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Realization of non-linear coherent states by photonic lattices

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In this paper, first, by introducing Holstein-Primakoff representation of α-deformed algebra, we achieve the associated non-linear coherent states, including $su(2)$ and $su(1,1)$ coherent states. Second, by using waveguide lattices with specific coupling coefficients between neighbouring channels, we generate these non-linear coherent states. In the case of positive values of α, we indicate that the Hilbert size space is finite; therefore, we construct this coherent state with finite channels of waveguide lattices. Finally, we study the field distribution behaviours of these coherent states, by using Mandel Q parameter. [http://dx.doi.org/10.1063/1.4923325]

I. INTRODUCTION

Undoubtedly, analogy between the physical systems has been one of the most important subject in physics which has twofold aspects: first, in theoretical aspect, it causes different theories to close each other; second, it helps to construct new experimental methods. It can be illustrated by analogy of media with curved space-time in general relativity[1–3] and classical analogy of quantum systems.[4–6] In fact, analogy between wave optics and quantum mechanics has been a highlighted subject, since the early developments of quantum mechanics.[7] This analogy is mathematically rooted in the similarity of dynamic process of par-axial optics domain and the dynamic quantum process as well as in an isomorphism between the time independent Schödinger equation and the Helmholtz equation.[8] Recently, analogy of quantum systems with classical optics is interesting, because of its enormous consequence both in fundamental quantum mechanics and in technological applications.

Optical analogy of quantum mechanics has been followed, at least, in three classes: the first one is related to some general issues in quantum mechanics and quantum information, such as Aharonov-Bohm and Bery phase, coherent control of quantum tunnelling, *etc*; the second one is related to mimics of some quantum optics simulations, such as Jaynes-Cummings model, Rabi Model, *etc*; the third one is related to solid-states physics, such as Bloch oscillations, Zener tunnelling, *etc*.[9] This study is focused on the first and second classes of them.

In fact, the dynamical process of light propagation in waveguide lattices are saturated with the Schödinger-like equation.[10] Due to such character, the generation of Glauber-Fock states, coherent states, $su(1,1)$ and $su(2)$ coherent states, by using the photonic lattices, have been studied.[11–17] In these cases, by considering the edge channel in the exited state, a displacement type $su(1,1)$ coherent states are realized in waveguide lattices. Also, the field distribution of any channel corresponds to

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number state and deformed number state, with regard to coupling coefficients among neighbouring channels of waveguide lattices.

As is known, coherent states for the harmonic oscillator were mathematically constructed, following Schrödinger, with the aim of finding the quantum counterparts of the classical points in the phase space.[18–20] Furthermore, coherent states have had central roles in quantum theory, such as quantum optics,[21] most quantization theories[22,23] and transition between quantum and classical mechanics,[24] *etc*. In addition, mathematically, trying to construct the non-linear coherent states leads to different definitions of non-linear coherent states, such as Gazeau-Klauder coherent states which are defined as the eigenstates of the deformed annihilator operators[24] and displacement type coherent states which are obtained through the operation of deformed displacement operators on their relevant reference states.[25]

Mathematically, α-deformed coherent states, as non-linear coherent states, are generalization of $su(2)$ and $su(1,1)$ coherent states.[26] In physical realization, α-deformed algebra describes harmonic oscillators confined at the center of a potential well, depending to sing of α with infinite or finite well.[26] In addition, this algebra can be realized the position-dependent mass harmonic oscillator as well as harmonic oscillators on the constant curvature surfaces, i.e., spherical and hyperbolic surfaces.[27,28]

In this paper, by defining the generalized Holstein-Primakoff representation of α-deformed algebra, we obtain the displacement type of α-deformed coherent states, as a special case of non-linear coherent states, for the first time. Then, by considering the coupling coefficients between neighbouring channels as $C_{k,k+1} \propto \sqrt{|\alpha| m\,(2j - sgn(\alpha)m)\,/2}$, where $j, m \in \mathbb{N}$, we generate the α-defoemed coherent states in the photonic lattices. Moreover, we demonstrate behaviours of field distributions in any channel waveguide lattices and consider the role of the coupling coefficient between neighbouring channels. Finally, we study the role of α, as a coupling coefficient between neighbouring channels, in statistical property of the field distributions in waveguide lattices and consider their field distribution behaviours, by examining the Mandel Q parameter.[29]

The paper is organized as follows: in section II, we introduce the generalized Holstein-Primakoff representation of α-deformed coherent states; in section III, we study generations of these coherent states, by using photonic lattices, and study the field distributions in these structures; in addition, we investigate the non-classical behaviours of these coherent states, by using the Mandel Q parameter; finally, section IV is devoted to some conclusions and remarks.

II. α-DEFORMED COHERENT STATES: GENERALIZED HOLSTEIN-PRIMAKOFF REPRESENTATION

Let us define the following generalized Holstein-Primakoff operators:

$$\begin{aligned}
\hat{A} &= \sqrt{\frac{|\alpha|}{2}}\left(2N - sgn(\alpha)\hat{a}^\dagger\hat{a}\right)^{1/2}\hat{a},\\
\hat{A}^\dagger &= \sqrt{\frac{|\alpha|}{2}}\hat{a}^\dagger\left(2N - sgn(\alpha)\hat{a}^\dagger\hat{a}\right)^{1/2},\\
\hat{M} &= \frac{\alpha}{2}\left(\hat{a}^\dagger\hat{a} - sgn(\alpha)N\right).
\end{aligned} \tag{1}$$

which $\alpha \in \mathbb{R}$ and $N \in \mathbb{N}$ are the arbitrary fixed numbers; $\hat{a}$ and $\hat{a}^\dagger$ are the harmonic oscillator annihilation and creation operators, respectively. These operators realize the α-deformed operators with following communication relations:[26]

$$[\hat{A}^\dagger, \hat{A}] = 2\hat{M},\ [\hat{M}, \hat{A}] = -\frac{1}{2}\alpha\hat{A},\ [\hat{M}, \hat{A}^\dagger] = \frac{1}{2}\alpha\hat{A}^\dagger. \tag{2}$$

Also, we can define generalized displacement operator, $\hat{D}_\alpha(\beta)$, as

$$\hat{D}_\alpha(\beta) = \exp\left[\beta\hat{A}^\dagger - \beta^*\hat{A}\right], \tag{3}$$

where $\beta = -|\beta|e^{-i\varphi}$. The normal ordered form of displacement operator (3) is given by,[26]

$$\hat{D}_\alpha(\beta) = e^{\zeta \hat{A}^\dagger} e^{\vartheta \hat{M}} e^{-\zeta^* \hat{A}}, \tag{4}$$

which the coefficients of the normal ordered form are given by,

$$\zeta = -e^{(-i\varphi)}\sqrt{\frac{2}{\alpha}}\tan(\sqrt{\frac{\alpha}{2}}\beta), \quad \vartheta = \frac{-4\alpha}{|\alpha|^2}\ln|\cos(\sqrt{\frac{\alpha}{2}}\beta)|. \tag{5}$$

In this case, if $\alpha = 2$ and $\alpha = -2$ are chosen, the relation (4) describes Gaussian decomposition of displacement operator of $su(2)$ and $su(1,1)$ algebras, respectively.

By using the relation (4), we can explicitly achieve the displacement type of α^+-deformed coherent states:

$$\begin{aligned}|\beta,\alpha^+\rangle &= \hat{D}_{\alpha^+}(\beta)|0\rangle \qquad \alpha > 0\\ &= \left[1+\tan^2(\sqrt{\frac{\alpha}{2}}\frac{|\beta|}{2})\right]^{-N}\sum_{n=0}^{2N}\binom{2N}{n}^{\frac{1}{2}} e^{-in\varphi}\tan^n(\sqrt{\frac{\alpha}{2}}\frac{|\beta|}{2})\,|n\rangle.\end{aligned} \tag{6}$$

Also, when α is negative, the displacement type of α^--deformed coherent states is given by

$$\begin{aligned}|\beta,\alpha^-\rangle &= \hat{D}_{\alpha^-}(\beta)|0\rangle, \qquad \alpha < 0\\ &= \left[1-\tanh^2(\sqrt{\frac{|\alpha|}{2}}\frac{|\beta|}{2})\right]^{2N}\sum_{n=0}^{\infty}\left(\frac{\Gamma(n+2N)}{n!\Gamma(2N)}\right)^{\frac{1}{2}} e^{-in\varphi}\tanh^n(\sqrt{\frac{\alpha}{2}}\frac{|\beta|}{2})|n\rangle.\end{aligned} \tag{7}$$

III. NON-LINEAR COHERENT STATES IN PHOTONIC LATTICES

The normalized modal field evolution can be described by the following set of coupled differential equations:

$$i\frac{dE_{j,m}}{dZ} + f(j,m)E_{m-1} + f(j,m+1)E_{m+1} = 0,\ E_{-1} = 0, \tag{8}$$

where $z = Z/\kappa$ is the actual propagation distance and κ is the coupling coefficient.[10] In this case, by choosing function $f(j,m)$ as the following relation

$$f(j,m) = \sqrt{\frac{|\alpha|}{2}m\,(2j - sgn(\alpha)m)}, \tag{9}$$

we can generate α-deformed coherent states in a photonic lattices so that the function $f(j,m)$ indicates the coupling coefficients among neighbouring channels of waveguide lattices as shown in the Figure 1. Also, the first channel of the waveguide lattices is chosen as exited channel, $|0\rangle$.

In addition, we study the Mandel Q parameter, as a parameter to monitor the nature of the density distribution.[29] The mandel Q parameter, which is defined by

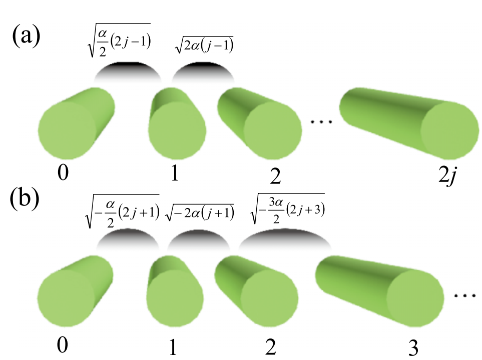

FIG. 1. Schematic views of a lattice of $2j+1$ and infinity waveguides, which are realization of α-deformed coherent states by positive and negative values of α, respectively, in plots (a) and (b).

$$Q_{|\beta,\alpha^{\pm}\rangle} = \frac{\langle \hat{n}^2\rangle_{|\beta,\alpha^{\pm}\rangle} - \langle \hat{n}\rangle^2_{|\beta,\alpha^{\pm}\rangle}}{\langle \hat{n}\rangle_{|\beta,\alpha^{\pm}\rangle}} - 1, \tag{10}$$

is negative for a sub-poissonian distribution (photon antibunching) and positive for a super-poissonian distribution (photon bunching) and $Q = 0$ stands for poissonian statistics.

A. Analogy of α^+-deformed coherent states

Consider a photonic lattices included $2j + 1$ channels, the coupling coefficients among neighbouring channels of waveguide lattices is given by $f(j,m) = \sqrt{\frac{\alpha}{2} m\,(2j - m)}$, $\alpha > 0$, and the first channel is chosen as the exited channel, $|0\rangle$. Thus, by using equation (8), we can generate an analogy with α^+-deformed coherent states,

$$|Z,\alpha^+\rangle = \left[1 + \tan^2\left(\sqrt{\frac{\alpha}{2}}\frac{Z}{2}\right)\right]^{-j} \sum_{m=0}^{2j} \binom{2j}{m}^{\frac{1}{2}} \tan^m\left(\sqrt{\frac{\alpha}{2}}\frac{Z}{2}\right)\, |m\rangle. \tag{11}$$

In this case, the amplitude field distribution E_m in the mth channel, at the distance Z, is given by,

$$E_m = \left[1 + \tan^2\left(\sqrt{\frac{\alpha}{2}}\frac{Z}{2}\right)\right]^{-j} \binom{2j}{m}^{\frac{1}{2}} \tan^m\left(\sqrt{\frac{\alpha}{2}}\frac{Z}{2}\right). \tag{12}$$

In Figures 2(a), 2(b) and 2(c), we depict the amplitude field distribution among the lattice when the first element is initially exited, i.e., $|0\rangle$, for different values of coupling constant α. These figures demonstrate that increasing of α causes field distributions to be localized in the center of the waveguide lattices.

For the state $|Z,\alpha^+\rangle$, by using equations (10) and (11), we find the Mandel Q parameter as $Q_{|Z,\alpha^+\rangle} = -\sin^2\left(\sqrt{\frac{\alpha}{2}}\frac{Z}{2}\right)$, saturated by the inequality $-1 \le Q_{|Z,\alpha^+\rangle} \le 0$, indicating sub-Poissonian statistics, apart from the trivial case, $Z = 0$, or $\alpha = 0$. In addition, at the distance $Z = \pi(2n - 1)\sqrt{2/\alpha}$, we obtain $Q_{|Z,\alpha^+\rangle} = -1$ as it is shown in Figure 3.

B. Analogy of α^--deformed coherent states

We can choose coupling function (9) as $f(j,m) = \sqrt{\frac{-\alpha}{2} m\,(2j + m)}$, $\alpha < 0$, which is realization of α^--deformed coherent states in photonic lattices with infinite channels. Also, we choose the first channel in exited state, $|0\rangle$. Thus, by using equation (8), we can generate an analogy of α^--deformed

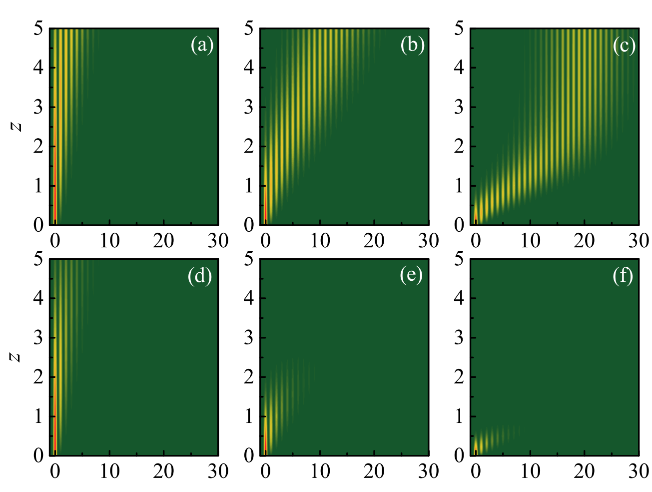

FIG. 2. Propagation dynamics of the classical nonlinear coherent state for 30 channels and $\alpha = 0.02$ in (a), $\alpha = 0.2$ in (b) and $\alpha = 2$ in (c). Also, propagation dynamics of them for $\alpha = -0.02$ in (d), $\alpha = -0.2$ in (e) and $\alpha = -2$ in (f).

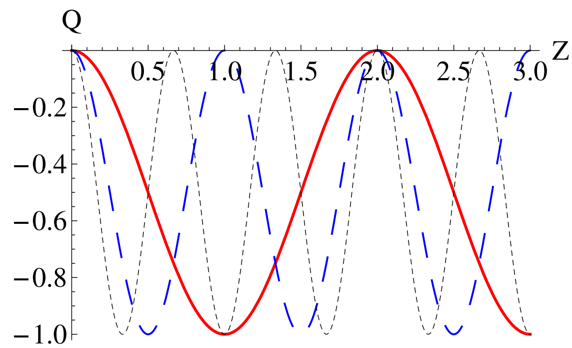

FIG. 3. Mandel Q parameter of α^+-deformed coherent state versus Z at $\alpha = \frac{\pi^2}{2}$ (solid line), $\alpha = 2\pi^2$ (dashed line) and $\alpha^2 = \frac{9\pi^2}{4}$ (dotted line).

coherent states,

$$|Z,\alpha^-\rangle = \left[1 - \tanh^2(\sqrt{\frac{-\alpha}{2}}\frac{Z}{2})\right]^{2j} \sum_{m=0}^{\infty} \left(\frac{\Gamma(m+2j)}{m!\Gamma(2j)}\right)^{\frac{1}{2}} \tanh^m\left(\sqrt{\frac{-\alpha}{2}}\frac{Z}{2}\right)|m\rangle. \tag{13}$$

In this case, the amplitude field distribution E_m in the mth channel, at the distance Z, is given by,

$$E_m = \left[1 - \tanh^2(\sqrt{\frac{-\alpha}{2}}\frac{Z}{2})\right]^{2j} \left(\frac{\Gamma(m+2j)}{m!\Gamma(2j)}\right)^{\frac{1}{2}} \tanh^m\left(\sqrt{\frac{-\alpha}{2}}\frac{Z}{2}\right). \tag{14}$$

Also, figures 2(d), 2(e) and 2(f), show the amplitude of the field distribution E_m, among the lattices, when the first element is initially exited, $|0\rangle$, for different values of α. These figures indicate that the field distributions are localized near the first channel when the coupling constant is decreased. In addition, by comparing plots (a) and (d), we indicate that the behaviour of the field distributions are the same, as a result of approaching α zero.

Also, after some simple calculation, we find the Mandel Q parameter for the state $|\beta,\alpha^-\rangle$ as $Q_{|Z,\alpha^-\rangle} = \sinh^2\left(\sqrt{\frac{-\alpha}{2}}\frac{Z}{2}\right)$. It is obvious we have $Q_{|Z,\alpha^-\rangle} \geqslant 0$, which indicates super-poissonian statistics unless we have either the trivial vacuum field case, $Z = 0$, or $\alpha = 0$.

In Figure 4, we show the effect of α on the variation of Mandel Q parameter for different values of Z. In this case, quantum behaviour of the system is increased by increasing of the α absolute value.

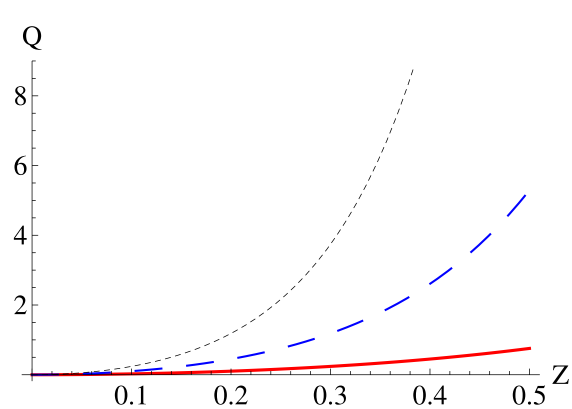

FIG. 4. Mandel Q parameter of α^--deformed coherent state versus Z at $\alpha = \frac{\pi^2}{2}$ (solid line), $\alpha = 2\pi^2$ (dashed line) and $\alpha^2 = \frac{9\pi^2}{4}$ (dotted line).

IV. CONCLUSIONS

In conclusion, we studied a generalization of Holstein-Primakoff representation of α-deformed harmonic oscillator as a non-linear algebra including $su(2)$ and $su(1,1)$ algebra, and associated coherent states. Besides, by using a photonic lattices with special choosing of coupling coefficients between neighbouring channels, we obtained α-deformed coherent states and studied their distributions of the fields. Finally, we considered the Mandel Q parameter as a criteria of non-classical property of these distributions. Specially, we indicated that positive α led to the sub-poissonian statistics while the negative α led to the super-poissonian statistics. Our findings will be very important to realization of non-linear coherent states, with non-classical behaviour, in classical optics, with photonic lattices.

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