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Realistic bus route model considering the capacity of the bus

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Abstract. This paper investigates the bus route behavior using a more realistic cellular automaton model in which the bus capacity is considered. It is shown that with the introduction of bus capacity, four new states appear compared with the previous works. The results enable us to conclude that the efficiency of the bus route system cannot be enhanced simply through increasing the number of buses. Moreover, it is pointed out that a proper value of the bus capacity can lead to the optimal configuration of the bus system.

PACS. 05.70.Fh Phase transitions: general studies – 64.60.-i General studies of phase transitions – 89.40.+k Transportation

1 Introduction

In the last few decades, traffic problems have attracted the interest of a community of physicists [1–4]. Among the phenomena under consideration, the bus route system is a typical many-body system of interacting buses and passengers. To understand the behavior of bus route system, various models have been proposed and studied, including cellular automaton (CA) models [5,6], time headway models [7,8], and car-following models [9,10].

In these models, if a bus is delayed by some fluctuation, the gap between it and its predecessor becomes larger because this bus has to pick up more passengers. During the period of delay, more passengers will be waiting for the bus. As a result, the bus will get further delayed. The slowly moving delayed bus will slow down the buses behind it. This causes the bus bunching. It has been found that the transition between an bus bunching state and a homogeneous state occurs with increasing the bus density. Thus, the bus behaviour exhibits the dynamical phase transition similar to the traffic flow [4].

However, in these models, the bus capacity is assumed to be infinite so that it can pick up all passengers waiting at the bus stops at a time. This obviously is not realistic. Thus, in this paper, we present a realistic bus route model considering the capacity of the bus. The model is presented in Section 2 and the simulation results are reported in Section 3. Section 4 gives the conclusions.

2 Model

In this section, we present a new CA model reflecting more realities of bus route system. This model is defined on a one-dimensional lattice with periodic boundary conditions. Each site represents a bus stop or a segment of road. We suppose there are N_s bus stops and every neighbouring bus stops are spaced uniformly by L road sites (see Fig. 1). Thus the total length of system is $L_m = (L+1)N_s$. Let i denote the number of bus, j the number of bus stop, M the bus capacity. From $t \to t+1$, the parallel update rules consist of 2 steps:

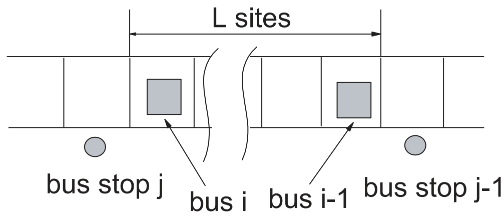

Fig. 1. The sketch of the bus route system.

1. Passengers arrival:
   $N_{ps}(j,t+1) = N_{ps}(j,t)+1$ for each bus stop site with probability λ. Here $N_{ps}(j,t)$ represents the number of passengers waiting at the bus stop j at time t, and λ is the passenger arrival rate.
2. Bus motion:
   (i) If the bus i is not at the bus stop site, then $v_i(t+1) = \min(1, gap_i(t))$ and $x_i(t+1) = x_i(t) + v_i(t+1)$. Here $v_i(t)$ and $x_i(t)$ are the velocity and position of bus i at time t, $gap_i(t)$ is the gap to the preceding bus $i-1$.
   We denote the number of passengers on bus i at time t as $N_{pb}(i,t)$. If site $x_i(t+1)$ is still not a bus stop site,

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then obviously $N_{pb}(i, t+1) = N_{pb}(i, t)$. However, if $x_i(t+1)$ is a bus stop site, then this means that the bus i pulls in the bus stop j at time $t+1$ under the assumption that position of bus stop j is $x_i(t+1)$. For the case, we suppose the number of passengers getting off the bus,

$$O = \mu \cdot N_{pb}(i, t);$$

accordingly the number of passengers getting on the bus:

$$I = \min(N_{ps}(j, t+1), M - (N_{pb}(i, t) - O)).$$

Thus, the number of passengers on the bus:

$$N_{pb}(i, t+1) = N_{pb}(i, t) - O + I,$$

and the number of passengers waiting at the bus stop j:

$$N_{ps}(j, t+1) = N_{ps}(j, t+1) - I.$$

The total time that the bus i must stay at the bus stop[1]:

$$T_{in}(i) = \mathrm{int}[\max(\gamma I, \delta O)] + 1. \qquad (1)$$

Here μ, γ, δ are proportional coefficients. We take $\gamma > \delta$, reflecting the fact that it will take more time getting on the bus than getting off it. Note that in (1), we suppose that each bus must stop at every bus stop even if passengers neither get off nor get on it.

(ii) If the bus i is at one bus stop site, then we check $T_{in}(i)$.

- If $T_{in}(i) > 0$, then the bus keeps up staying at the bus stop but $T_{in}(i) = T_{in}(i) - 1$. The number of the passengers on the bus does not change $N_{pb}(i, t+1) = N_{pb}(i, t)$, the position of the bus also does not change $x_i(t+1) = x_i(t)$.
- If $T_{in}(i) = 0$ and $gap_i(t) = 0$, then the bus cannot move. So $T_{in}(i) = 0$, $x_i(t+1) = x_i(t)$, $N_{pb}(i, t+1) = N_{pb}(i, t)$.
- If $T_{in}(i) = 0$ and $gap_i(t) > 0$, then the bus can move. So $x_i(t+1) = x_i(t) + 1$, $N_{pb}(i, t+1) = N_{pb}(i, t)$.

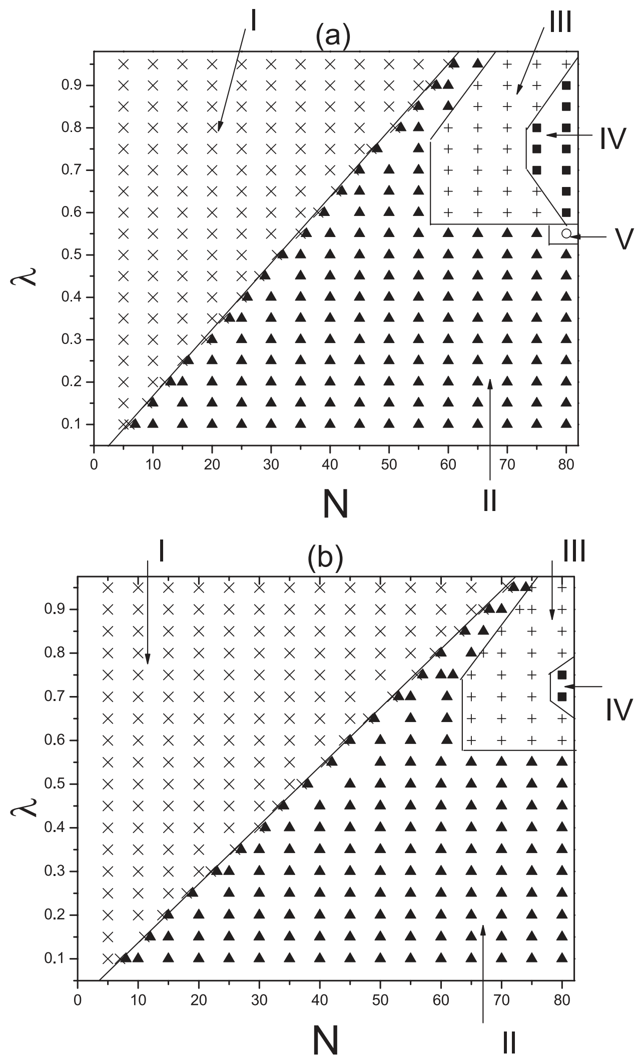

Fig. 2. The phase diagram of the bus system for (a) $M = 100$, (b) $M = 80$. Phase I corresponds to the insufficient transportation capability; in Phase II, the buses are bunching; in Phase IV, the phase separation occurs; Phase III is a coexistence of bunching and phase separation; in Phase V, the system is bistable. For $N \leq 80$, phase V does not exist in the case of $M = 80$.

3 Simulation and result

We carry out computer simulation at different values of λ under the periodic boundary conditions. In the simulations, the parameters are set: $M = 100$, $N_s = 40$, $L = 20$, $\mu = 0.2$, $\gamma = 0.5$, $\delta = 0.3$. From a realistic point of view, the maximum number of the buses is set to be twice of the number of the bus stops. Initially the buses are homogeneously distributed with no passengers and there is no passenger at the bus stops.

In Figure 2, the phase diagram in the (N, λ) space is shown, where N is the number of the buses. The phase diagram is classified into five regions.

Region I corresponds to the insufficient transportation capability, *i.e.*, the passengers cannot be picked up in time and the number of the passengers waiting at the bus stops will increase with time. This can be seen from Figure 3a. For the case, all the buses are full, so the bus bunching cannot develop. In Figure 4a, the space-time plot of the buses is shown. It can be seen that the buses are separated from each other without bunching.

In Figure 3b, $\lambda = 0.2$ and $N = 50$, the system is in region II, one can see that the passengers waiting at

[1] In reality, the number of the passengers on the bus is always changing in the time interval $(t+1, t+1+T_{in}(i))$. However, for simplicity, we assume that the bus stays at the bus stop for $T_{in}(i)$ time steps, but the passengers can get on and off the bus in one time step (*i.e.*, the number of the passengers on the bus does not change). Moreover, also for simplicity, we assume that the passengers arriving in the time interval $(t+1, t+1+T_{in}(i))$ do not get on the bus i (even if there is still space left on the bus) although the bus still stays at the bus stop.

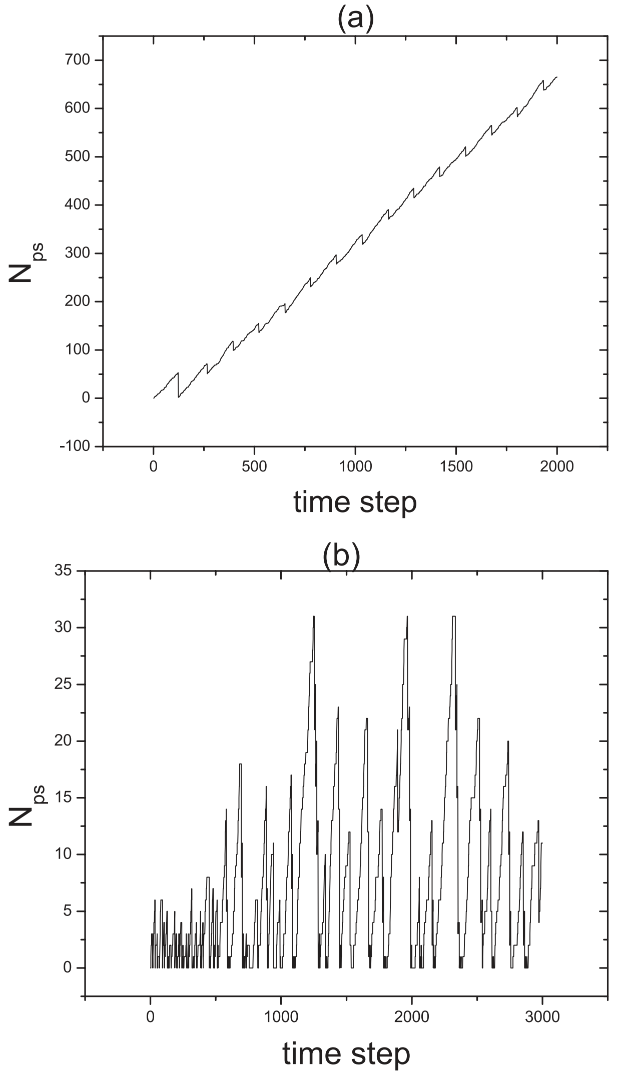

Fig. 3. The evolution of N_{ps}. Here N_{ps} denotes the number of passengers waiting at an arbitrarily chosen bus stop. (a) $\lambda = 0.5$, $N = 10$; (b) $\lambda = 0.2$, $N = 50$.

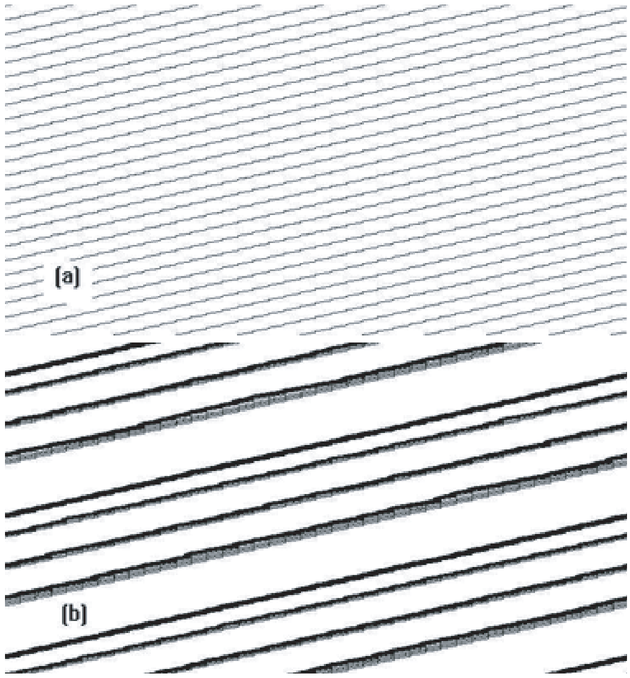

Fig. 4. The space time plot of the bus system corresponding to the parameters in Figure 3. The buses are moving from left to right, and the vertical direction (up) is (increasing) time. The vertical direction corresponds to 1000 time steps.

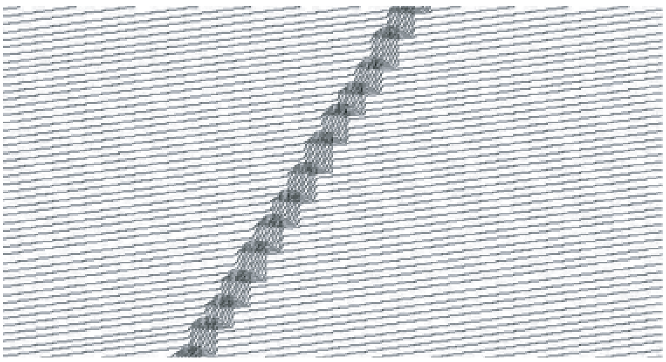

Fig. 5. The space time plot of the bus system for $\lambda = 0.75$ and $N = 80$. The vertical direction corresponds to 5000 time steps and only one trajectory of every four buses is shown for clarity reason.

the bus stops will not increase with the time, but fluctuates in certain range instead. This implies that the buses are enough to pick up the passengers in time. For the case, some buses carry more passengers and others carry less passengers. The buses carrying more passengers have to stay longer in the bus stops because both the number of passengers getting off and accordingly the number of passengers getting on are larger. Thus, the buses that carry less passengers will be slowed down by those carrying more, and as a consequence, the bus bunching occurs, see Figure 4b.

We study the average velocity v_{ave} of the buses. The simulations show that in both region I and region II, v_{ave} is a constant 0.656. This is explained as follows. In region I, all the buses are full; in region II, the leading bus in the bunching cluster is full. v_{ave} is determined by the velocities of these full buses, which depend on the time that they need to stop at the bus stops. The time can be calculated to be $\mathrm{int}(\gamma\mu M)+1$ from section II for the full buses, which is a constant. Therefore, v_{ave} remains a constant in both regions I and II.

Next we focus on region IV. In Figure 5, we show the space time plot of the system for $N = 80$, $\lambda = 0.75$. One can see that the state is quite different from the bunching. For the case, the system is in phase separation: some buses pile together with almost no gap between each other (phase 1) and other buses are well separated (phase 2). Due to the existence of phase 1, the average velocity of the system decreases compared with that in regions I and II.

In the phase diagram, region II and region IV are separated by region III. In this region, neither the

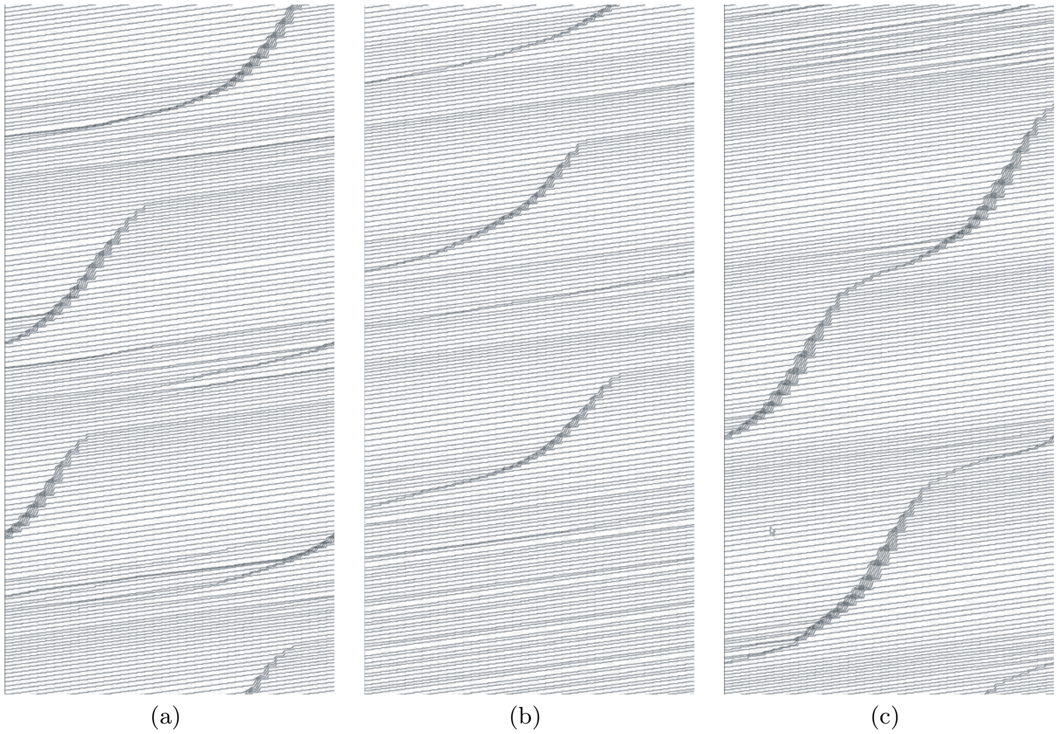

Fig. 6. The space time plot of the bus system, where $\lambda = 0.8$. $N = 65, 60, 70$ from left to right. The vertical direction corresponds to 20000 time steps and only one trajectory of every five buses is shown for clarity reason.

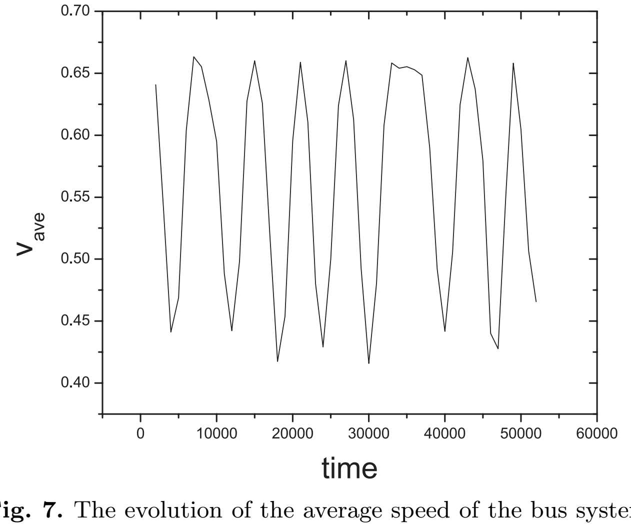

Fig. 7. The evolution of the average speed of the bus system corresponding to the parameters in Figure 6a.

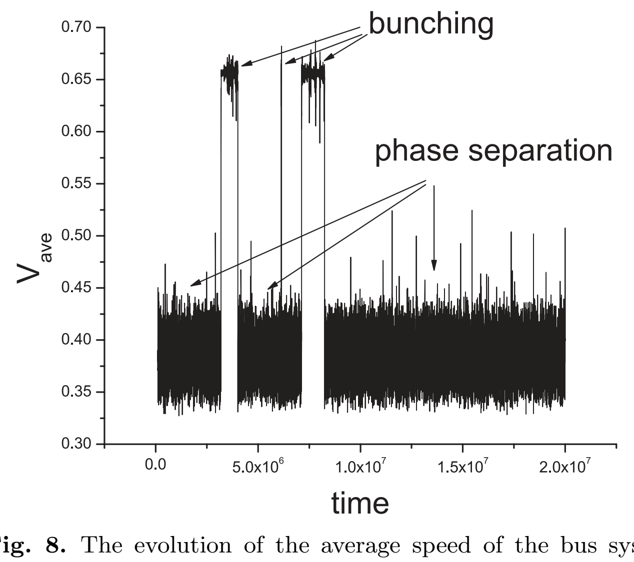

Fig. 8. The evolution of the average speed of the bus system for $\lambda = 0.6$ and $N = 70$. The transformation between the two states is erratic: even if starting from the same initial conditions, the evolution process may be different if the random seeds used are different.

bunching state nor the phase separation state can exists stably. In Figure 6a, we show the space time plot for $N = 65$, $\lambda = 0.8$. One can see that the system transforms between the bunching state and the phase separation state almost periodically. This can also be seen from Figure 7, when the phase separation is appearing, v_{ave} decreases and when the phase separation is disappearing, v_{ave} increases.

If N increases (decreases) with λ unchanged, one can see that the time ratio that the system stays in the phase separation state increases(decreases) (*cf.* Figs. 6b and c).

When λ is small, the situation is somewhat different. For the case, although neither the bunching state nor the phase separation state can exist stably in large time scale, the two states can exist stably in not so large time scale. For example, see Figure 8, where $\lambda = 0.6$ and $N = 70$. One can see that the phase separation state can exist for approximately 3,200,000 time steps, then it transforms into

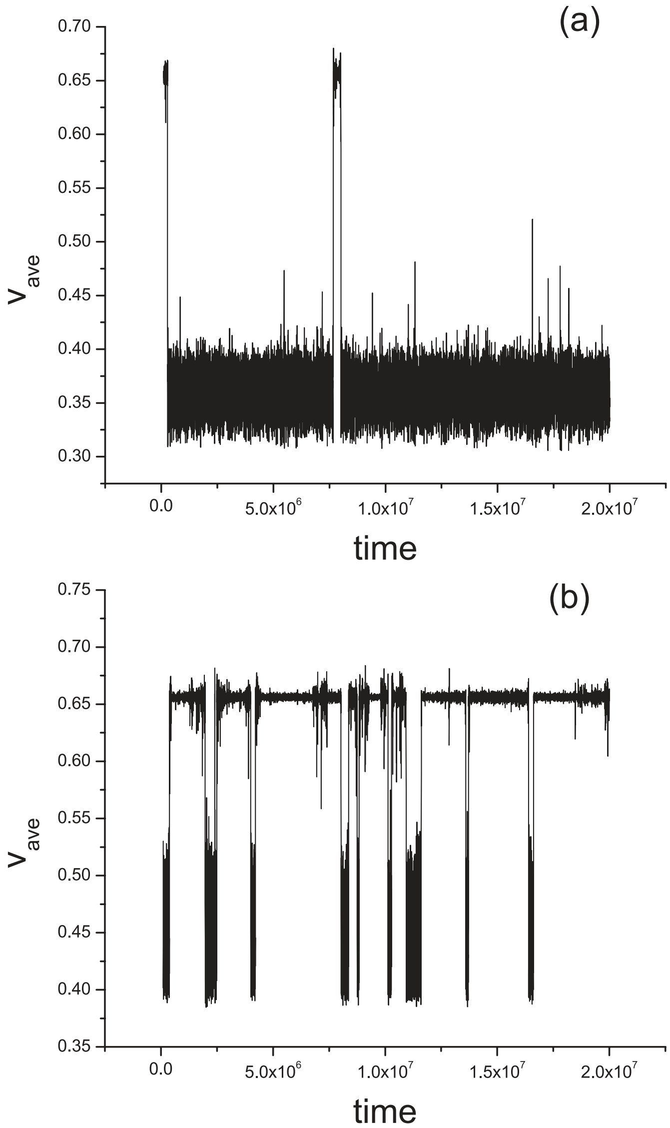

Fig. 9. The evolution of the average speed of the bus system for (a) $\lambda = 0.6$ and $N = 75$, (b) $\lambda = 0.6$ and $N = 65$.

the bunching state. The bunching state exists for approximately 800,000 time steps, then the system breaks down into phase separation state again. The process repeates and the system transforms between the two states.

The simulations show that transformation between the two states is erratic. The time that the system stays in either state cannot be determined. Even if starting from the same initial conditions, the evolution process may be different if the random seeds used are different. If one increases(decreases) N with λ unchanged, the system will generally stay longer(shorter) in the phase separation state (*cf.* Fig. 9).

Finally, in region 5, the system is bistable, either the bunching state or the phase separation state may be stable. We note that the system is also erratic: even if starting from the same initial conditions, the system may be in either bunching state or phase separation state if the random seeds used are different.

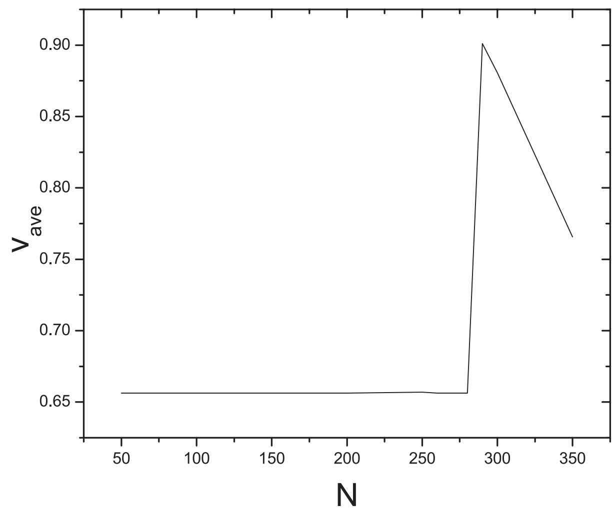

Fig. 10. The average speed of the bus system with no limit of N, where $\lambda = 0.2$.

In our simulations, there is no homogeneous state. This is because the maximum number of the buses is limited. In Figure 10, the plot of v_{ave} against N is shown without the limit of N. One can see that at $N \approx 290$, a first order transition occurs. v_{ave} suddenly increases and the system transits into the homogeneous state. However, a realistic bus service system will not have so many buses. So the study on homogeneous state has no realistic meaning.

In the original model of O'Loan *et al.* [5], the homogeneous phase arises when there is significantly less than one bus per stop. We argue this is due to that there is no space among the bus stops (*i.e.*, $L = 0$), which is not realistic.

We investigate the dependence of the phase diagram of the system on the bus capacity M. The simulations show that the phase structure is robust with respect to variation of M. With the decrease of M, phase I expands, phases II, III, IV, and V shrink. When M decreases to 80, phase V disappears (Fig. 2b) [2]. The simulations also show that in phases I and II, v_{ave} increases as M decreases. For $M = 80$, $v_{ave} = 0.700$ in phases I and II.

4 Conclusion and discussion

In conclusion, we have investigated the behavior of bus route system with a more realistic CA model, in which the bus capacity is considered. The simulations show that the phase diagram is classified into five regions: insufficient transportation capability; bus bunching; phase separation; coexistence of bunching and phase separation; bistable. Except bus bunching, the other four states are new findings due to the introduction of bus capacity.

From the simulations, one can conclude that the bus system is preferred to run in the region slightly right of the boundary between regions I and II, because under such

[2] It is likely that the phase V exists if the maximum number of the buses increases. But in this paper, the maximum number of the buses is restricted to 80.

situation the passengers can be picked up in time and simultaneously only a few buses are bunched.

We should note that the efficiency of the bus route system cannot be enhanced simply through increasing the number of buses. If λ is small, the increase of N will not increase the average speed of the system but resulting in much more bunched buses. If λ is large, the increase of N will lead the system into regions III or IV, which will decrease the average speed of the system.

Nevertheless, we notice that with the decrease of M, v_{ave} increases in phase II. This means that by decreasing M, the average speed of the system can be enhanced although more buses are needed. Thus, one needs to choose a proper value of M in order to reach an optimal configuration of the bus system.

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