Reasoning about collectively accepted group beliefs

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Abstract.
A proof-theoretical treatment of collectively accepted group beliefs is presented through a multi-agent sequent system for an axiomatization of the logic of acceptance. The system is based on a labelled sequent calculus for propositional multi-agent epistemic logic with labels that correspond to possible worlds and a notation for internalized accessibility relations between worlds. The system is contraction- and cut-free. Extensions of the basic system are considered, in particular with rules that allow the possibility of operative members or legislators. Completeness with respect to the underlying Kripke semantics follows from a general direct and uniform argument for labelled sequent calculi extended with mathematical rules for frame properties. As an example of the use of the calculus we present an analysis of the discursive dilemma.

1. Introduction

The study of collective attitudes has been in the focus of the philosophical literature concerned with *collective intentionality* (Gilbert, 1989; Searle, 1995; Tuomela, 2007). One outcome of this area of research has been an understanding of the nature of collectively accepted group beliefs and their importance in creating the social environment. Attempts have been made recently to formalize reasoning about such collective attitudes. One motivation comes from theoretical social sciences, especially theories of social choice that study aggregation of individual attitudes, especially preferences and judgements, into collective attitudes. Formal systems of logic have been used to gain a more precise understanding of the properties of these aggregation processes (see e.g. Pauly, 2007; Ågotnes et al., 2007). Another motivation comes from areas of application such as distributed artificial intelligence that aims at constructing multi-agent systems in which the agents can reason about the attitudes of other agents (Shoham and Leyton-Brown, 2009). Various multi-agent logics have been presented to this task. Most of them are multi-modal logics that extend traditional modal logics, in particular epistemic logic.

The focus has been until recently on individual attitudes and what are known as *summative* collective attitudes, which can be defined in terms of individual attitudes, in particular, shared beliefs, mutual beliefs, distributed knowledge, and common knowledge (Fagin et al., 1995). In recent work, also *non-summative* collective attitudes, such as *group beliefs*, have received attention (Gaudou et al., 2006a; Fischer and Nickles, 2006; Gaudou et al., 2008; Gaudou et al., forthcoming; Lorini et al., 2009). Group beliefs are taken to be collectively intentional attitudes that are based on what the group members accept as the group's belief (Gilbert, 1987; Tuomela, 1992). Thus, group beliefs do not reduce to individual beliefs but are properly attributed only to the collectivity. The fact that all the group members believe that A is neither sufficient nor necessary for a group belief that A. It is required for group belief that the group members take A to be true when they are acting in the group context, that is, that the individuals accept A when they are acting as group members. The distinction between belief and acceptance (Cohen, 1992) allows reasoning about individual and collective attitudes in their proper context without attributing contradictory beliefs to the agents. The concept of acceptance allows inferences about public commitments of agents, because from their communication only their acceptances can be inferred, not necessarily their beliefs (see e.g. Gaudou et al., 2006b for discussion).

In this paper, we present a sequent calculus system that allows to make proofs about collective attitudes. We take a formalization of this kind to be crucial for the implementation of reasoning about collective attitudes. We employ the general method for constructing modal sequent calculi presented by Negri (2005) (for an introduction to sequent calculus and more generally to structural proof theory see Negri and von Plato, 2001). The approach followed here is similar to the sequent system, presented by Hakli and Negri (2007), of multi-agent epistemic logic with knowledge operators $\mathcal{K}_a$ for individual agents $a \in G$ and an operator for distributed knowledge among agents in a group.

Here our focus is on group belief that we take to amount to a collective acceptance of a proposition by the group members to represent a view of the group (Gilbert, 1987; Tuomela, 1992; Hakli, 2006). Of the recent attempts to formalize such non-summative group beliefs (Gaudou et al., 2006a; Fischer and Nickles, 2006; Gaudou et al., 2008; Gaudou et al., forthcoming; Lorini et al., 2009), we have here selected the logic of acceptance (Lorini et al., 2009), which is formally sophisticated and quite faithful to philosophical accounts of group beliefs. In particular, it allows the possibility that agents accept different propositions in different social contexts. To borrow an example of coextensive groups from Margaret Gilbert (1987), the logic can be used to express that agents 1,2, and 3 qua members of the Food Committee accept that college members have to consume too much starch (proposition P), but they do not accept it qua members of the Library Committee. In the language of acceptance logic (which will be introduced in more detail later) this can be expressed as: $\sim\mathcal{A}_{\{1,2,3\}:FoodCommittee}\bot$ & $\mathcal{A}_{\{1,2,3\}:FoodCommittee}P$ & $\sim\mathcal{A}_{\{1,2,3\}:LibraryCommittee}P$. The proof-theoretical methods presented for the acceptance logic could be adapted for the other logics with minor modifications.

The paper is organized as follows: In Section 2, we recall the necessary proof-theoretical preliminaries. In Section 3, we present a sequent calculus system for the logic of acceptance. In Section 4, we study some extensions of the basic system. In Section 5, we give a direct proof of completeness for the system. In Section 6, we use the calculus to analyse a problem in judgement aggregation. We conclude and discuss related literature in Section 7.

2. Background on labelled sequent systems

To keep the presentation self-contained, we briefly recall in this section the background of our method (cf. Negri and von Plato, 1998; Negri and von Plato, 2001; Negri, 2005; Negri and von Plato, 2011, chapters 11 and 12) for the development of cut-free labelled systems for multi-modal logics.

For extensions of classical predicate logic, the starting point is the contraction- and cut-free sequent calculus **G3c** (for the rules, cf. Negri and von Plato, 2001; Troelstra and Schwichtenberg, 2000). We recall that all the rules of **G3c** are invertible and all the structural rules are admissible. Weakening and contraction are in addition *height-preserving-* (*hp-*) admissible, that is, whenever their premisses are derivable, so also is their conclusion, with at most the same derivation height (the *height* of a derivation is its height as a tree, that is, the length of its longest branch). Moreover, the calculus enjoys hp-admissibility of substitution of individual variables. Invertibility of the rules of **G3c** is also height-preserving (*hp-invertible*). For detailed proofs, consult Negri and von Plato (2001, chapters 3 and 4).

These remarkable structural properties of **G3c** are maintained in extensions of the logical calculus with suitably formulated rules that represent axioms for specific theories. Universal axioms are first transformed, through the rules of **G3c**, into a normal form that consists of conjunctions of formulas of the form $P_1\&\ldots\&P_m \supset Q_1 \vee \ldots \vee Q_n$, where all P_i, Q_j are atomic; then implication reduces to the succedent if $m = 0$, and the latter is $\bot$ if $n = 0$. The universal closure of any such formula is called a *regular* formula. We abbreviate the multiset $P_1, \ldots, P_m$ as $\overline{P}$. Each conjunct is then converted into a schematic rule, called the *regular rule scheme*, of the form

$$\frac{Q_1, \overline{P}, \Gamma \rightarrow \Delta \quad \dots \quad Q_n, \overline{P}, \Gamma \rightarrow \Delta}{\overline{P}, \Gamma \rightarrow \Delta}\,{\scriptstyle Reg}$$

By this method, all universal theories can be formulated as contraction- and cut-free systems of sequent calculi.

Negri (2003) extends the method to cover also *geometric theories*, that is, theories axiomatized by geometric implications. We recall that a *geometric formula* is a formula that does not contain $\supset$, $\neg$, or $\forall$, and a *geometric implication* is a sentence of the form $\forall\overline{z}(A \supset B)$ where A and B are geometric formulas. Geometric implications can be reduced to a normal form that consists of conjunctions of formulas, called *geometric axioms*, of the form

$$\forall\overline{z}(P_1 \& \dots \& P_m \supset \exists\overline{x}_1 M_1 \vee \dots \vee \exists\overline{x}_n M_n)$$

in which each P_i *is an atomic formula, each* M_j is a conjunction of atomic formulas $Q_{j_1}, \dots, Q_{j_{k_p}}$, and none of the variables in the vectors $\overline{x}_j$ are free in P_i. Without loss of generality, no x_i is free in any P_j. Note that regular formulas are degenerate cases of geometric implications, with neither conjunctions nor existential quantifications to the right of the implication. The *geometric rule scheme* for geometric axioms takes the form

$$\frac{\overline{Q}_1(\overline{y}_1/\overline{x}_1), \overline{P}, \Gamma \rightarrow \Delta \quad \dots \quad \overline{Q}_n(\overline{y}_n/\overline{x}_n), \overline{P}, \Gamma \rightarrow \Delta}{\overline{P}, \Gamma \rightarrow \Delta}\,{\scriptstyle GRS}$$

where $\overline{Q}_j$ and $\overline{P}$ indicate the multisets of atomic formulas $Q_{j_1}, \dots Q_{j_{k_j}}$ and $P_1, \dots, P_m$, respectively, and the eigenvariables $\overline{y}_1, \dots, \overline{y}_n$ of the premisses are not free in the conclusion. We use the notation $A(\overline{y}/\overline{x})$ to indicate A after the substitution of the variables $\overline{y}$ for the variables $\overline{x}$. In what follows, it will be enough to consider the the case in which the vectors of variables $\overline{y}_i$ consist of a single variable.

In order to maintain admissibility of contraction in the extensions with regular and geometric rules, the formulas $P_1, \dots, P_m$ in the antecedent of the conclusion of the scheme have, as indicated, to be repeated in the antecedent of each of the premisses. In addition, whenever an instantiation of free parameters in atoms produces a duplication (two identical atoms) in the conclusion of a rule instance, say $P_1, \dots, P, P, \dots, P_m, \Gamma \rightarrow \Delta$, there is of course a corresponding duplication in each premiss. The *closure condition* imposes the requirement that the rule with the duplication P, P contracted into a single P, both in the premisses and in the conclusion, be added to the system of rules.

For each axiom system, there is only a bounded number of possible cases of contracted rules to be added, very often none at all, so the condition is unproblematic.

The main result for such extensions is the following (Negri, 2003, Theorems 4 and 5):

Theorem 1. *The structural rules of* Weakening*,* Contraction *and* Cut *are admissible in all extensions of* **G3c** *with the geometric rule-scheme and satisfying the closure condition.* Weakening *and* Contraction *are hp-admissible.*

The method of extension of sequent calculi can be applied not only to the proof theory of specific theories such as lattice theory, arithmetic, and geometry (Negri and von Plato, 2011), but also to the proof theory of non-classical logics. Negri (2005) adds rules expressing properties of binary relations to a basic labelled sequent calculus for the normal modal logic **K** in such a way that complete systems for all the modal logics characterized by geometric frame conditions are obtained. The basic labelled sequent calculus is obtained by prefixing with labels the formulas in the rules of the sequent calculus for the propositional part of **G3c**. As initial sequents we take any of the form $x : P, \Gamma \rightarrow \Delta, x : P$ for atomic P. In each rule, the active and principal formulas are prefixed by the same label. This corresponds to the classical explanation of truth in Kripke semantics, flat on all the propositional logical constants. For instance, the rules for conjunction are

$$\frac{x : A, x : B, \Gamma \rightarrow \Delta}{x : A\&B, \Gamma \rightarrow \Delta}\, L\& \qquad \frac{\Gamma \rightarrow \Delta, x : A \quad \Gamma \rightarrow \Delta, x : B}{\Gamma \rightarrow \Delta, x : A\&B}\, R\&$$

and those for implication are

$$\frac{\Gamma \rightarrow \Delta, x : A \quad x : B, \Gamma \rightarrow \Delta}{x : A \supset B, \Gamma \rightarrow \Delta}\, L\supset \qquad \frac{x : A, \Gamma \rightarrow \Delta, x : B}{\Gamma \rightarrow \Delta, x : A \supset B}\, R\supset$$

The rules for the modal operator $\Box$ are obtained similarly from its semantical explanation in terms of possible worlds

$$x : \Box A \ \textit{iff} \ \textit{for all} \ \ y, \ xRy \ \textit{implies} \ \ y : A$$

that gives the rules

$$\frac{y : A, x : \Box A, xRy, \Gamma \rightarrow \Delta}{x : \Box A, xRy, \Gamma \rightarrow \Delta}\, L\Box \qquad \frac{xRy, \Gamma \rightarrow \Delta, y : A}{\Gamma \rightarrow \Delta, x : \Box A}\, R\Box$$

with the *variable condition* in $R\Box$ that y is *fresh*, i.e. not free in the conclusion.

The resulting sequent calculus, called **G3K**, gives a complete system for the basic normal modal logic **K**. This logic is characterized by arbitrary frames; correspondingly, there are no rules for the accessibility relation. The sequent calculi for extensions of **K** such as the modal logics **T, K4, KB, S4, B, S5** are obtained by adding to **G3K** the rules that express their *frame conditions*, i.e., the properties of the accessibility relation that characterize their frames. For instance, a sequent calculus for the modal logic **S4** is obtained by adding the rules for reflexivity and transitivity of the accessibility relation

$$\frac{xRx, \Gamma \to \Delta}{\Gamma \to \Delta}\,\mathit{Refl} \qquad \frac{xRz, xRy, yRz, \Gamma \to \Delta}{xRy, yRz, \Gamma \to \Delta}\,\mathit{Trans}$$

We recall the following properties of any extension **G3K*** of **G3K** with geometric rules for the frame conditions (Negri, 2005):

Theorem 2. *1. All sequents of the form* $x : A, \Gamma \to \Delta, x : A$ *are derivable in* **G3K***.

2. All sequents of the form $\to x : \Box(A \supset B) \supset (\Box A \supset \Box B)$ *are derivable in* **G3K***.

3. The substitution *rule*

$$\frac{\Gamma \to \Delta}{\Gamma(y/x) \to \Delta(y/x)}\,(y/x)$$

is hp-admissible in **G3K***.

4. The rules of Weakening

$$\frac{\Gamma \Rightarrow \Delta}{x : A, \Gamma \Rightarrow \Delta}\,LW \qquad \frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, x : A}\,RW \qquad \frac{\Gamma \Rightarrow \Delta}{xRy, \Gamma \Rightarrow \Delta}\,LW_R$$

are hp-admissible in **G3K***.

5. The Necessitation *rule*

$$\frac{\to x : A}{\to x : \Box A}\,Nec$$

is admissible in **G3K***.

6. For each frame condition, the corresponding modal axiom is derivable in **G3K***.

7. All the primitive rules of **G3K*** *are hp-invertible.*

8. The rules of Contraction

$$\frac{x : A, x : A, \Gamma \to \Delta}{x : A, \Gamma \to \Delta}\,L\text{-}Ctr \quad \frac{xRy, xRy, \Gamma \to \Delta}{xRy, \Gamma \to \Delta}\,L\text{-}Ctr_R \quad \frac{\Gamma \to \Delta, x : A, x : A}{\Gamma \to \Delta, x : A}\,R\text{-}Ctr$$

are hp-admissible in **G3K***.

9. The Cut *rule*

$$\frac{\Gamma \to \Delta, x : A \quad x : A, \Gamma' \to \Delta'}{\Gamma, \Gamma' \to \Delta, \Delta'}\,Cut$$

is admissible in **G3K***.

In multi-modal logics, there is not only one but many accessibility relations, each defining a corresponding modal operator. In multi-agent epistemic logics, the accessibility relations are indexed over a set of agents, and the modality defined by each of these is an individual's knowledge operator. The intersection of the accessibility relations gives then the accessibility relation for the modality of distributed knowledge. The results by Hakli and Negri (2007) exemplify the backbone of the method for multimodal logics: First we give the rules for the accessibility relations, including the rules for obtaining other accessibility relations from given ones, in the form of rules that follow the regular or the geometric rule scheme. Then we obtain the rules for the corresponding modalities from their explanation in terms of Kripke semantics. Once the structural properties are established, completeness with respect to a Hilbert-style axiomatization follows from the derivability of the characteristic axioms in the system.

3. The system G3KA

We shall follow the axiomatization for the logic of acceptance given by Lorini et al. (2009) but use a slightly different notation. We denote the collective acceptance of A by $\mathcal{A}_{g:i}A$. This formula means that if the agents in the set g function together within an institutional context i then in that context they accept as a group that A. On the other hand, group acceptance of falsity, $\mathcal{A}_{g:i}\bot$, means that the agents in g do not function together as members of i. A standard possible worlds semantics is considered, with W a non-empty set of possible worlds and $R_{g:i}$ the accessibility relations that correspond to the modal operators $\mathcal{A}_{g:i}$ for all sets g and institutional contexts i.

As our basic system we use the propositional part of the system **G3c** given by Negri and von Plato (2001) and extend it with the rules for modalities and acceptance relations as explained in the previous section. In complete analogy to the rules for $\Box$, we define the rules for the acceptance modality starting from their explanation in terms of relational semantics:

$$x \Vdash \mathcal{A}_{g:i}A \ \text{ iff } \ \forall y(xR_{g:i}y \to y \Vdash A)$$

The rules we obtain are the following:

$$\frac{xR_{g:i}y, \Gamma \longrightarrow \Delta, y : A}{\Gamma \longrightarrow \Delta, x : \mathcal{A}_{g:i}A}\ {\scriptstyle R\mathcal{A}_{g:i}} \qquad \frac{y : A, x : \mathcal{A}_{g:i}A, xR_{g:i}y, \Gamma \longrightarrow \Delta}{x : \mathcal{A}_{g:i}A, xR_{g:i}y, \Gamma \longrightarrow \Delta}\ {\scriptstyle L\mathcal{A}_{g:i}}$$

Rule $R\mathcal{A}_{g:i}$ has the variable condition that y must not appear in the conclusion.

Lorini et al. (2009) impose the following semantic constraints on the frames, where $R_{h:j}(x)$ denotes the set $\{z \in W \mid xR_{h:j}z\}$:

S.1 If $h \subseteq g$ and $y \in R_{h:j}(x)$, then $R_{g:i}(y) \subseteq R_{g:i}(x)$

S.2 If $h \subseteq g$ and $y \in R_{h:j}(x)$, then $R_{g:i}(x) \subseteq R_{g:i}(y)$

S.3 If $h \subseteq g$ and $R_{g:i}(x) \neq \emptyset$, then $R_{h:i}(x) \subseteq R_{g:i}(x)$

S.4 If $y \in R_{g:i}(x)$, then $y \in \bigcup_{k \in g} R_{k:i}(y)$

S.5 If $h \subseteq g$ and $R_{g:i}(x) \neq \emptyset$, then $R_{h:i}(x) \neq \emptyset$

Once the set-theoretic definitions have been unfolded, these constraints are converted into syntactic rules after the pattern of the regular rule scheme or of the geometric rule scheme recalled in the previous section:

$$\frac{xR_{g:i}z, h \subseteq g, xR_{h:j}y, yR_{g:i}z, \Gamma \longrightarrow \Delta}{h \subseteq g, xR_{h:j}y, yR_{g:i}z, \Gamma \longrightarrow \Delta}\ {\scriptstyle RS.1}$$

$$\frac{yR_{g:i}z, h \subseteq g, xR_{h:j}y, xR_{g:i}z, \Gamma \longrightarrow \Delta}{h \subseteq g, xR_{h:j}y, xR_{g:i}z, \Gamma \longrightarrow \Delta}\ {\scriptstyle RS.2}$$

$$\frac{xR_{g:i}z, h \subseteq g, xR_{g:i}y, xR_{h:i}z, \Gamma \longrightarrow \Delta}{h \subseteq g, xR_{g:i}y, xR_{h:i}z, \Gamma \longrightarrow \Delta}\ {\scriptstyle RS.3}$$

$$\frac{\{yR_{k:i}y, xR_{g:i}y, \Gamma \longrightarrow \Delta\}_{k \in g}}{xR_{g:i}y, \Gamma \longrightarrow \Delta}\ {\scriptstyle RS.4}$$

$$\frac{xR_{h:i}z, h \subseteq g, xR_{g:i}y, \Gamma \longrightarrow \Delta}{h \subseteq g, xR_{g:i}y, \Gamma \longrightarrow \Delta}\ {\scriptstyle RS.5}$$

Rule *RS.4* has a finite number of premisses, one for each element of the group g[1], and *RS.5* has the condition that z must not occur in the conclusion.

The closure condition (see Section 2) imposes that the contracted instances of rules RS.1–RS.3 be added to the system in order to obtain

[1] By using the geometric rule scheme with an eigenvariable ranging over elements of g, the rule can be generalized to the case in which the group is not given as a finite list.

full height-preserving admissibility of contraction. It turns out, for the reason explained below, that only the following rule has to be added:

$$\frac{yR_{g:i}y, g \subseteq g, xR_{g:i}y, \Gamma \rightarrow \Delta}{g \subseteq g, xR_{g:i}y, \Gamma \rightarrow \Delta} \; RS.2^*$$

We call the resulting system **G3KA**.

The contracted instances of rules RS.1 and RS.3 are instead of the following form:

$$\frac{g \subseteq g, xR_{g:i}y, xR_{g:i}y, \Gamma \rightarrow \Delta}{g \subseteq g, xR_{g:i}y, \Gamma \rightarrow \Delta} \; *$$

We observe that instances of the closure condition that are just like contractions on relational atoms need not be added because they are admissible:

Proposition 3. *Let R be a frame rule, R^* the contracted instance that arises from the closure condition. If R^* is an instance of contraction, it is hp-admissible in the system extended with those rules arising from the closure condition that are not instances of contraction.*

Proof. Suppose R^* is of the form

$$\frac{xRy, xRy, \Gamma \rightarrow \Delta}{xRy, \Gamma \rightarrow \Delta} \; R^*$$

We proceed by induction on the height of the derivation of the premiss of R^*. If it is an initial sequent, also the conclusion is. If it is a conclusion of a frame rule F with premiss, say, $xRy, xRy, \Gamma' \rightarrow \Delta$, we have by the inductive hypothesis $xRy, \Gamma' \rightarrow \Delta$. If both occurrences of xRy are principal in F and its contracted instance F^* is available, we apply the induction hypothesis and then F^*. If F^* is not available (because an instance of contraction), the same relational atom is repeated three times and the conclusion is obtained by applying the inductive hypothesis twice. The case in which not both occurrences of xRy are principal in F is treated by the inductive hypothesis followed by rule F. If it is derived by a logical rule, the duplication of relational atoms is found in the premiss(es) of the rule, so we can apply the inductive hypothesis and the logical rule (which uses at most one relational atom) to obtain $xRy, \Gamma \rightarrow \Delta$. □

If follows in particular that the contracted instances of rules $RS.1$ and $RS.3$ are admissible in the system with the only addition of $RS.2^*$, and there are no contractions in the system, not even on relational atoms.

Lorini et al. (2009) present an axiomatization of the logic of acceptance. The inference rules are the standard ones, *modus ponens* and *necessitation*, and the specific axioms to be added to those of basic multimodal logic (propositional tautologies and distribution axioms) are as follows:

PAccess $\mathcal{A}_{g:i}A \supset \mathcal{A}_{h:j}\mathcal{A}_{g:i}A$ if $h \subseteq g$.

NAccess $\sim\mathcal{A}_{g:i}A \supset \mathcal{A}_{h:j} \sim\mathcal{A}_{g:i}A$ if $h \subseteq g$.

Inc $(\sim\mathcal{A}_{g:i}\bot \wedge \mathcal{A}_{g:i}A) \supset \mathcal{A}_{h:i}A$ if $h \subseteq g$.

Unanim $\mathcal{A}_{g:i}(\bigwedge_{k\in g} \mathcal{A}_{k:i}A \supset A)$

Mon $\sim\mathcal{A}_{g:i}\bot \supset \sim\mathcal{A}_{h:i}\bot$ if $h \subseteq g$.

The above axioms correspond to the semantic constraints S.1, S.2, S.3, S.4, and S.5, respectively.

The following lemma, used for proposition 5 below, shows that in our system the enlargement of a group maintains pre-existing disagreements, unless additional assumptions such as the presence of authoritative members are added. Note that this lemma employs the monotonicity property (rule RS.5 corresponding to axiom **Mon** above) that was included in the axiomatization of Lorini et al. (2009) but dropped by de Boer et al. (2009) and Herzig et al. (2009). Adopting this property requires that the reading given to the modality operator $\mathcal{A}_{g:i}$ does not require g to be a fixed group but allows it to be a subset of a larger group within i (see de Boer et al., 2009).

Lemma 4. *The sequent* $h \subseteq g, x : \mathcal{A}_{h:i}\bot \ \rightarrow x : \mathcal{A}_{g:i}\bot$ *is derivable in* **G3KA**.

Proof. We have the following derivation

$$\dfrac{\dfrac{\dfrac{xR_{h:i}z, h \subseteq g, xR_{g:i}y, z:\bot, x : \mathcal{A}_{h:i}\bot \ \rightarrow y:\bot}{xR_{h:i}z, h \subseteq g, xR_{g:i}y, x : \mathcal{A}_{h:i}\bot \ \rightarrow y:\bot}\ {\scriptstyle L\mathcal{A}_{h:i}}}{h \subseteq g, xR_{g:i}y, x : \mathcal{A}_{h:i}\bot \ \rightarrow y:\bot}\ {\scriptstyle RS.5}}{h \subseteq g, x : \mathcal{A}_{h:i}\bot \ \rightarrow x : \mathcal{A}_{g:i}\bot}\ {\scriptstyle R\mathcal{A}_{g:i}}$$

where the topsequent is an instance of $L\bot$. □

Observe that the sequent that expresses persistence of agreement, obtained by replacing $\bot$ with an arbitrary formula A, is instead not derivable. This is seen by inspection of the small set of possible applicable rules at each step of the root-first proof search.

Proposition 5. *The axioms* PAccess*,* NAccess*,* Inc*,* Unanim*, and* Mon *are derivable in* **G3KA**.

Proof. Axiom *PAccess* can be derived in a root-first fashion, using the corresponding rule *RS.1*, as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{z : A, xR_{g:i}z, h \subseteq g, xR_{h:j}y, yR_{g:i}z, x : \mathcal{A}_{g:i}A \longrightarrow z : A}{xR_{g:i}z, h \subseteq g, xR_{h:j}y, yR_{g:i}z, x : \mathcal{A}_{g:i}A \longrightarrow z : A}\ L\mathcal{A}_{g:i}}{h \subseteq g, xR_{h:j}y, yR_{g:i}z, x : \mathcal{A}_{g:i}A \longrightarrow z : A}\ RS.1}{h \subseteq g, xR_{h:j}y, x : \mathcal{A}_{g:i}A \longrightarrow y : \mathcal{A}_{g:i}A}\ R\mathcal{A}_{g:i}}{h \subseteq g, x : \mathcal{A}_{g:i}A \longrightarrow x : \mathcal{A}_{h:j}\mathcal{A}_{g:i}A}\ R\mathcal{A}_{h:j}}{h \subseteq g \longrightarrow x : \mathcal{A}_{g:i}A \supset \mathcal{A}_{h:j}\mathcal{A}_{g:i}A}\ R\supset$$

The uppermost sequent is clearly derivable because it contains the same formula on both sides of the sequent arrow.

The derivation of axiom *NAccess* by rule *RS.2* is similar.

Axiom *Inc* can be derived using the corresponding rule *RS.3*, as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{z : A \ldots \longrightarrow y : \bot, z : A}{xR_{g:i}z, xR_{h:i}z, xR_{g:i}y, x : \mathcal{A}_{g:i}A, h \subseteq g \longrightarrow y : \bot, z : A}\ L\mathcal{A}_{g:i}}{xR_{h:i}z, xR_{g:i}y, x : \mathcal{A}_{g:i}A, h \subseteq g \longrightarrow y : \bot, z : A}\ RS.3}{xR_{g:i}y, x : \mathcal{A}_{g:i}A, h \subseteq g \longrightarrow x : \mathcal{A}_{h:i}A, y : \bot}\ R\mathcal{A}_{h:i}}{x : \mathcal{A}_{g:i}A, h \subseteq g \longrightarrow x : \mathcal{A}_{h:i}A, x : \mathcal{A}_{g:i}\bot}\ R\mathcal{A}_{g:i}}{x : \sim\mathcal{A}_{g:i}\bot, x : \mathcal{A}_{g:i}A, h \subseteq g \longrightarrow x : \mathcal{A}_{h:i}A}\ L\supset}{x : \sim\mathcal{A}_{g:i}\bot \mathbin{\&} \mathcal{A}_{g:i}A, h \subseteq g \longrightarrow x : \mathcal{A}_{h:i}A}\ L\&}{h \subseteq g \longrightarrow x : (\sim\mathcal{A}_{g:i}\bot \mathbin{\&} \mathcal{A}_{g:i}A) \supset \mathcal{A}_{h:i}A}\ R\supset$$

Here the (derivable) right premiss of $L\supset$ is an instance of $L\bot$ and has been left out.

Axiom *Unanim* is easily derivable by rule *RS.4*.

Finally, by propositional steps, the derivation of *Mon* reduces to that of the sequent $h \subseteq g, x : \mathcal{A}_{h:i} \bot \longrightarrow x : \mathcal{A}_{g:i} \bot$, so we conclude by Lemma 4. □

By an adaptation of the method illustrated in the previous section, we can prove that the system **G3KA** has the same good structural properties as the basic propositional calculus **G3c** it is built upon. In particular, we have:

Theorem 6. *All the rules of* **G3KA** *are hp-invertible and the structural rules of weakening, contraction, and cut admissible. Weakening and contraction are in addition hp-admissible.*

Proof. Routine. □

Proposition 7. *The rules of modus ponens and necessitation are admissible in* **G3KA**.

Proof. If the sequents $\rightarrow x : A$ and $\rightarrow x : A \supset B$ are derivable in **G3KA**, then by invertibility of the right rule for implication we derive $x : A \rightarrow x : B$ and by admissibility of cut we derive $\rightarrow x : B$.

If $\rightarrow w : A$ is derivable, then by substitution also $\rightarrow y : A$ is derivable for an arbitrary label y, and by weakening also $xR_{g:i}y \rightarrow y : A$ is derivable. A step of $R\mathcal{A}_{g:i}$ gives the conclusion $\rightarrow x : R_{g:i}A$. □

Corollary 8. *The system* **G3KA** *is a complete sequent calculus for the logic of acceptance in the axiomatization of Lorini et al. (2009).*

4. Extensions with legislators

In this section we study extensions of the basic system. In particular, we consider rules that allow the possibility of *operative members* or *legislators* who can accept views for the group on behalf of other group members. The axiom for legislators considered by Lorini et al. (2009) is

$$\mathcal{A}_{g:i}(\bigwedge_{k \in Leg(i)} \mathcal{A}_{k:i}A \supset A) \quad \mathbf{Leg}$$

where $Leg(i)$ is a finite non-empty set. We show that it corresponds to the frame property

$$\forall xy(xR_{g:i}y \supset \bigvee_{k \in Leg(i)} yR_{k:i}y) \quad \mathbf{FLeg}$$

This property gives, for $Leg(i) \equiv \{k_1, \ldots, k_n\}$, the n-premiss rule

$$\frac{yR_{k_1:i}y, xR_{g:i}y, \Gamma \rightarrow \Delta \;\ldots\; yR_{k_n:i}y, xR_{g:i}y, \Gamma \rightarrow \Delta}{xR_{g:i}y, \Gamma \rightarrow \Delta}\; \scriptstyle{RLeg}$$

We have:

Proposition 9. *The axiom for legislators is derivable in* **G3KA** *extended with rule* RLeg.

Proof. Starting root-first from the sequent to be derived, we have

$$\cfrac{\cfrac{\cfrac{\cfrac{\{xR_{g:i}y, yR_{k_j:i}y, y : \mathcal{A}_{k_1:i}A, \ldots, y : \mathcal{A}_{k_n:i}A \rightarrow y : A\}_{j=1,\ldots,n}}{xR_{g:i}y, y : \mathcal{A}_{k_1:i}A, \ldots, y : \mathcal{A}_{k_n:i}A \rightarrow y : A}\; \scriptstyle{RLeg}}{xR_{g:i}y, y : \bigwedge_{k \in Leg(i)} \mathcal{A}_{k:i}A \rightarrow y : A}\; \scriptstyle{L\&}}{xR_{g:i}y \rightarrow y : \bigwedge_{k \in Leg(i)} \mathcal{A}_{k:i}A \supset A}\; \scriptstyle{R\supset}}{\rightarrow x : \mathcal{A}_{g:i}(\bigwedge_{k \in Leg(i)} \mathcal{A}_{k:i}A \supset A)}\; \scriptstyle{R\mathcal{A}_{g:i}}$$

where the n premisses of rule for legislators are indexed over the set $\{k_1, \ldots, k_n\}$ of members of $Leg(i)$; one step of $L\mathcal{A}_{k_j:i}$ produces the derivable sequents

$$\{xR_{g:i}y, yR_{k_j:i}y, y : A, y : \mathcal{A}_{k_1:i}A, \ldots, y : \mathcal{A}_{k_n:i}A \longrightarrow y : A\}_{j=1,\ldots,n}$$

□

By the above, rule *RLeg* is sufficient to derive the legislator axiom *Leg*. This means, indirectly, that the frame condition *FLeg* is sufficient to validate the legislator axiom. In order to show that it is characteristic we prove the following:

Proposition 10. *The frame condition* FLeg *holds in the canonical model for the logic of acceptance extended with the legislator axiom* Leg.

Proof. Recall that the canonical accessibility relation is defined by

$$xR_{k:i}y \equiv \text{ for all } A, x \Vdash \mathcal{A}_{k:i}A \text{ implies } y \Vdash A$$

Suppose that the antecedent of *FLeg*, $xR_{g:i}y$, holds. By validity of *Leg*, we have that $y \Vdash \bigwedge_{k\in Leg(i)} \mathcal{A}_{k:i}A \supset A$, that is,

$$\text{if } y \Vdash \bigwedge_{k\in Leg(i)} \mathcal{A}_{k:i}A, \text{ then } y \Vdash A$$

By unfolding the forcing relation on the conjunction, the above can be rewritten as

$$\text{if } \bigwedge_{k\in Leg(i)} y \Vdash \mathcal{A}_{k:i}A, \text{ then } y \Vdash A$$

Observe that the antecedent of this implication is a conjunction, so by the classical tautology $A \& B \supset C$ if and only if $(A \supset C) \vee (B \supset C)$, it can be rewritten as

$$\bigvee_{k\in Leg(i)} (y \Vdash \mathcal{A}_{k:i}A \rightarrow y \Vdash A)$$

By arbitrariness of A and by the definition of the canonical accessibility relation the formula in parentheses gives $yR_{k:i}y$, so we have proved that the frame condition

$$\forall xy(xR_{g:i}y \supset \bigvee_{k\in Leg(i)} yR_{k:i}y)$$

holds in the canonical model. □

Corollary 11. *The legislator axiom* Leg *is canonical with respect to the frame condition* FLeg.

Similarly, the requirement that legislators of an institution i must function as members of i, expressed by Lorini et al. (2009) by the principle

$$\sim\mathcal{A}_{Leg(i):i}\perp \quad \mathbf{Leg_0}$$

corresponds to the geometric frame condition

$$\forall x\exists y.\ xR_{Leg(i):i}y \quad \mathbf{FLeg_0}$$

which is turned into the rule

$$\frac{xR_{Leg(i):i}y, \Gamma \rightarrow \Delta}{\Gamma \rightarrow \Delta}\ {\scriptstyle RLeg_0}$$

with the condition that y is not in the conclusion.

In fact, we have:

Proposition 12. *The axiom* Leg_0 *is derivable in* **G3KA** *extended with rule* RLeg_0.

Proof. We have the following derivation, where the topsequent is an instance of $L\perp$:

$$\cfrac{\cfrac{\cfrac{y:\perp, xR_{Leg(i):i}y, x:\mathcal{A}_{Leg(i):i}\perp\ \rightarrow x:\perp}{xR_{Leg(i):i}y, x:\mathcal{A}_{Leg(i):i}\perp\ \rightarrow x:\perp}\ {\scriptstyle L\mathcal{A}_{Leg(i):i}}}{x:\mathcal{A}_{Leg(i):i}\perp\ \rightarrow x:\perp}\ {\scriptstyle RLeg_0}}{\rightarrow x:\sim\mathcal{A}_{Leg(i):i}\perp}\ {\scriptstyle R\supset}$$

□

Conversely we have:

Proposition 13. *Any frame that validates axiom* Leg_0 *satisfies the frame condition* FLeg_0.

Proof. Observe that $\forall x.x \Vdash\sim \mathcal{A}_{Leg(i):i}\perp$ is classically equivalent to $\forall x\exists y.\ xR_{Leg(i):i}y$. □

Corollary 14. *Axiom* Leg_0 *is canonical with respect to the frame condition* FLeg_0.

5. Completeness

We have already proved completeness of our system through equivalence with the existing Hilbert-type system. In this section we shall give a direct proof of completeness with respect to Kripke semantics. The proof follows the pattern of the proof of completeness for extensions of the labelled sequent system for extensions of basic modal logic **G3K*** presented by Negri (2009). This in turn follows the Schütte-style method of reduction trees for predicate logic, as presented for a two-sided sequent calculus by Negri and von Plato (2001, section 4.4).

The idea pursued with the labelled sequent system is the same as in Kripke's (1963) original proof for tableaux, but instead of looking for a failed search of a countermodel, one looks directly for a proof. The presence of labels in the calculus is here fully exploited. To see whether a formula is derivable, one checks whether it is universally valid, that is, valid at an arbitrary world for an arbitrary valuation, $x \Vdash A$. This is translated to a sequent $\rightarrow x : A$ in our calculus. The rules of the calculus applied backwards give equivalent conditions until the atomic components of A are reached. It can happen that we find a proof, or that we find that a proof does not exist either because we reach a stage where no rule is applicable, or because we go on with the search forever. In the two latter cases the attempted proof itself gives a countermodel.

Theorem 15. *Let $\Gamma \rightarrow \Delta$ be a sequent in the language of* **G3KA** *extended with legislators. Then either it is derivable in* **G3KA**+*Rleg or it has a Kripke countermodel with properties Ref, Trans,* **S.1–S.5, FLeg**.

Proof. We define for an arbitrary sequent $\Gamma \rightarrow \Delta$ in the language of **G3KA** with legislators a reduction tree by applying the rules of **G3KA**+*Rleg* root-first in all possible ways. If the construction terminates we obtain a proof, else the tree becomes infinite. By König's lemma an infinite tree has an infinite branch that is used to define a countermodel to the endsequent.

1. Construction of the reduction tree: The reduction tree is defined inductively in stages as follows:

Stage 0 has $\Gamma \rightarrow \Delta$ at the root of the tree. Stage $n > 0$ has two cases:

Case I: If every topmost sequent is an initial sequent or a conclusion of $L\bot$ the construction of the tree ends.

Case II: If not every topmost sequent is an initial sequent or a conclusion of $L\bot$ we continue the construction of the tree by writing above those topsequents that are not initial, nor conclusions of $L\bot$ other sequents that are obtained by applying root-first the rules of **G3K*** whenever possible, in a given order.

There are 16 different stages, 6 for the propositional rules of the basic modal systems, 2 for the rules for each of the acceptance operators $\mathcal{A}_{k:i}$, 8 for the frame rules (*Ref*, *Trans*, **S.1**–**S.5**, **FLeg**). At stage $n = 16 + 1$ we repeat stage 1, at stage $n = 16 + 2$ we repeat stage 2, and so on for every n.

We start, for $n = 1$, with $L\&$: For each topmost sequent of the form

$$x_1 : B_1\&C_1, \ldots, x_m : B_m\&C_m, \Gamma' \to \Delta$$

where $B_1\&C_1, \ldots, B_m\&C_m$ are all the formulas in Γ with a conjunction as the outermost logical connective, we write

$$x_1 : B_1, x_1 : C_1, \ldots, x_m : B_m, x_m : C_m, \Gamma' \to \Delta$$

on top of it. This step corresponds to applying root-first m times rule $L\&$.

For $n = 2$, we consider all the sequents of the form

$$\Gamma \to x_1 : B_1\&C_1, \ldots, x_m : B_m\&C_m, \Delta'$$

where $x_1 : B_1\&C_1, \ldots, x_m : B_m\&C_m$ are all the labelled formulas in the succedent with a conjunction as the outermost logical connective. We write on top of them the 2^m sequents

$$\Gamma \to x_1 : D_1, \ldots, x_m : D_m, \Delta'$$

where D_i is either B_i or C_i and all possible choices are taken. This is equivalent to applying $R\&$ root-first successively with principal labelled formulas $x_1 : B_1\&C_1$, $\ldots$, $x_m : B_m\&C_m$.

We define in a similar way the reductions of stages 3–6 for disjunction and implication: they amount to the application of the corresponding left (right) rule to all the formulas of the antecedent (succedent) which have as outermost connective the connective in question.

For $n = 7$, we consider all topsequents with antecedent containing the labelled modal formulas $x_1 : \mathcal{A}_{g:i}B_1, \ldots, x_m : \mathcal{A}_{h:j}B_m$ and the relational atoms $x_1R_{g:i}y_1, \ldots, x_mR_{h:j}y_m$, and write on top of these sequents the sequents

$$y_1 : B_1, \ldots, y_m : B_m, x_1 : \mathcal{A}_{g:i}B_1, \ldots, x_m : \mathcal{A}_{h:j}B_m, x_1Ry_1, \ldots, x_mRy_m, \Gamma' \to \Delta$$

that is, apply m times rule $L\mathcal{A}_{g:i}$.

For $n = 8$, let $x_1 : \mathcal{A}_{g:i}B_1, \ldots, x_m : \mathcal{A}_{h:j}B_m$ be all the formulas with $\mathcal{A}_{g:i}$ as the outermost connective in the succedent of topsequents of the tree, and let Δ' be the other formulas. Let $z_1, \ldots, z_m$ be fresh variables, not yet used in the reduction tree, and write on top of each sequent the sequent

$$x_1R_{g:i}z_1, \ldots, x_mR_{h:j}z_m, \Gamma \to \Delta, z_1 : B_1, \ldots, z_m : B_m$$

that is, apply m times rule $R\mathcal{A}_{g:i}$.

Finally, for $n = 8 + j$, we consider in succession the frame rules and the rule * arising from the closure condition. Application of rule *Ref* consists in adding to the antecedent all the relational atoms $xR_{g:i}x$ and it can be shown that without loss of generality the application of this rule can be restricted to the finite number of instances in which x, g, and i are not arbitrary but belong to $\Gamma \rightarrow \Delta$. For *Trans*, consider all the sequents with a pair of atoms of the form $xR_{g:i}y$, $yR_{g:i}z$ in the antecedent and write on top of them the sequents with the atoms $xR_{g:i}z$ added. With a rule with eigenvariables, such as **RS.5**, the step adds all the atoms of the form $xR_{h:i}z$ with z a fresh variable, whenever $h \subseteq g, xR_{g:i}y$ are in Γ. Observe that because of height-preserving substitution of individual variables and height-preserving admissibility of contraction, once a rule with eigenvariables has been considered, it need not be instantiated again on the same principal formulas.

For any n, for each sequent that is neither initial, nor conclusion of $L\bot$, nor treatable by any one of the above reductions (we call the sequent a *dead-end*), we write the sequent itself above it.

If the reduction tree is finite, all its leaves are initial or conclusions of $L\bot$, and the tree, read from the leaves to the root, yields a derivation.

2. Construction of the countermodel: If the reduction tree is infinite, it has an infinite branch. Let $\Gamma_0 \rightarrow \Delta_0 \equiv \Gamma \rightarrow \Delta, \Gamma_1 \rightarrow \Delta_1 \ldots, \Gamma_i \rightarrow \Delta_i, \ldots$ be one such branch. Consider the sets of labelled formulas and relational atoms

$$\mathbf{\Gamma} \equiv \bigcup_{i>0} \Gamma_i \qquad \mathbf{\Delta} \equiv \bigcup_{i>0} \Delta_i$$

We define a Kripke model that forces all the formulas in $\mathbf{\Gamma}$ and no formula in $\mathbf{\Delta}$ and is therefore a countermodel to the sequent $\Gamma \rightarrow \Delta$.

Consider the frame K the nodes of which are all the labels that appear in the relational atoms in $\mathbf{\Gamma}$, with their mutual relationships expressed by the $xR_{g:i}y$'s in $\mathbf{\Gamma}$.

Clearly, the construction of the reduction tree is closed with respect to the frame rules of the system and therefore imposes the frame properties of the countermodel, in particular, for the system **G3KA**, the constructed frame is reflexive and transitive and satifies in addition **S.1**–**S.5**, **FLeg**.

The model is defined as follows: For all atomic formulas $x : P$ in $\mathbf{\Gamma}$, we stipulate that $x \Vdash P$ in the frame, and for all atomic formulas $y : Q$ in $\mathbf{\Delta}$ we stipulate that $y \nVdash Q$. Since no sequent in the infinite branch is initial, this choice can be coherently made, for if there were the same labelled atom in $\mathbf{\Gamma}$ and in $\mathbf{\Delta}$, then, since the sequents in the reduction tree are defined in a cumulative way, for some i there would be a labelled atom $x : P$ both in the antecedent and in the succedent of $\Gamma_i \rightarrow \Delta_i$.

We then show inductively on the weight of formulas that A is forced in the model at node x if $x : A$ is in $\mathbf{\Gamma}$ and A is not forced at node x if $x : A$ is in $\mathbf{\Delta}$. Therefore we have a countermodel to the endsequent $\Gamma \rightarrow \Delta$.

If A is $\bot$, it cannot be in $\mathbf{\Gamma}$ because no sequent in the branch contains $x : \bot$ in the antecedent, so it is not forced at any node of the model.

If A is atomic, the claim holds by the definition of the model.

If $x : A \equiv x : B\&C$ is in $\mathbf{\Gamma}$, there exists i such that $x : A$ appears first in Γ_i, and therefore, for some $l \geq 0$, $x : B$ and $x : C$ are in Γ_{i+l}. By the induction hypothesis, $x \Vdash B$ and $x \Vdash C$, and therefore $x \Vdash B\&C$.

If $x : A \equiv x : B\&C$ is in $\mathbf{\Delta}$, consider the step i in which the reduction for A applies. This gives a branching, and one of the two branches belongs to the infinite branch, so either $x : B$ or $x : C$ is in $\mathbf{\Delta}$, and therefore by the inductive hypothesis, $x \nVdash B$ or $x \nVdash C$, and therefore $x \nVdash B\&C$.

The cases of disjunction are dual to those of conjunction.

If $x : A \equiv x : B \supset C$ is in $\mathbf{\Gamma}$, then either $x : B$ is in $\mathbf{\Delta}$ or $x : C$ is in $\mathbf{\Gamma}$. By the inductive hypothesis, in the former case $x \nVdash B$, and in the latter $x \Vdash C$, so in both cases $x \Vdash B \supset C$.

If $x : A \equiv x : B \supset C$ is in $\mathbf{\Delta}$, then for some i, $x : B \in \Gamma_i$ and $x : C \in \Delta_i$, so by the inductive hypothesis $x \Vdash B$ and $x \nVdash C$, so $x \nVdash B \supset C$.

If $x : A \equiv x : \Box B$ is in $\mathbf{\Gamma}$, we consider all the relational atoms $xR_{g:i}y$ that occur in $\mathbf{\Gamma}$. If there is no such atom, then the condition that for all y accessible from x in the frame, $y \Vdash B$ is vacuously satisfied, and therefore $x \Vdash \mathcal{A}_{g:i}B$ in the model. Else, for any occurrence of $xR_{g:i}y$ in $\mathbf{\Gamma}$ we find, by the construction of the reduction tree, an occurrence of $y : B$ in $\mathbf{\Gamma}$. By the inductive hypothesis, $y \Vdash B$, and therefore $x \Vdash \mathcal{A}_{g:i}B$ in the model.

If $x : A \equiv x : \mathcal{A}_{g:i}B$ is in $\mathbf{\Delta}$, consider the step at which the reduction for $x : A$ applies. We then find $y : B$ in $\mathbf{\Delta}$ for some y with $xR_{g:i}y$ in $\mathbf{\Gamma}$. By the induction hypothesis, $y \nVdash B$, and therefore $x \nVdash A$.

$\square$

Observe that the use of the (non-constructive) König's lemma is motivated by the aim of maximal generality of the proof. In the construction given above, the case in which at some stage we reach a dead-end and the case in which the proof search proceeds forever both give an infinite branch because in the former case one repeats the same sequent. With termination of proof search, either direct or obtained via the pruning of looping branches (typical of S4-based systems such as **G3KA**), the construction is modified without the repetition in the latter case and the countermodel is built not on the infinite branch, but on the branch that leads to the dead-end.

Corollary 16. *If a sequent* $\Gamma \rightarrow \Delta$ *is valid in every Kripke model with the frame properties Ref, Trans,* **S.1–S.5, FLeg***, then it is derivable in the system* **G3KA**+*Rleg*.

6. Majority axiom and the discursive dilemma

Lorini et al. (2009) consider the following majority principle as a logical axiom for two sets of agents B and C such that $B \subseteq C$ and $|C \backslash B| < |B|$:

$$\mathcal{A}_{C:x}(\bigwedge_{i \in B} \mathcal{A}_{i:x}\phi \supset \phi) \quad \textbf{Majority}$$

The majority axiom can be dealt with in a similar way to the axiom for legislators, and corresponding rules are obtained by adding for any majority set a rule in which the set of legislators in rule *RLeg* is replaced by the majority set. However, extension of the logic with a majority principle may lead to inconsistent group views in situations exemplified by the *discursive dilemma* in which the views of the group members are distributed so that there is a majority for both the conclusion and the premisses that entail the negation of the conclusion (see List and Pettit, 2002).

For instance, consider a typical example, in which the conclusion *lia* (that a defendent is liable for a breach of contract) to be decided on has the logical form of a conjunction: (*act* & *obl*) (the defendent committed an act and was under an obligation not to). In the premiss-based approach the agents vote on each of the premisses *act* and *obl* separately and then the results of the two votings are conjoined: The conclusion *lia* will be supported if and only if there is a majority for both conjuncts. In the conclusion-based approach the agents will vote directly on *lia*. As shown in Table I, given certain individual voting profiles these procedures may lead to different results.

The discursive dilemma has been formalized using the logic of acceptance, and it was shown that it leads to an inconsistent view on the group level when a majority principle is used (de Boer et al., 2009). This can be shown using the sequent calculus system as well.

Table I. The discursive dilemma

	act	*obl*	*lia* ⊃⊂ (*act* & *obl*)	lia
Judge 1	Yes	No	Yes	No
Judge 2	No	Yes	Yes	No
Judge 3	Yes	Yes	Yes	Yes
Majority	Yes	Yes	Yes	Yes/No

Suppose first that the judges 1,2, and 3 act together as judges of the court and that they accept the equivalence $lia \supset\subset act \,\&\, obl$:

$$\sim\mathcal{A}_{123:c}\bot$$

$$\mathcal{A}_{123:c}(lia \supset\subset act \,\&\, obl)$$

Then the judges announce their opinions concerning *act* and *obl*:

$$\mathcal{A}_{123:c}\mathcal{A}_{1:c}(act \,\&\, obl)$$

$$\mathcal{A}_{123:c}\mathcal{A}_{2:c}(act \,\&\, \sim obl)$$

$$\mathcal{A}_{123:c}\mathcal{A}_{3:c}(\sim act \,\&\, obl)$$

Then the group accepts the majority principles concerning the premisses and the conclusion:

$$\begin{aligned} Maj \;=\; & \{\mathcal{A}_{123:c} \,\&_{\,i,j\in\{1,2,3\},i\neq j}((\mathcal{A}_{i:c}act \,\&\, \mathcal{A}_{j:c}act) \supset act),\\ & \{\mathcal{A}_{123:c} \,\&_{\,i,j\in\{1,2,3\},i\neq j}((\mathcal{A}_{i:c} \sim act \,\&\, \mathcal{A}_{j:c} \sim act) \supset\sim act)\\ & \{\mathcal{A}_{123:c} \,\&_{\,i,j\in\{1,2,3\},i\neq j}((\mathcal{A}_{i:c}obl \,\&\, \mathcal{A}_{j:c}obl) \supset obl),\\ & \{\mathcal{A}_{123:c} \,\&_{\,i,j\in\{1,2,3\},i\neq j}((\mathcal{A}_{i:c} \sim obl \,\&\, \mathcal{A}_{j:c} \sim obl) \supset\sim obl)\\ & \{\mathcal{A}_{123:c} \,\&_{\,i,j\in\{1,2,3\},i\neq j}((\mathcal{A}_{i:c}lia \,\&\, \mathcal{A}_{j:c}lia) \supset lia),\\ & \{\mathcal{A}_{123:c} \,\&_{\,i,j\in\{1,2,3\},i\neq j}((\mathcal{A}_{i:c} \sim lia \,\&\, \mathcal{A}_{j:c} \sim lia) \supset\sim lia)\} \end{aligned}$$

Let Γ_c denote the multiset consisting of the above assumptions. Now we can derive a contradiction from Γ_c as shown in the Appendix.

Attempts to overcome the problems with the majority principle in the context of the logic of acceptance were made by Herzig et al. (2009). This article presents ways to model a premiss-based procedure in which the group votes on the premisses and then use the connection rule ($act \,\&\, obl \supset\subset lia$) to infer the conclusion at the group level: First the logic is extended with an announcement operator and then it is shown that the group can remain consistent if the individual acceptances concerning the premisses (*act* and *obl*) and the connection rule are announced to the group, even when the group accepts the majority rules for both the premisses and the conclusion. However, this result is somewhat counterintuitive and seems to be a result of a technical choice to keep in the frame even those worlds that the group does not consider possible after announcements.

Herzig et al. (2009) also propose another solution based on the distinction between belief and acceptance (see e.g. Cohen, 1992). In general, it is possible for individuals to believe and accept different propositions and there is no necessary entailment to either direction. This is used in the analysis of the discursive dilemma by applying the majority principle to the judges' beliefs so that the group accepts a proposition if and only if a majority of the judges believe the proposition. The connection rule is only accepted by the group, but it must not be believed by the judges because otherwise the judges could infer the truth value of the conclusion (lia) in the scope of their beliefs, and the majority principle could be applied to the conclusion as well. However, this solution seems artificial since it is implausible to think that the judges would not believe the connection rule.

In our opinion, a better solution would be to use the majority principle not as a logical principle that automatically connects group acceptance and individual attitudes, but as a principle that is to be applied precisely to those propositions that the group decides to vote on. If a group decides to vote on the premisses, it accepts the majority principle concerning the premisses. If it instead decides to vote on the conclusion, it accepts the majority principle concerning the conclusion. This solution makes it possible to model situations of collective decision-making using the logic of acceptance without the need to introduce announcements nor to employ the distinction between belief and acceptance. Of course, this is not a solution to the dilemma itself, since the problem is that the selection of the voting procedure can determine the result. This is a general problem concerning voting mechanisms and cannot be solved using logical analysis. Where logic can help is in the analysis of different voting procedures because it can be used to study their consequences, for instance, whether accepting certain principles can lead to inconsistencies as in the example above.

In addition to the majority rule leading to inconsistency at the group level, also legislator rules that allow determining a group view on the basis of a proper subset of the group members seem to face related problems: They may lead to an inconsistency at the level of individuals. This can be seen by constructing a case in which the legislators accept a proposition, say A, and some non-legislators accept its negation. By the axiom for legislators, the group accepts A, and by axiom Inc we can then derive that all group members, even those who were against, accept the view A accepted by the legislators.

The problem does not appear with the *Unanim* rule that demands consensus among all group members. Even so, these problems seem to show that *Unanim* is not acceptable as an axiom, either. The purpose of axiom *Unanim* is to model the formation of a group view on the basis of consensus. Similarly, axioms *Leg* and *Maj* attempt to model the formation of a group view on the basis of majority voting or consensus among legislators, respectively. So the idea is to model collective decision-making, and the intuitive semantics of an acceptance operator $\mathcal{A}_{c:i}A$ would be something like "individual c votes for A as the group's view in context i".

However, the attempt to model formation of a group view clashes with the attempt to model what *follows* from the adoption of a view by a group. It is a generally accepted principle concerning group views that when a group accepts a view, then every group member accepts that view when operating as a member of the group. This idea is encoded in axiom *Inc*, but it does not fit with the intuitive semantics suggested above, because now we are speaking of individual acceptance *after* the formation of the group view whereas previously we were thinking about acceptance in the voting situation, that is, *before* the formation of the group view. These two senses of acceptance cannot be modelled simultaneously without either using different modalities for pre- and post-voting views, e.g., by using different context variables, or using some kind of a dynamic or temporal logic that allows changes in views. The reason that *Unanim* does not lead to inconsistent acceptances is that it requires that everyone agrees and thus nobody will have to change one's mind.

One will thus have to choose which aspect of collective acceptance one wants to model with the logic of acceptance: Focus either on what follows from existing group views or study the formation of group views. In the former case, one can have axioms *PAccess*, *NAccess*, *Inc*, and *Mon* but not axioms that derive group views from individual acceptances. In the latter case, one can have any axiom that allows deriving group views from individual, *Unanim*, *Leg* or *Maj*, but one should not then include axiom *Inc* that allows deriving individual views from the collective view. The former approach suits, for instance, for multi-agent systems in which agents reason about the commitments made in different institutional contexts. The latter could be used for reasoning about what propositions groups will accept on the basis of individual acceptances, and perhaps also to some extent to meta-level reasoning about the properties of different aggregation procedures in the spirit of judgement aggregation logics (Ågotnes et al., 2007; Pauly, 2007).

7. Conclusion and future work

We have presented here a system of sequent calculus for the logic of acceptance and proved its completeness with respect to an existing axiomatization of the logic. Because of the explicit use of labels, completeness with respect to the characterizing class of frames can be established also in a direct way: For every sentence of the logic either a proof is found or a countermodel is obtained directly from the failed proof search. The completeness proof is constructive as long as a bound to proof search can be established, something that has to be checked case by case and clearly cannot be established for all possible extensions.

We can also show how the search space can be limited by methods of proof analysis in order to obtain decision procedures. Owing to the invertibility of the rules, cut-freeness, and bounded search space, our calculus permits to make conclusions not only about derivability but also about underivability of certain propositions and to study the sources of inconsistencies, which is not possible in the axiomatic approach. The methods presented can be adapted to the treatment of other non-summative collective attitudes that are based on collective acceptance beside group beliefs, for instance, group goals and collective preferences. This will be left for future work.

Other works that use modal logic as the basic formal language for reasoning about judgement aggregation are (Pauly, 2007) and (Ågotnes et al., 2007). The former studies questions of social choice from the perspective of judgment aggregation by providing an axiomatization of various voting procedures, dictatorship, majority voting, and consensus voting. The axiomatization is given in a language that is syntactically minimal, in the sense that it allows only expression for propositions for which there is collective consensus or rejection. The latter paper achieves significant expressive power that permits, among other things, the treatment of social welfare functions, Condorcet's paradox, and Arrow's theorem, by the use of a logical language with quantification at several levels, that is, over alternatives, preference profiles, and over agents. Both works propose an axiomatic treatment, and thus differ from our approach that is instead explicitly proof-theoretical.

A closely related approach (that we found after a preliminary version of this paper was submitted) is a tableau system for the logic of acceptance presented by de Boer et al. (2009). Compared to our system, a tableau proof can be regarded as a single-sided sequent calculus proof, with formulas only in the antecedent, and with trees proceeding from root to leaves, and aiming at a check for satisfiability, whereas a sequent proof in a labelled system is a check for validity. By the duality in a classical framework between the unsatisfiability of a formula and the validity of its negation, the two approaches are dual of each other. The tableau system of de Boer et al. (2009) operates on labelled formulas and accessibility relations and has labels ranging over natural numbers whereas our system does not assume any underlying implicit structure on the set of labels, but imposes it with suitable properties of an explicit accessibility relation. Similarly to ours, the system of de Boer et al. (2009) has both rules for the logical connectives and for the acceptance attitudes, and rules for the accessibility relations. The former are directly justified by the Kripkean semantics of the logic of acceptance, the latter by the frame properties that correspond to the proper axioms of acceptance logic. The successive steps in the tableau construction starting with $n : \phi$ amount to the root-first proof of the sequent $n : \phi \vdash$, or equivalently of $\rightarrow n : \neg\phi$. If no closed tableau for ϕ exists then the proof search of the corresponding sequent fails, so $\neg\phi$ has a countermodel, and ϕ is satisfiable. To summarize, a closed tableau corresponds to a proof in our system (where all branches lead to initial sequents), whereas an open tableau gives a countermodel. Finally, de Boer et al. (2009) present a direct completeness proof (in the same spirit of ours and of the general methodology presented by Negri, 2009). Soundness is established by observing that all the rules of the tableau system considered preserve satisfiability, and completeness by showing that if no tableau closes, then a countermodel is found. The construction of the countermodel is performed on the basis of a *saturated* tableau, the analog of the reduction tree presented in the proof of Theorem 14.

Our method covers in addition the treatment of the logic of acceptance augmented with operative members. We have shown in Section 4 that our rule system provides a heuristics for finding the frame conditions corresponding to a certain modal axiom. In the specific case of the axioms for legislators such frame properties turn out to be geometric implications, and the corresponding rules to be added to the system are rules that follow the geometric rule scheme, characterized by the presence of eigenvariables.

The expressive power of our method covers all the frame conditions of the form of geometric implications and it is still an open question whether it can be extended to include all the Sahlqvist fragment (even if there are extensions to conditions that are not first-order and thus beyond the Sahlqvist fragment). We observe that, by a simple argument, the method covers all the *displayable* modal logic. By Kracht's results (cf. Wansing, 1998, Thm. 4.20) displayable extensions of basic modal logic are characterized by *primitive* frame conditions, that is, frame conditions of the form $(\forall)(\exists)A$ where the quantifiers are restricted by the frame accessibility relation R and its inverse R^{-1} and A is built from atomic formulas of the form $x = y$, xRy, $xR^{-1}y$ through conjunctions and disjunctions, and at least one of x and y is not in the scope of an existential quantifier. Through standard conversions of first order logic, formulas of the form

$$\forall x_1(At_1(x_1) \supset (\forall x_2 At_2(x_1, x_2) \supset \ldots \exists y_1(Bt_1(y_1) \& (\exists y_2 Bt_2(y_2) \& \ldots))))$$

get converted to the form

$$\forall x_1 \forall x_2 \ldots \forall x_n(At_1(x_1) \& At_2(x_1, x_2) \supset \exists y_1 \exists y_2 Bt_1(y_1) \& Bt_2(y_1, y_2) \ldots)$$

and therefore primitive frame conditions convert to the canonical form of geometric implications. Observe that not every geometric implication satisfies the additional conditions on variables dictated by primitive frame conditions, but those that are needed in our context do. This is seen by inspecting the way in which the geometric implications are determined on the basis of the modal axioms to be derived. Of the two variables in relational/equality atoms, one is the variable that corresponds to a (universal) label in the formula to be proved, whereas the existential label is the one that licenses additional steps. This is probably better seen by looking at the rule that corresponds to geometric implications, where the existential quantifier gets replaced by a variable condition: If both labels in an atom were bound by the existential quantifier they would be both fresh in the geometric rule scheme and thus never active in a derivation.

Appendix

Let G denote the set of agents $\{1, 2, 3\}$. The institution c is omitted since it remains the same throughout the proof. Also set-theoretical expressions $G \subseteq G, \{1\} \subseteq G, \{2\} \subseteq G$, and $\{3\} \subseteq G$ are omitted.

$$\dfrac{}{\ldots, z : act, z : obl, \ldots \;\rightarrow\; z : act, \ldots}\;Ax$$

$$\dfrac{\ldots, z : act, z : obl, \ldots \;\rightarrow\; z : act, \ldots}{\ldots, z : act \,\&\, obl, \ldots \;\rightarrow\; z : act, \ldots}\;L\&$$

$$\dfrac{\ldots, z : act \,\&\, obl, \ldots \;\rightarrow\; z : act, \ldots}{yR_1 z, \ldots, y : \mathcal{A}_1(act \,\&\, obl), \ldots \;\rightarrow\; z : act, \ldots}\;L\mathcal{A}_1$$

$$\dfrac{yR_1 z, \ldots, y : \mathcal{A}_1(act \,\&\, obl), \ldots \;\rightarrow\; z : act, \ldots}{\ldots \;\rightarrow\; y : \mathcal{A}_1 act, \ldots}\;R\mathcal{A}_1 \qquad \dfrac{\textit{similarly}}{\ldots \;\rightarrow\; y : \mathcal{A}_2 act, \ldots}\;R\mathcal{A}_2$$

$$\dfrac{\ldots \;\rightarrow\; y : \mathcal{A}_1 act, \ldots \qquad \ldots \;\rightarrow\; y : \mathcal{A}_2 act, \ldots}{\ldots \;\rightarrow\; y : \mathcal{A}_1 act \,\&\, \mathcal{A}_2 act, \ldots}\;R\& \qquad (i)$$

$$\dfrac{\ldots \;\rightarrow\; y : \mathcal{A}_1 act \,\&\, \mathcal{A}_2 act, \ldots \qquad (i)}{y : lia \supset (act \,\&\, obl), y : (act \,\&\, obl) \supset lia, \ldots, y : (\mathcal{A}_1 act \,\&\, \mathcal{A}_2 act) \supset act, \ldots \;\rightarrow\; \ldots}\;L\supset$$

$$\dfrac{y : lia \supset (act \,\&\, obl), y : (act \,\&\, obl) \supset lia, \ldots, y : (\mathcal{A}_1 act \,\&\, \mathcal{A}_2 act) \supset act, \ldots \;\rightarrow\; \ldots}{xR_G y, y : lia \supset\subset (act \,\&\, obl), y : \mathcal{A}_1(act \,\&\, obl), \ldots \;\rightarrow\; y : \bot, x : \bot}\;L\&$$

$$\dfrac{xR_G y, y : lia \supset\subset (act \,\&\, obl), y : \mathcal{A}_1(act \,\&\, obl), \ldots \;\rightarrow\; y : \bot, x : \bot}{xR_G y, x : \mathcal{A}_G(lia \supset\subset (act \,\&\, obl)), x : \mathcal{A}_G \mathcal{A}_1(act \,\&\, obl), \ldots \;\rightarrow\; y : \bot, x : \bot}\;L\mathcal{A}_G{}^*$$

$$\dfrac{xR_G y, x : \mathcal{A}_G(lia \supset\subset (act \,\&\, obl)), x : \mathcal{A}_G \mathcal{A}_1(act \,\&\, obl), \ldots \;\rightarrow\; y : \bot, x : \bot}{\ldots \;\rightarrow\; x : \mathcal{A}_G \bot, x : \bot}\;R\mathcal{A}_G \qquad \dfrac{}{x : \bot \;\rightarrow\; x : \bot}\;L\bot$$

$$\dfrac{\ldots \;\rightarrow\; x : \mathcal{A}_G \bot, x : \bot \qquad x : \bot \;\rightarrow\; x : \bot}{x : \sim\mathcal{A}_G \bot \ldots \;\rightarrow\; x : \bot}\;L\supset$$

Proof continues in a root-first fashion from (i):

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{}{z : act, z : obl, \ldots \;\rightarrow z : obl}\;{\scriptstyle Ax}
      }{z : act \,\&\, obl, \ldots \;\rightarrow z : obl}\;{\scriptstyle L\&}
    }{yR_1z, \ldots, y : \mathcal{A}_1(act \,\&\, obl), \ldots \;\rightarrow z : obl, \ldots}\;{\scriptstyle L\mathcal{A}_1}
  }{\ldots \;\rightarrow y : \mathcal{A}_1 obl, \ldots}\;{\scriptstyle R\mathcal{A}_1}
  \qquad
  \dfrac{\textit{similarly}}{\ldots \;\rightarrow y : \mathcal{A}_3 obl, \ldots}\;{\scriptstyle R\mathcal{A}_3}
}{
  \dfrac{\ldots \;\rightarrow y : \mathcal{A}_1 obl \,\&\, \mathcal{A}_3 obl, \ldots \qquad (ii)}{y : act, \ldots, y : ((\mathcal{A}_1 obl \,\&\, \mathcal{A}_3 obl) \supset obl), \ldots \;\rightarrow y : \bot, x : \bot}\;{\scriptstyle L\supset}
}\;{\scriptstyle R\&}
$$

Proof continues from (ii):

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{
              \dfrac{
                \dfrac{
                  \dfrac{
                    \dfrac{
                      \dfrac{}{z : lia \ldots \;\rightarrow z : lia, z : \bot, \ldots}\;{\scriptstyle Ax}
                    }{\ldots \;\rightarrow z : lia, z :\sim lia, \ldots}\;{\scriptstyle R\supset}
                    \quad
                    \dfrac{
                      \dfrac{
                        \dfrac{}{z : act, z : obl, \ldots \;\rightarrow z : obl, \ldots}\;{\scriptstyle Ax}
                        \quad
                        \dfrac{}{z : \bot, \ldots \;\rightarrow \;\ldots}\;{\scriptstyle L\bot}
                      }{z : act, z : obl, z : act, z :\sim obl, \ldots \;\rightarrow z :\sim lia, \ldots}\;{\scriptstyle L\supset}
                    }{z : act \,\&\, obl, z : act, z :\sim obl, \ldots \;\rightarrow z :\sim lia, \ldots}\;{\scriptstyle L\&}
                  }{z : act, z :\sim obl, z : lia \supset (act \,\&\, obl), z : (act \,\&\, obl) \supset lia, \ldots \;\rightarrow z \sim lia, \ldots}\;{\scriptstyle L\supset}
                }{z : act \,\&\, \sim obl, z : (lia \supset\subset (act \,\&\, obl)), \ldots \;\rightarrow z :\sim lia, \ldots}\;{\scriptstyle L\&^*}
              }{z : (lia \supset\subset (act \,\&\, obl)), \ldots, yR_2z, \ldots, y : \mathcal{A}_2(act \,\&\, \sim obl) \rightarrow z :\sim lia, \ldots}\;{\scriptstyle L\mathcal{A}_2}
            }{xR_Gz, \ldots, x : \mathcal{A}_G(lia \supset\subset (act \,\&\, obl)), \ldots \;\rightarrow z :\sim lia, \ldots}\;{\scriptstyle L\mathcal{A}_G}
          }{yR_Gz, \ldots, xR_Gy, \ldots \;\rightarrow z :\sim lia, \ldots}\;{\scriptstyle RS.1}
        }{yR_Gy, yR_2z, \ldots \;\rightarrow z :\sim lia, \ldots}\;{\scriptstyle RS.3}
      }{yR_2z, \ldots, xR_Gy, xR_Gy, \ldots \;\rightarrow z :\sim lia, \ldots}\;{\scriptstyle RS.2^*}
    }{\ldots \;\rightarrow y : \mathcal{A}_2 \sim lia, \ldots}\;{\scriptstyle R\mathcal{A}_2}
    \qquad
    \dfrac{\textit{similarly}}{\ldots \;\rightarrow y : \mathcal{A}_3 \sim lia, \ldots}\;{\scriptstyle R\mathcal{A}_3}
  }{\ldots \;\rightarrow y : \mathcal{A}_2 \sim lia \,\&\, \mathcal{A}_3 \sim lia, \ldots \qquad (iii)}\;{\scriptstyle R\&}
}{y : obl, y : act, \ldots, y : ((\mathcal{A}_2 \sim lia \,\&\, \mathcal{A}_3 \sim lia) \supset\sim lia), \ldots \;\rightarrow y : \bot, x : \bot}\;{\scriptstyle L\supset}
$$

Proof continues from (iii):

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{}{y : obl, y : act, \ldots \;\rightarrow y : act, \ldots}\;{\scriptstyle Ax}
      \quad
      \dfrac{}{y : obl, \ldots \;\rightarrow y : obl, \ldots}\;{\scriptstyle Ax}
    }{y : obl, y : act, \ldots \;\rightarrow y : act \,\&\, obl, \ldots}\;{\scriptstyle R\&}
    \quad
    \dfrac{}{y : lia, \ldots \;\rightarrow y : lia, \ldots}\;{\scriptstyle Ax}
  }{y : obl, y : act, \ldots, y : (act \,\&\, obl) \supset lia, \ldots \;\rightarrow y : lia, \ldots}\;{\scriptstyle L\supset}
  \quad
  \dfrac{}{y : \bot, \ldots \;\rightarrow \;\ldots}\;{\scriptstyle L\bot}
}{y :\sim lia, y : obl, y : act, \ldots \;\rightarrow y : \bot, x : \bot}\;{\scriptstyle L\supset}
$$

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References

Cohen, L. J.: 1992, *An Essay on Belief & Acceptance*. New York, USA: Oxford University Press.

de Boer, M., A. Herzig, T. de Lima, and E. Lorini: 2009, 'Tableaux for acceptance logic'. In: M. Baldoni, J. Bentahar, J. Lloyd, and M. B. van Riemsdijk (eds.): *Declarative Agent Languages and Technologies: Seventh International Workshop, DALT 2009, Workshop Notes*. pp. 17–32.

Fagin, R., J. Y. Halpern, Y. Moses, and M. Y. Vardi: 1995, *Reasoning about Knowledge*. MIT Press.

Fischer, F. A. and M. Nickles: 2006, 'Computational opinions'. In: *Proceedings of the 17th European Conference on Artificial Intelligence (ECAI'06)*. pp. 240–244.

Gaudou, B., A. Herzig, and D. Longin: 2006a, 'Grounding and the expression of belief'. In: *Proceedings of the Tenth International Conference on Principles of Knowledge Representation and Reasoning*. pp. 221–229, AAAI Press.

Gaudou, B., A. Herzig, and D. Longin: forthcoming, 'Group belief and grounding in conversation'. In: A. Trognon et al. (ed.): *Language, Cognition, Interaction*. Presses Universitaires de Nancy.

Gaudou, B., A. Herzig, D. Longin, and M. Nickles: 2006b, 'A new semantics for the FIPA agent communication language based on social attitudes'. In: *Proceedings of the 17th European Conference on Artificial Intelligence (ECAI'06)*. pp. 245–249.

Gaudou, B., D. Longin, E. Lorini, and L. Tummolini: 2008, 'Anchoring institutions in agents' attitudes: Towards a logical framework for autonomous multi-agent systems'. In: *Proceedings of 7th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2008)*. pp. 728–735.

Gilbert, M.: 1987, 'Modelling collective belief'. *Synthese* **73**, 185–204.

Gilbert, M.: 1989, *On Social Facts*. Routledge.

Hakli, R.: 2006, 'Group beliefs and the distinction between belief and acceptance'. *Cognitive Systems Research* **7**, 286–297.

Hakli, R. and S. Negri: 2007, 'Proof theory for distributed knowledge'. In: *Computational Logic in Multi-Agent Systems. 8th International Conference (CLIMA-VIII), LNCS 5056*. pp. 100–116, Springer.

Herzig, A., T. de Lima, and E. Lorini: 2009, 'On the dynamics of institutional agreements'. *Synthese (Knowledge, Rationality & Action)* **171**, 321–355 (923–957).

Kripke, S.: 1963, 'Semantical analysis of modal logic I. Normal modal propositional calculi'. *Zeitschrift f. Math. Logik und Grund. d. Math.* **9**, 67–96.

List, C. and P. Pettit: 2002, 'Aggregating sets of judgments: An impossibility result'. *Economics and Philosophy* **18**, 89–110.

Lorini, E., D. Longin, B. Gaudou, and A. Herzig: 2009, 'The logic of acceptance: Grounding institutions on agents' attitudes'. *Journal of Logic and Computation* **19**(6), 901–940.

Negri, S.: 2003, 'Contraction-free sequent calculi for geometric theories with an application to Barr's theorem'. *Archive for Mathematical Logic* **42**, 389–401.

Negri, S.: 2005, 'Proof analysis in modal logic'. *Journal of Philosophical Logic* **34**, 507–544.

Negri, S.: 2009, 'Kripke completeness revisited'. In: G. Primiero and S. Rahman (eds.): *Acts of Knowledge: History, Philosophy and Logic*. College Publications, pp. 233–266.

Negri, S. and J. von Plato: 1998, 'Cut elimination in the presence of axioms'. *The Bulletin of Symbolic Logic* **4**, 418–435.

Negri, S. and J. von Plato: 2001, *Structural Proof Theory*. Cambridge University Press.

Negri, S. and J. von Plato: 2011, *Proof Analysis: A Contribution to Hilbert's Last Problem*. Cambridge University Press, in press.

Pauly, M.: 2007, 'Axiomatizing collective judgements in a minimal logical language'. *Synthese* **158**, 233–250.

Searle, J. R.: 1995, *The Construction of Social Reality*. New York, USA: The Free Press.

Shoham, Y. and K. Leyton-Brown: 2009, *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. New York: Cambridge University Press.

Troelstra, A. S. and H. Schwichtenberg: 2000, *Basic Proof Theory (2nd ed.)*. New York, NY, USA: Cambridge University Press.

Tuomela, R.: 1992, 'Group beliefs'. *Synthese* **91**, 285–318.

Tuomela, R.: 2007, *The Philosophy of Sociality*. Oxford University Press.

Wansing, H.: 1998, *Displaying Modal Logic*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Ågotnes, T., W. van der Hoek, and M. Wooldridge: 2007, 'Reasoning about judgment and preference aggregation'. In: *Proceedings of the Sixth International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS 2007)*. pp. 554–561.