

# Reasoning on UML Class Diagrams is EXPTIME-hard

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## Abstract

UML is the de-facto standard formalism for software design and analysis. One of the most important components of UML are *class diagrams*, which model the information on the domain of interest in terms of objects organized in classes and relationships between them. To support the design of large-scale industrial applications, CASE tools should be equipped with automated reasoning capabilities in order to detect relevant formal properties of UML diagrams, such as inconsistencies or redundancies. However, reasoning on UML class diagrams is fairly complex: in this paper we show that it is EXPTIME-hard.

## 1 Introduction

UML (Unified Modeling Language) [25] is the de-facto standard formalism for the analysis and design of software. One of the most important components of UML are *class diagrams*, which model the information on the domain of interest in terms of objects organized in classes and relationships between them<sup>1</sup>.

Several works propose to describe UML class diagrams using various kinds of formal systems [18, 17, 22, 10]. Using such formal systems, one can potentially reason on UML class diagrams, and formally prove properties of interest through inference. In order to select the appropriate kind of formal tool for UML class diagrams, a fundamental question needs to be addressed: What is the computational complexity of reasoning on UML class diagrams? That is, independently of the particular formal tool adopted for describing such diagrams, how difficult is it to reason about them from the computational point of view?

In this paper we address this question resorting to results developed through the years in Description Logics (DLs) [1]. It is well known that DLs are logics that admit decidable reasoning and that are specifically designed for the conceptual representation of an application domain in terms of classes and relationships between classes. Representing conceptual data models by means of DLs has gathered consensus over the years, cf. [6, 7, 15, 24, 26, 13, 16, 12, 20, 21, 14, 23].

Our contribution is to show that reasoning on UML class diagrams is EXPTIME-hard even under fairly restrictive assumptions, namely: only binary associations, only

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<sup>1</sup>In this paper we deal with UML class diagrams for the *conceptual perspective*, as opposed to the *implementation perspective*, see, e.g., [19].

minimal multiplicity constraints, generalizations with disjointness and covering constraints. We get this result by exhibiting a polynomial reduction from reasoning in the basic DL  $\mathcal{ALC}^2$  [1], which is EXPTIME-complete. In particular, we show that every  $\mathcal{ALC}$  knowledge base can be expressed as a UML class diagram preserving soundness and completeness of reasoning. This possibility is quite surprising, since UML class diagrams apparently have very limited means to express negative and disjunctive information, namely disjointness and covering constraints in generalization hierarchies. Note that we do not consider arbitrary OCL [25] constraints, which in their full generality have the same power of full first-order logic and hence lead to undecidability. Instead,  $\mathcal{ALC}$  is equipped with unrestricted negation and disjunction. That is, it is able to treat negative information in the same way as positive one, and to reason by cases to fully take into account disjunctive information.

The rest of the paper is organized as follows. In Section 2 we give some preliminary notions on DLs that we use later on. In Section 3 we present our EXPTIME-hardness result for reasoning on UML class diagrams. Finally, in Section 4, we draw some conclusions. Throughout the paper we assume that the reader is familiar with UML class diagrams [25].

## 2 The Description Logics $\mathcal{ALC}$ and $\mathcal{ALC}^-$

$\mathcal{ALC}$  is a standard DL, that represents knowledge in terms of concepts (classes) and roles (binary relations). Let  $A$  and  $P$  denote atomic concepts and atomic roles respectively. An  $\mathcal{ALC}$  concept  $C$  is built according to the following syntax:

$$C ::= A \mid \neg C \mid C_1 \sqcap C_2 \mid \exists P.C$$

We also introduce the standard abbreviations:  $C_1 \sqcup C_2$  for  $\neg(\neg C_1 \sqcap \neg C_2)$  and  $\forall P.C$  for  $\neg \exists P.\neg C$ . An  $\mathcal{ALC}$  Knowledge Base (KB) is constituted by a finite set of *inclusion assertions* of the form  $C_1 \sqsubseteq C_2$ , with  $C_1$  and  $C_2$  arbitrary concept expressions. We also consider *primitive* inclusion assertions, i.e., assertions of the form  $A \sqsubseteq C$ , where  $A$  is an atomic concept and  $C$  is an arbitrary concept. A *primitive KB* is constituted by a finite set of primitive inclusion assertions.

As usual in DLs, the semantics of  $\mathcal{ALC}$  is specified through the notion of interpretation. An *interpretation*  $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$  of an  $\mathcal{ALC}$  KB  $\mathcal{K}$  is constituted by an *interpretation domain*  $\Delta^{\mathcal{I}}$  and an *interpretation function*  $\cdot^{\mathcal{I}}$  that assigns to each concept  $C$  a subset  $C^{\mathcal{I}}$  of  $\Delta^{\mathcal{I}}$  and to each role  $P$  a subset  $P^{\mathcal{I}}$  of  $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ , such that the following conditions are satisfied:

$$\begin{aligned} A^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} & (C_1 \sqcap C_2)^{\mathcal{I}} &= C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}} \\ (\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} & (\exists P.C)^{\mathcal{I}} &= \{a \in \Delta^{\mathcal{I}} \mid \exists b. (a, b) \in P^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\} \end{aligned}$$

An interpretation  $\mathcal{I}$  *satisfies* an inclusion assertion  $C_1 \sqsubseteq C_2$  if  $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$ . An interpretation that satisfies all assertions in a KB  $\mathcal{K}$  is called a *model* of  $\mathcal{K}$ . A KB  $\mathcal{K}$  is *satisfiable* if there exists a model of  $\mathcal{K}$ . A concept  $C$  is *satisfiable* in a KB  $\mathcal{K}$  if there is a model  $\mathcal{I}$  of  $\mathcal{K}$  such that  $C^{\mathcal{I}}$  is non-empty. An assertion  $\alpha$  is *logically implied* by

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<sup>2</sup>In this paper when we mention reasoning in a DL, we always intend reasoning over a knowledge base expressed in that DL.

$\mathcal{K}$  if all models of  $\mathcal{K}$  satisfy  $\alpha$ . It can be shown that all these reasoning tasks, namely KB satisfiability, concept satisfiability in a KB, and logical implication, are mutually reducible (in polynomial time). In spite of its simplicity, reasoning in  $\mathcal{ALC}$  KBs is EXPTIME-complete [1].

The DL  $\mathcal{ALC}^-$  is obtained from  $\mathcal{ALC}$  by dropping intersection, and restricting the assertions to be primitive. Therefore, an  $\mathcal{ALC}^-$  concept  $C$  is built as follows:

$$C ::= A \mid \neg A \mid A_1 \sqcup A_2 \mid \exists P.A \mid \forall P.A$$

where  $A$  denotes an atomic concept and  $P$  denotes an atomic role. An  $\mathcal{ALC}^-$  KB is a finite set of primitive  $\mathcal{ALC}^-$  inclusion assertions, i.e., inclusion assertions of the form  $A \sqsubseteq C$  where  $C$  is an  $\mathcal{ALC}^-$  concept. The semantics of  $\mathcal{ALC}^-$  constructs and KBs is that of  $\mathcal{ALC}$ . For  $\mathcal{ALC}^-$ , we can define KB satisfiability, concept satisfiability in a KB, and logical implication, as for  $\mathcal{ALC}$ .

### 3 Hardness of reasoning on UML class diagrams

The design quality of UML class diagrams can be improved by checking relevant properties, such as, for example, consistency of the whole class diagram, class consistency, class subsumption, class equivalence [7, 15, 8]. From the formal point of view, the reasoning tasks necessary for checking these properties are mutually reducible to each other. Hence in the following, without loss of generality, we focus on class consistency only. Specifically, we show that class consistency in UML class diagrams with only disjointness and covering constraints is EXPTIME-hard. We prove the claim by a reduction from concept satisfiability in  $\mathcal{ALC}$  KBs, which is EXPTIME-hard [1]. For lack of space we report only proof sketches. The full proofs may be found in [4]. We proceed in two steps:

1. First, we show that we can restrict the attention to atomic concept satisfiability in  $\mathcal{ALC}^-$  KBs.
2. Then, we describe a reduction from atomic concept satisfiability in  $\mathcal{ALC}^-$  KBs to class consistency in UML class diagrams.

For point (1), we resort to known results [9, 11].

**Lemma 3.1 ([9])** *Concept satisfiability in an  $\mathcal{ALC}$  KB can be linearly reduced to atomic concept satisfiability in a primitive  $\mathcal{ALC}$  KB.*

*Proof (sketch).* Let  $\mathcal{K}$  be an  $\mathcal{ALC}$  KB and  $C$  an  $\mathcal{ALC}$  concept. It is easy to see that  $C$  is satisfiable in  $\mathcal{K}$  if and only if  $A_C$  is satisfiable in the KB consisting of the following two assertion:

$$\begin{aligned} A_T &\sqsubseteq \bigcap_{C_1 \sqsubseteq C_2 \in \mathcal{K}} (\neg C_1 \sqcup C_2) \sqcap \bigcap_{1 \leq i \leq n} \forall P_i.A_T \\ A_C &\sqsubseteq A_T \sqcap C \end{aligned}$$

where  $A_C$  and  $A_T$  are new atomic concepts and  $P_1, \dots, P_n$  are all atomic roles appearing in  $\mathcal{K}$  and  $C$ . □

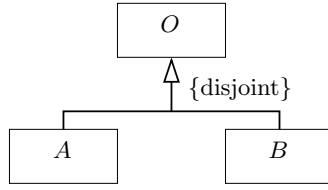


Figure 1: UML encoding of the assertion  $A \sqsubseteq \neg B$

Below we assume, without loss of generality, that primitive  $\mathcal{ALC}$  KBs are in negation normal form. Indeed, every primitive  $\mathcal{ALC}$  KB can be put in negation normal form in linear time.

Given a primitive  $\mathcal{ALC}$  KB  $\mathcal{K}$  (in negation normal form), we construct a primitive  $\mathcal{ALC}^-$  KB  $\mathcal{K}'$  by recursively replacing each  $\mathcal{ALC}$  assertion in  $\mathcal{K}$  that is not already a (primitive)  $\mathcal{ALC}^-$  assertion as follows:

- $A \sqsubseteq C_1 \sqcap C_2$  is replaced by  $A \sqsubseteq C_1$  and  $A \sqsubseteq C_2$ ;
- $A \sqsubseteq C_1 \sqcup C_2$  is replaced by  $A \sqsubseteq A_1 \sqcup A_2$ ,  $A_1 \sqsubseteq C_1$  and  $A_2 \sqsubseteq C_2$ , where  $A_1$  and  $A_2$  are new atomic concepts;
- $A \sqsubseteq \forall P.C$  is replaced by  $A \sqsubseteq \forall P.A_1$  and  $A_1 \sqsubseteq C$ , where  $A_1$  is a new atomic concept;
- $A \sqsubseteq \exists P.C$  is replaced by  $A \sqsubseteq \exists P.A_1$  and  $A_1 \sqsubseteq C$ , where  $A_1$  is a new atomic concept.

Notice that the number of such replacements is linear, since for each occurrence of an  $\mathcal{ALC}$  construct in  $\mathcal{K}$  at most one replacement is done. The following result holds.

**Lemma 3.2** ([11]) *An atomic concept  $A_0$  is satisfiable in a primitive  $\mathcal{ALC}$  KB  $\mathcal{K}$  if and only if  $A_0$  is satisfiable in the (primitive)  $\mathcal{ALC}^-$  KB  $\mathcal{K}'$  obtained as above.*

Next, we reduce concept satisfiability in a primitive  $\mathcal{ALC}^-$  KB  $\mathcal{K}'$  to class consistency in an UML class diagram  $\mathcal{D}$ . For each atomic concept  $A$  in  $\mathcal{K}'$ , we introduce a class  $A$  in  $\mathcal{D}$ . Additionally, we add a class  $O$  that generalizes (possibly indirectly) all classes in  $\mathcal{D}$ .  $O$  is also used to specify disjointness among classes (see later). For each atomic role  $P$ , we introduce an association  $P$ , involving the class  $O$  twice. Intuitively, using  $O$  in such a way, we do not constrain in any way the classes to which the component instances of  $P$  may belong. More classes and associations, as well as generalizations between  $O$  and the new classes, are added below as needed.

The assertions in the  $\mathcal{ALC}^-$  KB  $\mathcal{K}'$  are encoded in the class diagram  $\mathcal{D}$  as follows:

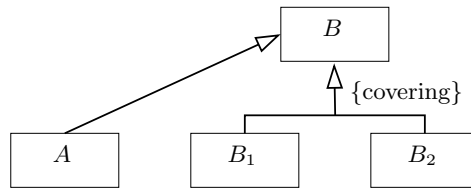


Figure 2: UML encoding of the assertion  $A \sqsubseteq B_1 \sqcup B_2$

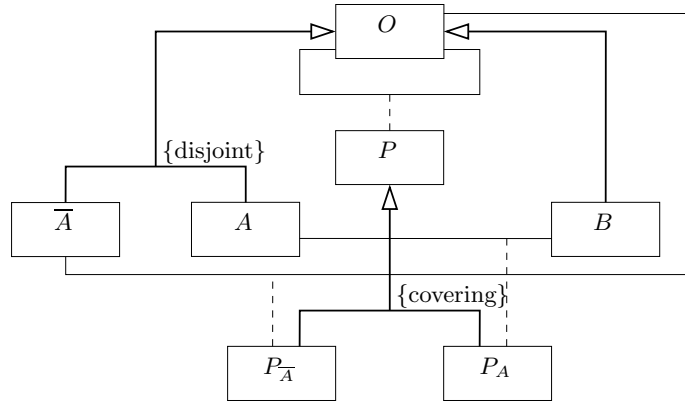


Figure 3: UML encoding of the assertion  $A \subseteq \forall P.B$

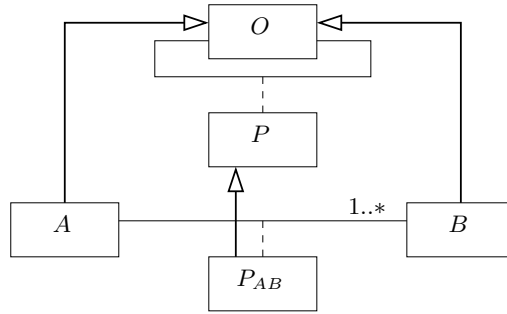


Figure 4: UML encoding of the assertion  $A \subseteq \exists P.B$

- For each assertion of the form  $A \subseteq B$ , we introduce a generalization between the classes  $A$  and  $B$  (where  $A$  is the subclass).
- For each assertion of the form  $A \subseteq \neg B$ , we construct the hierarchy in Figure 1, exploiting the superclass  $O$  to express disjointness between  $A$  and  $B$ .
- For each assertion of the form  $A \subseteq B_1 \sqcup B_2$ , we introduce an auxiliary class  $B$ , and construct the hierarchy in Figure 2. Intuitively, being  $B$  a covering of  $B_1$  and  $B_2$ , and  $A$  a subclass of  $B$ , it follows that  $A$  is a subclass of the union of  $B_1$  and  $B_2$ .
- For each assertion of the form  $A \subseteq \forall P.B$ , we introduce a new class  $\bar{A}$  and two new binary associations  $P_A$  and  $P_{\bar{A}}$  and we construct the portion of diagram in Figure 3, where  $A$  and  $\bar{A}$  are disjoint and there is a generalization with covering constraint between  $P$  and its children  $P_A$  and  $P_{\bar{A}}$ . Note that  $A$  and  $B$  are the components of  $P_A$ , whereas  $\bar{A}$  and  $O$  are the components of  $P_{\bar{A}}$ . Intuitively, the diagram enforces that each instance of  $A$  participating to  $P$  is in fact participating to  $P_A$ , and hence associated via  $P$  to an instance of  $B$ .
- For each assertion of the form  $A \subseteq \exists P.B$ , we introduce a new binary association  $P_{AB}$  and we construct the portion of diagram shown in Figure 4. Note the proper

multiplicity constraint 1.\* on the participation of  $A$  to  $P_{AB}$ <sup>3</sup>. Intuitively, this implies that for each instance of  $A$ , there exists an instance of  $B$  related to it through  $P_{AB}$ , and hence through  $P$ .

**Lemma 3.3** *Given a primitive  $\mathcal{ALC}^-$  KB  $\mathcal{K}'$ , the size of the UML class diagram  $\mathcal{D}$  constructed as above is linear in the size of  $\mathcal{K}'$ .*

**Lemma 3.4** *An atomic concept  $A$  is satisfiable in an  $\mathcal{ALC}^-$  KB  $\mathcal{K}'$  if and only if the class  $A$  is consistent in the UML class diagram  $\mathcal{D}$  constructed as above.*

*Proof (sketch).* We detail the proof only for  $\mathcal{ALC}^-$  assertions of the form  $A \sqsubseteq \forall P.B$  and  $A \sqsubseteq \exists P.B$ .

“ $\Leftarrow$ ” Let  $\mathcal{J} = (\Delta^{\mathcal{J}}, \cdot^{\mathcal{J}})$  be an instantiation for  $\mathcal{D}$ . We show that  $\mathcal{J}$  is also a model of all assertions in  $\mathcal{K}'$ .

- Each assertion of the form  $A \sqsubseteq \forall P.B$  in  $\mathcal{K}'$  corresponds, in  $\mathcal{D}$ , to the sub-diagram in Figure 3.  $\mathcal{J}$  populates it according to the following constraints:

$$\begin{array}{ll}
 A^{\mathcal{J}} \subseteq O^{\mathcal{J}} & P_A^{\mathcal{J}} \subseteq \overline{A}^{\mathcal{J}} \times O^{\mathcal{J}} \\
 \overline{A}^{\mathcal{J}} \subseteq O^{\mathcal{J}} & P_A^{\mathcal{J}} \subseteq A^{\mathcal{J}} \times B^{\mathcal{J}} \\
 A^{\mathcal{J}} \cap \overline{A}^{\mathcal{J}} = \emptyset & P_A^{\mathcal{J}} \subseteq P^{\mathcal{J}} \\
 B^{\mathcal{J}} \subseteq O^{\mathcal{J}} & P_A^{\mathcal{J}} \subseteq P^{\mathcal{J}} \\
 P^{\mathcal{J}} \subseteq O^{\mathcal{J}} \times O^{\mathcal{J}} & P_A^{\mathcal{J}} \subseteq P_A^{\mathcal{J}} \cup P_{\overline{A}}^{\mathcal{J}}
 \end{array}$$

From the constraints above, we get that  $P_A^{\mathcal{J}} \cap P_{\overline{A}}^{\mathcal{J}} = \emptyset$ . Therefore, if  $x \in A^{\mathcal{J}}$  then for all  $x' \in O^{\mathcal{J}}$  if  $(x, x') \in P^{\mathcal{J}}$  then  $(x, x') \in P_A^{\mathcal{J}}$  and therefore  $x' \in B^{\mathcal{J}}$ , i.e.,  $A^{\mathcal{J}} \subseteq \{x \in O^{\mathcal{J}} \mid \forall x' \in O^{\mathcal{J}}. (x, x') \in P^{\mathcal{J}} \supset x' \in B^{\mathcal{J}}\}$ .

- Each assertion of the form  $A \sqsubseteq \exists P.B$  in  $\mathcal{K}'$  corresponds, in  $\mathcal{D}$ , to the sub-diagram shown in Figure 4.  $\mathcal{J}$  populates it and satisfies the constraints  $A^{\mathcal{J}} \subseteq O^{\mathcal{J}}$ ,  $B^{\mathcal{J}} \subseteq O^{\mathcal{J}}$ ,  $P^{\mathcal{J}} \subseteq O^{\mathcal{J}} \times O^{\mathcal{J}}$ ,  $P_{AB}^{\mathcal{J}} \subseteq P^{\mathcal{J}}$ ,  $P_{AB}^{\mathcal{J}} \subseteq A^{\mathcal{J}} \times B^{\mathcal{J}}$ , and for each  $x \in A^{\mathcal{J}}$  we have that  $\#\{x' \in \Delta^{\mathcal{J}} \mid (x, x') \in P_{AB}^{\mathcal{J}}\} \geq 1$  (mandatory participation constraint). From these we get that for each  $x \in A^{\mathcal{J}}$  there exists  $x' \in O^{\mathcal{J}}$  such that  $(x, x') \in P^{\mathcal{J}}$  and  $x' \in B^{\mathcal{J}}$ , i.e.,  $A^{\mathcal{J}} \subseteq \{x \in O^{\mathcal{J}} \mid \exists x' \in O^{\mathcal{J}}. (x, x') \in P^{\mathcal{J}} \wedge x' \in B^{\mathcal{J}}\}$ .

“ $\Rightarrow$ ” Let  $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$  be a model of  $\mathcal{K}'$  with  $A^{\mathcal{I}} \neq \emptyset$ . We show that it can be seen as an instantiation of  $\mathcal{D}$ , once we assign a suitable extension to the auxiliary classes and roles introduced in the construction of  $\mathcal{D}$ . First, we define  $O^{\mathcal{I}} = \Delta^{\mathcal{I}}$ .

- For each assertion of the form  $A \sqsubseteq \forall P.B$  in  $\mathcal{K}'$ , we have a fragment of  $\mathcal{D}$  as in Figure 3. Let us define:

$$\begin{array}{l}
 - \overline{A}^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus A^{\mathcal{I}} \\
 - P_A^{\mathcal{I}} = \{(x, x') \in P^{\mathcal{I}} \mid x \in A^{\mathcal{I}}\} \\
 - P_{\overline{A}}^{\mathcal{I}} = \{(x, x') \in P^{\mathcal{I}} \mid x \in \overline{A}^{\mathcal{I}}\}
 \end{array}$$

<sup>3</sup>In fact, in the case where we also have the assertion  $A \sqsubseteq \forall P.B$  for some  $B$ , instead of proceeding as in Figure 4, we can simply add the cardinality constraint 1.\* to the association  $P_{AB}$  in Figure 3.

Then, by  $A^{\mathcal{I}} \subseteq \{x \in \Delta^{\mathcal{I}} \mid \forall x' \in \Delta^{\mathcal{I}}. (x, x') \in P^{\mathcal{I}} \supset x' \in B^{\mathcal{I}}\}$ , we get:

$$\begin{array}{ll}
A^{\mathcal{I}} & \subseteq O^{\mathcal{I}} \\
\overline{A}^{\mathcal{I}} & \subseteq O^{\mathcal{I}} \\
A^{\mathcal{I}} \cap \overline{A}^{\mathcal{I}} & = \emptyset \\
B^{\mathcal{I}} & \subseteq O^{\mathcal{I}} \\
P^{\mathcal{I}} & \subseteq O^{\mathcal{I}} \times O^{\mathcal{I}}
\end{array}
\qquad
\begin{array}{ll}
P_A^{\mathcal{I}} & \subseteq A^{\mathcal{I}} \times B^{\mathcal{I}} \\
P^{\mathcal{I}} & \subseteq P_A^{\mathcal{I}} \cup P_{\overline{A}}^{\mathcal{I}} \\
P_A^{\mathcal{I}} & \subseteq P^{\mathcal{I}} \\
P_{\overline{A}}^{\mathcal{I}} & \subseteq P^{\mathcal{I}}
\end{array}$$

thus correctly capturing the fragment of  $\mathcal{D}$ .

- For each assertion of the form  $A \sqsubseteq \exists P.B$  in  $\mathcal{K}'$ , we have a fragment of  $\mathcal{D}$  as in Fig. 4. Let us define  $P_{AB}^{\mathcal{I}} = \{(x, x') \in P^{\mathcal{I}} \mid x \in A^{\mathcal{I}}\}$ . Then, by  $A^{\mathcal{I}} \subseteq \{x \in \Delta^{\mathcal{I}} \mid \exists x' \in \Delta^{\mathcal{I}}. (x, x') \in P^{\mathcal{I}} \wedge x' \in B^{\mathcal{I}}\}$ , we get that for each  $x \in A^{\mathcal{I}}$  we have  $\#\{x' \in \Delta^{\mathcal{I}} \mid (x, x') \in P_{AB}^{\mathcal{I}}\} \geq 1$ , and we have that such an instantiation is correct for the fragment of  $\mathcal{D}$ .  $\square$

By Lemmata 3.1, 3.2, 3.3, 3.4, and EXPTIME-hardness of reasoning in  $\mathcal{ALC}$  knowledge bases [1], we get our hardness result.

**Theorem 3.5** *Class consistency in UML class diagrams is EXPTIME-hard.*

To show our hardness result, we have made no closure assumptions on the diagram. In UML class diagrams, closure assumptions of two forms are considered: (i) all classes not in the same hierarchy are a priori disjoint, and (ii) each object must be an instance of a *single* most specific class. Instead, in our framework we allow for two classes (possibly in a hierarchy) to have common instances, even when they do not have a common subclass.

It is easy to see that the results above continue to hold even if in UML class diagrams we make the assumption of mutual disjointness of classes that are not in a hierarchy. Indeed, it suffices to add to the diagram an auxiliary subclass for each pair of classes that correspond to concepts that are not trivially disjoint (here multiple inheritance is called in). In this way the default disjointness assumption will have no impact on the instantiation of the part of the diagram that is built as above.

## 4 Conclusions

We have shown that reasoning on UML class diagrams can be quite a complex task, since it is EXPTIME-hard. In [4] we show also that it can be reduced to reasoning in a DL KB, and thus is also in EXPTIME. Note that, in doing this, we have considered restricted forms of OCL constraints, which in their full generality have the same power of full first-order logic and hence lead to undecidability. This result suggests that it is highly desirable to provide automated reasoning support for detecting relevant properties of the diagram. The experimental results we reported in [3, 5, 4], while certainly limited and not providing a definitive answer, indicate that current state-of-the-art DL-based systems could serve as a core reasoning engine in advanced CASE tools, even if more research is needed in order to deal with complex UML class diagrams, and to perform reasoning tasks that involve reasoning on keys and identification constraints.

Finally, it is worth noting that the results presented here hold also for other conceptual modeling formalisms typically used in software engineering and databases.

In particular, the EXPTIME-completeness result applies to the Entity-Relationship model enhanced with ISA on entities and relationships [2].

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