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Recent Developments in Structural Sensitivity Analysis

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ABSTRACT

The present paper reviews recent developments in two major areas of structural sensitivity analysis: sensitivity of static and transient response; and sensitivity of vibration and buckling eigenproblems. Recent developments from the standpoint of computational cost, accuracy, and ease of implementation are presented.

In the area of static response, current interest is focused on sensitivity to shape variation and sensitivity of nonlinear response. Two general approaches are used for computing sensitivities: differentiation of the continuum equations followed by discretization, and the reverse approach of discretization followed by differentiation. It is shown that the choice of methods has important accuracy and implementation implications.

In the area of eigenproblem sensitivity, there is a great deal of interest and significant progress in sensitivity of problems with repeated eigenvalues. The paper raises the issue of differentiability and continuity that is inherent to the repeated eigenvalue case.

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INTRODUCTION

The past few years saw vigorous activity in sensitivity analysis concerned with the calculation of derivatives of engineering systems response with respect to problem parameters. Some of the engineering fields include control systems (e.g. Herrera-Vaillard et al., 1986, Freudenberg et al., 1982), flow of chemically reaching systems (e.g. Raiszadek and Dwyer, 1985, Reuven et al., 1986), supersonic flow (e.g. Wacholder and Dayan, 1984) and heat conduction in solids (e.g. Santos, 1988). There is also interest in interdisciplinary sensitivity calculations (Sobiesczanski-Sobieski, 1988 and Sobiesczanski-Sobieski, et al. 1988) and in automating the process of differentiating complex algebraic expressions (Wexler, 1987). For structural applications, sensitivity derivatives are not only important for design optimization, but also for system identification (e.g. Ibrahim, 1987) and statistical structural analysis (e.g. Nakagiri, 1987, Liu et al., 1988). There has been great interest in developing methods for structural sensitivity analysis. Recent surveys by Adelman and Haftka (1986) and Haftka and Grandhi (1986) provide reviews of the field up to 1985. However, the strong activity in the structural sensitivity analysis since then has resulted in significant developments. The purpose of the present paper is to review some of this recent progress.

One important recent development is an emphasis on implementing sensitivity calculations in general-purpose structural analysis programs (e.g. Choi et al., 1988, Prasad and Emerson, 1982, Giles and Rogers, 1982, Camarda and Adelman, 1984, Herendeen et al., 1986, Nagendra and Fleury, 1987). These programs tend to be large and cumbersome, so that ease-of-implementation becomes a major issue in considering competing sensitivity algorithms. The implementation effort must be weighed against the performance of the algorithms as reflected in their accuracy and computational efficiency. The present paper considers trade-offs between ease-of-implementation and performance.

The paper is divided into two major sections. The first deals with both the static and transient response and the second deals with eigenvalue problems. This division is motivated by the fact that eigenvalue-sensitivity problems arising in vibration and damping problems require a different class of solution methods from those used in static and dynamic response sensitivity analysis.

SENSITIVITY OF STATIC AND TRANSIENT RESPONSE

Finite-Difference Implementation

The easiest method to implement for calculating response derivatives is the finite difference approach. Consider, for example, a displacement field $U(x)$ which depends on a structural parameter x. The derivative U' at $x = x_o$ can be approximated by first-order forward differences as

$$U' = \frac{U(x_o + \Delta x) - U(x_o)}{\Delta x} - \frac{\Delta x}{2} U''(x_o + \zeta \Delta x) \qquad (1)$$

$$0 \leq \zeta \leq 1$$

The first term in Eq. (1) is the forward-difference approximation, and the second term is the truncation error. To minimize the truncation error it is desirable to reduce the step-size Δx. However, a small step size amplifies the algorithmic and round-off errors in $U(x_o)$ and $U(x_o + \Delta x)$ – the so-called condition errors. This is the step-size dilemma whereby a large step size generates large truncation errors and a small step size large condition errors. It is possible to find an optimum step-size (see, Gill et al., 1983 and Iott et al., 1985). However, in some cases no step size gives acceptable errors. In that case, it is recommended that the central-difference approximation be used

$$U' = \frac{U(x_o + \Delta x) - U(x_o - \Delta x)}{2\Delta x} - \frac{(\Delta X)^2}{6} U'''(x_o + \zeta\Delta x) \qquad (2)$$

$$0 \leq \zeta \leq 1$$

The second term in Eq. (2) is the truncation error associated with the central-difference approximation. The central difference approximation typically allows a larger Δx for the same value of the truncation error, and so alleviates problems associated with large condition errors.

The problem of large condition errors can be particularly severe when U is obtained via an iterative process. Consider, for example, a case where U is obtained from the iterative solution of a system of algebraic equations

$$F(U,x) = 0 \qquad (3)$$

representing for example the equations of static structural equilibrium. Assume that $\bar{U}_o$ is the solution for the displacement obtained for $x = x_o$ when the iterative process is deemed to have converged. It is tempting to start the solution for $x = x_o+\Delta x$ from $\bar{U}_o$, but this can lead to large condition errors in the derivatives. The reason is that the iteration simultaneously accounts for the change in x and also for the residual error in $\bar{U}_o$. Haftka (1985) suggested a solution for this difficulty. For $x+\Delta x$, instead of re-solving Eq. (3), we solve

$$F(U,x_o + \Delta x) - F(\bar{U}_o,x_o) = 0 \qquad (4)$$

for an approximation to $U(x_o + \Delta x)$.

The finite-difference calculation of derivatives is easy to implement and is, therefore, quite popular. There is, however, a general impression that the approach is very computationally expensive as compared to analytical and semi-analytical approaches. This is true for static problems where most of the computational cost is associated with displacement field solution. However, in static problems where stress recovery is a major computational ingredient, and in transient response, the forward-difference method is competitive (e.g. Haftka and Malkus, 1981), and is the method of choice provided the accuracy is acceptable.

Discrete Analytical and Semi-Analytical Sensitivity

Most widely-used structural analysis programs discretize the equations of equilibrium using assumed displacement fields. In the static linear case these equations are written as

$$KU = F \tag{5}$$

where K is the stiffness matrix and F the load vector. We can differentiate Eq. (15) to obtain

$$KU' = -K'U + F' \tag{6}$$

The solution of Eq. (6) for U' is the discrete version of the direct method. For calculating the derivative of a function of U, g(U) it may be more efficient to use the adjoint method

$$g' = -\Lambda^T(K'U - F') \tag{7}$$

where the adjoint vector Λ is the solution of

$$K\Lambda = \left(\frac{dg}{dU}\right)^T \tag{8}$$

Belegundu (1985) showed that Λ can be interpreted as the Lagrange multiplier for the constraint of Eq. (5) when g' is calculated, so that

$$\Lambda = \frac{dg}{dF} \tag{9}$$

There have been several enhancements and generalizations of the discrete approach in recent years. Atrek (1985) suggested a simplification for truss structures. Mota Soares and Pereira Leal (1987) generalized it to mixed elements using the Hellinger-Reissner variational functional. Nguyen

(1987a,b) formulated the sensitivity calculations when multilevel substructuring is employed. Ryu et al. (1985) present a generalization of the direct and adjoint methods to nonlinear analysis using the tangent stiffness matrix.

One major concern receiving much attention is the calculation of the right-hand-side of Eq. (6) the so-called "pseudo load". If this load is applied to the structure then the resulting displacement field is equal to U'. The calculation of this vector is cumbersome even for some sizing variables (e.g. Yuan and Wu, 1988). It is particularly difficult for shape design variables, because analytical derivatives for K' are not easy to obtain. Wang et al. (1985), Braibant and Fleury (1984) and Braibant (1986) obtained derivatives of the stiffness matrix for general shape variables. However, for implementation in general-purpose structural analysis packages a simple and easy-to-implement approach was required - the semi-analytical method.

The semi-analytical method is based on finite-difference evaluations of K' and F' in the calculation of the pseudo load. Because it combines ease of implementation with computational efficiency it has become a very popular method and is implemented in NASTRAN (Nagendra and Fleury, 1987) EAL (via runstreams, Camarda and Adelman, 1987) and other finite element programs. While the method has been used for many years, the name "semi-analytical" was coined only recently. It is also called the quasi-analytical method (Cheng and Yingwei, 1987). The derivative K' can be calculated at the system level, or the pseudo load can be calculated at the element level (e.g. Rajan and Budiman, 1987, Belegundu and Rajan, 1988).

The semi-analytical method works very well for sizing-type variables such as cross-sectional areas of bars or plate thicknesses. However, for shape design variables, the truncation error associated with the finite-difference approximation of K' can be substantial. Mild problems were reported by Cheng and Yingwei (1987) for truss and 3-D solid examples, by Yang and Botkin (1986) for plane stress problems, and very large errors were encountered for beam problems by Barthelemy et al. (1986) and Pedersen et al. (1987). The source of the problem was traced by Barthelemy and Haftka (1988) to the basic concept of the pseudo load. As noted before, it is the load that must be applied to the structure to produce the sensitivity field. In many cases of shape variation the sensitivity field is not a reasonable displacement field for the structure and its boundary conditions. For example, for beam- or plate-like structures the sensitivity U' to a length dimension is dominated by shear rather than bending. To produce these unlikely shear-dominated fields the pseudo load has to include large self-cancelling components. Small truncation errors in these components then get amplified into large errors in U'. Barthelemy and Haftka (1988) provide an error index that can be used to detect cases with large errors and correct the errors in some instances. It should be noted, however, that even for shape variables there are many cases where the semi-analytical method provides excellent accuracy (e.g. Liefooghe et al., 1988).

The semi-analytical method was also applied to transient problems and found to work well for sizing-type design variables (Greene and Haftka, 1988). Similarly, the method was successfully applied to aerodynamic sensitivity calculations (Murthy and Kaza, 1988).

The use of critical point constraints for design subject to constraints on transient response attracts interest because of its efficiency for sensitivity calculations (e.g. Haftka and Kamat, 1985, Grandhi et al., 1986, Arora and Hsieh, 1986, Tseng and Arora, 1988, Greene and Haftka, 1988).

Other aspects of transient sensitivity calculation investigated recently include the effect of general boundary conditions (Hsieh and Arora, 1985), and the calculation of derivatives of transition times (Chang and Chou, 1988).

Variational and Continuum-Based Sensitivity

Most general-purpose structural analysis programs are not easily amenable to implementation of discrete sensitivity calculations. The calculation of the pseudo load typically requires intimate knowledge of and access to the source code of these programs. But these source codes are typically inaccessible and very complex. Therefore there is great interest in sensitivity calculations based on pre- or post-processing operations which require only minimal knowledge of the structural analysis code. This is typically afforded by sensitivity formulations which operate on the equations of structural response before they are discretized. Often this is accomplished via a virtual-work formulation of the equations of equilibrium.

This approach is particularly simple for calculating sensitivities with respect to sizing or stiffness variables. We write the strain-displacement, stress-strain and equilibrium equations as

$$\begin{aligned} \varepsilon &= L_1(u) \\ \sigma &= D(\varepsilon - \varepsilon^i) \\ \sigma \cdot \delta\varepsilon &= f \cdot \delta u \end{aligned} \qquad (10)$$

where u, ε and σ denote the displacement, strain and stress field, respectively, L_1 is a linear differential operator, D is the material stiffness matrix, ε^i the initial strain field, f is the applied load field, and a dot between two quantities denotes a scalar product integrated over the structure. If ε and σ are interpreted as generalized strain and stress fields Eq. (10) applies to one and two dimensional elements such as beams and shells as well as to three dimensional solids. We can now differentiate Eq. (10) with respect to a stiffness parameter to get

$$\varepsilon' = L_1(u')$$
$$\sigma' = D[\varepsilon' + D^{-1}D'(\varepsilon - \varepsilon^i)] \qquad (11)$$
$$\sigma' \cdot \delta\varepsilon = 0$$

where a prime denotes a derivative with respect to the stiffness parameters. Comparison of Eqs. (10) and (11) indicates that the sensitivity field $u', \varepsilon', \sigma'$ can be obtained by loading the structure by an initial strain field equal to $-D^{-1}D'(\varepsilon - \varepsilon^i)$. This applies regardless of the structural analysis program used for the analysis. Barthelemy et al. (1988) demonstrated this approach for truss, plane stress and plate problems, using the EAL finite-element program and the FASOR shell-of-revolution code. Equation (11) represents the direct approach to the derivative calculation. Application of the adjoint approach typically leads to integrals that can be calculated by adding adjoint loads and post-processing the output of the structural analysis program. This has been demonstrated by Barthelemy et al. (1988), Chenais (1987, 1988), Choi and Seong (1986a and 1986b), Chon (1987), Dopker and Choi (1987), Haftka and Mróz (1986), Santos and Choi (1987) and by Choi et al. (1988) using several finite element programs including EAL and ANSYS. Similar implementation of the direct and adjoint method for thermal sensitivity calculations in ANSYS was presented by Santos (1988).

There also have been several derivations of continuum-based sensitivity analysis which did not address implementation issues, including Dems and Mróz (1985), Mróz (1987) and Sokolowski and Zolesio (1987). These continuum derivations have also been extended beyond linear elasticity to thermoelasticity problems (Dems, 1987b, Dems and Mróz, 1987, Meric, 1986, 1987b) and thermal problems (Dems, 1986, 1987a, Hou and Sheen, 1988).

The focus of attention in variational and continuum-based sensitivity is presently shifting to nonlinear and transient problems. Most papers address geometrical nonlinearity including Arora and Wu (1987), Barthelemy et al. (1988), Choi and Santos (1987), Haber (1987), while others including Mróz et al. (1985), Cardoso and Arora (1987), Wu and Arora (1987), Tsay and Arora (1988) and Szefer et al. (1988) include material nonlinearity, albeit for elastic behavior. Another type of nonlinearity is that introduced by unilateral constraints. Sensitivity analysis for plates with unilateral constraints was performed by Bendsoe et al. (1985), Sokolowski and Zolesio (1987) and Bendsoe and Sokolowski (1987). Sensitivity calculations in transient response has been addressed by Dems and Mróz (1987), Meric (1987c, 1988) and Wuu et al. (1986).

Shape Sensitivity Accuracy Problems

Sensitivity derivatives with respect to shape appear to be much more prone to accuracy problems than calculations of sensitivity with respect to sizing variables. When the discrete approach is employed, these accuracy problems manifest themselves in the semi-analytical method as noted in the section of the discrete approach. In continuum based derivations the adjoint method typically leads to surface integrals (e.g. Choi and Haug, 1983, Dems and Mróz, 1984). These integrals have been found to be poorly evaluated by finite element programs because such programs rarely provide accurate boundary values for stresses and strains. The difficulty can be particularly acute in problems of variations in interface boundaries because of the very high stress gradients often encountered near such boundaries (see Dems and Haftka, 1988).

It is possible to move surface integrals away from interface boundaries by utilizing conservation rules (see Dems and Mróz, 1986), but this is a problem-dependent remedy which may not be easy to implement in general-purpose codes. Instead, the standard approach is to transform the surface integrals to domain integrals which are more adequately handled by finite element programs (see for example, Choi, 1987, Choi and Seong, 1986, Hou et al., 1986, Yang and Botkin, 1987). The domain integral approach requires the definition of a shape "velocity" field in the entire domain which is not unique. Approaches which limit the shape change to regions near the boundary are computationally more efficient, however, such a choice may compromise accuracy. Seong and Choi, 1987, studied the tradeoff inherent in the selection of the depth of the region affected by boundary changes. The problem is encountered also for discrete sensitivity calculation, and Botkin (1988) reports using only one-element-deep sensitivity calculations.

While domain integrals have proved to be more accurate than surface integrals for adjoint shape sensitivity, it is not clear that they completely eliminate accuracy problems. It has been shown (e.g. Yang and Botkin, 1987) that these methods are equivalent to the discrete approach. It can be expected, therefore, that problems which have large errors with the semi-analytical method may also be sensitive to the details of the numerical implementation of the domain integrals.

Boundary element methods provide high accuracy of response on the boundary and are, therefore, ideally suited for the surface adjoint method. Consequently, there is growing interest in the use of boundary element methods for calculating shape sensitivity (e.g. Mota Soares et al., 1987, Meric, 1987a, Hou and Sheen, 1988, Kwak and Choi, 1988, Kane and Saigal, 1988, Barone and Yang, 1988).

SENSITIVITY ANALYSIS FOR EIGENVALUE PROBLEMS

Real Symmetric Eigenvalue Problems - Distinct Eigenvalues

The discretized eigenvalue problem associated with linear vibration or buckling is

$$K\phi - \lambda M\phi = 0 \tag{12}$$

where K is the stiffness matrix, and ϕ is the vibration or buckling mode shape. For vibration problems M is the mass matrix and λ the square of the frequency. For buckling problems M is the geometric stiffness matrix and λ the buckling load. The eigenvector ϕ is typically normalized as

$$\phi^T M\phi = 1 \tag{13}$$

When the eigenvalues are distinct each derivative is given by

$$\lambda' = \phi^T(K' - \lambda M')\phi \tag{14}$$

where the derivatives of K and M are often calculated by finite differences (a semi-analytical implementation, e.g. Sutter et al., 1986). To obtain the derivative of the eigenvector we can use the direct method and differentiate Eq. (12) to obtain

$$(K - \lambda M)\phi' = -(K' - \lambda M')\phi + \lambda' M\phi \tag{15}$$

Equation (15) is singular, and cannot be solved directly. Nelson (1976) developed a solution procedure which begins with a temporary replacement of the normalization condition, Eq. (13), by a condition that the largest component of ϕ is unity,

$$\phi_m = 1 \tag{16}$$

Removing the mth row and column from Eq. (15) results in a nonsingular equation which is solved for a particular solution $\bar{\phi}'$. The complementary solution to Eq. (15) is ϕ therefore the general solution to Eq. (15) subject to the condition of Eq. (16) is

$$\phi' = \bar{\phi}' + c\phi \tag{17}$$

The undetermined coefficient c can be obtained by substituting Eq. (17) into the derivative of Eq. (13).

The derivative of the eigenvectors can also be obtained by the modal (adjoint) approach (e.g. Fox and Kapoor, 1968) whereby we expand ϕ' in terms of the eigenvectors. Thus

$$\phi' = \sum c_i \phi^i \tag{18}$$

For large problems the modal approach is of practical value only if a good approximation to ϕ' can be obtained with a relatively small number of terms in Eq. (18). Recently, Wang (1985) proposed a modification of the modal method to accelerate the convergence by adding a so-called psuedo-static term to the expansion of Eq. (18) to obtain a modified modal method

$$\phi' = K^{-1}\left[-(K' - \lambda M')\phi + \lambda' M\phi\right] + \sum_i d_i \phi^i \tag{19}$$

This approach is analogous to the mode acceleration method in structural dynamics. Sutter et al. (1986) reported substantial improvements in convergence of this modified modal method over the regular modal method. For example, the derivative of the first mode shape of a finite element model of a cantilever beam with respect to the root thickness was obtained using 21 modes with the modal method and only two modes with the modified modal method.

It is also possible to use an iterative approach to calculate the derivative of eigenvectors. The basic iterative process was suggested by Rudisill and Chu (1975) for the system

$$A\phi - \lambda\phi = 0 \tag{20}$$

as

$$\lambda^{(k)} = \phi^T A'\phi + \phi^T(A - \lambda I)\phi^{(k)'}$$
$$\phi^{(k+1)'} = [(A' - \lambda^{(k)} I)\phi - A\phi^{(k)'}]/\lambda \tag{21}$$

Where $\lambda^{(k)}$, $\phi^{(k)'}$ are the kth iterate for λ and ϕ'. The iteration converges, albeit slowly for the largest eigenvalue of A (Andrew, 1978), and can be applied to the vibration or buckling problem in Eq. (12) by using $A = K^{-1}M$. Recently Tan (1986, 1987) proposed ways of accelerating the convergence and applying the process to other than the largest eigenvalue.

For the buckling problem the calculation of M' is a computational issue, because the geometric stiffness matrix, M, depends on the prebuckling stresses. An adjoint method which avoids the need to calculate the sensitivity of the prebuckling stress field was proposed by Nogis (1986). The adjoint field Λ satisfies the equation

$$K\Lambda = -\left[\frac{\partial(\phi^T M\phi)}{\partial U_p}\right]^T \tag{22}$$

where U_p denotes the prebuckling displacements and ϕ the buckling mode. Using Λ we get

$$\lambda' = \phi^T K'\phi + \lambda\Lambda^T K' U_p + \lambda\phi^T M'\phi \tag{23}$$

Equation (23) is valid only for the case of a load vector independent of x.

There has also been substantial work on the sensitivity analysis of the eigenproblem using a continuum or variational approach (see Haug, Choi and Komkov, 1986 for an excellent discussion). For the buckling problem, the adjoint field can be shown to be identical to the second-order postbuckling field introduced by Koiter (Haftka et al., 1988). Implementation of buckling sensitivity in general purpose structural analysis programs are reported by Haftka, Cohen and Mróz (1988) and Cohen and Haftka (1988). Pierre (1987) developed a procedure for accounting for the effect of changing natural boundary conditions in a general problem and demonstrated it by rod and beam examples.

For the case of vibration (and buckling) eigenvalue problems of one-dimensional structures, a recent paper has developed an expression for the derivative of the nodal location of the mode shape, see Pritchard et al. (1988). Denoting the mode shape as $\phi(x,v)$ where x is the coordinate and v a design variable, the equation for the nodal location x_n is

$$\frac{\partial x_n}{\partial v} = - \frac{\partial\phi/\partial v}{\partial\phi/\partial x}\Big]_{x=x_n} \tag{24}$$

Finally, in buckling problems it is possible to calculate the buckling load without using eigenvalues (e.g. Haftka, 1983). This can be of particular interest for calculating the sensitivity of limit-load type buckling (Kamat, 1987).

General Eigenvalue Problems

The damped vibrations of structures, including the effects of aerodynamic forces and active control systems, result in non-hermitian eigenvalue problem with complex eigenvalues and eigenvectors. The vibration problem is complex for rotating structures. Typically an eigenvector ϕ of the damped system is written as a linear combination of the undamped modes ϕ^i

$$\phi = \sum q_i \phi^i \tag{25}$$

where only a small number of modes are often required for a good approximation. In the case of point actuators it may be desirable to augment the vibration modes by "actuator modes", see Sandridge and Haftka (1987).

The reduced order eigenvalue problem obtained by using Eq. (25) may be written as

$$(A-\lambda I)Q = 0 \tag{26}$$

where A is, in general, complex. The normalization condition for this problem is more important than in the real symmetric case. A condition of the form of

$$Q^TQ = 1 \tag{27}$$

is not acceptable as can be seen from the case $Q = (1,i)$. A discussion of normalization conditions and their effects on eigenvector derivatives is given by Murthy and Haftka (1988) and Lim et al. (1987).

Efficiency and implementation considerations are different in the general case because typically the eigenvalue problem, Eq. (26), is small and dense while the eigenvalue problem of Eq. (12) is large and sparse. It is reasonable for the solution of a small general eigenproblem to calculate all the eigenvectors and use the adjoint (modal) approach as in Chen and Pan (1986). An efficiency study of the direct versus the adjoint approach is given by Murthy and Haftka (1988). An approach similar to Nelson's method based on the generalized Benrose inverse (but which does not preserve bandedness) was suggested by Chen and Wei (1985). Rajan et al. (1986) provide a discussion of derivative calculations on the case of rotor bearing systems.

For the general case it is often desirable to calculate singular values rather than eigenvalues. A singular value σ of a matrix A satisfies

$$AU = \sigma V \qquad A^*V = \sigma U \tag{28}$$

Where an asterisk denotes the hermitian transpose and the singular vectors U and V are normalized as

$$U^*U = 1 \qquad V^*V = 1 \tag{29}$$

Freudenberg et al. (1982) show that

$$\sigma' = R_e[V^*A'U] \tag{30}$$

Singular value sensitivities have been used by Mukhopadhyay and Newsom (1984) and Herrera-Vaillard et al. (1986) for studying the sensitivity and robustness of control systems.

Repeated Eigenvalues

For the real symmetric case a generalization of Nelson's method which preserves the bandedness of the matrix was obtained by Ojalvo (1987) and amended by Mills-Curran (1988) and Dailey (1988). These methods compute the derivatives of the m eigenvectors corresponding to eigenvalues of multiplicity m. As stated by Dailey, when the eigenvalues are repeated and a design variable is perturbed, the eigenvectors "split" into as many as m distinct eigenvectors. We seek the derivatives of these distinct eigenvectors which "appear" with design variable perturbation. Using Dailey's notation, define the eigenvalue problem

$$KX = \Lambda MX \tag{31}$$

where X contains the m eigenvectors cited previously

$$\Lambda = \lambda I \tag{32}$$

λ is the repeated eigenvalue

I is the identity matrix of order m

The eigenvectors which appear as a result of the splitting are contained in a matrix denoted Z which is related to X as follows

$$Z = X\Upsilon \tag{33}$$

where Υ is a set of orthogonal vectors to be determined. The technique for calculating Z' as contained in Dailey is outlined next. The vector Υ and the derivative of the multiple eigenvalue Λ' are obtained as solutions of the following eigenvalue problem

$$D\Upsilon = \Upsilon\Lambda' \tag{34}$$

where

$$D = X^T(K' - \lambda M')X \tag{35}$$

Next in a manner analogus to Nelson (1976) let

$$Z' = V + ZC \tag{36}$$

where V is the solution to

$$(K - \lambda M)V = (\lambda M' - K')Z + MZ\Lambda' \tag{37}$$

(numerically obtained by removing the m rows and columns associated with the m largest components of Z) and C is a matrix which is obtained as the solution to the equation

$$C\Lambda' - \Lambda'C + \frac{1}{2}\Lambda'' = -V^TMZ - Z^TMV - Z^TM'Z \tag{38}$$

Equation (38) which requires substantial algebraic manipulations for its derivations, determines the matrix C and the matrix of second derivatives of the eigenvalues Λ''. Fortunately Λ'' is diagonal and $C\Lambda' - \Lambda'C$ always has zero on the diagonal. Therefore we can solve for the matrix C separate from Λ'' and the latter matrix only needs to be calculated if it is needed for some other purpose.

For the case of general matrices, first treated by Lancaster (1964), Lim et al. (1988) suggest the use of singular value decomposition for eigenvector derivative calculation. Juang et al. (1988) provide a proof of existence of derivatives of multiple eigenvalues and eigenvectors for nondefective analytic matrices. They differentiate between the cases where the derivatives of the eigenvalues are repeated or nonrepeated and provide an algorithm for calculation of eigenvector derivatives in both cases.

The modal method was also generalized to the case of multiple eigenvalues by Chen and Pan (1986).

Before leaving the topic of derivatives associated with repeated eigenvalues, we note the limited utility of such derivatives. For example the eigenproblem is differentiable in terms of a single parameter but not as a function of several. This may be demonstrated by the example where the matrix

$$A = \begin{bmatrix} 2 + y & x \\ x & 2 \end{bmatrix} \tag{39}$$

The eigenvalues of A are

$$\lambda_{1,2} = 2 + y/2 \pm \sqrt{x^2 + y^2/4} \tag{40}$$

At $x = y = 0$ the eigenvalues are repeated and $\partial\lambda/\partial x = \pm 1$, $\partial\lambda/\partial y = 0,1$. However the eigenvalues are not differentiable as a function of both x and y, that is the relation

$$d\lambda = \frac{\partial\lambda}{\partial x}\,dx + \frac{\partial\lambda}{\partial y}\,dy \tag{41}$$

does not hold. Therefore, the utility of the partial derivatives is questionable. The eigenvectors are also discontinuous at (0,0). This can be checked by noting that at $(\varepsilon,0)$ the eigenvectors are (1,0) and (0,1) and at $(0,\varepsilon)$ they are (1,1) and (1,-1) no matter how small ε is.

Nonlinear Eigenvalue Problems

In flutter and nonlinear vibration problems we encounter eigenvalue problems of a more general form. Bindolino and Mantegazza (1987) consider the aeroelastic response which produces a transcendental eigenvalue problem of the form

$$A(\lambda)U = 0 \tag{42}$$

When differentiated

$$A\,\frac{dU}{dx} + \frac{d\lambda}{dx}\,\frac{\partial A}{\partial\lambda}\,U = -\,\frac{\partial A}{\partial x}\,U \tag{43}$$

Using the normalizing condition of Eq. (16) we obtain

$$\frac{du_m}{dx} = 0 \tag{44}$$

Equations (43) and (44) can be solved together for dU/dx and $d\lambda/dx$. Instead, Bindolino and Mantegazza suggest the use of the adjoint method, using the left eigenvector V satisfying

$$V^T A = 0$$

$$v_m = 1 \tag{45}$$

to obtain

$$\frac{d\lambda}{dx} = -\frac{V^T \frac{\partial A}{\partial x} U}{V^T \frac{\partial A}{\partial \lambda} U} \tag{46}$$

Jankovic (1988) used the direct approach to obtain higher-order derivatives of the eigenvalues and eigenvectors. Hou et al. (1985) and Hou et al. (1987) consider nonlinear vibrations leading to the eigenvalue problem of the form

$$(K + G(U))U - \lambda MU = 0 \tag{47}$$

with the normalization condition of Eq. (13). Differentiating Eq. (47) we get

$$(K_T - \lambda M)U' - \lambda' MU = [G'(U) + \lambda M]U \tag{48}$$

where the tangent stiffness matrix K_T is

$$K_T = K + \frac{\partial G}{\partial U} U \tag{49}$$

Equation (48) can now be solved using Nelson's method. It is also possible to use the left eigenvector satisfying

$$V^T(K_T - \lambda M) = 0 \tag{50}$$

to get

$$\lambda' = -\frac{V^T[G'(U) + \lambda M']U}{U^T MU} \tag{51}$$

REFERENCES

Adelman, H.M. and Haftka, R.T., 1986, "Sensitivity Analysis for Discrete Structural Systems," AIAA Journal, 24, pp. 823-832.

Andrew, A.L., 1978, "Convergence of an Iterative Method for Derivatives of Eigensystems," Journal of Computational Physics, 26, pp. 107-112.

Arora, J.S. and Hsieh, C.C., 1986, "Algorithms for Point-Wise State Variable Constraints in Structural Optimization," Computers and Structures, 22, No. 3, pp. 225-238.

Arora, J.S. and Wu, C.C., 1987, "Design Sensitivity Analysis and Optimization of Nonlinear Structures," in Computer Aided Optimal Design: Structures and Mechanical Systems (C.A. Mota Soares, Editor), Springer-Verlag, pp. 589-604.

Atrek, E., 1985, "Theorems of Structural Variations: A Simplification," International Journal for Numerical Methods in Engineering, 21, pp. 481-485.

Barone, M.R. and Yang, R.-J., 1988, "Boundary Integral Equations for Recovery of Design Sensitivity in Shape Optimization," General Motors Research Laboratories Report, GMR-5816, Warren, Michigan, to appear in AIAA Journal.

Barthelemy, B., Chon, C.T. and Haftka, R.T., 1986, "Sensitivity Approximation of Static Structural Response," paper presented at the First World Congress of Computational Mechanics, Austin, Texas, Sept. 6-8.

Barthelemy, B. and Haftka, R.T., 1988, "Accuracy Analysis of the Semi-Analytical Method for Shape Sensitivity Calculation," AIAA Paper 88-2284, Proceedings of the AIAA/ASME/ASCE/AHS/ASC 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, Part 1, pp. 572-581.

Barthelemy, B., Haftka, R.T. and Cohen, G.A., 1988, "Physically Based Sensitivity Derivatives for Structural Analysis Programs," Computational Mechanics, in press.

Belegundu, A.D., 1985, "Lagrangian Approach to Design Sensitivity Analysis," ASCE Journal of Engineering Mechanics, 111, No. 5, pp. 680-695.

Belegundu, A.D. and Rajan, S.D., 1988, "Shape Optimal Design Using Isoparametric Elements," AIAA Paper 88-2300, Proceedings, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, Part 2, pp. 696-701.

Bendsoe, M.P., Olhoff, N. and Sokolowski, J., 1985, "Sensitivity Analysis of Elasticity with Unilateral Constraints," Journal of Structural Mechanics, 13, pp. 201-222.

Bendsoe, M.P. and Sokolowski, J., 1987, "Sensitivity Analysis and Optimal Design of Elastic Plates with Unilateral Point Supports," Mechanics of Structures and Machines, 15, 3, pp. 383-393.

Bindolino, G. and Mantegazza, P., 1987, "Aeroelastic Derivatives as a Sensitivity Analysis of Nonlinear Equations," AIAA Journal, 25, No. 8, pp. 1145-1146.

Botkin, M.E., 1988, "Shape Optimization with Multiple Loading Conditions and Mesh Refinement," AIAA Paper 88-2299, Proceedings of the AIAA/ASME/ASCE/AHS/ASC 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, Part 2, pp. 689-695.

Braibant, V. and Fleury, C., 1984, "Shape Optimal Design Using B-Splines," Computer Methods in Applied Mechanics and Engineering, 44, pp. 247-267.

Braibant, V., 1986, "Shape Sensitivity by Finite Elements," Journal of Structural Mechanics," 14, No. 2, pp. 209-228.

Camarda, C.J. and Adelman, H.M., 1984, "Static and Dynamic Structural Sensitivity Derivatives Calculations in the Finite-Element Based Engineering Analysis Language (EAL) System," NASA TM-85743.

Cardoso, J.B. and Arora, J.S., 1987, "Design Sensitivity Analysis of Nonlinear Structural Response," in Sensitivity Analysis in Eingineering (H.M. Adelman and R.T. Haftka, Editors), NASA CP-2457, to be published in AIAA Journal.

Chang, C-O. and Chou, C-S., 1988, "Dynamic Analysis and Optimal Design of the Viscous Ring Nutation Damper for a Freely Precessing Gyroscope," AIAA Paper 88-2264, Proceedings, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, Part 1, pp. 411-419.

Chen, S-H. and Pan, H.H., 1986, "Design Sensitivity Analysis of Vibration Modes by Finite Element Perturbation," Proceedings of the 4th International Modal Analysis Conference, Los Angeles, CA, I, pp. 38-43.

Chen, S-Y. and Wei, F-S., 1985, "Systematic Approach for Eigensystem Analysis," AIAA Paper 85-0635, Proceedings of the AIAA/ASME/ASCE/AHS 26th Structures, Structural Dynamics and Materials Conference, Orlando, Florida, April 15-17, Part 2, pp. 178-183.

Chenais, D., 1987, "Shape Optimization in Shell Theory: Design Sensitivity of the Continuous Problem," Eng. Opt. 11, pp. 289-303.

Chenais, D., 1988, "Optimal Design of Midsurface of Shells: Differentiability Proof and Sensitivity Computation," Applied Mathematics and Applications, to be published.

Cheng, G. and Yingwei, L., 1987, "A New Computation Scheme for Sensitivity Analysis," Engineering Optimization, 12, pp. 219-234.

Choi, K.K., 1987, "Shape Design Sensitivity Analysis and Optimal Design of Structural Systems," in Computer Aided Optimal Design: Structures and Mechanical Systems (C.A. Mota Soares, Editor), Springer-Verlag, pp. 439-492.

Choi, K.K. and Haug, E.J., 1983, "Shape Design Sensitivity Analysis of Elastic Structures," Journal of Structural Mechanics, 11, No. 2, pp. 231-269.

Choi, K.K. and Santos, J.L.T., 1987, "Design Sensitivity Analysis of Non-Linear Structural Systems Part I: Theory," International Journal for Numerical Methods in Engineering, 24, pp. 2039-2055. Part II, Numerical Method, to appear, 1988.

Choi, K.K., Santos, J.L.T. and Frederick, M.C., 1988, "Implementation of Design Sensitivity Analysis with Existing Finite Element Codes," ASME Journal of Mechanisms, Transmissions, and Automation in Design.

Choi, K.K. and Seong, H.G., 1986a, "A Domain Method for Shape Design Sensitivity of Built-Up Structures," Computer Methods in Applied Mechanics and Engineering, 57, pp. 1-15.

Choi, K.K. and Seong, H.G., 1986b, "Design Component Method for Sensitivity Analysis of Built-Up Structures," Journal of Structural Mechanics, 14, No. 3, pp. 379-399.

Chon, C.T., 1987, "Sensitivity of Total Stain Energy of a Vehicle Structure to Local Joint Stiffness," AIAA Journal, 25, No. 10, pp. 1391-1395.

Cohen, G.A. and Haftka, R.T., 1988, "Sensitivity of Buckling Loads of Anisotropic Shells of Revolution to Geometric Imperfections and Design Changes," submitted to Computers and Structures.

Dailey, R.L., 1988, "Eigenvector Derivatives with Repeated Eigenvalues," AIAA Journal, to be published.

Dems, K., 1986, "Sensitivity Analysis in Thermal Problems - I: Variation of Material Parameters within a Fixed Domain," Journal of Thermal Stresses, 9, pp. 303-324.

Dems, K., 1987a, "Sensitivity Analysis in Thermal Problems - II: Structure Shape Variation," 10, No. 1, pp. 1-16.

Dems, K., 1987b, "Sensitivity Analysis in Thermoelasticity Problems," Computer Aided Optimal Design: Structures and Mechanical Systems (C.A. Mota Soares, Editor), Springer-Verlag, pp. 563-572.

Dems, K. and Haftka, R.T., 1988, "Two Approaches to Sensitivity Analysis for Shape Variation of Structures," Mechanics of Structures and Machines.

Dems, K. and Mróz, Z., 1984, "Variational Approach by Means of Adjoint Systems to Structural Optimization and Sensitivity Analysis - II Structural Shape Variation," International Journal of Solids and Structures, 20, No. 6, pp. 527-552.

Dems, K. and Mróz, Z., 1985, "Variational Approach to First- and Second-Order Sensitivity Analysis of Elastic Structures," International Journal for Numerical Methods in Engineering, 21, pp. 637-661.

Dems, K. and Mróz, Z., 1986, "On a Class of Conservation Rules Associated with Sensitivity Analysis in Linear Elasticity," International Journal of Solids and Structures, 22, No. 7, pp. 737-758.

Dems, K. and Mróz, Z., 1987, "Variational Approach to Sensitivity Analysis in Thermoelasticity," Journal of Thermal Stresses, 10, pp. 283-306.

Dopker, B. and Choi, K.K., 1987, "Sizing and Shape Sensitivity Analysis using a Hybrid Finite Element Code," Finite Elements in Analysis and Design, 3, pp. 315-331.

Fox, R.L. and Kapoor, M.P., 1968, "Rates of Change of Eigenvalues and Eigenvectors," AIAA Journal, 6, No. 12, pp. 2426-2429.

Freudenberg, J.S., Looze, D.P. and Cruz, J.B., 1982, "Robustness Analysis using Singular Value Sensitivities," Int. J. Control, 35, No. 1, pp. 95-116.

Giles, G.L. and Rogers, J.L., Jr., 1982, "Implementation of Structural Response Sensitivity Calculations in a Large Scale Finite-Element Analysis System," AIAA Paper 82-0714, Proceedings of AIAA/ASME/ASCE/AHS 23rd Structures, Structural Dynamics and Materials Conference, New Orleans, LA, May, Part II, pp. 348-359.

Gill, P.E., Murray, W., Saunders, M.A. and Wright, M.H., 1983, "Computing Forward-Difference Intervals for Numerical Optimization," SIAM J. Sci and Stat. Comput., 4, No. 2, pp. 310-321.

Grandhi, R.V., Haftka, R.T. and Watson, L.T., 1986, "Efficient Identification of Critical Stresses in Structures Subject to Dynamic Loads," Computers and Structures, 22, No. 3, pp. 373-386.

Greene, W.H. and Haftka, R.T., 1988, "Computational Aspects of Sensitivity Calculations in Transient Structural Analysis," NASA TM-100589.

Haug, E.J., Choi, K.K. and Kemkov, V., 1986, Design Sensitivity Analysis of Structural Systems, Academic Press.

Haber, R.B., 1987, "A New Variational Approach to Structural Shape Design Sensitivity Analysis," in Computer Aided Optimal Design: Structures and Mechanical Systems (C.A. Mota Soares, Editor), Springer-Verlag, pp. 573-588.

Haftka, R.T., 1983, "Design for Temperature and Thermal Buckling Constraints Employing Non-Eigenvalue Formulation," Journal of Spacecraft and Rockets, 20, No. 4, pp. 363-367.

Haftka, R.T., 1985, "Sensitivity Calculations for Iteratively Solved Problems," International Journal for Numerical Methods in Engineering, 21, pp. 1535-1546.

Haftka, R.T., Cohen, G.A. and Mróz, Z., 1988, "Derivatives of Buckling Loads and Vibration Frequencies with Respect to Stiffness and Initial Strain Parameters," submitted to Journal of Applied Mechanics.

Haftka, R.T. and Grandhi, R.V., 1986, "Structural Shape Optimization - A Survey," Computer Methods in Applied Mechanics and Engineering, 57, pp. 91-106.

Haftka, R.T. and Kamat, M.P., 1985, Elements of Structural Optimization, Martinus Nijhoff, pp. 167-169.

Haftka, R.T. and Malkus, D.S., 1981, "Calculation of Sensitivity Derivatives in Thermal Problems by Finite Differences," International Journal for Numerical Methods in Engineering, pp. 1811-1821.

Haftka, R.T. and Mróz, Z., 1986, "First- and Second-Order Sensitivity Derivatives of Linear and Nonlinear Structures," AIAA Journal, 24, pp. 1187-1192.

Herendeen, D.L., Hoesly, R.L., Johnson, E.H. and Venkayya, V.B., 1986, "Astros - An Advanced Software Environment for Automated Design," AIAA Paper 86-0856 presented at the AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics and Materials Conference, San Antonio, Texas, May 19-21, Part 1, pp. 59-66.

Herrera-Vaillard, A., Paduano, J. and Downing, D., 1986, "Sensitivity Analysis of Automatic Flight Control Systems Using Singular-Value Concepts," Journal of Guidance, Control and Dynamics, 9, No. 6, pp. 621-626.

Hou, J.W., Chen, J.L. and Sheen, J.S., 1986, "Computational Method for Optimization of Structural Shapes," AIAA Journal, 24, No. 6, pp. 1005-1012.

Hou, J.W., Mei, C. and Xue, Y.X., 1987, "On the Design Sensitivity Analysis of Beams Under Free and Forced Vibrations," AIAA Paper 87-0936, presented at the 28th SDM, April.

Hou, J.W. and Sheen, J.S., 1988, "On the Design Velocity Field in the Domain and Boundary Methods for Shape Optimization," AIAA Paper 88-2238, Proceedings, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20.

Hsieh, C.C. and Arora, J.S., 1985, "Structural Design Sensitivity Analysis with General Boundary Conditions: Dynamic Problem," International Journal for Numerical Methods in Engineering, 21, pp. 267-283.

Ibrahim, R.A., 1987, "Structural Dynamics with Parameter Uncertainties," Applied Mechanics Reviews, 40, No. 3, pp. 309-328.

Iott, J., Haftka, R.T. and Adelman, H.M., 1985, "Selecting Step Sizes in Sensitivity Analysis by Finite Differences," NASA TM 86382, August.

Jankovic, M.S., 1988, "Analytical Solution for the nth Derivatives of Eigenvalues and Eigenvectors for a Nonlinear Eigenvalue Problem," AIAA Journal, 26, 2, pp. 204-205.

Juang, J-N., Ghaemmaghami, P. and Lim, K.B., 1988, "On the Eigenvalues and Eigenvector Derivatives of a Non-Defective Matrix," AIAA Paper 88-2352, Proceedings, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, Part 3, pp. 1124-1131.

Kamat, M.P., 1987, "Optimization of Shallow Arches Against Instability using Sensitivity Derivatives," Finite Elements in Analysis and Design, 3, pp. 277-284.

Kane, J.H. and Saigal, S., 1988, "Design Sensitivity Analysis of Solids Using BEM," Journal of Engineering Mechanics (to appear).

Kwak, B.M. and Choi, J.H., 1988, "Numerical Implementation of Shape Design Sensitivity Analysis by Boundary Integral Equation for Fillet Design Problem," Computational Mechanics '88 (S.N. Atluri and G. Yagawa, Editors), Springer Verlag, 2, pp. 45.V.1 - 45.V.2.

Lancaster, P., 1964, "On Eigenvalues of Matrices Dependent on a Parameter," Numensche Mathematik, 6, 5.

Liefooghe, D., Shyy, Y.K. and Fleury, C., "Shape Sensitivity Analysis Using Low and High Order Finite Elements," AIAA Paper 88-2341, Proceedings, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, Part 3, pp. 1656-1666.

Lim, J. and Chopra, I., 1987, "Design Sensitivity Analysis for an Aeroelastic Optimization of a Helicopter Blade," AIAA Paper 87-0923 Proceedings of the AIAA Dynamics Specialists Conference, Monterey, CA, April, Part 2B, pp. 1093-1102.

Lim, K.B., Juang, J-N. and Ghaemmaghami, P., 1988, "Eigenvector Derivative of Repeated Eigenvalue with Distinct Eigenvalue Derivatives using Singular Value Decomposition", Journal of Guidance, Control and Dynamics, to appear.

Lim, K.B., Junkins, J.L. and Wang, B.P., 1987, "A Re-Examination of Eigenvector Derivatives," Journal of Guidance, Control and Dynamics, 10, No. 6, pp. 581-587.

Liu, W.K., Besterfield, G., Lawrence, M. and Belytscko, T., 1988, "Use of Adjoint Methods in the Probabilistic Finite Element Approach to Fracture Mechanics," AIAA Paper 88-2420, Proceedings, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, 3, pp. 1631-1634.

Meric, R.A., 1986, "Material and Load Optimization of Thermoelastic Solids, Part I: Sensitivity Analysis," Journal of Thermal Stresses, 9, pp. 359-372.

Meric, R.A., 1987a, "Boundary Elements in Shape Design Sensitivity Analysis of Thermoelastic Solids," in Computer Aided Optimal Design: Structures and Mechanical Systems (C.A. Mota Soares, Editor), Springer-Verlag, pp. 643-652.

Meric, R.A., 1987b, "Sensitivity Analysis for a General Performance Criterion in Micropolar Thermoelasticity," International Journal of Engineering Science, 25, No. 3, pp. 265-276.

Meric, R.A., 1987c, "Dynamic Sensitivity Analysis of Material and Load Parameters in Nonlocal Elasticity," International Journal of Engineering Science, 25, No. 7, pp. 915-922.

Meric, R.A., 1988, "Shape Design Sensitivity Analysis of Dynamic Structures," AIAA Journal, 26, 2, pp. 206-212.

Mills-Curran, W.C., 1988, "Calculation of Derivatives for Structures with Repeated Eigenvalues," AIAA Journal, to be published.

Mota Soares, C.A., Pereira Leal, R. and Choi, K.K., 1987, "Boundary Elements in Shape Optimal Design of Structural Components," in Computer Aided Optimal Design: Structures and Mechanical Systems (C.A. Mota Soares, Editor), Springer-Verlag, pp. 605-632.

Mota Soares, C.A. and Pereira Leal, R., 1987, "Mixed Elements in the Sensitivity Analysis of Structures," Eng. Opt. 11, pp. 227-237.

Mróz, Z., 1987, "Sensitivity Analysis and Optimal Design with Account for Varying Shape and Support Conditions," in Computer Aided Optimal Design: Structures and Mechanical Systems (C.A. Mota Soares, Editor), Springer-Verlag, pp. 407-438.

Mróz, Z. and Haftka, R.T., 1988, "Sensitivity of Buckling Loads and Vibration Frequencies of Plates," in the Josef Singer Anniversary Volume, Springer-Verlag.

Mróz, Z., Kamat, M.P. and Plaut, R.H., 1985, "Sensitivity Analysis and Optimal Design of Nonlinear Beams and Plates," Journal of Structural Mechanics, 13, Nos. 3 and 4, pp. 245-266.

Mukhopadhyay, V. and Newsom, J.R., 1984, "A Multiloop System Stability Margin Study using Matrix Singular Values," Journal of Guidance, Control and Dynamics, 7, Sept.-Oct., pp. 582-587.

Murthy, D.V. and Haftka, R.T., 1988, "Derivatives of Eigenvalues and Eigenvectors of a General Complex Matrix," International Journal for Numerical Methods in Engineering, 26, pp. 293-311.

Murthy, D.V. and Kaza, K.R.V., 1988, "Application of a Semianalytical Technique for Sensitivity and Analysis of Unsteady Aerodynamic Computations," AIAA Paper 88-2377, Proceedings, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, Part 3, pp. 1307-1316.

Nagendra, G.P. and Fleury, C., 1987, "Sensitivity and Optimization of Composite Structures using MSC/NASTRAN," in Sensitivity Analysis in Engineering (H.N. Adelman and R.T. Haftka, Editors), NASA CP-2457, pp. 147-166.

Nakagiri, S., 1987, "Fluctuation of Structural Response Why and How," JSME International Journal, 30, March, pp. 369-374.

Nelson, R.B., 1976, "Simplified Calculation of Eigenvector Derivatives," AIAA Journal, 14, No. 9, pp. 1201-1205.

Nguyen, D.T., 1987a, "Multilevel Substructuring Sensitivity Analysis," Computers and Structures, 25, No. 2, pp. 191-202.

Nguyen, D.T., 1987b, "Practical Implementation of an Accurate Method for Multilevel Design Sensitivity Analysis," AIAA Paper 87-0718, presented at the AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, Monterey, CA, April, Part I, pp. 76-87.

Nogis, R., 1986, "Design Sensitivity Analysis for Buckling Load of Elastic Structures," ZAMM, Z. Agnew, Math. Mech., 66, No. 4, pp. 775-776.

Ojalvo, I.U., 1987, "Efficient Computation of Mode-Shape Derivatives for Large Dynamic Systems," AIAA Journal, 25, No. 10, pp. 1386-1390.

Pedersen, P., Cheng, G. and Rasmussen, J., 1987, "On Accuracy Problems for Semi-Analytical Sensitivity Analysis," DCAMM report 367, Technical University of Denmark.

Pierre, C., 1987, "Eigensolution Perturbation for Systems with Perturbed Boundary Conditions," Journal of Sound and Vibration, 112, No. 1, pp. 167-172.

Prasad, B. and Emerson, J.F., 1982, "A General Capability of Design Sensitivity for Finite Element Systems," AIAA Paper 82-0680, Proceedings of the AIAA/ASME/ASCE/AHS 23rd Structures, Structural Dynamics and Materials Conference, New Orleans, LA, May, Part II, pp. 175-186.

Pritchard, J.I., Adelman, H.M. and Haftka, R.T., 1987, "Sensitivity Analysis and Optimization of Nodal Point Placement for Vibration Reduction," Journal of Sound and Vibration, 119, pp. 277-289.

Raiszadek, F. and Dwyer, H.A., 1985, "Sensitivity Analysis of Turbulent Variable Density Round Jet and Diffusion Flame Flows," in Progress in Astronautics and Aeronautics, 105 (J.R. Bowen, J.C. Leyer, and R.I. Soloukhin, Eds.), pp. 3-17, presented at the 10th International Colloquium on Dynamics of Explosions and Reactive Systems, CA, August.

Rajan, M., Nelson, H.D. and Chen, W.J., 1986, "Parameter Sensitivity in the Dynamics of Rotor Bearing Systems," ASME Journal of Vibration, Acoustics, Stress and Reliability in Design, 108, April, pp. 197-206.

Rajan, S.D. and Budiman, J., 1987, "A Study of Two-Dimensional Plane Elasticity Finite Elements for Optimal Design," Mechanics of Structures and Machines, 15, No. 2, pp. 185-207.

Reuven, Y., Smooke, M.D. and Rabitz, H., 1986, "Sensitivity Analysis of Boundary Value Problems: Application to Nonlinear Reaction - Diffusion Systems," Journal of Computational Physics, 64, pp. 27-55.

Rudisill, C.S. and Chu, Y.Y., 1975, "Numerical Methods for Evaluating the Derivatives of Eigenvalues and Eigenvectors," AIAA Journal, 13, pp. 834-837.

Ryu, Y.S., Hariran, M. and Wu, C.C. and Arora, J.S., 1985, "Structural Design Sensitivity of Nonlinear Response," Computers and Structures, 21, No. 1/2, pp. 245-255.

Sandridge, C.A. and Haftka, R.T., 1987, "Accuracy of Derivatives of Control Performance using a Reduced Structural Model," AIAA Paper 87-0905, Proceedings, AIAA/ASME/ASCE/AHS 28th Structures, Structural Dynamics and Materials Conference, and AIAA Dynamics Specialists Conference, Part B, pp. 622-628.

Santos, J.L.T., "Sizing Design Sensitivity Analysis of Linear Steady Heat Transfer Systems using Established FE Codes," to be published in Numerical Heat Transfer.

Santos, J.L. and Choi, K.K., 1987, "Design Sensitivity Analysis of Nonlinear Structural Systems with an Established Finite Element Code," presented at the ANSYS Conference, Newport Beach, CA, March 31-April 3.

Seong, H.G. and Choi, K.K., 1987, "Boundary Layer Approach to Shape Design Sensitivity Analysis," Journal of Structural Mechanics, 15, 2, pp. 241-263.

Sobiesczanski-Sobieski, J., 1988, "On the Sensitivity Complex, Internally Coupled Systems," AIAA Paper 88-2378 presented at the AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April.

Sobiesczanski-Sobieski, J., Bloebaum, C.L. and Hajela, P., 1988, "Sensitivity of Control Augmented Structure Obtained by a System Decomposition Method," AIAA Paper 88-2205 presented at the AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April.

Sokolowski, J. and Zolesio, J.P., 1987, "Shape Design Sensitivity of Plates and Plane Elastic Solids under Unilateral Constraints," Journal of Optimization Theory and Applications, 54, No. 2, pp. 361-382.

Sutter, T.R., Camarda, C.J., Walsh, J.L. and Adelman, H.M., 1986, "A Comparison of Several Methods for the Calculation of Vibration Modes Shape Derivatives," AIAA Paper 86-0873-CP presented at the AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics and Materials Conference, San Antonio, Texas, May.

Szefer, G., Mróz, Z. and Demkowicz, L., 1988, "Variational Approach to Sensitivity Analysis in Nonlinear Elasticity," Arch. Mech., in press.

Tan, R.C.E., 1986, "Accelerating the Convergence of an Iterative Method for Derivatives of Eigensystems," Journal of Computational Physics, 67, pp. 230-235.

Tan, R.C.E., 1987, "Computing Derivatives of Eigensystems by the Vector ϵ-Algorithm," IMA Journal of Numerical Analysis, 7, pp. 484-495.

Tsay, J.J. and Arora, J.S., 1988, "Variational Methods for Design Sensitivity Analysis of Nonlinear Response with History Dependent Effects," Computational Mechanics '88 (S.N. Atluri and G. Vagawa, Editors), Springer Verlag, 2, pp. 45.vi.

Tseng, C.H. and Arora, J.S. 1988, "Optimum Design of Systems for Dynamic and Control using Sequential Quadratic Programming," AIAA Paper 88-2303, Proceedings, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, Part 2, pp. 739-748.

Wacholder, E. and Dayan, J., 1984, "Application of the Adjoint Sensitivity Method to the Analysis of a Supersonic Ejector," ASME Journal of Fluid Engineering, 106, Dec., pp. 425-429.

Wang, B.P., 1985, paper presented at a work-in-progress session, AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics and Materials Conference, San Antonio, Texas, May.

Wang, S-Y., Sun, Y. and Gallagher, R.H., 1985, "Sensitivity Analysis in Shape Optimization of Continuum Structures," Computers and Structures, 20, No. 5, pp. 855-867.

Wexler, A.S., 1987, "Automatic Evaluation of Derivatives," Applied Mathematics and Computation, 24, pp. 19-46.

Wu, C.C. and Arora, J.S., 1987, "Design Sensitivity Analysis of Nonlinear Response using Incremental Procedure," AIAA Journal, 25, 8, pp. 1118-1125.

Wuu, T.L., Becker, R.G. and Polak, E., 1986, "On the Computation of Sensitivity Functions of Linear Time-Invariant System Responses via Diagonalization," IEEE Transactions on Automatic Control, AC-31, No. 12, Dec., pp. 1141-1143.

Yang, R.J. and Botkin, M.E., 1986, "Comparison Between the Variational and Implicit Differentiation Approaches to Shape Design Sensitivities," AIAA Journal, 24, No. 6, pp. 1027-1032.

Yang, R.J. and Botkin, M.E., 1987, "Accuracy of the Domain Material Derivative Approach to Shape Design Sensitivity," AIAA Journal, 25, No. 12, pp. 1606-1610.

Yuan, K. and Wu, C., 1988, "Sensitivity Calculations for the Numerically Integrated Degenerate Axisymmetric Shell Element," AIAA Paper 88-2285, Proceedings, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, Part 1, pp. 582-589.

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16. Abstract

This paper reviews recent developments in two major areas of structural sensitivity analysis: sensitivity of static and transient response; and sensitivity of vibration and buckling eigenproblems. Recent developments from the standpoint of computational cost, accuracy, and ease of implementation are presented. In the area of static response, current interest is focused on sensitivity to shape variation and sensitivity of nonlinear response. Two general approaches are used for computing sensitivities: differentiation of the continuum equations followed by discretization, and the reverse approach of discretization followed by differentiation. It is shown that the choice of methods has important accuracy and implementation implications. In the area of eigenproblem sensitivity, there is a great deal of interest and significant progress in sensitivity of problems with repeated eigenvalues. In addition to reviewing recent contributions in this area, the paper raises the issue of differentiability and continuity associated with the occurrence of repeated eigenvalues.

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