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ABSTRACT

Solitons are stable localized wave packets that can propagate long distance in dispersive media without changing their shapes. As particle-like nonlinear localized waves, solitons have been investigated in different physical systems. Owing to potential applications in optical communication and optical signal processing systems, optical solitons have attracted intense interest in the past three decades. To experimentally study the formation and dynamics of temporal optical solitons, fiber lasers are considered as a wonderful nonlinear system. During the last decade, several kinds of theoretically predicted solitons were observed experimentally in fiber lasers. In this review, we present a detailed overview of the experimentally verified optical solitons in fiber lasers, including bright solitons, dark solitons, vector solitons, dissipative solitons, dispersion-managed solitons, polarization domain wall solitons, and so on. An outlook for the development on the solitons in fiber lasers is also provided and discussed.

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I. INTRODUCTION

Solitons, also called solitary waves, are a specific kind of wave that exists in nonlinear systems.^{1,2} Solitons have been theoretically predicted for more than 50 years and have been observed in different physical systems. During the past decades, solitons have been intensively investigated in the field of nonlinear optics.^{3–13} Owing to the simplicity, easy, and precise control on the experimental parameters, ultrafast fiber lasers are considered a perfect platform to study the optical solitons. Various kinds of solitons have been experimentally investigated in the fiber laser systems. In the following text, we review recent progress on optical solitons in fiber lasers. Specifically, we emphasize on several types of solitons generated in fiber lasers and demonstrate their unique properties and characteristics. The review starts with a brief introduction to the historic development of solitons and basic theory on solitons in Sec. I. We then discuss the bright and dark solitons described by nonlinear Schrödinger equation (NLSE) in mode locked fiber lasers in Sec. II. In Sec. III, we review vector soliton formation in fiber lasers. In Sec. IV, we review dissipative solitons in fiber lasers. In Sec. V, breathers and dispersion-managed solitons are reviewed. In Sec. VI, multiple soliton interactions and dynamics are

reviewed. We briefly review polarization domain wall (DW) and its potential applications in Sec. VII. Finally, we discuss the challenges and future directions on this topic in Sec. VIII.

A. Brief history of solitons

The phenomenon of solitons was first observed by John Scott Russell in a canal of Scotland in 1834. In his report, a soliton was defined as a special kind of wave that can propagate over a long distance without distortion. In physics, a soliton is a self-reinforcing localized wave packet that maintains its shape while it propagates at a constant velocity. The study of solitons covers a large area of nonlinear physics scenarios, including fluid dynamics, plasma physics, optics, biological and atmospheric systems, nonlinear fiber optics, and so on. In 1965, soliton behavior in media described by the Korteweg–de Vries equation (KdV equation) was reported by Zabusky and Kruskal.¹⁴ In 1967, analytical solution of the KdV equation was developed by inverse scattering transformation.¹⁵ This solution was extended to soliton generation by Peter Lax who proposed Lax pairs and the Lax equation. Solitons are, by definition, unaltered in shape

and speed by a collision with other solitons. So, solitary waves on a water surface are not exactly solitons as after the interaction of two solitary waves, they change a bit in amplitude, and an oscillatory residual is left behind.¹⁶ Solitons are also studied in quantum mechanics, thanks to the fact that they could provide a new foundation through de Broglie's unfinished program, known as "double solution theory" or "nonlinear wave mechanics." This theory, developed by de Broglie in 1927 and revived in the 1950s, is the natural continuation of de Broglie's ideas developed between 1923 and 1926, which extended the wave-particle duality introduced by Einstein for the light quanta, to all the particles of matter.

B. Optical solitons

In optics, an optical soliton refers to an optical field that does not change during propagation because of a delicate balance between linear and nonlinear effects in the optical media.¹⁷ According to the formation mechanism, optical solitons can be classified as temporal solitons or spatial solitons. Temporal optical solitons are formed due to the combined effects of the refractive nonlinearity and the pulse dispersion, while spatial solitons are formed due to the combined effects of the nonlinearity and the beam diffraction.

The first theoretical prediction on the optical solitons shaping in optical fibers comes from Hasegawa and Tappert in 1973.^{18,19} In their prediction, optical solitons in fibers are mathematically described by the nonlinear Schrödinger equation (NLSE). NLSE and KdV equation have the same structure of Lax pair. The conventional temporal optical solitons governed by nonlinear Schrödinger equation can be basically classified as the bright solitons, which are characterized as intensity peaks in the time domain, and the dark solitons which are characterized as intensity dips embedded in a continuous wave background. Based on NLSE, optical bright solitons are supported when the group velocity dispersion (GVD) of the fiber is anomalous, while optical dark solitons are supported when the GVD of the fiber is normal.

Owing to bending and strains in single mode fibers, two orthogonal polarization modes are supported when light is propagating in it. Solitons with two coupled polarization components are named as vector solitons. Optical solitary wave propagation in birefringent optical fibers was first theoretically studied by Menyuk.²⁰ It is demonstrated that above a certain pulse intensity level, two orthogonally polarized solitons with different center wavelengths could couple together and propagate at the same group velocity. Coupled NLSEs have been employed to describe the solitons in the single mode fibers.

Based on the above-mentioned theory, it is natural to develop pulse fiber lasers as a platform to generate the solitons. Optical soliton formation in fiber lasers is a result of the mutual interaction among the cavity dispersion, fiber nonlinearity, laser gain saturation, and gain bandwidth filtering. Strictly speaking, the solitons in fiber lasers are dissipative solitons which are governed by the Ginzburg-Landau Equation (GLE). However, NLSE solitons could also be obtained in fiber lasers under certain conditions as long as the gain is balanced by losses and the gain bandwidth filtering effect could be ignored. Thus, both GLE and NLSE can be considered to describe the soliton generation in fiber lasers. Moreover, owing to the birefringence of the single mode fibers, vector solitons are also supported in single mode fiber lasers. Vector solitons in fiber lasers are mathematically described by the coupled GLEs or NLSEs.

Solitons can be experimentally generated by using passive mode locking technique in fiber lasers. Mode locking produces an initial optical pulse in the cavity. The pulse is then shaped into solitons under the balance between the dispersion and nonlinearity, gain, and losses. Based on the method bright solitons governed by the NLSE, dissipative solitons governed by the GLE, bright-bright solitons governed by the coupled NLSEs, and bright-bright vector dissipative solitons governed by the coupled GLEs were experimentally verified and studied. Moreover, in the nonmode locking regime, several other kinds of solitons that were theoretically predicted under certain conditions have also been experimentally observed, such as dark solitons, polarization domain wall solitons, vector dark-bright solitons, and so on. Besides, another intrinsic feature of solitons in fiber lasers is multiple soliton formation and soliton interactions. A typical pattern of multiple soliton interactions is bound solitons or also named as soliton molecules. In Secs. II–VIII, we will present a comprehensive review of theoretical and experimental investigations on the solitons formed in fiber lasers.

II. FUNDAMENTAL SOLITON GENERATION IN FIBER LASERS

When propagating in optical fibers, the properties of optical pulses are undergoing complicated changes due to chromatic dispersion and intermodal delay effects. Typical physical effects influencing pulses include fiber dispersion and nonlinearity.

Dispersion is an intrinsic feature of materials. In optical fibers, dispersion is defined as the dependence of the phase velocity on the optical frequency or the propagation mode. In single mode optical fibers (SMFs), there are mainly two types of dispersions that need to be considered, namely, the chromatic dispersion and the polarization mode dispersion. The chromatic dispersion of an optical medium is the phenomenon that the phase velocity and group velocity of light wave depend on the optical frequency. Generally, there are two sources of chromatic dispersion, namely, the material dispersion and the waveguide dispersion. Material dispersion depends on the materials used to make the optical fiber. For most fibers, material dispersion is the principle component of chromatic dispersion. The origin of chromatic dispersion is related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of bound electrons. In other words, the refractive index of optical media is a function of optical frequency of the light wave.

Nonlinear effects in optical fibers are originated from the nonlinear response of polarization to a strong optical field, which is another factor that must be taken into account in the study of pulse propagation in optical fibers. As optical fibers are made of silica, which has an amorphous microstructure, the lowest-order nonlinear effects in optical fibers are the third-order effects. In optical fibers, there are various types of third-order nonlinear effects, such as optical Kerr effect, third-harmonic generation, four-wave mixing, stimulated Raman scattering, and stimulated Brillouin scattering. The phase matching condition must be fulfilled for the occurrence of some third-order nonlinear effects. The optical Kerr effect is one of the most common nonlinear effects in optical fibers because the phase match condition for optical Kerr effect is automatically fulfilled. Thus, optical Kerr effect is always considered in the study of nonlinear pulse propagation in optical fibers. Optical Kerr effect originates from the intensity dependence of the refractive index which is given by

$$n(I) = n_0 + n_2 I, \quad (1)$$

where I is the optical intensity, n_0 is the linear index of refraction, and n_2 is the nonlinear refractive index of the medium. The value of n_2 can be measured with the z -scan technique.²¹ The optical Kerr effect has an instantaneously occurring nonlinear response and corresponds to the intensity variation of the light. Thus when we consider intense pulse propagation in optical fibers, both fiber dispersion and nonlinearity must be considered. Derived from the well-known Maxwell equation with several approximations, optical pulse propagation in single mode fibers is described by the nonlinear Schrodinger equation as follows:

$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + i\gamma|A|^2 A, \quad (2)$$

where A is the slow varying envelope of the optical pulse, γ is the nonlinear parameter, β_2 is the second order dispersion coefficient which corresponds to the group velocity dispersion (GVD) effect. It is noted that the higher order dispersion effects are omitted. NLSE gives a simple description over the propagation of an optical pulse in optical fibers, which takes into account both the GVD and nonlinear effects. The nonlinear effects always generate a positive frequency chirp in the pulse and the GVD can introduce either positive or negative chirp, depending on the sign of GVD. If GVD introduces a negative chirp, i.e., the $\beta_2 < 0$, the frequency chirp can be eliminated by the balance between the nonlinear Kerr effect and the GVD. Under this condition, the soliton solution can be obtained by solving the NLSE using the inverse scattering method. Depends on the signature of β_2 , NLSE has different soliton solutions. The soliton solutions in the optical fibers will be discussed later.

Considering pulse propagation in fiber lasers, apart from the propagation in passive single mode fibers, pulse propagation in the gain fibers and the laser output loss must also be taken into account. Thus, the Ginzburg–Landau equation is used to describe the pulse propagation in fiber lasers

$$\frac{\partial A}{\partial z} = -\beta_1 \frac{\partial A}{\partial t} - \frac{i}{2}\beta_2 \frac{\partial^2 A}{\partial t^2} + i\gamma|A|^2 A + \frac{g-\alpha}{2}A + \frac{g}{2\Omega_g} \frac{\partial^2 A}{\partial t^2}, \quad (3)$$

where g represents the gain and α represents the loss, and Ω_g is the bandwidth of the laser gain. In a fiber laser system, if the effect of gain and gain bandwidth can be neglected, the equation will be simplified to NLSE.

A. Bright solitons and passive mode locking technique

Mathematically, according to the NLSE, a bright soliton is a solution of the NLSE when $\beta_2 < 0$, namely, the fiber dispersion is anomalous. The fundamental bright soliton solution is given by²²

$$A(z, t) = \sqrt{2\gamma}\eta \exp\{-4i(\xi^2 - \eta^2)z - 2i\xi t + i\varphi\} \text{sech}(2\eta(t - t_0) + 8\eta\xi z), \quad (4)$$

where η , ξ , φ , and t_0 are all constants and $\beta_2 = -2$.

As shown in Fig. 1, in the context of optical fibers, bright solitons are in the form of short pulses with high peak intensities. It is noted that the bright soliton solution of NLSE describes the pulse propagation in optical fibers that are a conservative system, where no gain and losses exist. In fiber lasers, pulse propagation experiences gain and

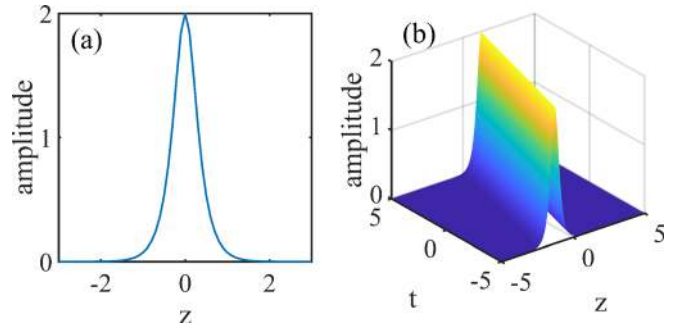


FIG. 1. A typical bright soliton solution of Eq. (4) with $\gamma = 2$, $\eta = 1/2$, $\xi = 1$, and $t_0 = 0$. (a) Bright soliton solution. (b) Evolution of a bright soliton.

losses; NLSE is thus extended to Ginzburg–Landau equation where gain and gain bandwidth effects have to be taken into account. However, in certain conditions, the solitons in fiber lasers can also be described by NLSE as long as the gain is balanced by the losses and the gain bandwidth effect can be ignored.

A typical method that can generate bright solitons in fiber lasers is based on the mode locking technique. A comprehensive review on the mode locking techniques in lasers can be found in Ref. 23. Depending on the mode locking methods used, laser mode locking can be classified as active mode locking and passive mode locking. Active mode-locking of a fiber laser is typically obtained by inserting a modulator into the cavity to modulate either the amplitude or the phase of the intracavity optical field at a frequency that equals to integer multiples of the cavity longitude mode spacing.²⁴ Active mode locking is a preferred choice for producing ultralow-jitter pulses at high repetition rates as high as tens of GHz with pulse duration in the picosecond range. However, as there are too many pulses in the lasers, which share the energy of the intra cavity laser beam, the energy of each pulse is weak and the pulse width is correspondingly broad. Moreover, a broad pulse width also means low peak power. Due to the low peak power of the active mode-locked pulses, nonlinear self-phase modulation (SPM) is so weak that soliton shaping is almost impossible in most actively mode locked lasers. Moreover, drop-out problem also exists in the actively mode-locked fiber lasers owing to the gain competition between the pulses.^{25,26} To overcome the shortcomings of active mode locking, passive mode locking of lasers is therefore proposed which may solve these problems.

Passive mode-locking is a widely used mode-locking technique which does not require any externally modulated media or devices^{27–29} but only employs a saturable absorber (SA) in the fiber laser cavity to realize the mode-locking. A typical passively mode locked fiber laser which can generate solitons is shown in Fig. 2. By inserting an SA into the laser cavity, passive mode locking is capable of producing ultra-short optical pulses, because SAs are able to modulate the resonator losses much faster than any electronic modulator. A SA usually refers to an optical device or medium that exhibits an intensity-dependent transmission,³⁰ which selectively absorbs low intensity light and transmits light with sufficiently high intensity. SAs in passively mode-locked lasers could be divided into two categories: Artificial SAs based on nonlinear light interference and real SAs based on material's nonlinear optical absorption property. A typical artificial saturable absorber is the nonlinear polarization rotation (NPR) technique,

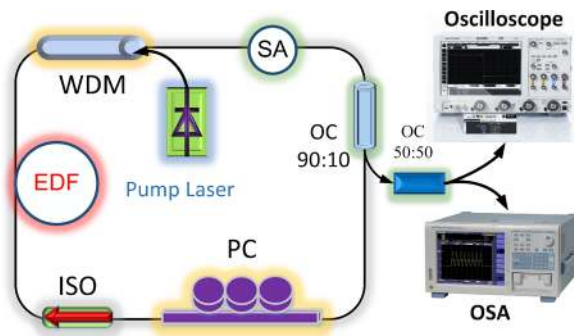


FIG. 2. A typical schematic diagram of a soliton fiber ring laser operating at 1550 nm based on passive mode locking technique. EDF: Erbium-doped fiber; WDM: Wavelength division multiplexer; PC: Polarization controller; OC: Optical coupler; ISO: Isolator; and OSA: Optical spectrum analyzer.

which is normally realized by inserting a polarizer in the fiber cavity. Thus, the solitons generated are linear polarized. These linear polarized bright solitons are defined as scalar solitons to distinguish with vector solitons. NPR technique was first successfully employed to mode lock fiber lasers in 1992. Since then solitons formed in fiber lasers mode-locked by the NPR technique have been intensively investigated.^{31–35} A great number of related interesting properties have been revealed and the corresponding physical mechanisms have been explained, such as Kelly sidebands,³⁶ multiple soliton formation,^{37–39} soliton energy quantization,^{39–41} and soliton interaction.^{42–44} For the real saturable absorbers, semiconductor saturable absorber mirrors (SESAMs) discovered by Keller *et al.* in the 1990s are widely used in fiber lasers.⁴⁵ Apart from the SESAMs, there are several other types of mode lockers discovered recently, like carbon nanotubes,^{46–66} graphene,^{67–105} black phosphorus,^{106–127} and other graphene-like two-dimensional materials.^{128–135} Discovery of the 2D materials based saturable absorbers significantly enhanced the study of the ultrashort pulse lasers and solitons in fiber lasers. Bright solitons have been so far observed in mode locked fiber lasers with various kinds of saturable absorbers.

Bright solitons governed by the NLSE are the first fundamental solitons verified experimentally in fiber laser systems. With different mode locking techniques, one can obtain the mode locked pulses with different pulse widths and stabilities in a fiber laser, but as long as solitons are formed, the influence of the specific mode locking technique on the general properties of solitons is subtle. In general, mode-locking can be considered as an effective approach to generate bright solitons in fiber lasers. As the mode-locking technique develops, bright solitons with new features and properties could be further discovered.

B. Dark solitons

Dark solitons are local pulses that appear as “holes” on a continuous wave background. A dark soliton is formed in optical fibers with normal dispersion and is also governed by the NLSE. In 1973, by using the inverse scattering method, Zakharov and Shabat obtained the dark soliton solutions with the boundary condition $|A(x, t)| \rightarrow 1$, $A_t \rightarrow 0$ (A_t refers to the first-order partial derivative of A with respect to t) as $t \rightarrow \pm\infty$ and they can be written as¹³⁶

$$A(z, t) = \sqrt{\frac{2(\lambda + iv)^2 + \exp\{2v(t - t_0 - 2\lambda z)\}}{\gamma}} \frac{1}{1 + \exp\{2v(t - t_0 - 2\lambda z)\}}, \quad (5)$$

where $\lambda^2 + v^2 = 1$. A typical dark soliton solution is shown in Fig. 3.

According to the minimum pulse intensity, dark solitons can be divided into black solitons and gray solutions. The soliton shown above is known as a gray soliton. The black solitons with the boundary condition were given by Menza and Gallo in 2007.¹³⁷ In 2007, Takhtajan and Faddeev got the dark N-soliton solutions.¹³⁸ Gredeskul and Kivshar theoretically predicted that different from the NLSE type of bright solitons, whose formation requires a certain fixed pulse intensity threshold, the formation of the NLSE type of dark solitons could have no threshold.¹³⁹ A comprehensive review on the dark solitons, which includes the physical origin and properties of dark solitons in optical fibers, was finished by Kivshar in 1998.¹⁴⁰

In addition of the above-mentioned theoretical studies, there are numerical studies on the dark solitons in optical fibers^{141–143} and fiber lasers.^{144–146} However, experimental observation of dark solitons in optical fibers and fiber lasers are still challenging. In optical fibers, dark solitons were first experimentally observed by utilizing specially shaped antisymmetric input pulses at a wavelength of ~ 617 nm in 1988.¹⁴⁷ In fiber lasers, it is well known that mode-locking technique can be used to generate “bright pulses.” However, there is no similar technique to generate dark pulses. Nevertheless, according to Kivshar’s prediction, in a normal dispersion cavity fiber laser a small intensity dip embedded in a high intensity continuous wave background could evolve into dark solitons. In 2009, a scalar dark pulse train was first observed in a continuous wave fiber laser by Zhang *et al.*¹⁴⁸ Although the measured optical spectrum of the laser emission exhibited clear characteristics of the dark solitons numerically simulated for the fiber laser, the measured dark pulses had a broad pulse width which could not be verified as dark solitons, as pointed out by Coen and Sylvestre.¹⁴⁹ The dark solitons related to the measured optical spectra would have several picosecond pulse width, which cannot be detected by the low speed detecting system used in the experiment. Thanks to the rapid advance of the ultrafast optoelectronics technology, Tang *et al.* repeated the experiment using an ultrahigh-speed real-time electronic detection system in 2013 and revealed the detailed properties of the dark solitons formed in fiber lasers.¹⁵⁰ According to Tang *et al.*, a small initial intensity dip in the laser cavity could be evolved into dark solitons, and with a high speed detection system these dark solitons can be experimentally observed.¹⁵⁰ The dark

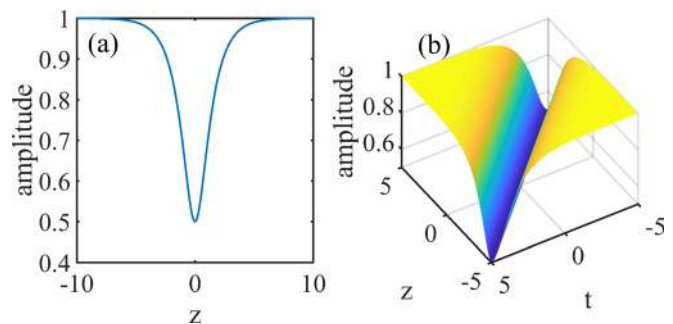


FIG. 3. A typical dark soliton described by Eq. (5) with $v = \frac{\sqrt{3}}{2}$, $\lambda = -\frac{1}{2}$, and $\gamma = -2$. (a) Dark soliton profile. (b) Evolution plot of the dark soliton.

solitons were always automatically formed in the cavity once the intra-cavity laser intensity was sufficiently strong, which is different from the bright solitons formation in a fiber laser, where in order to form the bright solitons a mode locked pulse needs to be formed in prior. As the dark solitons can be easily formed, and in a fiber ring laser there are plenty sources, e.g., the environmental noise and the mode beating, that could cause weak intensity dips, many dark solitons were simultaneously formed in the laser cavity. The dark solitons are randomly embedded in the laser emission. However, due to their narrow pulse width they are undetectable with even a moderate (<15 GHz) speed detection system. Under sufficiently strong pumping where dark solitons have bunched together and formed giant dark pulses, their existence would also become detectable even with 1 GHz bandwidth detection systems, as reported previously.¹⁴⁸ Tang further study the dark soliton formation in fiber lasers without antisaturable absorber and verified that dark soliton formation is a general feature of fiber laser.¹⁵¹ In 2014, a 280 GHz dark soliton fiber laser was reported which opens the study of high repetition rate dark solitons.¹⁵² The modulation instability induced in a fiber loop cavity was adopted to increase the repetition rate of the dark solitons. The repetition rate of the dark solitons was tunable by adjusting the power of the pump laser. This finding may enhance the dark soliton's potential for optical communication applications in the future.

In general, study of dark solitons in the fiber lasers is still at its early stage. Comparing to the bright solitons, dark soliton pulses show more stable behavior under perturbations, including amplifier noise, fiber losses, and so on. The dark solitons are also less sensitive to the backgrounds and lower pulse shape distortion. Study of the dark solitons would be a promising field of the optical solitons and may find applications in optical signal processing, optical computing and optical communications, and so on.

III. VECTOR SOLITONS IN FIBER LASERS

Vector solitons generally refer to solitons that have multiple mutually coupled components. In fiber optics, vector solitons can be formed in weakly birefringent single mode optical fibers (SMFs), where the light propagation is mathematically described by the coupled NLSEs. Theoretically, different forms of vector solitons, such as the bright-bright solitons, dark-bright solitons, and dark-dark solitons, have been predicted for the coupled NLSEs. Mathematically, coupled NLSEs are integrable if the nonlinear terms would have the same magnitude of coefficients. When both of the nonlinear terms are focusing, the integrable case is known as the focusing Manakov model, where the bright-bright soliton solutions can be found.¹⁵³ When both of the nonlinear terms are defocusing, the integrable case is called the defocusing Manakov model, where the dark-bright solitons¹⁵⁴ and dark-dark solitons^{154–156} can be found. If the nonlinear terms are a mixture of focusing and defocusing, then bright-bright solitons,^{157,158} dark-bright solitons,¹⁵⁹ and dark-dark solitons^{154,156,160–163} can be found.

The light propagation in weakly birefringent single mode fiber is described by the coupled NLSEs as follows:

$$\begin{aligned}\frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + i\gamma \left(|u|^2 + \frac{2}{3} |v|^2 \right) u + i\frac{\gamma}{3} v^2 u^*, \\ \frac{\partial v}{\partial z} &= i\beta v + \delta \frac{\partial v}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 v}{\partial t^2} + i\gamma \left(|v|^2 + \frac{2}{3} |u|^2 \right) v + i\frac{\gamma}{3} u^2 v^*,\end{aligned}\quad (6)$$

where u and v are the two normalized slowly varying pulse envelopes along the slow and the fast axes, u^* and v^* represent their conjugates, $2\beta = 2\Delta n/\lambda$ is the wave-number difference, and $2\delta = 2\beta\lambda/2\pi c$ is the group velocity difference. The additional terms v of the first term on the right hand side refers to the cross phase modulation effect. The second term of the right hand side refers to the effect of four-wave mixing.

Furthermore, if a light pulse is propagating in a weakly birefringent cavity fiber laser where the laser gain and losses must also be considered, the NLSE can be further extended to the coupled GLEs as follows:

$$\begin{aligned}\frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + \frac{\beta_3}{6} \frac{\partial^3 u}{\partial t^3} + i\gamma \left(|u|^2 + \frac{2}{3} |v|^2 \right) u \\ &\quad + i\frac{\gamma}{3} v^2 u^* + \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 v}{\partial t^2} + \frac{\beta_3}{6} \frac{\partial^3 v}{\partial t^3} + i\gamma \left(|v|^2 + \frac{2}{3} |u|^2 \right) v \\ &\quad + i\frac{\gamma}{3} u^2 v^* + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2}.\end{aligned}\quad (7)$$

The coupled GLEs are traditionally used to model the vector soliton formation and dynamics in dissipative nonlinear systems. The equations can also be used to study the vector soliton formation in fiber lasers. However, it is to note that a soliton circulating in a laser cavity is also subjected to the actions of other cavity components and feedback. These effects are not considered in the coupled GLEs above.

A. Bright-bright vector solitons

Bright-bright vector solitons refer to the vector solitons with bright pulse in both of the two polarization components. In 1973, Manakov studied the Manakov system

$$\begin{aligned}iu_t + u_{xx} + (|u|^2 + |v|^2)u &= 0 \\ iv_t + v_{xx} + (|u|^2 + |v|^2)v &= 0\end{aligned}\quad (8)$$

and obtained the bright-bright soliton solution as

$$\begin{aligned}(u, v)^T(x, t) &= 2\eta \operatorname{sech}[2\eta(x + 4\zeta t - x_0)] e^{-2i\zeta x - 4i(\zeta^2 - \eta^2)t} \\ &\quad \times \frac{1}{\sqrt{|\alpha_1|^2 + |\alpha_2|^2}} (\alpha_1, \alpha_2)^T,\end{aligned}\quad (9)$$

where the two real parameters ζ and η determine the velocity and amplitude, respectively, x_0 is the coordinate at $t = 0$, and α_1 and α_2 are two complex constants. A typical bright-bright vector soliton is shown in Fig. 4.

Experimentally, bright-bright vector solitons can be simply generated in passively mode-locked fiber lasers, which has been intensively studied.^{71,84,89,93,98–100,109,144,164–219} To generate the vector solitons, all the fibers and passive components of the mode locked fiber lasers have to be polarization insensitive. The main challenge for achieving vector soliton operation of a fiber laser is to find an appropriate saturable absorber that has polarization insensitive saturable absorption. In fiber lasers, SESAM is the earliest proposed SA which has a polarization independent saturable absorption; vector soliton

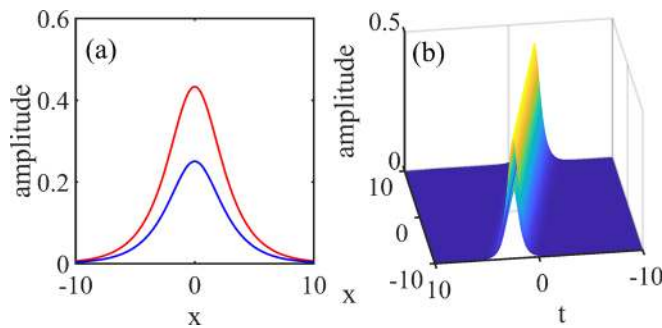


FIG. 4. The bright-bright soliton described by Eq. (9) with $\alpha_1 = \sqrt{3}$, $\alpha_2 = 1$, $\eta = 0.5$, $\xi = 1$, $x_0 = 0$. (a) Bright-bright soliton profiles. (b) Evolution plot of the bright-bright vector soliton.

properties in SESAM fiber lasers have been verified and intensively investigated.^{20–24}

In the last decade, carbon based SAs, represented by carbon nanotubes, with low cost and ultrafast recovery time were proposed and applied in the ultrafast lasers. Since 2009, graphene and graphene-like 2D materials as a group of novel wideband saturable absorbers with low cost, high damage threshold, large modulation depth, and easy fabrication were proposed and widely studied in the mode-locked fiber lasers. Similar to SESAM, these SAs are also polarization independent which supports vector soliton formation in fiber lasers. So far, vector solitons have been observed in ultrafast fiber lasers passively mode locked by carbon nanotubes,^{181,187} graphene,^{71,84,89,93,98–100,198} PbS quantum dots,¹⁸³ black phosphorus,¹⁰⁹ and so on. Vector solitons are not only observed in ytterbium-doped fiber lasers and erbium-doped fiber lasers but also in mid-infrared mode locked fiber lasers.^{183,204,208,217}

It is noted that bright-bright vector solitons produced by the coupled NLSEs may be asymmetric, with different energies and propagation constants in the two components. Bright-bright vector solitons with internal dynamics were studied by Kaup *et al.*²²⁰ Indeed, depending on the net cavity birefringence and coupling strength between the two components, experimentally the two orthogonal polarization components of the formed vector solitons could have different phases and intensities group velocities.

In a weak linear birefringence cavity, vector solitons will experience coherent energy exchange caused by the four wave mixing that was observed by Zhang *et al.*¹⁷³ In a birefringent fiber laser cavity induced solitons could also be formed.²²¹

Besides, depending on the cavity birefringence and cross-polarization coupling strength, vector solitons formed can be classified as polarization locked vector solitons, polarization rotation vector solitons, and group-velocity locked vector solitons and so on. Vector solitons with a uniform, nonevolving polarization state during the propagation is referred to as polarization locked vector solitons or phase locked vector solitons.^{167,168,172,181,222–224} Polarization locked vector solitons were first experimentally reported and investigated by Cundiff *et al.* in a SESAM mode locked fiber laser.²²² They presented a comprehensive study on polarization locked vector solitons in fiber lasers in 2000,^{167,168} which reveals that the two orthogonal polarization components of the polarization locked vector solitons have $\pm\pi/2$ relative phases. In 2008, high order polarization locked vector solitons in a SESAM mode locked fiber laser were reported.¹⁷²

Polarization rotation vector solitons are theoretically predicted by Afanasjev.²²⁵ The polarization rotation period of a polarization rotation vector soliton formed in a fiber laser is equal to (or a multiple of) the period of the cavity round trip time. Although the total soliton intensity is still uniform, the pulse intensity along two orthogonal polarization components varies with a certain period. The intensity of the pulses typical alters among two or several values, which is different from that of polarization locked vector solitons. Polarization rotation is a general feature of the vector solitons in fiber lasers, which has been observed in erbium-doped fiber lasers mode locked by SESAM,^{174,178} graphene,⁹⁹ carbon nanotube,²¹³ and so on. Sergeyev *et al.* investigated the polarization dynamics of vector solitons in a carbon nanotube saturable absorber mode locked fiber laser and it was found that the solitons have a locked and processing polarization states.²²⁶

Another type of vector solitons is the group velocity locked vector solitons. Formed in a highly birefringent fiber, the two orthogonally polarized components of the vector soliton can trap and overcome the group-velocity difference through the nonlinear cross coupling.²²⁷ This type of vector solitons has been observed experimentally in fiber transmission systems. Group velocity locked vector solitons (GVLVSs) are reported as high order GVLVS,^{228,229} dissipative GVLVS,^{200,230,231} and bound state of GVLVS²¹⁴ and so on.

B. Dark-bright vector solitons

In 1997, Sheppard and Kivshar¹⁵⁴ investigated the NLSEs

$$i \frac{\partial \vec{e}}{\partial z} + \frac{1}{2} \frac{\partial^2 \vec{e}}{\partial t^2} - |\vec{e}|^2 \vec{e} = \vec{0}, \quad (10)$$

where z and t are distance and time coordinates, respectively, and $\vec{e} = (e_+, e_-)^T$ refers to the transversely polarized light. By Hirota's bilinear method, the authors derived the dark-bright soliton solutions as the form of

$$\begin{aligned} e_+ &= \tau \{ i \sin \phi + \cos \phi \tanh[a(t - bz)] \} e^{i \tau t + i [\frac{c^2}{2} + \tau^2] z}, \\ e_- &= \sqrt{\tau^2 \cos^2 \phi - a^2} \operatorname{sech}[a(t - bz)] e^{i \tau t + i [\frac{b^2}{2} - \tau^2] z}, \end{aligned} \quad (11)$$

where a , b , c , and τ are constants satisfying $a^2 + (c^2 - b^2) \leq \tau^2$ and $\phi = \arctan \frac{c-b}{a}$. A typical dark-bright vector soliton is shown in Fig. 5.

For the mixed case, Vijayajayanthi, Kanna, and Lakshmanan got the dark-bright soliton solutions in 2008.¹⁵⁹ The existence of dark-bright solitons in the single mode fibers is first theoretically predicted

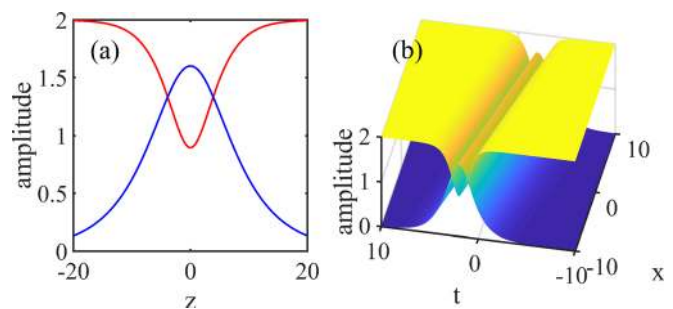


FIG. 5. The dark-bright soliton solution of Eq. (11) with $a = 0.8$, $b = -0.2$, $c = 0.2$, and $\tau = 2$. (a) Bright-bright soliton solution. (b) Evolution plot of bright soliton.

by Christodoulides.²³² Based on the coherently coupled NLSEs, he pointed out that under a weak birefringence, the dark-bright vector solitons can exist in different dispersion regimes with different properties. In normal dispersion regime, the dark soliton plays a dominant role, and in the anomalous dispersion regime the bright soliton dominates. Their predictions provide a guideline for the experimental study on the dark-bright solitons in fiber lasers.

According to the theoretical prediction of Christodoulides, there are two crucial conditions for the dark-bright soliton formation in fiber lasers. First, the birefringence of the fiber cavity must be sufficiently weak so that the two polarization components of the light can be strongly coupled, and second, the energy of the intra cavity should be sufficiently high to support the formation of dark solitons. Similar to the case of scalar dark soliton, the main challenge in the dark-bright soliton experiment study is how to detect the existence of the dark solitons as they are difficult to be observed in the time domain. Although there are several experimental observations in fiber lasers,^{233–244} in which they claim that the “dark-bright pulse” was obtained. However, these reported pulses were not verified as dark-bright solitons. Owing to the limitation of the detection system, there are still no systematic experimental studies on the formation of the dark-bright solitons in fiber lasers, to the best of the authors’ knowledge.

To generate the dark-bright vector solitons, both dark pulse and bright pulse must be automatically obtained in the cavity. Therefore, proper cavity dispersion must be chosen. Based on the previous experiments, it was found that in all anomalous dispersion cavities, dark solitons were difficult to be achieved, while bright pulses cannot be shaped into solitons without mode locking in all normal dispersion cavities. A dispersion-managed cavity is thus necessary in order to achieve the dark-bright soliton operation of the fiber lasers. The authors have experimentally investigated dark-bright solitons in the fiber lasers. We found that stable dark-bright solitons could be formed in fiber lasers when the net cavity dispersion is set near the net zero point. Technically, this was achieved with a dispersion-managed laser cavity. Another essential condition for dark-bright soliton formation is low birefringence of the cavity so that the coherent coupling could occur between the two polarizations of the light in the fiber. A near zero dispersion regime is preferred to keep the balance between the dark component and the bright component. Fulfilling the above conditions, a stable dark-bright soliton pulse train can be generated. Through the above experimental studies, the mechanism of the dark-soliton generation is also revealed. The dark-bright soliton formation is due to the strong cross polarization coupling of light in the fiber lasers, which coincides with the theoretical prediction of Christodoulides.²³² The generation of stable dark-bright soliton pulses provides a potential on the study of the vector dark-bright solitons and other vector solitons for optics communications.

C. Dark-dark vector solitons

Dark-dark soliton solutions appear in the case of defocusing Manakov model and the mixed focusing and defocusing nonlinearity. For the first case, Radhakrishnan and Lakshmanan studied the coupled equations as follows:

$$\begin{aligned} iu_z - u_{tt} + 2\mu(|u|^2 + |v|^2)u &= 0, \\ iv_z - v_{tt} + 2\mu(|u|^2 + |v|^2)v &= 0, \end{aligned} \quad (12)$$

by Hirota’s bilinear method in 1995.¹⁵⁵ They derived the dark-dark solitons

$$\begin{aligned} u(z, t) &= -\frac{\tau_1}{2} \left[(1 + Z_g) - (1 - Z_g) \tanh \left(\frac{P_1 t - \Omega_1 z + \xi_1^0}{2} \right) \right] \\ &\quad \times e^{i[l_1 t - (\lambda - l_1^2)z + \psi_1^0]}, \\ v(z, t) &= -\frac{\tau_2}{2} \left[(1 + Z_h) - (1 - Z_h) \tanh \left(\frac{P_2 t - \Omega_2 z + \xi_2^0}{2} \right) \right] \\ &\quad \times e^{i[l_2 t - (\lambda - l_2^2)z + \psi_2^0]}, \end{aligned} \quad (13)$$

where $l_1, l_2, \psi_1^0, \psi_2^0, P_1, \Omega_1$, and ξ_1^0 are all real constants, τ_1, τ_2, Z_g , and Z_h are all complex constants, and $|Z_g| = |Z_h| = 1$. A typical dark-dark vector soliton is shown in Fig. 6.

For the second case, Ohta, Wang, and Yang solved the N-dark-dark soliton solutions by Hirota’s bilinear method in 2011.¹⁶⁰ Vector dark solitons in optical fibers have been theoretically predicted for two decades.²⁴⁵ Vector dark pulses were reported by Zhang *et al.*²⁴⁶ In 2015, Guo predicted the energy exchange of dark solitons in fiber lasers.¹⁴⁴ However, the soliton properties of the pulses have not been confirmed owing to the low speed detecting system. Dark vector solitons so far have not been experimentally verified in the fiber laser systems. It would be a potential study direction on the soliton fiber lasers.

IV. DISSIPATIVE SOLITONS IN FIBER LASERS

Dissipative solitons are localized structures of an electromagnetic field that are balanced through an energy exchange with the environment in the presence of nonlinearity, dispersion, or diffraction.²⁴⁷ The concept of dissipative solitons is not restricted to optics, which can be extended to physics, biology, and medicine and so on. The most prominent feature of dissipative solitons is that they exist only when there is a continuous energy supply from an external source. In the fiber lasers, conventional solitons are formed due to the balance between anomalous fiber dispersion and self-phase modulation (SPM), whose properties are determined by the nonlinear Schrödinger equation, while dissipative solitons are formed as a result of the mutual nonlinear interactions among the normal cavity dispersion, SPM, effective gain bandwidth filtering, and gain saturation. Properties of the dissipative solitons are determined by the Ginzburg–Landau equation. As a result of different soliton formation mechanism, the dissipative solitons

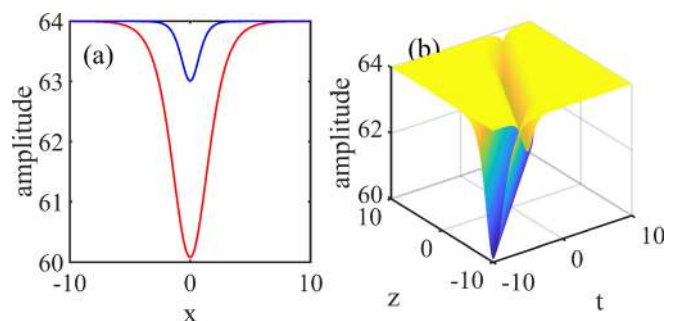


FIG. 6. The dark-dark soliton solution of Eq. (13) with $\tau_1 = 4\sqrt{2} + 4\sqrt{2}i$, $\tau_2 = 4\sqrt{2} - 4\sqrt{2}i$, $Z_g = \cos \frac{1}{2} + i \sin \frac{1}{2}$, $Z_h = \cos \frac{1}{4} + i \sin \frac{1}{4}$, $P_1 = \Omega_1 = \Omega_2 = 1$, $\Omega_2 = 2$, and $\xi_1^0 = \xi_2^0 = 0$. (a) Dark-dark vector soliton profiles. (b) Evolution plot of the dark-dark vector soliton.

exhibit remarkably distinct properties from those of the conventional solitons.

A. Ginzburg-Landau equation and dissipative soliton solution

The Ginzburg-Landau equation, which is a nonintegrable system, can be seen as the nonlinear Schrödinger equation with gain and loss. In 2016, the bright solitons and dark solitons were demonstrated by Mirzazadeh *et al.*²⁴⁸ The earlier study of the bright-soliton-like solutions was given by Pereira and Stenflo in 1977.²⁴⁹ The works of Liu, Li, and Tian demonstrated the dark-soliton-like solutions in 2009.²⁵⁰

Considering the Ginzburg-Landau equation, If $g=0$ and $\alpha=0$, the equation returns back to the NLSE. According to the study of Pereira and Stenflo,²⁴⁹ an exact solution of GLE in the anomalous dispersion regime is a chirped hyper-secant pulse which is given by

$$\mu(\zeta, \tau) = N_s \operatorname{sech}(p\tau) \exp[iK_s \zeta - iq \ln(\cosh(p\tau))], \quad (14)$$

where $\zeta = \frac{z|\beta_2|}{T_0^2}$, $\tau = \frac{T}{T_0}$, $u = (\frac{\gamma T_0^2}{|\beta_2|})^{1/2}$. N_s , p , q , and K_s are constants depending on the parameter $d = \frac{gT_0^2}{|\beta_2|}$. Equation (14) is a dissipative soliton solution. Solitons whose dynamics are described by one-dimensional GLE are known as the scalar dissipative solitons. Vector dissipative solitons are governed by a coupled GLE.

Strictly speaking, solitons in the fiber lasers are all dissipative owing to the existence of the gain fiber and losses. As NLSE based bright solitons are not supported in all normal dispersion regime, NLSE type solitons were experimentally studied in mode locked fiber lasers with anomalous dispersion. While dissipative solitons are intensively studied in the mode-locked fiber lasers with all normal dispersion.²⁵¹ Dissipative solitons in the fiber lasers are initially named as gain-guided solitons when they were first observed in fiber lasers with net normal dispersion.^{252–255} In 2012, Grelu and Akhmediev reviewed the recent development of dissipative solitons in lasers.²⁴⁷ Therefore, we will not discuss it further here.

B. Flat-top dissipative soliton

Dissipative solitons are closely related to the high-energy pulses in the fiber lasers owing to its intrinsic feature.^{256–267} As the development of the dissipative soliton research, an interesting pattern called dissipative soliton resonance (DSR) was reported. A DSR pulse is a complex of two interacting dissipative fronts. Under small gain conditions, two fronts are closely connected, forming a plain dissipative soliton. With increasing gain, the resonance effect limits the growth of peak power, while allowing for two fronts moving apart from each other. The pulse generates a plane wave in the center, which strongly binds two fronts together. The central plane wave extends, and the distance between the two fronts grows linearly and infinitely with the energy supply, whereas the fronts themselves do not change. The central plane wave and the fronts were also found to feature different chirps: A moderately low linear chirp throughout the extended central plane wave and large linear chirps across both fronts.

Dissipative soliton resonance is an interesting phenomenon in fiber lasers with dissipative solitons and has been frequently observed and reported.^{90,188,218,257,268–328} The concept of dissipative soliton resonance was first proposed by Chang in 2008.^{269,270} They theoretically predicted existence of dissipative soliton resonance in a mode-locked

fiber laser, which enables an almost infinite boost in the pulse energy without wave breaking.²⁷⁸ They also predicted that DSR operation could be obtained in both anomalous and normal dispersion regions.^{269,272} Experimentally in the fiber lasers, DSR phenomenon leads to the formation of flat-top dissipative solitons. In 2009, Wu *et al.* observed DSR operation in a 1.56 μm normal-dispersion erbium-doped fiber laser mode-locked by nonlinear polarization rotation.²⁷³ The pulse width of the as observed flat top solitons increased as the pump power increased. It was claimed that the DSR occurs when the pulse peak is clamped in the laser cavity. In 2012, Luo *et al.* reported the pulse dynamics operating in the DSR region of a ring laser, which showed that DSR phenomenon transits a sech-like pulse to flat top dissipative soliton as the input power increased.²⁷⁶ In 2014, Mei *et al.* demonstrated the generation of width- and amplitude-tunable DSR pulses by a dual-pump passive mode-locked erbium-doped fiber laser.³²⁵ A numerical study on the DSR in the fiber laser showed that DSR-type flat-top dissipative soliton generation is attributed to the peak power clamping effect by inducing spectral filtering into the laser cavity.²⁸⁸

DSR type flat-top pulses were continuously studied because of their high pulse energy. Krzempek *et al.* successfully realized high-power pulse generation in figure-8 Er:Yb double-clad fiber laser with a 2.3 μJ pulse energy and a 455 ns pulse width.²⁹³ Semaan *et al.* further increased the DSR pulse energy to as high as $\sim 10 \mu\text{J}$ with a pulse width of 416 ns from figure-8 double-clad Er:Yb fiber laser.²⁹⁷ Du *et al.* proposed a short-length nonlinear optical loop mirror to significantly boost the peak power of the DSR pulse and achieved 100 ps DSR pulses at a wavelength of 1.56 μm with an average power of 1.2 W and a peak power of 700 W.³⁰⁸ DSR pulses in 2 μm all-fiber laser were recently reported by Xu *et al.* with 6.19 ns pulse width and 19.51 nJ pulse energy²⁹⁰ and they further obtained 1.96 μm DSR pulses with a tunable pulse width of 3.74–72.19 ns and a peak power of only 0.56 W.³⁰¹ Du *et al.* reported direct generation of a high power and large energy dissipative soliton resonance in a thulium-doped double-clad fiber laser with pulse energy of 353 nJ.³²⁴ Recently, DSR is reported in a holmium doped fiber laser,³²⁹ which further proves that DSR is an intrinsic feature of the mode locked fiber lasers.

In the fiber lasers, another flat-top solitons based on GLE, which was named as kink-antikink bound states, was theoretical predicted by Malomed.³³⁰ Experimentally, this type of soliton was observed together with polarization domain walls in the fiber lasers, which will be further discussed in Sec. VII.

V. BREATHERS: DISPERSION-MANAGED SOLITONS

Breather as an essential extension of the concept of fundamental single-soliton state is an important class of solutions in nonlinear Schrodinger equation. Breather is the periodic solution with two types: Standing ones and traveling ones. The first was obtained by Kuznetsov.³³¹ In 1979, Ma studied the focusing NLS equation (2) and derived the regular breather³³² as follows:

$$A(z, t) = e^{i\omega z} \left(d + \frac{4\nu b}{d^2} - \frac{4i\zeta}{d^2} (\mu e^{i\omega z} + \mu^* e^{-i\omega z}) \right) \frac{4}{d^2} e^{2i\nu t} + \frac{\zeta^2 |\mu|^2}{d^2} e^{-2i\nu t} + \frac{4b}{d^2 \nu}, \quad (15)$$

where

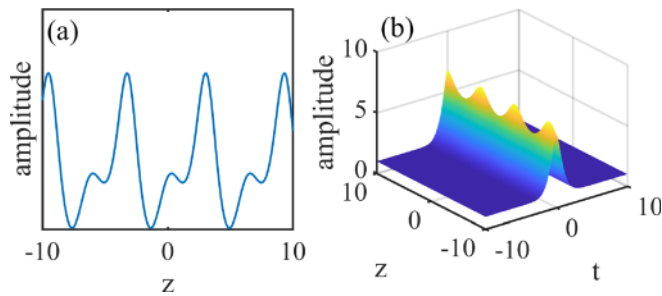


FIG. 7. The breather solution of Eq. (15) with $\omega = 1$, $d = 1$, $\mu = 1$, $\xi = 4$, $v = 1$, and $z_0 = 0$. (a) Breather solution. (b) Evolution plot of breather.

$$\mu(t) = \mu(0)e^{(-4iv\xi - 2id^2)z}, \quad \frac{\xi|\mu|^2}{4v^2} = e^{4vz_0}, \quad b = i(\mu e^{i\omega z} - \mu^* e^{-i\omega z}).$$

Later, Akhmediev also derived the breather solution in 1987.³³³ Recently, dynamics of nonlinear Schrödinger breathers in a potential trap was reported by Malomed *et al.*³³⁴ A typical breather solution is shown in Fig. 7.

Experimentally, two-soliton or N-soliton breathers are intrinsically unstable in a fiber laser due to the perturbations of the cavity components and gain and losses, so far they have not been observed in fiber lasers; therefore, we did not specifically discuss the case. In the mode locked fiber lasers, breathers were generated in the oscillator with dispersion-management technique. As discussed in Sec. II, conventional solitons governed by NLSE are generated and studied in the fiber laser with all anomalous dispersion. While the dissipative solitons are studied in all-normal dispersion cavity. Breathers are studied in a dispersion-managed cavity, in which fibers with negative dispersion and positive dispersion are distributed. The generated pulses are compressed and stretched in different dispersion regime, which is named as stretch pulse generation in fiber lasers. Stretch pulse technique, first proposed by Tamura *et al.*,³³⁵ by using the dispersion management, is capable to support the generation of ultrashort pulse with pulse duration of <100 fs or large single pulse energy >1 nJ in the fiber laser cavity. Dispersion managed soliton is intensively studied in mode locked fiber lasers.^{87,177,196,336–358} A detailed review on the dispersion-managed solitons in fiber lasers was reported by Turitsyn *et al.* at 2012.³⁵⁹

VI. SOLITON DYNAMICS: MULTIPLE SOLITON INTERACTIONS AND BOUND STATES OF SOLITONS

Soliton dynamics has been a subject of intensive research in fiber lasers.^{38,360,361} Harmonic mode-locking,³⁶² period doubling,³⁶³ and soliton pairs^{364,365} are examples of phenomena that deserved substantial theoretical and experimental efforts. In the case of vector soliton, Menyuk *et al.* showed that two orthogonally polarized components of pulse propagating in a birefringent nonlinear environment can be coupled and propagate with equal group velocity.²⁰ This phenomenon underlies the formation of vector solitons in the laser cavity. In multiple-pulse regime typical for cavities with anomalous dispersion, the vector soliton interaction could lead to a bunch development propagating as an entity at fundamental repetition rate. It was found that contrary to scalar soliton bunch behavior, the vector solitons exhibit periodic-like contractive and repulsive motion within the bunch. The

slow response of absorption from mode locker induces an attractive force between vector solitons resulting in a tight bunch formation with temporal separation between pulses at a picoseconds scale.¹⁷⁹ Therefore, it was established that details of absorption recovery could be an instrumental for vector soliton control.

In this section, we will briefly review the multiple soliton formation, and interactions and formation of bound solitons, which is a typical pattern owing to the effect of soliton interactions.

A. Multiple soliton interactions

Multiple soliton formation in mode locked fiber lasers is a well-known phenomenon and has been extensively investigated in the past.^{43,44,366–371} It has been shown that various mechanisms could lead to the formation of multiple solitons. These mechanisms include the wave-breaking effect, the effective spectral filter effect, the soliton peak clamping effect, and the soliton shaping of dispersive waves. Indeed, in previous experimental studies on the multiple solitons formed in the fiber lasers, people have observed various modes of multiple soliton operation, such as soliton bunches, soliton collisions,³⁷² vibration of soliton pairs,³⁶⁵ restless solitons,¹⁷⁹ bound state of solitons,^{373,374} and so on. Some of these effects can be traced back as a result of the direct soliton interaction of the dissipative solitons, or the dispersive waves mediated NLSE type of soliton interaction. Therefore, based on the different features of the multiple soliton operation of a fiber laser, one can get an insight into the properties of the formed solitons. Recently, a novel form of multiple soliton operation named as “soliton rain” was observed by Chouli *et al.*^{375–377} It was shown that the multiple soliton formation in a fiber laser could even manifest the process of the rain-drop formation in the nature, or in another word, the multiple soliton interaction in a fiber laser follows the universal statistics of the many body systems.

To study the multiple vector soliton formation, SESAM was initially employed in the fiber laser cavity. In an experiment with a SESAM as the passive mode locker, Zhao *et al.* have observed a state of so-called bunched restless vector solitons.¹⁷⁹ Vector soliton bunching controlled by SESAMs with different recovery times was also experimentally investigated by Gumenyuk *et al.*³⁷⁸ Recently, the mode locking of fiber lasers with atomic-layer graphene based saturable absorbers has attracted considerable attention of research.^{84,89,93,98–100,191,198}

B. Bound states of solitons

Bound states of solitons, also named as bound soliton or soliton molecules, are referred to that two or more fundamental solitons bind tightly together in the temporal or spatial domain through the direct soliton interaction. The solitons in the state have not only fixed separations but also fixed phase differences, and the assembly of the solitons behaves like a new super-soliton. The formation of bound solitons can be attributed to the multiple soliton interactions.

In 1994, Haelterman and Sheppard studied the coupled NLSEs and showed the existence of bound vector solitary waves which had the possibility to increase the bandwidth of transmission lines.³⁷⁹ Theoretically, formation of bound states of conventional solitons in the extended nonlinear Schrödinger equation systems was first predicted by Malomed.^{380,381} Afanasjev *et al.* theoretically studied the stability of bound state of dissipative solitons in the Ginzburg–Landau equations.³⁸² Akhmediev *et al.* also studied the formation of bound

states of solitons in the complex Ginzburg–Landau equation system.^{383,384} Formation of bound states of solitons in fiber lasers has attracted considerable interest experimentally. It is anticipated that the soliton fiber lasers could serve as an ideal testbed for the study on bound solitons. So far, there have been plenty of reports on bound solitons^{72,75,80,86,100,110,130,134,187,348,371,374,385–442} or claimed as soliton molecules^{70,74,78,96,131,189,209,214,354,443–463} in mode locked fiber lasers. The bound states of solitons were first experimentally observed in a fiber laser mode locked with the nonlinear polarization rotation technique.^{374,385} Later, with the development of novel material based real saturable absorber (SA) mode locking techniques, such as the carbon nanotube mode locking and 2D-nano-materials mode locking, formation of bound states of solitons has also been observed in fiber lasers mode locked with the carbon nanotubes,^{410,421,423,464,465} graphene,^{70,72,74,75,78,80,86,96,100} MoS₂,^{130,131,134} black phosphorus,^{109,110} and so on. As these saturable absorbers may be polarization independent, bound states of vector solitons were also reported in fiber lasers.^{100,187,189,209,214,464} Mou *et al.* reported the existence of bound vector solitons in a carbon nanotube mode locked fiber laser.⁴⁶⁴ Luo *et al.* observed the group velocity locked vector soliton molecules in a SESAM mode locked fiber laser.²¹⁴ Bound vector solitons were also found with properties of soliton interaction in the laser cavity. In 2016, Song reported the experimental observation of coexistence and interactions between vector and bound vector solitons in a fiber laser passively mode locked by graphene.¹⁰⁰ Owing to the bound soliton interactions, bound-bound soliton was also found experimentally.⁴⁶⁵

VII. POLARIZATION DOMAIN WALL AND DOMAIN WALL SOLITONS

Domain wall (DW) refers to a topological defect that connects two stable static states of a physical system. Formation of the DWs is recognized as a spontaneous symmetry-breaking phase transition in a variety of contexts, which are ubiquitous and generic in the entire field of nonlinear physics ranging from ferromagnetism theory to optics and Bose-Einstein condensate, DNA fluctuations, deoxyribonucleic acid, and string theory. In mathematics, domain wall also appears as kinks, in close analogy with the celebrated kink solutions of the Sine-Gordon equation. In 1982, Boiti, Laddomada, and Pempinelli solved kink solitons by applying the Bäcklund transformation to the NLSE.⁴⁶⁶ The single-kink soliton solution is given by

$$A(z, t) = 2\sigma_0 \tanh \left[\sqrt{2}\sigma_0(t - t_0 + 4\zeta_0 z) \right] e^{i[-2\zeta_0 t - 4(\sigma_0^2 + \zeta_0^2)z - \theta_0]}, \quad (16)$$

where σ_0 and ζ_0 are two real parameters, the real constants θ_0 , t_0 fix the location of the traveling wave at the initial time, $\beta = -2$, and $\gamma = -1$. A kink soliton solution is shown in Fig. 8. It is noted that the kink solitons are different from the dark solitons shown in Fig. 3. As shown in Figs. 3 and 8, kink soliton has different boundary conditions as $x \rightarrow \pm\infty$ or $t \rightarrow \pm\infty$. While the dark soliton has the same boundary conditions as $x \rightarrow \pm\infty$ or $t \rightarrow \pm\infty$. By a method based on the association of the Painlevé test theory and Hirota's bilinear technique, Pelap and Faye obtain the kink solitons of Ginzburg–Landau equation in 2004.⁴⁶⁷

In the field of nonlinear optics, Zakharov and Mikhailov first theoretically predicted the existence of stable optical domain walls.⁴⁶⁸ Formation of optical domain walls is independent of dispersion, purely through the interaction between the polarization states of two counter-

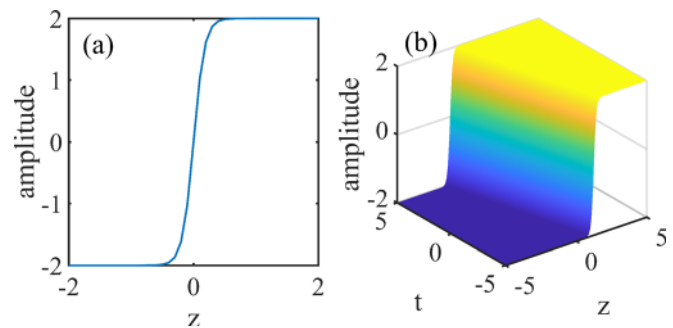


FIG. 8. The kink soliton solution of Eq. (16) with $\sigma_0 = \zeta_0 = 1$, $\theta_0 = t_0 = 1$. (a) Kink soliton solution. (b) Evolution plot of kink soliton.

propagating electromagnetic waves.⁴⁶⁸ They pointed out that in analog to the formation of the magnetic domains in the ferromagnetic materials, the cross-interaction between two counter-propagating optical beams in a third-order nonlinear medium could lead to the formation of optical polarization domains. Physically, the formation of polarization domains is due to that a Hamiltonian system in the stable state must have the minimum energy. Associated with the formation of polarization domains, it turns out that the domain walls that separate the polarization domains are a stable localized structure. They constitute a fundamentally new type of solitary waves, known as the polarization domain wall solitons.⁴⁶⁹ In optical fibers, Wabnitz and Daino analytically predicted the possibility of generating domain walls as a result of polarization switching,⁴⁷⁰ and Pitois experimentally confirmed the prediction.⁴⁷¹ Later, Haelterman, Sheppard, and Malomed found that the dynamics of domain wall soliton could be well encapsulated by the incoherently coupled NLSEs where both the nonlinear polarization coupling and normal dispersion were taken into account.^{173,174} In 1990, the domain wall of coupled Ginzburg–Landau equation was obtained by Malomed, Nepomnyashchy, and Tribelsky.⁴⁷² In 1994, Haelterman and Sheppard obtained the domain walls by studying the coupled NLSEs.⁴⁷³ Malomed also theoretically studied the formation of domain walls between two traveling waves,⁴⁷⁴ and the dynamical properties of domain walls formed in twist single mode fibers.⁴⁷⁵ It was shown that domain walls between linearly polarized beams could also be possible.

Experimental observation of kink solitons in fiber lasers does not reply on the dispersion of the fiber cavity. And the kink solitons observed have good long-term stability. It is different from the NLSE dark solitons, which were only supported in normal dispersion regime and difficult to be observed. Experimental demonstration of the polarization domains is as early as 1995 when Kockaert *et al.* experimentally demonstrated the generation of polarization domains using a spun fiber.⁴⁷⁶

So far, a few experimental observations on the polarization domain formation and polarization domain wall solitons in fiber lasers are reported.^{180,246,320,477–483} Zhang *et al.* reported the experimental observation of a kind of periodic fast polarization switching between the two orthogonal linear eigen-polarizations of an erbium-doped fiber ring laser.²⁴⁶ The experimental studies on the phenomenon suggested that it could be due to the polarization domain formation in the laser. In their experiment, the detection system is of 1 GHz bandwidth to monitor the polarization resolved laser emission. Limited by the

bandwidth of the detection system, some important features of the polarization dynamics of the laser could not be observed. In 2014, Tang *et al.* experimentally and theoretically studied the polarization domain formation and domain dynamics in a quasi-isotropic cavity fiber laser.⁴⁸³ It is shown experimentally that the polarization domain formation is a general feature of the quasi-isotropic cavity fiber lasers; it can occur either under the incoherent or coherent coupling between the two orthogonal linear polarization modes of the lasers. Lecaplain *et al.* present a simple theoretical model that explains polarization switching in fiber ring lasers operating with a normal path-averaged dispersion and a typical intermediate level of birefringence. The proposed polarization dynamics is based on a type of polarization domain wall structures and was named as polarization domain wall complex in fiber lasers.⁴⁷⁹

Recently, Gilles *et al.* experimentally demonstrated the existence of a universal class of polarization domain walls in the form of localized polarization knots in conventional optical fibers.⁴⁸⁴ It is demonstrated that how trapping energy in a well-defined train of polarization domain walls allows undistorted propagation of polarization knots at a rate of 28 GHz along a 10 km length of normally dispersive optical fiber. These results constitute the first experimental observation of kink-antikink solitary wave propagation in nonlinear fiber optics, which may find important applications for the polarization domain walls in the future.

VIII. SUMMARY AND OUTLOOK

In summary, we have presented divergent aspects of the physics of optical solitons in fiber lasers, including analytical, numerical, and experimental results. They demonstrate a number of interesting properties of these nonlinear waves which can exist on a background being characterized by a nontrivial phase distribution of the field. In many cases discussed in this review, these solitary waves not only can be described analytically and numerically for a variety of nonlinear models but also, and this is the most amazing fact, be verified, with relatively good accuracy, by employing the fiber lasers as a platform.

In the last decade, development of two-dimensional materials has enhanced the investigation on solitons in fiber lasers. Searching for and employing new materials with strong nonlinear properties may sufficiently speed up this process. Recently, a novel technique named time-stretch spectroscopy was applied in the analysis of the ultrafast lasers.^{485,486} This technique has been applied for the analysis of pulse growth dynamics,⁴⁸⁷ soliton molecules,⁴⁸⁸ soliton burst,⁴⁸⁹ dissipative solitons,⁴⁹⁰ dissipative soliton molecules and interactions,⁴⁹¹ rogue waves,⁴⁹² and so on. It can be anticipated that time-stretch dispersive Fourier transform technique will be a powerful tool for the study of soliton in fiber lasers in the future.

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