and intensity dynamics were detected using fast photo diodes with 20 GHz analog bandwidth. The detector output was recorded using a real-time oscilloscope with 40 GSamples/s and an analogue bandwidth of 16 GHz.

The feedback laser is a fiber coupled edge emitting discrete mode laser with a side mode-suppression ratio exceeding 40 dB. It was temperature stabilized to ± 0.02 K, resulting in a stability of the SLM emission of ± 0.16 GHz. This drift in emission wavelength, however, occurs only on 10s of seconds timescales. Therefore, this drift is irrelevant for the optical spectra, which are recorded on timescales shorter than 1 ms. The diode current was controlled via a Thorlabs LDC8001 module with a current noise of < 0.1 μ A.

As an optical reference source, we utilized the tunable laser of the HROSA (Aragon photonics BOSA) with a linewidth of 1 MHz.

Before applying equation (3), it is crucial to characterize and to compensate for the effect of different intensities impinging on the individual detectors.

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Author contributions

D.B. designed the experiment, D.B. performed the experiment, D.B., X.P. and M.C.S. analyzed the data, I.F. supervised the research, D.B. and I.F. wrote the paper.

Additional information

Competing financial interest: The authors declare no competing financial interests.

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