

Real-Time Traveler Information for Optimal Adaptive Routing in Stochastic Time-Dependent Networks

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Abstract Real-time information can enable travelers to adapt to changing traffic conditions and make better routing decisions in uncertain networks. In this paper, a generic description of real-time online information is provided based on three schemes using partial online information and one scheme with no online information. A theoretical analysis shows that more error-free information is always better than (or at least as good as) less information for optimal adaptive routing in flow-independent networks. A heuristic algorithm is designed for the optimal adaptive routing problem for all the four information schemes, based on a set of necessary conditions for optimality. The effectiveness of the heuristic algorithm is shown to be satisfactory over the tested random networks. This study is of interest for traveler information system evaluation and design.

Keywords: Traveler Information; Stochastic Time-Dependent Network; Adaptive Routing; Value of Information; Routing Policy

1. Introduction

An advanced traveler information system (ATIS) provides travelers with real-time traffic condition information to enable better routing decisions. In order to assess the effects of an ATIS, a comprehensive model is needed to consider travelers' decisions and the demand-supply interaction under the influence of ATIS. This paper examines the demand aspect of the problem, and describes the optimal routing decisions a traveler can make using real-time information on realized travel times and the overall benefits obtained. No demand-supply interaction is modeled in this paper, i.e., travel times are not affected by travelers' choices. In order to better evaluate how the traveler uses real-time information to make decisions, it is assumed that he/she has already formed a clear perception of travel time.

In order to evaluate different information schemes, decision makers' behaviors must be consistent. The realized benefit of information depends greatly on how decision makers use the available information. For example, an in-vehicle GPS unit with up-to-date traffic information does not provide any benefit if the driver chooses to ignore it, while a radio message announcing a traffic jam can help the traveler divert if he/she chooses to alter his/her route. In this paper, it is assumed that a traveler optimizes an objective function (e.g., minimal travel time, minimal variability) and makes a series of optimal routing decisions based on time-of-day and realized link travel times during the trip. The value of traveler information is therefore defined as the difference between optimal routing outcomes (e.g., minimal expected travel time) that a traveler would obtain with and without the information.

This definition of value of information is comparable to that employed in decision analysis and information economics. Marschak and Miyasawa (1968) offer the following definition:

“An information system is a set of potential messages to be received by the decision maker. It is characterized by the statistical relation of the messages to the payoff-relevant events, and also by the message cost. Neglecting this cost, the (gross) value of an information system for a given user is the (gross) payoff that he would obtain, on the average, if he would respond to each message by the most appropriate decision.”

Numerous studies have been conducted on the value of traveler information using the above definition, with explicit modeling of the probabilistic relation of the information to the benefits, including Arnott et al. (1996, 1999), Levinson (2003), Denant-Boemont and Petiot (2003), Chorus et al. (2006) and de Palma and Picard (2006).

Traveler information can be characterized in a number of ways, including quantitative or qualitative, historical, prevailing (realized) or predictive, and level of noise. This paper focuses on the scope of information in time and space using quantitative error-free information that reveals realized link travel times without error. Gao and Chabini (2006) studied perfect online information that approximates an ideal in-vehicle system by providing information on all links at all time periods up to decision time. However, realistic information situations are generally limited in scope temporally and/or spatially, and so it is called partial online information. For example, a variable message sign (VMS) is usually fixed in one location and can only provide information to travelers who pass it on their route. Radio broadcasts can provide information to travelers anywhere within the radio coverage, but the scope is usually limited to major highways and arterials. Temporal limitations are also a concern; for example, radio traffic reports can be delayed for 15 minutes, making travelers commuting at 8:00am only aware of traffic conditions up to 7:45am. The Internet can also be a source of pre-trip traffic information, but is likely unavailable *en route*.

In this paper, three schemes of partial online information are introduced: delayed global information, global pre-trip information and up-to-date radio information. Compared with perfect online information, the first two are limited temporally and the last, spatially. This paper provides: 1) a theoretical proof stating that more error-free information is always better (or at least as good as less information) for optimal adaptive routing in a flow-independent stochastic time-dependent (STD) network; 2) an analysis of the optimal adaptive routing problem with partial and no online information indicating that Bellman's principle of optimality does not apply, and the proposal of a set of necessary conditions for optimality; and 3) a heuristic algorithm based on the necessary conditions with polynomial running time and satisfactory effectiveness, tested computationally.

The paper is organized as follows. In Section 2, a literature review is presented in two areas: value of traveler information and optimal routing policy problems. Section 3 defines an optimal routing policy problem in an STD network for partial online information schemes. Section 4 presents a theoretical proof of the non-negative value of error-free traveler information. In Section 5, Bellman's principle of optimality is shown to be invalid for the schemes with partial and no online information. A set of necessary conditions for optimality is then proposed and proved. A heuristic algorithm is designed based on the necessary condition and computational test results are presented. Section 6 gives conclusions and future research directions.

2. Literature Review

Over the last two decades, numerous studies have been conducted on traveler information. Accurately representing various types of information situations in a network has been a major concern in traveler information research. Under a traffic equilibrium framework, some (e.g., Hall, 1996; Yang, 1998; Bottom, 2000; Levinson, 2003; Dong *et al.*, 2006) have assumed that travelers with access to ATIS have complete information, which may be unrealistic. In Mahmassani and Jayakrishnan (1991), Hall (1996) and Engelson (2003), travelers are assumed to switch routes based on instantaneous path travel times, rather than ones they will actually experience. This assumption circumvents the need to retrieve future link travel times. In Yin and Yang (2003) and Lo and Szeto (2004), the imperfection of various ATIS is represented by introducing random errors to true path travel times, with varying degrees of error associated with different information systems. Under a dynamic process framework, information can be included in travelers' learning processes to represent traffic conditions from the previous day or time period (e.g., Ben-Akiva *et al.*, 1991; Friesz *et al.*, 1994; Emmerink *et al.*, 1995; Jha *et al.*, 1998; Mahmassani and Liu, 1999). A common shortcoming of these studies is that the information representation cannot be directly related to real life situations, e.g., the spatially or temporally limited information systems discussed in Section 1.

Other researchers have conducted theoretical studies on simplified networks. Arnott et al. (1999) examined the effects of online information in a two-link network with random capacities under equilibrium in both departure time and route, using the bottleneck model to calculate congested travel times. Rigorous studies of zero information, full information, and imperfect information are carried out. Other studies working on simplified networks include Arnott *et al.* (1991, 1996), Emmerink *et al.* (1998), de Palma and Picard (2006) and Chorus et al. (2006). Denant-Boemont and Petiot (2003) evaluated travel information value using human subjects' willingness to pay in an experimental setting with limited mode and route choices.

It is difficult to compare the results in a highly simplified network to those in a typical network. While the optimal choice problem can be solved by observation in a simplified network, algorithms are needed for a general network. Two possible types of routing problems exist in stochastic networks: non-adaptive and adaptive. Non-adaptive routing determines a fixed path at the origin that is followed regardless of the realizations of the stochastic network. In contrast, adaptive routing considers intermediate decision nodes, and a next link (or sub-path) is chosen based on collected information at each decision node. Adaptive routing is better than (or at least as good as) non-adaptive routing, since the latter can be viewed as a constrained version of the former. In this review, the term "routing policy" is used to denote the adaptive routing process. The review focuses on problems in time-dependent (as opposed to static) networks, as summarized in Table 1 with various assumptions on link stochastic dependencies and information access.

In the studies of schemes with no time-wise or link-wise dependencies and no online information, marginal distributions of link travel times are used and the routing is only adaptive to arrival times at decision nodes (hence the term time-adaptive). Hall (1986) studied for the first time the time-dependent version of the optimal routing policy problem, showing that in an STD network, routing policies are more effective than paths. Based on the concept of decreasing order of time, Chabini (2000) produced a dynamic programming algorithm, which is optimal in the sense that no algorithms with better worst-time complexity exist. The algorithm is later described in Gao (2005). Miller-Hooks and Mahmassani (2000) developed a label-correcting algorithm,

which Miller-Hooks (2001) compared with the dynamic programming algorithm (Chabini, 2000) computationally. Yang and Miller-Hooks (2004) extended the study to a signalized network.

Table 1. Taxonomy of the optimal routing policy problems

Information Network	Perfect online information	Partial online information	No online information (time-adaptive)
No time-wise or link-wise dependency		Opasanon and Miller-Hooks (2006)	See the note below*
Complete dependency	Gao and Chabini (2002, 2006)	This paper	This paper
Partial dependency		Psaraftis and Tsitsiklis (1993), Kim <i>et al.</i> (2005), Boyles (2006)	

*: Hall (1986), Miller-Hooks and Mahmassani (2000), Pretolani (2000), Chabini (2000), Miller-Hooks (2001), Bander and White (2002), Nielson *et al.* (2003), Yang and Miller-Hooks (2004), Fan *et al.* (2005), Fan and Nie (2006), Pretolani *et al.* (2009).

Pretolani (2000) used a hyper-path formulation of the adaptive routing problem based on arrival times. Bander and White (2002) designed a heuristic approach that terminates with an optimal solution if one exists. Fan *et al.* (2005) maximized the probability of arriving on time with continuous probability density functions on link travel times. Fan and Nie (2006) later explored algorithmic issues for the same problem. Nielson *et al.* (2003) studied the bicriterion time-adaptive problem.

In the case of partial online information, Opasanon and Miller-Hooks (2006) studied the multicriterion adaptive routing problem with information on traversed link travel times in a statistically independent network. Later, Pretolani *et al.* (2009) went on to distinguish between time-adaptive and history-adaptive routing in a multicriterion optimization context.

Psaraftis and Tsitsiklis (1993) examined networks using a methodology where link costs evolve as Markov processes and travelers learn the current state of the Markovian chain at any time. The network is assumed to be acyclic to enable the design of a polynomial-time algorithm. Kim *et al.* (2005) studied the problem in a general Markovian network with a wider information

range. In Boyles (2006), conditional probabilities of adjacent link travel costs are utilized and travelers are assumed to remember only the travel time on the last link they traverse. The objective function is a general piece-wise polynomial function of arrival time at the destination.

Gao and Chabini (2002, 2006) studied routing policy problems in a general STD network with both time-wise and link-wise dependency and perfect online information. This paper expands upon past research by examining the optimal routing policy problem in a general STD network with partial or no online information. A heuristic, rather than exact, algorithm is designed and employed based on a set of necessary conditions for optimality.

3. Problem Definition

3.1. The Network

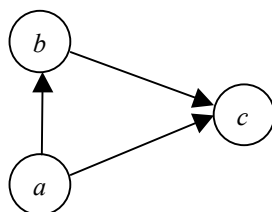


Fig. 1. An illustrative small network

Table 2. Support points for the small network ($p_1 = p_2 = p_3 = 1/3$)

Time	Link	C^1	C^2	C^3
0	(a, b)	1	1	1
	(b, c)	2	2	1
	(a, c)	3	3	2
1	(a, b)	1	1	2
	(b, c)	1	2	1
	(a, c)	3	2	2

Let $G=(N,A,T, \tilde{C})$ denote an STD network. N is the set of nodes and A is the the set of links, with $|A|=m$. Assume there is at most one directional link from node j to k , denoted as (j,k) . T is the set of time periods $\{0,1,\dots,K-1\}$. A support point is defined as a distinct value (vector of values) that a discrete random variable (vector) can take. A probability mass function (PMF) of a random variable (vector) is a combination of support points and the associated probabilities. In

this paper, a symbol with a \sim over it is a random variable (vector), while the same symbol without the \sim is the associated support point. The travel time on each link (j,k) at each time t is a random variable $\tilde{C}_{jk,t}$ with a finite number of discrete, positive and integral support points. The time periods from 0 to $K-1$ are denoted as dynamic, while those beyond $K-1$ are static. In practice, the peak hour period is generally modeled as dynamic, while off-peak (when the traffic is more stable) is modeled as static. $\{C^1, \dots, C^R\}$ is the universal set of network support points for the joint probability distribution of all link travel times at all times, where C^r is a vector of time-dependent link travel times with a dimension of $K \times m$, $r=1, 2, \dots, R$. In the remainder of the paper, a support point is used for the joint distribution of all links at all times, unless otherwise specified. $C_{jk,t}^r$ is the travel time on link (j,k) at time t in the r^{th} support point, with probability p_r , and $\sum_{r=1}^R p_r = 1$. The travel time on a given link (j,k) at any time $t > K-1$ is equal to that of time $K-1$ for any support point: $C_{jk,t}^r = C_{jk,K-1}^r, \forall (j,k), \forall t > K-1, \forall r$.

An example network is shown in Figure 1 and Table 2 with three nodes, three links and two time periods. There are three support points, each with a probability of 1/3, for the joint distribution of six travel time random variables (links (a, b) , (b, c) and (a, c) over time periods 0 and 1). Travel times beyond time 1 are the same as those at time 1 in each of the three support points.

The framework and methods developed in this paper can be extended to networks with turn penalties by modifying the framework to add additional links corresponding to turning movements. As the focus of this paper is on imperfect information, we limit our discussion to a basic network without turn penalties.

The discrete distributions of link travel times are assumed for the convenience of defining routing policies (Section 3.4), which are based on realized travel times. Even if the underlying travel time distribution is continuous, in order to define a routing policy with a finite number of states, the distribution must be made discrete. The extension of the routing policy definition to a continuous travel time distribution is a challenging task and will be included in future work.

3.2. Online Information

Let H be a trajectory of (node, time) pairs a traveler could experience in the network to the current node j and time t : $H = \{(j_0, t_0), \dots, (j, t)\}$, where j_0 is the origin and t_0 is the departure time. Denote the information coverage over links and time periods as $Q \subseteq A \times T$. Information is represented as travel time realizations on time-dependent links of Q . No predictive information is assumed, i.e., Q cannot contain elements beyond the current time t . It is assumed that there is no error in revealing the true travel times. An information scheme is defined as mapping from a trajectory H to information coverage Q , meaning that information depends on traversed locations and times. Here are some examples of online information schemes with a trajectory $H = \{(j_0, t_0), \dots, (j, t)\}$:

- Perfect online information: $Q(H) = A \times \{0, 1, \dots, t\}$ (all links up to the current time t)
- Delayed global information with time lag Δ : $Q(H) = A \times \{0, 1, \dots, t - \Delta\}$ (all links up to Δ units of time ago)
- Global pre-trip information with departure time t_0 : $Q(H) = A \times \{0, 1, \dots, t_0\}$ (all links up to the departure time t_0)
- Up-to-date radio information on $B \subseteq A$: $Q(H) = B \times \{0, 1, \dots, t\}$ (a subset of links up to the current time t)
- No online information: $Q(H) = \emptyset$ (no information on any link at any time)

The example in Figure 1 and Table 2 is used to illustrate different information schemes. At time 0, a traveler with perfect online information knows the travel time realizations of $\{\tilde{C}_{ab,0}, \tilde{C}_{bc,0}, \tilde{C}_{ac,0}\}$: either $\{1, 2, 3\}$ or $\{1, 1, 2\}$; a traveler with global information with one unit of time lag (LAG1) does not yet have any information on the realization of travel time; a traveler with global pre-trip information with departure time 0 has the same knowledge as the traveler with perfect online information; a traveler with up-to-date radio information on link (a, b) knows the travel time realization of $\tilde{C}_{ab,0}$ that is 1; and a traveler with no online information does not have any realization of travel time whatsoever. At time 1, a traveler with perfect online information

knows the travel time realizations of $\{\tilde{C}_{ab,0}, \tilde{C}_{bc,0}, \tilde{C}_{ac,0}, \tilde{C}_{ab,1}, \tilde{C}_{bc,1}, \tilde{C}_{ac,1}\}$, which could be each of the three support points; a traveler with delayed information knows what happened at time 0 and gains the same information as with perfect online information at time 0; a traveler with pre-trip information does not gain any more information *en route* and thus his/her information remains unchanged; a traveler with radio information knows the travel time realization of $\{\tilde{C}_{ab,0}, \tilde{C}_{ab,1}\}$ that could be $\{1,1\}$ or $\{1,2\}$; and a traveler with no online information still has no realization of travel time. At time 2, only the traveler with delayed information will gain more useful information, as he/she now knows what happened in time 1. A traveler with perfect online, pre-trip or radio information does not gain any more useful information, because of the static period assumption. A traveler with no online information does not gain any more information by definition. Note that routing under the no online information scheme could still be adaptive to the arrival time at each decision node, which is random due to random travel times.

3.3. Event Collection

The concept of event collection is generalized from that in Gao and Chabini (2006) to a general information scheme. Let \tilde{C}_Q be the vector of random travel times of time-dependent links in Q . For a given support point C_Q , there exists one or more support points of the whole network that are expansions of C_Q . In other words, for any possible revealed link travel times of Q , a set of compatible support points can be identified. Such a set is defined as an event collection, EV . It can be viewed as the conditional joint distribution of link travel times given realized link travel times in the coverage Q . With more information collected, the information coverage Q grows and the size of EV decreases or remains unchanged. When EV becomes a single support point each, a deterministic network (not necessarily static) is revealed to travelers. If a traveler has perfect online information, the network becomes deterministic no later than the start of static period $K-1$. If travelers have less than perfect online information, the network may remain stochastic beyond the dynamic period.

All possible event collections with the information coverage Q , denoted as $EV(Q)$, can be generated by performing a partition of $\{C^l, \dots, C^R\}$ based on \tilde{C}_Q . $EV(Q) = \{EV_1, EV_2, \dots\}$, where $C_{jk,t}^r$ is invariant over $r \in EV_i$, $\forall ((j,k),t) \in Q$, $\forall i$, and $\exists ((j,k),t) \in Q$ such that $C_{jk,t}^r \neq C_{jk,t}^{r'}$, for $r \in EV_i$, $r' \in EV_j$, $\forall j \neq i$. In other words, support points in an EV are indistinguishable in terms of revealed travel times of Q , but are distinctive from those in another EV . All possible event collections for a given information scheme can be generated in preprocessing.

The generation of event collection can be carried out in an increasing order of time, as the information is error-free and later information will not contradict earlier information. An example from Figure 1 and Table 2 is shown here for a traveler with up-to-date radio information on link (a,b) . Since the information coverage in question depends only on the current time t and not the whole trajectory, $Q(H)$ is simplified as $Q(t)$ and $EV(Q)$ as $EV(t)$. At time 0, the information coverage can be defined as $Q(0) = \{(a,b)\} \times \{0\}$. The travel time on link (a,b) at time 0 is 1 for all three support points, so the partition yields only one event collection and $EV(0) = \{\{C^1, C^2, C^3\}\}$. At time 1, the information coverage $Q(1) = \{(a,b)\} \times \{0,1\}$ where the incremental information is on $\{(a,b)\} \times \{1\}$. The partition can be carried out on $EV(0)$ based on travel time realizations of link (a,b) at time 1, which can be either 1 or 2. Therefore, $EV(1) = \{\{C^1, C^2\}, \{C^3\}\}$. In the static period, no more useful information is available, so $EV(t) = \{\{C^1, C^2\}, \{C^3\}\}$, for all $t > 1$. The same logic can be applied to other information schemes.

3.4. The Decision and the Optimal Routing Policy Problem

It is assumed that travelers make decisions only at nodes. The decision of what node k to take next is based on the state defined as a triplet $\{j, t, EV\}$, where j is the current node, t the current time, and EV the current event collection.

Definition 1. (Routing Policy) A routing policy μ is a mapping from state to decision, for all possible states and all possible next nodes of a given state, $\mu : \{j, t, EV\} \rightarrow k$.

A routing policy can be visualized as a contingency table with as many rows as the number of possible combinations of node, time and event collection. For each combination, a

next node is given. A path (e.g., as defined in Ahuja *et al.*, 1993) is a purely topological concept and a special case for routing, such that the same next node is given regardless of the time and event collection. The travel time by following a routing policy (sometimes termed *routing policy travel time*) from any origin and departure time to a destination is a random variable, with one realization in each support point. The routing policy travel time can then be represented as a list of travel times in all support points with the associated probabilities. The routing policy itself can also be viewed as a collection of paths with the associated probabilities.

A routing policy is defined based on event collections, not support points, and an event collection includes a number of support points compatible with revealed information at the decision node and time. An event collection can be considered equivalent to a support point only in a scheme where a traveler is omnipotent and knows exactly what will happen in each day at the beginning of the day. Because this is impossible, a set of possible support points must be considered, although the set size will likely decrease over time during the trip with more information collected, as described in Section 3.3.

Definition 2. (Optimal routing policy problem) The optimal routing policy problem in an STD network is to find the routing policy that optimizes an objective function of routing policy travel times over all support points to a given destination, from a given origin and departure time.

Let $e_{\mu}(j,t)$ be the objective function (to be minimized) of following routing policy μ from origin node j at departure time t to a given destination. The optimal objective function value $e^*(j,t) = \min_{\mu} e_{\mu}(j,t)$.

Note that an optimal routing policy is not necessarily *ex post* optimal for any given support point (day), but is optimal on average over all possible support points.

The objective function could be used to describe expected travel time, travel time variance, expected travel time schedule delay, or a combination of a number of criteria. The discussions in Section 4 are not restricted to a particular objective functional form. However, objective functional form does affect algorithm design and as such only expected travel time is evaluated in Section 5.

Given an information scheme, a partition of the universal support point set $\{C^I, \dots, C^R\}$ at (j, t) provides the initial set of event collections $EV(Q(j,t))$. If the objective function is additive

over support points, e.g., in the case of expected travel time or expected schedule delay, an optimal routing policy for the initial universal set of support points is also optimal for any of the initial event collections. In this case, finding an optimal routing policy for the universal set of support points is equivalent to finding an optimal routing policy for each of the initial event collections, and as such Section 5 deals with optimal routing policies with regard to initial event collections. However this is not necessarily true for a non-additive objective function, such as travel time variance, and in such cases, an optimal routing policy problem cannot be broken down to a number of similar problems with initial event collections.

4. Theoretical Analysis of the Value of Information

Optimal routing outcomes are compared under two information schemes (1 and 2) in the same network with different coverage.

Assumption 1. For any trajectory H , information scheme 2 has a larger coverage Q_2 than that of information scheme 1, Q_1 , that is, $Q_1(H) \subseteq Q_2(H)$.

Definition 3. (S_1 contains S_2). Let S_1 and S_2 be two partitions of a set S . S_1 is said to contain S_2 if for any $y \in S_2$, there exists $z \in S_1$, such that $y \subseteq z$. In other words, any element of S_2 is a subset of one and only one element of S_1 , and any element of S_1 is the union of one or more elements of S_2 . See Figure 2 for a schematic representation.

S	a	b	c	d	e	f	g	h
S_1	a	b	c	d	e	f	g	h
S_2	a	b	c	d	e	f	g	h

Fig. 2. A schematic view of S_1 containing S_2

Lemma 1. With Assumption 1, $EV(Q_1)$ contains $EV(Q_2)$ for any trajectory H .

Proof. $EV(Q_1)$ and $EV(Q_2)$ are partitions of the set of support points $\{C^l, \dots, C^R\}$. By definition, for any $EV_2 \in EV(Q_2)$, travel times on time-dependent links of Q_2 are invariant across support points in EV_2 . Since $Q_1 \subseteq Q_2$, travel times on time-dependent links of Q_1 are also invariant

across support points in EV_2 . Therefore there must exist $EV_1 \in \mathbf{EV}(Q_1)$ such that $EV_2 \subseteq EV_1$.

Q.E.D.

With Lemma 1, we can proceed to compare the optimal objective function values under two different information schemes. Note that two travelers under different information schemes generally do not have the same starting information coverage and, therefore, have different initial sets of event collections, even with the same origin and departure time. For example, a radio may only report travel times on highways, while a pre-trip information source (e.g. a website) may report travel times on both highways and arterial roadways. There are two initial event collections under radio information with the highway being normal or congested, and four initial event collections under pre-trip information, with the additional combinations with the arterial being normal or congested. The comparison of the two information schemes is based on performance over all of the possible initial event collections under each scheme, i.e. all support points.

Theorem 1. With Assumption 1, the optimal objective function value under information scheme 2 is no worse than that under information scheme 1, for the same origin j_0 and departure time t_0 , that is,

$$e_2^*(j_0, t_0) \leq e_1^*(j_0, t_0), \quad \forall j_0 \in N, \forall t_0 \in T.$$

Proof. Given any optimal routing policy μ_1 under information scheme 1, an equivalent feasible routing policy μ_2 under information scheme 2 can be constructed as follows: at the original node j_0 and departure time t_0 , partition the universal set of support points based on the two information schemes to obtain the initial event collection sets: $\mathbf{EV}(Q_1(j_0, t_0))$ and $\mathbf{EV}(Q_2(j_0, t_0))$. For any $EV_2 \in \mathbf{EV}(Q_2(j_0, t_0))$, according to Lemma 1 there must exist $EV_1 \in \mathbf{EV}(Q_1(j_0, t_0))$, such that $EV_2 \subseteq EV_1$. We can then set $\mu_2(j_0, t_0, EV_2) = \mu_1(j_0, t_0, EV_1)$. As μ_1 and μ_2 give exactly the same next node under any support point, they produce the same trajectory under any support point at the next decision node. Let the arrival at the next node j occur at time t , then the information coverage Q_1 is a subset of Q_2 from the same trajectory $\{(j_0, t_0), (j, t)\}$. By Lemma 1, $\mathbf{EV}(Q_1)$ contains $\mathbf{EV}(Q_2)$, therefore we can set $\mu_2(j_0, t_0, EV'_2) = \mu_1(j_0, t_0, EV'_1)$, $\forall EV'_2 \in \mathbf{EV}(Q_2)$, $EV'_2 \subseteq EV'_1$. The process continues and a routing policy μ_2 is constructed with exactly the same

trajectory as μ_1 under any support point, and thus the same objective function value, i.e.,

$$e_{\mu_2}(j_0, t_0) = e_{\mu_1}(j_0, t_0) = e_1^*(j_0, t_0).$$

Since μ_2 is a feasible routing policy under information scheme 2, the optimal objective function value under scheme 2 is at least as good as that from μ_2 by definition, i.e.,

$$e_2^*(j_0, t_0) \leq e_{\mu_2}(j_0, t_0).$$

Thus, the optimal objective function value under scheme 2 is at least as good as that under scheme 1, that is, $e_2^*(j_0, t_0) \leq e_1^*(j_0, t_0)$ **Q.E.D.**

Theorem 1 indicates that with larger information coverage throughout the trip, a traveler has more flexibility in every decision node based on a finer partition of the possible outcomes (support points). For example, instead of having to choose a next node based on whether the highway is congested, a traveler can now make the decision based on whether both the highway and arterial are congested. A traveler can always ignore the additional information on arterial roadways and use only highway information. Therefore, optimal actions under larger information coverage are at least as good as those under smaller information coverage.

Theorem 1 also applies when only a subset of the universal set of support points is used to evaluate routing policies. The proof is the same with the universal set replaced by the subset.

The theorem can be alternatively stated as follows: more error-free information is always better than (or at least as good as) less information for adaptive routing in a flow-independent network. It is consistent with Marschak and Miyasawa (1968)'s Theorem 11.3 regarding noiseless information systems: if two information systems are noiseless and one is finer than (in this paper's terminology, contained by) the other, it can never have smaller value than the other for any payoff function defined on a given set of events. However, the decision problem in Marschak and Miyasawa (1968) is single-staged, and Theorem 1 extends the result to a multi-staged routing decision situation in a network context.

5. Solutions to the Partial and No Online Information Schemes

Theorem 1 provides a theoretical comparison between two information schemes; however it is applicable only when coverage is larger or no smaller in both spatial and temporal dimensions. In reality, an information scheme can have larger coverage in one dimension but smaller coverage in the other. In order to evaluate the value of traveler information empirically for more complicated situations, computer algorithms are needed.

Because a routing policy has a random travel time, there exist multiple optimization criteria. Expected travel time is used in the remainder of the paper, as generally it is the primary criterion in routing choices. Other criteria regarding travel reliability, such as expected schedule delay and travel time variance, will be explored in future research.

In this section, it is shown that Bellman's principle of optimality does not hold for the three partial or no online information schemes even with additive objective functions. For these schemes, a heuristic algorithm is designed and applied.

In all the schemes studied, the information coverage Q is determined by the current time, instead of the whole trajectory, therefore $EV(t)$ is used instead of $EV(Q)$. Time lag Δ in delayed information, departure time t_0 in pre-trip information and radio coverage B in radio information are treated as exogenous system parameters. In pre-trip information with departure time t_0 , $EV(t) = EV(t_0)$, $\forall t \geq t_0$.

Except for delayed information, no additional useful information is available in all other schemes during the static period, i.e., Q does not grow beyond $K-1$, because either no information is provided (pre-trip and no online information), or additional information will not enlarge Q (radio and perfect online information). In the case of delayed information, a traveler continues receiving information in the static period until $K-1+\Delta$, at which time $Q=A \times T$. Let T^* denote the time beyond which a traveler receives no more useful information and Q remains unchanged. We then have $T^*=K-1+\Delta$ for delayed information, and $T^*=K-1$ for all other four information schemes.

5.1. Bellman's Principle of Optimality

Proposition 1. Bellman's principle of optimality does not hold for the delayed, pre-trip, radio or no online information schemes with additive objective functions. In other words, if μ^* is optimal for a given initial event collection EV_0 at (j_0, t_0) , and (j, t, EV) is an intermediate state during the execution of μ^* , then the remainder of μ^* is not necessarily optimal for the initial state (j, t, EV) .

Discussion. This can be shown through an example in Figure 3 and Table 3. Note that only relevant link travel times are shown. The travel time on link (d, c) is always 0 and not listed. No online information is assumed, such that the routing decision only depends on the arrival time at each decision node, i.e, $EV = \{C^1, C^2\}$ at any node and time. The primary problem is finding a minimum expected travel time routing policy from node a to c for departure time 0.

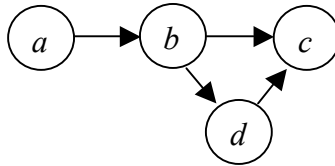


Fig. 3. An illustrative small network

Table 3. Support points for the small network ($p_1 = p_2 = 1/2$)

Time	Link	C^1	C^2
0	(a, b)	1	2
1	(b, c)	1	10
	(b, d)	3	3
2	(b, c)	10	1
	(b, d)	3	3

Link (a, b) has two possible travel times at time 0: 1 and 2, therefore the arrival time at node b can be either 1 or 2. As there are two alternatives to go from node b to c at each of the two possible arrival times, altogether there are four routing policies, listed in Table 4 along with the corresponding expected travel times.

Table 4. Routing policies from node a at time 0

	At node a	At node b		Expected travel time
		Arrival time 1	Arrival time 2	
Routing policy 1	Node b	Node c	Node c	2.5
Routing policy 2	Node b	Node c	Node d	3.5
Routing policy 3	Node b	Node d	Node c	3.5
Routing policy 4	Node b	Node d	Node d	4.5

The optimal routing policy from node a to c at departure time 0 is therefore $a-b-c$ (actually a path). However, the optimal routing policy from node b to c at either departure time 1 or 2 is not the policy $b-c$ with mean travel time $0.5(1+10)$, but $b-d-c$ with mean travel time 3.

The key here is the treatment of the travel time on link (b, c) . The travel time of 10 on link (b, c) can never be realized if the traveler leaves node a at time 0, due to the stochastic dependency between link (a, b) and (b, c) . However if b is the origin, then the travel time of 10 is possible and should be taken into account. If link travel times are time-wise and link-wise independent, Bellman's optimality principle will hold and the problem in the no online information scheme reduces to those studied by Miller-Hooks and Mahmassani (2000), Chabini (2000) and Miller-Hooks (2001).

Examples for the three partial online information schemes can be constructed similarly. If j is an origin with EV , the calculation of expected travel time from j is not conditional on the past and thus includes all support points in EV . However, if j is an intermediate node, the calculation must be conditional on the traversed link travel times from the origin to the current node, which are not necessarily covered by online information. Since link travel times are stochastically dependent, the conditional expected travel time may differ from the unconditional one. Examples can be constructed resulting in different optimal policies based on whether or not the node is an origin. Details of these examples are not presented due to space limit.

Bellman's principle of optimality is valid for the perfect online information scheme (stated formally later in Proposition 4). Note that in this scheme, online information includes everything that happened in the past, including the traversed link travel times to any intermediate node. Therefore the expected travel time with perfect online information does not depend on whether the node is an origin.

5.2. Necessary Conditions for Optimality

Proposition 1 indicates that we cannot generate an optimal routing policy by combining the optimal next node and the optimal policy from the next node. The necessary conditions for the optimal solutions are presented in Proposition 2. Any feasible solution to the optimal routing policy problem must provide an upper bound on the minimal expected travel time; however, a solution that satisfies the necessary conditions for optimality provides a tighter upper bound than an arbitrary solution. Therefore a heuristic algorithm is proposed to solve for the necessary conditions, and its effectiveness (in terms of closeness to optimal solutions) is evaluated computationally. The heuristic algorithm is a generalization of the algorithm for the perfect online information problem in Gao and Chabini (2006), with a distinction in the major recursive equation.

Let $e_\mu(j,t,EV)$ be the expected travel time to the destination node d by following routing policy μ , if the departure from origin node j happens at time t with the event collection EV . For each support point (at the end of a day), a routing policy is manifested as a path with a deterministic travel time. We thus define $S_\mu(j,t,r)$ as the travel time to the destination node d if support point r is realized with an exit from node j (origin or intermediate) at time t by following routing policy μ . The relationship between $e_\mu(j,t,EV)$ and $S_\mu(j,t,r)$ is as follows:

$$e_\mu(j,t,EV) = \sum_{r \in EV} S_\mu(j,t,r) \Pr(r | EV) \quad (1)$$

where $\Pr(r | EV) = p_r / \sum_{i \in EV} p_i$ is the probability of support point r given EV . Note that the

algorithm in Gao and Chabini (2006) for perfect online information uses only $e_\mu(j,t,EV)$ because, in this scheme, Bellman's principle of optimality is valid, while $S_\mu(j,t,r)$ must be used to correctly calculate expected travel times for partial and no online information schemes.

For a given time t and support point r , there is one and only one corresponding event collection $EV(t,r)$, since $EV(t)$ is a partition of the universal set of support points. This ensures that the next node of routing policy μ at (j,t,r) can be uniquely retrieved as $\mu(j,t,EV(t,r))$, and $S_\mu(j,t,r)$ can be obtained by executing μ in support point r . In the example of Figure 1 and Table 2, for a traveler with radio information on (a,b) , the routing decision at node a and time 0 can only be made based on the event collection $\{C^1, C^2, C^3\}$. Let $\mu\{a,0,\{C^1, C^2, C^3\}\} = c$. The travel

time by following routing policy μ starting from node a at time 0 is a random variable with different outcomes for different support points: $S_\mu(a,0,C^1)=3$, $S_\mu(a,0,C^2)=3$, and $S_\mu(a,0,C^3)=2$.

The recursive relationship between S_μ at node j and the succeeding node k by following μ is critical to solving the optimal routing policy problem. $S_\mu(j,t,r)$ is defined for a trip leaving node j at time t . For all the information schemes except for pre-trip, the information coverage is not a function of departure time, and thus event collections at time t and node j are the same whether j is an origin or intermediate node. In this case,

$$S_\mu(j,t,r) = C_{jk,t}^r + S_\mu(k,t + C_{jk,t}^r, r), \text{ where } k = \mu(j,t, EV(t,r)). \quad (2)$$

With perfect online information, the travel time on the next link (j,k) at time t , $C_{jk,t}^r$ is the same for all support points in a given EV (denoted as $\pi_{jk,t}^{EV}$), and thus inputting an expectation of both sides of (2) over EV produces the following:

$$\begin{aligned} e_\mu(j,t, EV) &= \sum_{r \in EV} S_\mu(j,t,r) \Pr(r | EV) \\ &= \sum_{r \in EV} (\pi_{jk,t}^{EV} + S_\mu(k,t + \pi_{jk,t}^{EV}, r)) \Pr(r | EV) \\ &= \pi_{jk,t}^{EV} + \sum_{EV' \in EV(t + \pi_{jk,t}^{EV})} \sum_{r \in EV'} S_\mu(k,t + \pi_{jk,t}^{EV}, r) \Pr(r | EV') \Pr(EV' | EV) \\ &= \pi_{jk,t}^{EV} + \sum_{EV' \in EV(t + \pi_{jk,t}^{EV})} e_\mu(k,t + \pi_{jk,t}^{EV}, EV') \Pr(EV' | EV) \end{aligned} \quad (3)$$

where $k = \mu(j,t, EV)$. In the third equality, support points at a later time $t + \pi_{jk,t}^{EV}$ are re-partitioned into finer event collections EV' . In the fourth equality, support point travel times in each EV' are summarized as the expected travel time.

Such a relationship between expected travel times at adjacent nodes generally does not exist for partial or no online information schemes, since the derivation in (3) depends on the fact that the travel time on the next link is included in the information coverage at the next node.

For the pre-trip information scheme, the information coverage depends on the departure time, and therefore is unclear in which event collection r is appropriate at a given time t . A different variable $S_\mu(j,t,r;t_0)$ can be defined as the travel time from node j (origin or intermediate) and time t to the destination node if support point r is realized by following routing policy μ ,

with a departure time t_0 . $EV(t,r;t_0)$ then gives a unique event collection at time t corresponding to support point r with a departure time t_0 , resulting in a recursive relationship:

$$S_{\mu}(j,t,r;t_0) = C_{jk,t}^r + S_{\mu}(k,t + C_{jk,t}^r,r;t_0) \text{ where } k = \mu(j,t,EV(t,r;t_0)).$$

$e_{\mu}(j,t,EV)$ the expected travel time from the origin j with departure time t can then be written with $t_0=t$: $e_{\mu}(j,t,EV) = \sum_{r \in EV} S_{\mu}(j,t,r;t) \Pr(r | EV)$.

We propose the following system of recursive equations to solve the perfect online, delayed, radio and no online information schemes based on the recursive equation in (2).

$$e_{\mu^*}(j,t,EV) = \min_{k \in A(j)} \left\{ \sum_{r \in EV} (C_{jk,t}^r + S_{\mu^*}(k,t + C_{jk,t}^r,r)) \Pr(r | EV) \right\} \quad (4)$$

$$\mu^*(j,t,EV) = \arg \min_{k \in A(j)} \left\{ \sum_{r \in EV} (C_{jk,t}^r + S_{\mu^*}(k,t + C_{jk,t}^r,r)) \Pr(r | EV) \right\} \quad (5)$$

$$\forall j \in N \setminus \{d\}, \forall t, \forall EV \in EV(t)$$

where $A(j)$ the set of downstream nodes out of node j . The boundary conditions are:

- a) At the destination: $S_{\mu^*}(d,t,r) = 0$, $\mu^*(d,t,EV) = d$, $\forall t, \forall EV \in EV(t)$, $\forall r \in EV$.
- b) Beyond T^* : $\mu^*(j,t \geq T^*, EV) = \mu^*(j, T^*, EV)$, $\forall j, \forall EV \in EV(T^*)$, $T^* = K-1 + \Delta$ for delayed information, and $T^* = K-1$ for the other three schemes (radio, perfect and no online information).

Note that, $S_{\mu^*}(j,t,r) = C_{jk^*,t}^r + S_{\mu^*}(k^*,t + C_{jk^*,t}^r,r)$, where $k^* = \mu^*(j,t,EV(j,t))$. $S_{\mu^*}(d,t,r)$ is the travel time of the solution routing policy μ^* in support point r , not the minimum travel time calculated using a deterministic shortest path algorithm in support point r . $S_{\mu^*}(d,t,r)$ is obtained by executing μ^* after μ^* is generated.

For the pre-trip scheme, a similar system of equations can be solved to obtain a solution from all nodes and all possible event collections, but with departure time t_0 only.

Proposition 2. Conditions (4) and (5) are necessary for μ^* to be an optimal routing policy for all possible initial states for the perfect online, delayed, radio and no online information schemes.

Proof. Trivially, if the boundary conditions at the destination node are not satisfied, μ^* is not optimal.

At time period T^* and beyond, the information coverage includes all links at all time periods. Therefore there are R event collections, each with one support point. The optimal routing policy beyond T^* is not a function of time t , as travel times and event collections do not change over time. $\mu^*(j, t \geq T^*, EV) = \mu^*(j, T^*, EV)$, $\forall j, \forall EV \in EV(T^*)$. Conditions (4) and (5) become

$$e_{\mu^*}(j, T^*, \{r\}) = \min_{k \in A(j)} \{C_{jk, T^*}^r + e_{\mu^*}(k, T^*, \{r\})\} \quad (6)$$

$$\mu^*(j, T^*, \{r\}) = \arg \min_{k \in A(j)} \{C_{jk, T^*}^r + e_{\mu^*}(k, T^*, \{r\})\} \quad (7)$$

$$\forall j \in N \setminus \{d\}, \forall r$$

plus boundary conditions. These are the optimality conditions of a static shortest path problem in a deterministic network where link travel times are C_{jk, T^*}^r , $\forall(j, k)$. If μ^* is optimal, it must manifest as a shortest path in each deterministic network defined by a support point beyond T^* , and thus (6) and (7) must be satisfied.

Assume by contradiction that (4) and (5) are not satisfied for some state with a departure time earlier than T^* . Let (j, t, EV) be such a state. Therefore there must exist an outgoing node $k \in A(j)$, such that

$$\sum_{r \in EV} (C_{jk, t}^r + S_{\mu^*}(k, t + C_{jk, t}^r, r)) \Pr(r | EV) < \sum_{r \in EV} (C_{jk^*, t}^r + S_{\mu^*}(k^*, t + C_{jk^*, t}^r, r)) \Pr(r | EV)$$

A different routing policy μ can be constructed such that $\mu(j, t, EV) = k$, and $\mu = \mu^*$ for all other states. The following is obtained:

$$\begin{aligned} e_{\mu}(j, t, EV) &= \sum_{r \in EV} S_{\mu}(j, t, r) \Pr(r | EV) = \sum_{r \in EV} (C_{jk, t}^r + S_{\mu}(k, t + C_{jk, t}^r, r)) \Pr(r | EV) \\ &= \sum_{r \in EV} (C_{jk, t}^r + S_{\mu^*}(k, t + C_{jk, t}^r, r)) \Pr(r | EV) \\ &< \sum_{r \in EV} (C_{jk^*, t}^r + S_{\mu^*}(k^*, t + C_{jk^*, t}^r, r)) \Pr(r | EV) = e_{\mu^*}(j, t, EV) \end{aligned}$$

The third equality is due to the fact that μ and μ^* are the same at all times later than t . The other equalities come from the definition of the expected travel time (1) and the recursive relationship between S_{μ} at node j and the succeeding node k by following routing policy μ (2).

The equation contradicts with the fact that μ^* is optimal, therefore (4) and (5) must be satisfied for $t < T^*$. **Q.E.D.**

Proposition 3. Conditions (4) and (5) are not sufficient for optimality in the delayed, radio or no online information schemes.

Proof. This can be shown through the same example in Proposition 1. Solving the equations at node b at either time 0 or 1 gives $b-d-c$ as the solution because it gives smaller expected travel time (over the event collection for no online information, which contains all support points) than $b-c$ does. The equation at node a is trivial, as there is only one outgoing link. The solution is then $a-b-d-c$ (routing policy 4), which is not optimal. **Q.E.D.**

Similar conclusions can be drawn for the pre-trip information scheme following Propositions 2 and 3 with the modified routing policy travel time variables.

Proposition 4. Conditions (4) and (5) are sufficient and necessary for μ^* to be an optimal routing policy for all possible initial states in the perfect online information problem, and equivalent to the optimality conditions in Gao and Chabini (2006).

Proof. With perfect online information, $C_{jk,t}^r$ is the same for all support points in a given EV , and thus taking expectations of both sides of (4) over EV and changing (5) accordingly gives the optimality conditions in Gao and Chabini (2006), similar to the derivation in (3). The sufficiency and necessity of (4) and (5) then follows from the optimality of the conditions in Gao and Chabini (2006). **Q.E.D.**

Note that the optimality conditions for the perfect online information scheme in Gao and Chabini (2006) are neither sufficient nor necessary for optimality in the partial or no online information schemes, as the recursive equation is complicated by imperfect information.

5.3. Algorithm DOT-PART

In this section a heuristic algorithm is designed to solve the system of equations (4) and (5). The evaluation of $e_{\mu^*}(j,t,EV)$ only depends on $S_{\mu^*}(j,t',r)$ from a later time $t' > t$, due to the positive and integral link travel time assumption. Therefore the problems can be solved in a decreasing order of time, making use of the acyclic property of the network in the time dimension (Chabini, 1998).

At time T^* and beyond, any deterministic static shortest path algorithm can be used to compute $e_{i^*}(j, t, \{r\})$, $\forall j \in N$, $\forall t \geq T^*$, $\forall r$. The procedure to generate event collections carries out partitions of the universal set of support points in an increasing order of time. At time t , a partition is made on $EV(t-1)$ based on each (link, time) pair in the incremental information coverage, $Q(t) \setminus Q(t-1)$. Note that Q is written as a function of t , because in all the five information schemes, Q only depends on t , not the trajectory.

Generate_Event_Collection

$$D = \{C^L, \dots, C^R\}$$

If information scheme = no online, $EV(t) \leftarrow D$, $t = 0$ to $K-1$, **STOP**.

For $t = 0$ to T^*

If information scheme = perfect online, $Q(t) = A \times \{0, 1, \dots, t\}$

If information scheme = delayed, $Q(t) = A \times \{0, 1, \dots, t - \Delta\}$

If information scheme = pre-trip, $Q(t) = A \times \{0\}$

If information scheme = radio, $Q(t) = B \times \{0, 1, \dots, t\}$

$Q(-1) = \emptyset$ //a proxy for convenience of representation

For $t = 0$ to T^*

For each (link, time) pair $((j, k), t') \in Q(t) \setminus Q(t-1)$

For each disjoint subset $S \in D$

$D' \leftarrow$ A partition of S based on $\tilde{C}_{jk, t'}$

$D \leftarrow$ Union of all D'

$EV(t) \leftarrow D$;

Algorithm DOT-PART

(Generic for perfect online, delayed, pre-trip with departure time 0, radio and no online information schemes)

Initialization

Step 1:

If information scheme = delayed, $T^* = K - 1 + \Delta$; else $T^* = K - 1$.

Construct $EV(t)$, $t=0, \dots, T^*$ by calling Generate_Event_Collection.

Step 2:

Compute $e_{\mu^*}(j, T^*, \{r\})$ and $\mu^*(j, T^*, \{r\})$, $\forall j \in N$, $\forall EV \in EV(T^*)$ with a static deterministic shortest path algorithm in a network where link travel times are those at time T^* for support point r .

Compute $S_{\mu^*}(j, T^*, r)$ by executing μ^* in the original static stochastic network, $\forall j \in N$, $\forall r \in EV$; set $S_{\mu^*}(j, t > T^*, r) = S_{\mu^*}(j, T^*, r)$

Step 3:

$e_{i^*}(j, t, EV) \leftarrow +\infty$, $\forall j \in N \setminus \{d\}$, $\forall t < T^*$, $\forall EV \in EV(t)$

$$e_{\mu^*}(d, t, EV) \leftarrow 0, S_{\mu^*}(d, t, r) \leftarrow 0, \forall t < T^*, \forall EV \in EV(t), \forall r \in EV$$

Main Loop

For $t = T^*-1$ down to 0 and for each $EV \in EV(t)$

For each link $(j, k) \in A$

$$temp = \sum_{r \in EV} (C_{jk,t}^r + S_{\mu^*}(k, t + C_{jk,t}^r, r)) \Pr(r | EV)$$

If $temp < e_{\mu^*}(j, t, EV)$ then

$$e_{\mu^*}(j, t, EV) = temp$$

$$\mu^*(j, t, EV) = k$$

For each $r \in EV$ and each $j \in N$

$$k^* = \mu^*(j, t, EV)$$

$$S_{\mu^*}(j, t, r) = C_{jk^*,t}^r + S_{\mu^*}(k^*, t + C_{jk^*,t}^r, r)$$

According to Propositions 2 and 3, Algorithm DOT-PART is exact for the perfect online information scheme. It generates approximate solutions with all initial states for delayed, radio and no online information schemes, and with departure time 0 for pre-trip information scheme. In order to solve pre-trip scheme with all departure times, a loop over all departure times t_0 has to be added outside the main loop, and the main loop will be executed from T^*-1 to t_0 (not shown in the algorithm statement).

Following a similar analysis as in Gao and Chabini (2006), Algorithm DOT-PART (including Generate_Event_Collection) has a time complexity of $O(mKR \ln R + R \times SSP)$ and $O(mK^2R \ln R + R \times SSP)$ for pre-trip information, where SSP is the time complexity of the static deterministic shortest path algorithm. The algorithm is strongly polynomial in R , the number of support points. For real life applications, travel time observations on all (random) links from each day can be viewed as one support point. Such data are available with the advent of advanced sensor and surveillance technologies, such as GPS and probe vehicles. The number of support points might seem exponential to the number of links, however, if we consider the high stochastic dependencies among link travel times and use observations from each day as a support point, we can safely have several years' data with the number of support points in the thousands, similar to the number of links in a medium-sized network and much less than its exponential. Running time tests are conducted with randomly generated networks that confirm the complexity analysis. The reader is referred to Gao and Huang (2009) for a detailed account of the running time test results.

5.4. Computational Tests

The objectives of the computational tests are to 1) systematically investigate the effectiveness of the heuristic Algorithm DOT-PART in generating optimal solutions to the partial and no online information schemes; and 2) study the (approximate) value of information empirically as a complement to the theoretical study in Section 4.

Algorithm DOT-PART provides upper bounds on the minimal expected travel times in partial and no online information schemes by generating feasible solutions. However, the upper bound can be made arbitrarily loose by constructing an example similar to that in Proposition 1. Our main focus is evaluating average effectiveness through systematic testing over a large number of problem instances. We do not have an exact algorithm that will solve for partial or no online information schemes. However, Theorem 1 states that the optimal solution under a perfect online information scheme is at least as good as the optimal solution under any partial or no online information scheme, since the former coverage is larger with any given trajectory.

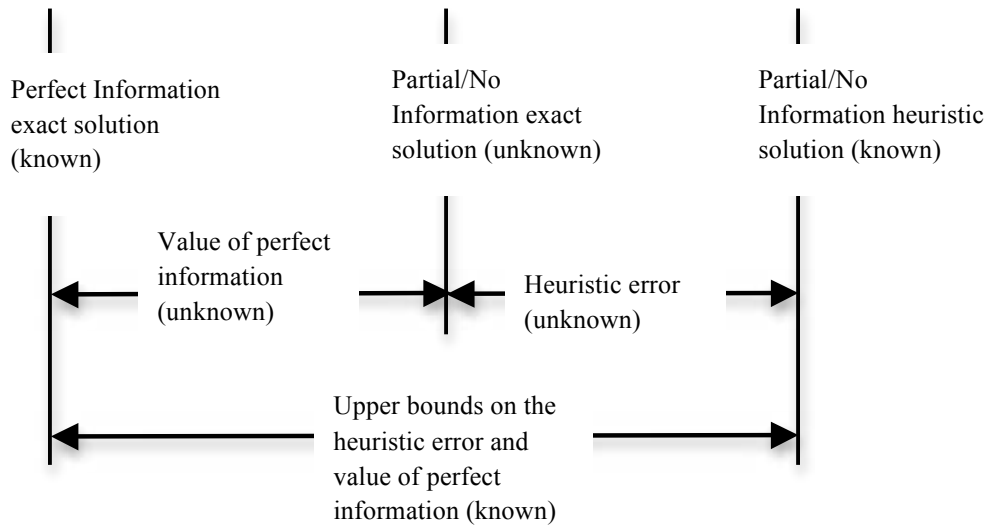


Fig. 4. Relationships between heuristic and exact solutions

Therefore the optimal solution with perfect online information, which can be computed exactly by Algorithm DOT-PART, provides a lower bound of the optimal solution with any partial or no online information. The error of the heuristic algorithm, which is the difference between the

unknown exact solution to a partial or no online information scheme and the heuristic solution, is then limited by the difference between the perfect online information solution and the heuristic solution. Furthermore, we can also view the same difference as an upper bound on the value of perfect information compared to partial or no online information. A schematic view of these relationships for any given partial or no online information scheme is shown in Figure 4.

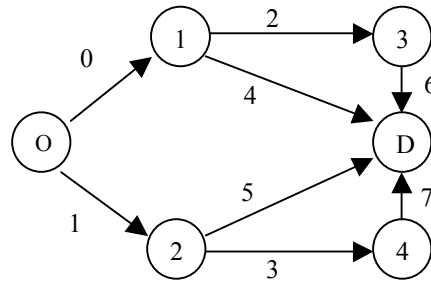


Fig. 5. The test network

The first test network is shown in Figure 5 with six nodes and eight directed links. There are diversion possibilities at nodes O, 1 and 2. The link travel time distribution is generated through an exogenous simulation with the mesoscopic supply simulator of DynaMIT (Ben-Akiva *et al.*, 2001). The study period is from 6:30am to 8:00am. The time resolution is 1 minute for departures and arrivals at intermediate nodes, and there are 90 time periods in total. The travel time is in seconds. The demand between the origin and destination is low from 6:30am to 7:00am and increases later in the day. There are random incidents in the network that result in 37 support points. Details of the network can be found in Gao (2005).

Algorithm DOT-PART is run for the three partial online, no online and perfect online information schemes to derive the (upper bounds of) minimum expected travel times for each scheme from node O to D for all departure times and all event collections. The results are aggregated by departure time and include expectations over all event collections at a given time.

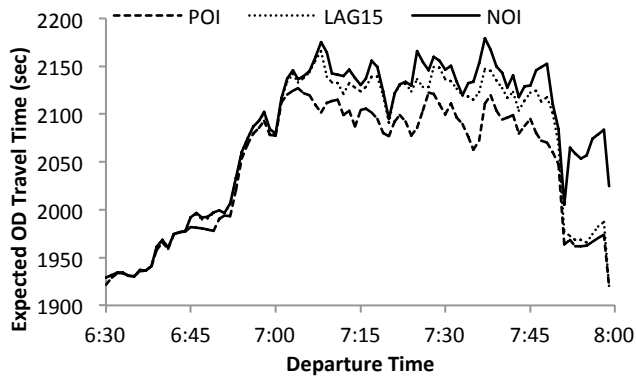


Fig. 6. Results for the 15-min delayed (LAG15) vs. perfect (POI) and no online information (NOI)

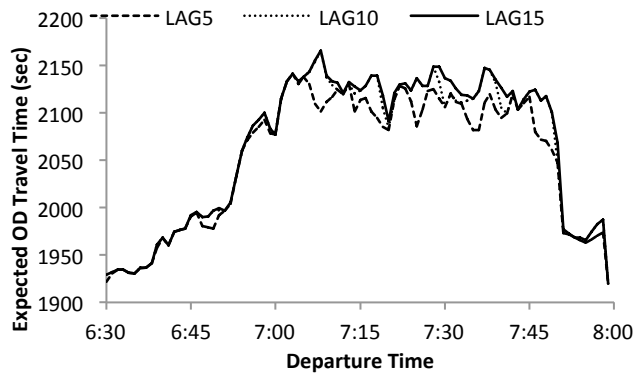


Fig. 7. Results for delayed information with 5 (LAG5), 10 (LAG10) and 15-min time lags

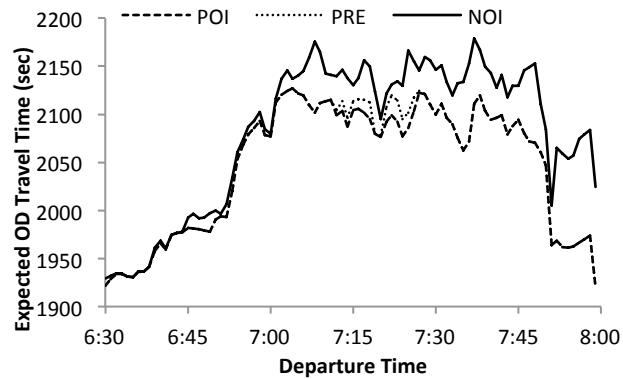


Fig. 8. Results for pre-trip (PRE) vs. perfect and no online information

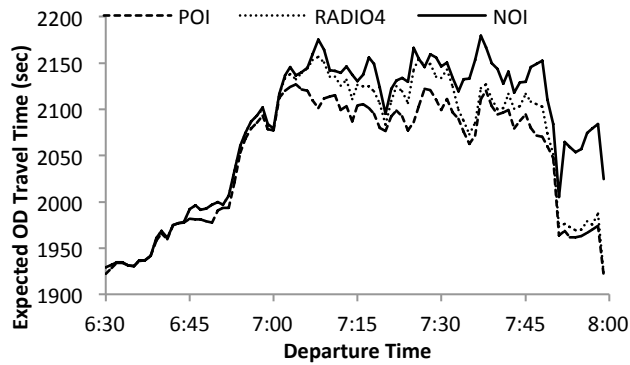


Fig. 9. Results for radio on link 4 vs. perfect and no online information

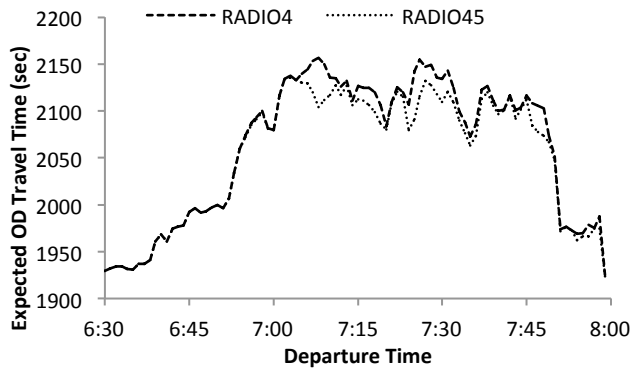


Fig. 10. Results for radio information with different radio coverage

Figures 6 through 10 show the expected OD travel times for the no online, 5-min delay, 10-min delay, 15-min delay, pre-trip, radio on link 4 and radio on links 4 and 5 schemes. RADIO4 indicates that only traffic condition information on link 4 is available and RADIO45 indicated condition information on links 4 and 5. It is shown that the upper bounds generated by Algorithm DOT-PART are relatively tight, within 3% of the (unknown) exact solution. It is also shown that in the specific settings, global pre-trip information is nearly as good as perfect online information. It is of note that although the solutions to the partial and no online information schemes are not exact, they do exhibit the trend that more error-free information is better in a flow-independent network. For example, in Figure 7 expected travel time with delayed information decreases when the delay decreases from 15 to 10 minutes and from 10 to 5 minutes; in Figure 10, expected travel times with radio covering both links 4 and 5 are better than those

with radio covering only link 4. However this should not be viewed as a verification of Theorem 1 as the solutions are approximate.

Additional tests conducted on larger randomly generated networks were used to investigate the effectiveness of the heuristic algorithm. The random network generator takes the following as inputs: 1) the number of nodes; 2) the number of links; and 3) the number of time periods. Four levels of the number of nodes are considered: 50, 100, 250, and 500. The number of links is defined as three times the number of nodes, i.e., 150, 300, 750, and 1500. Three levels of the duration of the peak period are considered: 25, 50, and 100 time intervals. Other parameters include the number of support points fixed at 300, the range of link travel time fixed as $[0, 10]$, and the maximum in-degree and out-degree fixed as 5. The topology of the network is randomly generated. The travel time on each link at each time interval for each support point is generated from a uniform distribution within the fixed range. More details on the random network generation can be found in Gao (2005).

There are 12 different combinations of inputs, and 10 random networks are generated for each combination. Table 5 shows the upper bounds of heuristic errors, defined as the percentage difference between the results from the partial and no online information schemes and the results from the perfect online information scheme. The results are averaged over all departure times

Table 5. Upper bounds of heuristic errors (% difference from perfect online information)

<i>Nodes</i>	<i>Links</i>	<i>Time Periods (K)</i>	No Online	Pre-trip	Delayed by 0.5K	Delayed by 0.25K	Radio on one link
50	150	25	40.3	0	14.9	6.1	2.2
50	150	50	26.6	0	11.2	4.2	0.5
50	150	100	22.3	0	10.5	4.9	0.3
100	300	25	13.8	0	5.3	2.3	0.9
100	300	50	24.4	0	10.5	4.1	0.6
100	300	100	26.0	0	12.8	6.1	0.4
250	750	25	31.4	0	12.0	5.1	1.8
250	750	50	33.9	0	14.3	5.6	0.8
250	750	100	27.0	0	12.4	5.6	0.3
500	1500	25	21.6	0	6.5	2.3	0.8
500	1500	50	26.5	0	11.4	4.5	0.7
500	1500	100	28.8	0	13.3	6.0	0.3
Average			26.9	0	11.2	4.7	0.8

(except for the pre-trip scheme, where only departure time 0 results are reported) and all origins to a single destination for each network, and then averaged over the 10 networks. The radio information scheme covers only one link, randomly sampled 10 times for each of the 10 random networks. Thus in the radio column, the results are averages over 100 runs.

Algorithm DOT-PART performs better than predicted by the theoretical worst case (arbitrarily large errors), with errors within 15% for partial online schemes and 30% for no online information scheme. Note that these are upper bounds of errors, and the heuristic algorithm may perform better in practice. Future research is needed to design an exact algorithm and perform a more comprehensive evaluation of the effectiveness of the algorithm. Future research avenues include evaluating the effectiveness of the heuristic algorithm with real-world data, an important step towards its practical application.

The data in the test scenarios again confirm that more error-free information is better in a flow-independent network. For example, information delayed for $0.25K$ units of time produces smaller expected travel time than information delayed for $0.5K$ units of time, which in turn does better than no online information. Pre-trip information is as good as perfect online information in all test scenarios, and up-to-date radio information is almost as good. Note that pre-trip information as defined in this paper as real-time information on realized travel times at the time of departure (up-to-date when it is first available) is different from historical information. On the other hand, delayed information seems to perform less successfully than pre-trip information. This might suggest that delays are more detrimental to the value of information than spatial limitations.

In order to investigate how Algorithm DOT-PART performs heuristically, we have also developed a brute-force method that enumerates all routing policies to find ones that are optimal. This method only works for very small network, as the number of routing policies grows exponentially with the problem size.

As defined in Definition 1, a routing policy is a mapping from state to decision, for all possible states and all possible next nodes out of a given state. If there are, on average, Q outgoing links for each node, N nodes in total in the network, and X event collections over all time periods, the maximum number of routing policies is Q^{NX} .

The computational tests agree with the analysis. The largest network in which we can conduct the brute-force method is the no online information scheme, with 6 nodes, 18 links, 10 time periods and 10 support points. The computational results show that, in such small networks, the heuristic algorithm DOT-PART gives the same optimal solutions as the exact solutions given by the brute-force method for all information variants.

6. Conclusions and Future Directions

In this paper a generic representation of online information in a general stochastic network is developed, based on three types of information schemes: delayed global information, global pre-trip information, and up-to-date radio information on a subset of links. The scope limitations of an information system on both the temporal and spatial dimensions are considered. A theoretical proof of the non-negative value of error-free traveler information for adaptive routing in a flow-independent stochastic network is presented. It is shown that Bellman's principle of optimality does not apply to the optimal routing policy problem in schemes with partial or no online information. A heuristic algorithm is then designed based on a set of necessary conditions for optimality and is tested for effectiveness.

Other information schemes will be studied in the future, e.g., VMS, which is one of the most common types of ATIS. Because VMS is trajectory-based rather time-based, the optimality problem is more complex than those discussed in this paper. This increases the complexity needed in the algorithm design. The noise level of the information can also be evaluated, and then the information is no longer error-free. Theoretical studies can be conducted to establish the conditions (if any) under which noisy information systems are comparable.

Predictive information (Bovy and van der Zijpp, 1999; Bottom, 2000; and Dong et al., 2006) that provides estimates of future travel times is not explicitly studied under the online information framework in this paper. One can easily build a mathematical information scheme where the coverage $Q(t)$ contains realized travel times beyond t , and the analyses and algorithm described in this paper are applicable. The more fundamental question is whether an analysis framework built upon error-free information assumption is good for predictive information.

Although the error in measuring realized travel times can be reasonably assumed to approach zero with the ever-increasing accuracy of traffic surveillance systems, the same cannot be said for predictive information. Therefore the effort to model predictive information should be connected with research on the incorporation of noisy information.

As previously mentioned, the interaction between demand and supply must be considered to assess the value of real-time information with a large market penetration of information. In a congested un-priced network, information could be detrimental, as shown in Gao (2005) and others (e.g., Arnott et al., 1991, 1999, Levinson, 2003). The next logical step would be a study of the value of various types of information systems in a congested network. An equilibrium dynamic traffic assignment model or a day-to-day dynamic process model should be applied.

Another interesting avenue for future research is the theoretical quantification of the value of traveler information as a function of an array of information system and network characteristics. This would enable the cross comparison of different types of information systems. For example, *is up-to-date spatially-limited information better than delayed global information?* Although answers can be obtained computationally as shown in Section 5.4, a theoretical solution would provide valuable insights and guidelines for optimal investment in ATIS.

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Appendix: Notation

Network

- N : set of nodes
- A : set of links, with $|A|=m$
- (j,k) : directional link from node j to k
- T : set of time periods $\{0, 1, \dots, K-1\}$
- j_0 : origin node
- t_0 : departure time
- T^* : the time beyond which a traveler receives no more useful information
- $A(j)$: set of downstream nodes out of node j

Stochasticity

- \tilde{C} : vector of time-dependent link travel time random variables with a dimension of $K \times m$
- $\tilde{C}_{jk,t}$: travel time random variable on link (j,k) at time t
- C^r : network support point r , a vector of time-dependent link travel times with a dimension of $K \times m$
- R : number of network support points for the joint distribution of all links at all time periods
- p_r : probability of support point r
- \tilde{C}_Q : vector of random travel times of time-dependent links in Q (see the information group for the definition of Q)
- C_Q : support point of \tilde{C}_Q
- $\Pr(r|EV)$: probability of support point r given EV

Information

- H : trajectory of consecutive (node, time) pairs
- Q : information coverage over links and time periods, $Q \subseteq A \times T$
- $Q(H)$: information scheme defined as a mapping from H to Q
- Δ : time lag of delayed information
- EV : event collection, set of network support points compatible with the realized link travel times
- $EV(Q)$: set of all the possible event collections with information coverage Q

$EV(t)$: set of all possible event collections at time t

**Routing
policy**

$\{j, t, EV\}$: state comprising of current node j , time t and event collection EV

μ : $\{j, t, EV\} \rightarrow k$, routing policy mapping from states to next nodes

$\mu(j, t, EV)$: next node on routing policy μ out of the current state $\{j, t, EV\}$

$e_r(j, t)$: objective function value (to be minimized) of following routing policy μ from origin node j at departure time t to a given destination over all support points

$e^*(j, t)$: $\min_r e_r(j, t)$, optimal objective function value

μ^* : optimal routing policy

$e_r(j, t, EV)$: expected travel time to the destination node by following routing policy μ , if the departure from origin node j happens at time t with the event collection EV

$S_r(j, t, EV)$: travel time to the destination node if support point r is realized with an exit from node j (origin or intermediate) at time t by following routing policy μ

$EV(t, r)$: unique event collection corresponding to support point r at time t for the delayed, radio, no online and perfect online information schemes

$S_r(j, t, EV; t_0)$: travel time to the destination node if support point r is realized with an exit from node j (origin or intermediate) at time t by following routing policy μ , with a departure time from the origin at t_0 for the pre-trip information scheme

$EV(t, r; t_0)$: unique event collection at time t corresponding to support point r with a departure time from the origin at t_0