Reconciling Steady-State Kalman and Alpha-Beta Filter Design

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The deterministic design of the "Alpha-Beta" filter and the stochastic design of its Kalman counterpart are placed on a common basis. The first step is to find the continuous-time filter architecture which transforms into the alpha-beta discrete filter via the method of "impulse invariance." This yields relations between filter bandwidth and damping ratio and the coefficients, α and β. In the Kalman case, these same coefficients are related to a defined stochastic signal-to-noise ratio and to a defined normalized tracking error variance. These latter relations are obtained from a closed form, unique, positive-definite solution to the matrix Ricatti equation for the tracking error covariance. A nomograph is given, relating the stochastic and deterministic designs.

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INTRODUCTION

The traditional application for the alpha-beta filter is tracking moving objects. This second-order filter became popular in the late 1950s and early 1960s, when analog implementation was still the rule [1, 2]. A variation increased the filter order to three, to yield better performance for objects with greater dynamics [3, 4]. Filter development continued throughout the 70s [5–7].

During the 1970s, design techniques for linear tracking filters shifted from the classical, deterministic, exemplified by the Alpha-Beta filter, to the stochastic, popularized by the Kalman filter [8–10]. The latter filter gained wide acceptance, coincident with the popularization of state-space modeling and analysis techniques [11]. The Kalman filter found its greatest acceptance in high-order tracking problems, with both time-varying and time-invariant (Wiener) filter implementations.

With the advent of the Kalman filter, it was realized that the optimum time-invariant second-order Kalman filter for tracking position and velocity had the same architecture as the deterministic alpha-beta filter. All that differed between the two implementations was the method for designing the two adjustable "tuning" coefficients of the filters. The Kalman design was based on statistical properties of an assumed signal and noise generating model, while the alpha-beta design was based on classical deterministic response criteria, such as bandwidth and damping ratio.

This work ties together the Kalman and alpha-beta design, showing how they are related, and comparing them on a common basis. Methods are given for using design criteria from both the deterministic and stochastic realms in a single filter design. For instance, for a (Type-1) alpha-beta filter of prescribed (sampling time)$*$(noise bandwidth) product, it is shown what is the equivalent stochastic data model specifications for which the filter is Kalman optimal, and the resulting deterministic performance parameters.

THE KALMAN/ALPHA-BETA ARCHITECTURE

The approach taken here to the alpha-beta filter is through the steady-state Kalman filter. A particular generating model is assumed for the received signal and noise, such that the resulting Kalman architecture is the same as that for the alpha-beta filter.

The impulse-invariant transformation model for the discrete-time received signal-noise generation model is shown in Fig. 1. The received process $z(k)$ is the sum of signal $y(k)$ plus noise $v(k)$ where k is sample number. Both $y(k)$ and $v(k)$ are assumed to be Gaussian, zero-mean, independent, with known standard deviations σ_y and σ_v, respectively. The noise $v(k)$ is assumed to be white, whereas $y(k)$ is correlated from sample to sample.

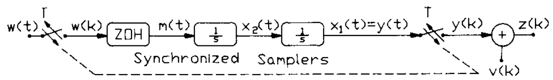

Fig. 1. Signal-noise generator; impulse-invariant model.

Fig. 1 shows that the discrete-time signal $y(k)$ is derived by sampling the output of a continuous model. The model is a double integrator driven by a sampler and zero-order-hold (ZOH). The driving process $w(t)$ is "almost white," which yields a white sampled process $w(k)$, zero-mean, of known standard deviation, σ_w.

The model state $x_1(t)$ which is also the output, is taken to be a "position variable," for the tracking problem. Then, the $x_2(t)$ state is a "velocity variable."

The continuous-time state-variable equations for the model of Fig. 1 are

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A \cdot \mathbf{x}(t) + \mathbf{b} \cdot m(t) \\ y(t) &= \mathbf{h}^{\mathrm{T}} \cdot \mathbf{x}(t); \qquad 0 \le t < \infty \\ m(t) &= w(kT); \qquad kT \le t < (k+1)T \end{aligned} \tag{1}$$

where boldface denotes vector quantity, superscript T denotes vector transpose, and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \qquad \mathbf{b} \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \qquad \mathbf{h} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

From the model of Fig. 1, using the method of impulse invariance [12], the continuous-time equations are transformed to discrete time as

$$\begin{aligned} \mathbf{x}(k+1) &= F \cdot \mathbf{x}(k) + \mathbf{d} \cdot w(k) \\ y(k) &= \mathbf{h}^{\mathrm{T}} \cdot \mathbf{x}(k), \qquad k = 0,1,2,\ldots,\text{sample number} \\ z(k) &= y(k) + v(k) \end{aligned} \tag{2}$$

where

$$F = \exp(A \cdot T) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

and

$$\mathbf{d} = \int_0^T (\exp(A \cdot q)) \cdot \mathbf{b} \cdot dq = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}.$$

The steady-state Kalman filter (fixed-gain) for the data generating model of (2) is

$$\begin{aligned} \hat{\mathbf{x}}(k) &= F \cdot \hat{\mathbf{x}}(k-1) + \mathbf{g} \cdot e(k), \qquad \mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \\ e(k) &= z(k) - \mathbf{h}^{\mathrm{T}} \cdot F \cdot \hat{\mathbf{x}}(k-1), \qquad k = 1,2,3,\ldots \\ \hat{y}(k) &= \mathbf{h}^{\mathrm{T}} \cdot \hat{\mathbf{x}}(k) \end{aligned} \tag{3}$$

where $\hat{\mathbf{x}}(\cdot)$ and $\hat{y}(\cdot)$ are the filtered estimates of $\mathbf{x}(\cdot)$ and $y(\cdot)$, respectively. The vector $\mathbf{g}$ is the Kalman gain, which is obtained from the solution of the steady-state discrete Ricatti equation.

$$\begin{aligned} P^{-1} &= [F \cdot P \cdot F^{\mathrm{T}} + \mathbf{d} \cdot \mathbf{d}^{\mathrm{T}} \cdot \sigma_w^2]^{-1} + \mathbf{h} \cdot \mathbf{h}^{\mathrm{T}}/\sigma_v^2 \\ \mathbf{g} &= P \cdot \mathbf{h}/\sigma_v^2 \end{aligned} \tag{4}$$

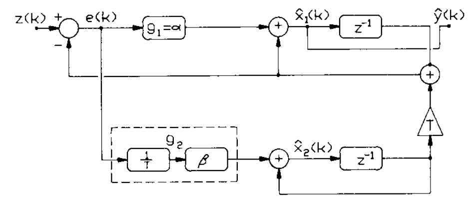

Fig. 2. Kalman (alpha-beta) filter.

and P is the steady-state tracking error covariance matrix, defined by

$$P = E[\mathbf{x}(k) - \hat{\mathbf{x}}(k)][\mathbf{x}(k) - \hat{\mathbf{x}}(k)]^{\mathrm{T}}.$$

The filter corresponding to (3) is diagrammed in Fig. 2.

The Kalman filter in the above figure is also the alpha-beta filter, under the identification,

$$\begin{aligned} g_1 &= \alpha \\ g_2 &= \beta/T. \end{aligned} \tag{5}$$

IMPULSE-INVARIANT ALPHA-BETA INVERSE MODEL

The physical interpretation of the alpha-beta filter is based on the assumption that the signal being tracked, $y(k)$, is a position variable. With this assumption, $\hat{x}_1(k)$ is, dimensionally, position, and $\hat{x}_2(k)$ is velocity. Then, the filter residual, $e(k)$, is, dimensionally, position prediction error. Thus, $\alpha \cdot e(k)$ is a correction to the prediction, $\hat{x}_1(k-1) + T \cdot \hat{x}_2(k-1)$, which yields the current estimate, $\hat{x}_1(k) = \hat{y}(k)$. Also, $\beta \cdot e(k)/T$ is a velocity correction, based on dividing the position prediction error by the time between samples, to get a velocity prediction error.

Now, if α and β are computed via g_1 and g_2, using σ_w^2 and σ_v^2, then the design is well founded in the Kalman sense. However, if a stochastic generating model is not available, how else could α,β be determined?

It is noted that the alpha-beta filter architecture is that of a discrete-time Kalman filter, derived from the impulse-invariant transformation of the position/velocity generator. Suppose that the continuous-time inverse of the alpha-beta filter is found via impulse invariance. Then, α and β may be derived from a standard specification of damping ratio and bandwidth (undamped natural frequency) for the continuous-time filter. This is the approach to be followed here.

Discretizing the filter of Fig. 3, by impulse invariance, using a sampling interval of T seconds (the same value as the feedback gain from state, $\hat{x}_2(\cdot)$) yields the same discrete architecture as Fig. 2, with

$$\begin{aligned} g_1 &= b_1 \cdot T + b_2 \cdot T^2/2 \\ g_2 &= b_2 \cdot T. \end{aligned} \tag{6}$$

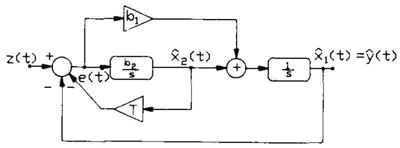

Fig. 3. Impulse-invariant inverse of alpha-beta filter.

Now, in Fig. 3, let

$$b_1 = \omega_n(2\delta - \omega_n \cdot T)$$
$$b_2 = \omega_n^2. \tag{7}$$

Then, the continuous-time transfer function is

$$Y(s)/Z(s) = K(s-z)/(s^2 + 2\delta \cdot \omega_n \cdot s + \omega_n^2)$$
$$K = \omega_n \cdot (2\delta - \omega_n \cdot T)$$
$$z = -\omega_n/(2\delta - \omega_n \cdot T)$$

and δ, ω_n have their usual interpretation, as damping ratio and undamped natural (radian) frequency.

Substituting (7) into (6) produces

$$g_1 = b_1 \cdot T + b_2 \cdot (T^2/2) = (\omega_n \cdot T)(2\delta - \omega_n \cdot T/2) = \alpha$$
$$g_2 \cdot T = b_2 \cdot T^2 = (\omega_n \cdot T)^2 = \beta.$$

Thus, α and β are related to δ, ω_n and T on a well-founded basis.

KALMAN DESIGN

The Kalman gain parameter **g** is obtained from the solution of the alternate gain equations [13] in normalized form.

$$[P_N^{-1} - \mathbf{h}_N \cdot \mathbf{h}_N^{\mathrm{T}}]^{-1} = F_N \cdot P_N \cdot F_N^{\mathrm{T}} + \mathbf{d}_N \cdot \mathbf{d}_N^{\mathrm{T}} \cdot (\sigma_w^2/\sigma_v^2)$$
$$\mathbf{g} = N \cdot P_N \cdot \mathbf{h}_N \tag{8}$$

where N is a matrix for transforming the original generating model state $\mathbf{x}(k)$ into an equivalent state [14], $\mathbf{x}_n(k)$, having the same impulse response. The $\mathbf{d}_N$, F_N, and $\mathbf{h}_N$ are the elements of the transformed generating model. The transformed steady-state tracking error covariance matrix is P_N. (See Appendix A.)

$$N = \begin{bmatrix} 1 & 0 \\ 0 & 1/T \end{bmatrix}; \qquad \mathbf{d}_N = \begin{bmatrix} T/2 \\ T \end{bmatrix};$$
$$F_N = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}; \qquad \mathbf{h}_N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$P_N = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}.$$

Using the Matrix Inversion Lemma [15], the matrix covariance equation may be written is coupled scalar form

$$p_{11}/(1-p_{11}) = p_{11} + 2 \cdot p_{12} + p_{22} + r$$
$$p_{12}/(1-p_{11}) = p_{12} + p_{22} + 2 \cdot r \tag{9}$$
$$(p_{22} - p_{11} \cdot p_{22} + p_{12}^2)/(1-p_{11}) = p_{22} + 4 \cdot r$$

where

$$r = ((\sigma_w \cdot T^2/2)/\sigma_v)^2.$$

These coupled nonlinear equations may be combined by substitution into a single quartic in p_{11} [16].

$$x^4 + 2rx^3 + r(r-18)x^2 + 2r(16-r)x + r(r-16) = 0 \tag{10}$$

where $x = p_{11}$.

The quartic may be factored into a product of two quadratics [16].

$$\left[x^2 + \left(r + 4\sqrt{r}\right)x - \left(r + 4\sqrt{r}\right)\right]$$
$$\times \left[x^2 + \left(r - 4\sqrt{r}\right)x - \left(r - 4\sqrt{r}\right)\right] = 0. \tag{11}$$

There exists a unique solution (of the four) of (11), satisfying $0 < p_{11}$ and $0 < p_{11} \cdot p_{22} - p_{12}^2$ such that P_N is a positive-definite matrix. It is given as [17].

$$p_{11} = \left(\left(r + 4\sqrt{r}\right)/2\right)\left(\sqrt{1 + 4/\left(r + 4\sqrt{r}\right)} - 1\right)$$
$$p_{12} = (p_{11}^2 + r(1-p_{11}))/(2-p_{11}) \tag{12}$$
$$p_{22} = p_{11} \cdot p_{12}/(1-p_{11}) - 2r.$$

Then,

$$g_1 = p_{11} = \alpha$$
$$g_2 = p_{12}/T = \beta/T \qquad \text{or} \qquad \beta = p_{12}. \tag{13}$$

The quantity r upon which the gains depend, has the dimensions of a signal-to-noise ratio. Assuming that $y(k)$ is a position variable (say, in meters), then $v(k)$ is measurement noise (also in meters). The quantity, $\sigma_w \cdot T^2/2$ (in the numerator of r) has the same dimension as $y(k)$ or $v(k)$ and may be interpreted as the random perturbation of position in one sample interval, due to the (assumed white) acceleration $w(k)$.

The actual tracking error variance is defined by

$$\sigma_t^2 = E(y(k) - \hat{y}(k))^2 = \sigma_v^2 \cdot \mathbf{h}_N^{\mathrm{T}} \cdot P_N \cdot \mathbf{h}_N.$$

Thus, the tracking error, normalized by measurement noise, is

$$\sigma_t^2/\sigma_v^2 = p_{11} = \left(\left(r + 4\sqrt{r}\right)/2\right)\left(\sqrt{1 + 4/\left(r + 4\sqrt{r}\right)} - 1\right). \tag{14}$$

To be an optimum Kalman design, the actual signal-to-noise ratio must be equal to that value of r which is used to determine the gains g_1 and g_2 (or α and β). If the actual value of σ_w^2 and/or σ_v^2 are different than assumed, the actual tracking error will be different from that calculated via (14). In case the actual signal and noise environments are not known exactly, a rule of thumb is to overestimate the signal-to-noise ratio if the signal level varies, and underestimate the ratio if the noise varies.

Kalman design, then, starts with choosing a signal-to-noise ratio r. Alternatively, a normalized tracking error σ_f^2/σ_v^2 may be chosen, and r solved for, numerically. Then, α and β are solved for. Finally T is chosen.

DETERMINISTIC DESIGN

From the relations between the Kalman version of the alpha-beta filter and its continuous-time impulse-invariant inverse, the following design equations are obtained.

$$\beta = (\omega_n \cdot T)^2$$
$$\alpha = 2 \cdot \delta \cdot \sqrt{\beta} - \beta/2. \tag{15}$$

Also, from the continuous-time transfer function, is obtained a relation for the product of filter noise bandwidth [18] (in hertz) times sampling interval (in seconds).

$$B_N T = \left(\sqrt{\beta}/8\delta\right)\left(1 + \left(2\delta - \sqrt{\beta}\right)^2\right). \tag{16}$$

The transfer function may also be manipulated to show that the filter is Type-1.

Now, filter stability and impulse invariance require

$$\text{Invariance} : 0 \leq \alpha \leq 1 : \text{Stability}.$$

Also, consideration of aliasing and analog-to-digital (A/D) distortion lead to an approximate Nyquist criterion constraint, with respect to ω_n, of

$$\sqrt{\beta} = \omega_n T \leq \pi.$$

Now, it turns out that when the gains g_1, g_2, (or α, β) are computed via Kalman, the filter always realizes $\delta = 1/\sqrt{2}$ [8]. For a deterministic design with Kalman-optimum damping ratio,

$$0 \leq \alpha \leq 1; \qquad \delta = 1/\sqrt{2}.$$
$$0 \leq \beta \leq 2; \qquad \text{Kalman-optimum damping.}$$

Deterministic design consists of choosing the $B_N T$ product and δ to yield β. Then, solve for α.

DESIGN NOMOGRAPH

A single design nomograph is created to handle the Kalman design or the deterministic design with Kalman-optimum damping ratio (0.707). This nomograph is shown here as Fig. 4.

The nomograph is based on the fact that the filter gain coefficient α is identical to the normalized tracking error σ_f^2/σ_v^2 variance. The other coefficient β is related to α by (15), and may, therefore, be

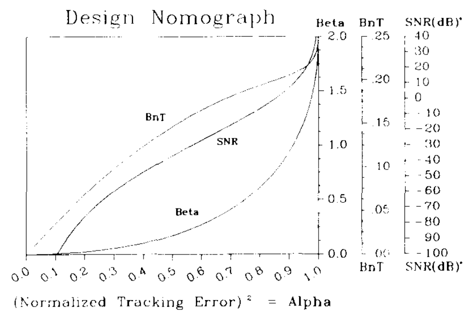

Fig. 4. Design nomograph.

plotted as a function of α. Likewise, the normalized filter bandwidth $B_N \cdot T$ may be plotted as an implicit function of α, through its relation to β, as given in (16). Finally, the signal-to-noise ratio r may be plotted in decibels (SNRDB) as a function of α, through the relations of (13) and (14).

For deterministic design ($\delta = 0.707$) the nomograph is entered via $B_N \cdot T$, and read downward to obtain β and α. Also obtained is the value of SNRDB for which the deterministic design is Kalman optimum. For Kalman design, the nomograph is entered via either SNRDB or normalized tracking error. β is then obtained from the nomograph, as well as the corresponding value of $B_N \cdot T$.

EXAMPLE

Assume 1) an automobile using a Global Positioning System (GPS) receiver as a source of position measurements, 2) a horizontal measurement accuracy of 32 ft, rms, with samples available at a rate of 10/s, and 3) a random, driver-generated lateral acceleration of $\frac{1}{10}$-gravity, or 3.2 ft/s^2. Thus, is specified

$$\sigma_w = 3.2; \qquad \sigma_v = 32; \qquad T = 0.1.$$

Next, calculate

$$\text{SNR} = (\sigma_w/\sigma_v)^2 = 10^{-2}$$

and

$$\text{SNR(dB)} = 10 * \log_{10}(\text{SNR}) = -20.$$

Now, enter the nomograph at the value -20 on the SNR(dB) scale. Pass a horizontal and mark the SNR curve. Pass a vertical through the SNR mark and mark the $B_n T$ and beta curves and the alpha scale. Read the marked alpha scale to obtain

$$\alpha = 0.68.$$

Pass horizontals through the $B_n T$ and beta marks and mark the $B_n T$ and beta scales. Read the marked scales to obtain

$$B_n T = 0.175, \qquad \beta = 0.375.$$

Next, given the sample interval, $T = 0.1$, calculate

$$B_n = B_nT/T = 0.175/0.1 = 1.75 \text{ Hz}$$

and

$$g_2 = \beta/T = 0.375/0.1 = 3.75.$$

Finally, the rms (random) tracking error is obtained as

$$\sigma_t = \sigma_v \cdot \sqrt{\alpha} = 32 \cdot \sqrt{0.68} = 26.4 \text{ ft rms}$$

APPENDIX. STATE NORMALIZATION

Define a normalized state vector $\mathbf{x}_N(k)$ according to

$$\mathbf{x}(k) = N \cdot \mathbf{x}_N(k), \qquad N = \begin{bmatrix} 1 & 0 \\ 0 & 1/T \end{bmatrix}.$$

Then, the Kalman generating model becomes

$$\mathbf{x}_N(k+1) = F_N \cdot \mathbf{x}_N(k) + \mathbf{d}_N \cdot w(k);$$
$$y(k) = \mathbf{h}_N^{\mathrm{T}} \cdot \mathbf{x}_N(k)$$

where

$$F_N = N^{-1} \cdot F \cdot N = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix};$$
$$\mathbf{d}_N = N^{-1} \cdot \mathbf{d} = (1/\sigma_v) \begin{bmatrix} T^2/2 \\ T^2 \end{bmatrix} \tag{17}$$
$$\mathbf{h}_N^{\mathrm{T}} = \mathbf{h}^{\mathrm{T}} \cdot N = \sigma_v \cdot [1,0].$$

The σ_v factors are inserted to also normalize the steady-state error covariance, as well as the state vector. The normalized model, given above, may be easily shown to have the same impulse response as the original Kalman generating model, given in (1) and (2).

The Kalman filter equations for the normalized state are

$$\hat{\mathbf{x}}_N(k) = F_N \cdot \hat{\mathbf{x}}_N(k-1) + \mathbf{g}_n \cdot e_N(k), \qquad \mathbf{g}_N = N^{-1} \cdot \mathbf{g}$$
$$e_N(k) = z(k) - \mathbf{h}_N^{\mathrm{T}} \cdot F_N \cdot \hat{\mathbf{x}}_N(k-1)$$
$$y(k) = \mathbf{h}_N^{\mathrm{T}} \cdot \hat{\mathbf{x}}_N(k).$$

The corresponding doubly normalized steady-state error covariance equation in alternate form is

$$(P_N^{-1} - \mathbf{h} \cdot \mathbf{h}^{\mathrm{T}})^{-1} = F_N \cdot P_N \cdot F_N^{\mathrm{T}} + \mathbf{q} \cdot \mathbf{q}^{\mathrm{T}},$$
$$\mathbf{q} = (\sigma_w \cdot T^2/2)/\sigma_v) \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Note that in the covariance equation, the quantity $\mathbf{h}$ is from the original generating model, not the normalized model.

Applying the Matrix Inversion Lemma [15] yields

$$P_N + P_N \cdot \mathbf{h} \cdot \mathbf{h}^{\mathrm{T}} \cdot (I - P_N \cdot \mathbf{h} \cdot \mathbf{h}^{\mathrm{T}})^{-1} \cdot P_N$$
$$= F_N \cdot P_N \cdot F_N^{\mathrm{T}} + \mathbf{q} \cdot \mathbf{q}^{\mathrm{T}}$$

which may be expanded to scalar form to yield (9).

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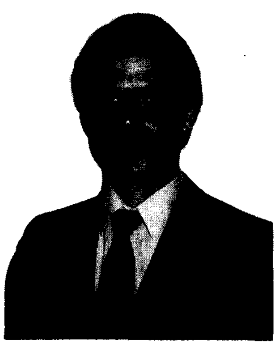

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