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Reconciling the cosmic age problem in the $R_h = ct$ universe

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Abstract Many dark energy models fail to pass the cosmic age test. In this paper, we investigate the cosmic age problem associated with nine extremely old Global Clusters (GCs) and the old quasar APM 08279+5255 in the $R_h = ct$ universe. The age data of these oldest GCs in M31 are acquired from the Beijing–Arizona–Taiwan–Connecticut system with up-to-date theoretical synthesis models. They have not been used to test the cosmic age problem in the $R_h = ct$ universe in previous literature. By evaluating the age of the $R_h = ct$ universe with the observational constraints from the type Ia supernovae and the Hubble parameter, we find that the $R_h = ct$ universe can accommodate five GCs and the quasar APM 08279+5255 at redshift $z = 3.91$. But for other models, such as ΛCDM, the interacting dark energy model, the generalized Chaplygin gas model, and holographic dark energy model, cannot accommodate all GCs and the quasar APM 08279+5255. It is worthwhile to note that the age estimates of some GCs are controversial. So, unlike other cosmological models, the $R_h = ct$ universe can marginally solve the cosmic age problem, especially at high redshift.

1 Introduction

Many astronomical observations, such as type Ia supernovae (SNe Ia) [1–3], the cosmic microwave background (CMB) [4–6], gamma-ray bursts [7,8] and the large-scale structure (LSS) [9], indicate that the universe is undergoing an accelerated expansion, which suggests that our universe may have an extra component like dark energy. The nature of this dark energy is still unknown, but the simplest and most interesting candidate is the cosmological constant [10]. This model may be consistent with most astronomical observations. The latest observation shows that the present cosmic age is about $t_0 = 13.82$ Gyr in the ΛCDM model [6], but it still suffers from the cosmic age problem [11,12]. The cosmic age problem is that some objects are older than the age of the universe at its redshift z. In previous literature, many cosmological models have been tested by the old quasar APM 08279+5255 with age 2.1 ± 0.3 Gyr at $z = 3.91$ [13,14], such as the ΛCDM [13,15], the $\Lambda(t)$ model [16], the interacting dark energy models [12], the generalized Chaplygin gas model [17,18], the holographic dark energy model [19], braneworld models [20–22], and the conformal gravity model [23]. But all of these models have a serious age problem except the conformal gravity model, which can accommodate this quasar at 3σ confidence level [23].

In this paper, we will use the old quasar APM 08279+5255 at redshift $z = 3.91$ and the nine extremely old Global Clusters [24,25] to investigate the cosmic age problem in the $R_h = ct$ universe. The data of these nine extremely old GCs listed in Table 1 are acquired from the Beijing–Arizona–Taiwan–Connecticut system with up-to-date theoretical synthesis models. The evolutionary population synthesis modeling has become a powerful tool for the age determination [26,27]. In [24,25], they get the ages of those GCs by using multi-color photometric CCD data and comparing them with up-to-date theoretical synthesis models. But the ages of GCs derived by different authors based on different measurements using same method are not always consistent [24]. We find that the nine of those GCs can give stronger constraints on the age of universe than the old quasar APM 08279+5255. Those nine extremely old GCs have been used to test the cosmic age problem in dark energy models in previous work and many dark energy models have a serious age problem [12]. The $R_h = ct$ universe is a model of the cosmos which is closely restricted by the cosmological principle and Weyl's postulate [28]. In the $R_h = ct$ universe, the gravitational horizon R_h is always equal to ct. The $R_h = ct$ universe can fit the SNe Ia data well [29], explain the growth of high-z quasars [30], and account for the apparent absence in the CMB angular correlation [31]. As we discussed above, many

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Table 1 The properties of the nine extremely old Global Clusters from [24,25]

GC's no.	GC	Age	Reference
1	B239	14.50 ± 2.05	[24]
2	B050	16.00 ± 0.30	[24]
3	B129	15.10 ± 0.70	[24]
4	B144D	14.36 ± 0.95	[25]
5	B024	15.25 ± 0.75	[25]
6	B260	14.30 ± 0.50	[25]
7	B297D	15.18 ± 0.85	[25]
8	B383	13.99 ± 1.05	[25]
9	B495	14.54 ± 0.55	[25]

cosmological models cannot pass the age test. But whether the $R_h = ct$ universe suffers from the cosmic age problem is still unknown.

The structure of this paper is as follows. In Sect. 2, we introduce the $R_h = ct$ universe. In Sect. 3, we give the constraints on the $R_h = ct$ universe from SNe Ia and $H(z)$ data. Then we will test the $R_h = ct$ universe with the nine extremely oldest GCs and the old quasar APM 08279+5255. The age test in other cosmological models is presented in Sect. 4. Conclusions will be given in Sect. 5.

2 The $R_h = ct$ universe

The $R_h = ct$ universe is a model of the cosmos which is closely restricted by the cosmological principle and Weyl's postulate [28,32]. For a certain age of the universe t, there is a limiting observable distance $R_h(t)$, which is called the cosmic horizon. Any signal beyond the cosmic horizon cannot be observed by us. The horizon is defined as

$$R_h = \frac{2GM(R_h)}{c^2}, \tag{1}$$

where $M(R_h)$ is the total mass enclosed within R_h [28,33]. From Eq. (1), we can find that the cosmic horizon is a Schwarzschild radius. If we write the energy density of matter as ρ, then $M(R_h) = 4\pi R_h^3 \rho/(3c^2)$, so it yields

$$R_h = \sqrt{(3c^4/(8\pi G\rho))}. \tag{2}$$

The expansion of the universe is calculated from the Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3c^2} - \frac{kc^2}{a^2}, \tag{3}$$

where $H = \dot{a}/a$ is the Hubble parameter, a is the scale factor, k is the spatial curvature constant and $k = -1, 0$ and $+1$ corresponds to an open, flat, and closed universe, respectively. If we assume the universe is flat, from Eqs. (2) and (3), we have $H = c/R_h$. For the $R_h = ct$ universe, we have $R_h = ct$. We obtain

$$H = \frac{\dot{a}}{a} = \frac{1}{t}, \tag{4}$$

where t is the age of universe. Solving Eq. (4) with $a = \frac{1}{1+z}$ and the initial condition $H = H_0$ when $z = 0$, one can get

$$H = H_0(1+z). \tag{5}$$

The luminosity distance in the $R_h = ct$ universe is [33]

$$d_L = (1+z)R_h(t_0)\ln(1+z) = \frac{c(1+z)}{H_0}\ln(1+z), \tag{6}$$

where t_0 is the age of the local universe.

3 Observational constraints on the $R_h = ct$ universe

In this section, we constrain the $R_h = ct$ universe using the Union 2.1 SNe Ia data [34] and the observed Hubble parameter data $H(z)$. The SNe Ia distance moduli and the value of $H(z)$ reported in the literature depend on the specific cosmological model, i.e., ΛCDM. When we use them to constrain other cosmological models, the original data must be re-analyzed. Wei et al. [35] derived the SNe Ia distance moduli in the $R_h = ct$ universe. For the Hubble parameter data, we choose 19 model-independent data from [36]. Then we test the model with the nine extremely old GCs in M31 and the old quasar APM 08279+5255 based on the principle that any object is younger than its local universe.

3.1 Constraining the $R_h = ct$ universe with SNe Ia and $H(z)$ data

SNe Ia supernovae are considered as the best standard candles to measure the distance and investigate the expansion of the universe. The Hubble parameter $H(z)$ reveals the expansion of the universe directly. So we use the SNe Ia and $H(z)$ data to constrain the $R_h = ct$ universe. The Union 2.1 sample contains 580 SNe Ia supernovae at a redshift less than 1.5 [34,37,38]. Wei et al. re-calculate those SNe Ia distance moduli in the $R_h = ct$ universe and give their redshift z_i, distance modulus $\mu_{obs}(z_i)$, and its corresponding error σ_i. The theoretical distance modulus is defined as

$$\mu_{th}(z_i) = 5\log_{10} d_L(z_i) + 25. \tag{7}$$

Table 2 The best-fit values of the Hubble constant H_0 in the $R_\mathrm{h} = ct$ universe

Observations	$H_0/(\mathrm{km\ s^{-1}\ Mpc^{-1}})$	$\chi^2_{min}/\mathrm{dof}$
SNe Ia	70.01 ± 0.40	0.99
SNe Ia + $H(z)$	69.83 ± 0.40	1.01

We can get the theoretical distance modulus $\mu_{th}(z_i)$ for each SN Ia from Eqs. (6) and (7). The χ^2 for SNe Ia is

$$\chi^2_{SN}(H_0) = \sum_{i=1}^{i=580} \frac{(\mu_{th}(z_i) - \mu_{obs}(z_i))^2}{\sigma_i^2}. \tag{8}$$

So χ^2_{SN} has only one parameter, H_0. We can get the best-fit H_0 by minimizing χ^2_{SN} (see Table 2). Reference [29] also found that the $R_\mathrm{h} = ct$ universe can well fit the Union 2.1 sample.

The Hubble parameter values we use are obtained from previous published literature [39–45]. These Hubble parameter data are complied in [46]. In [36], 19 model-independent values have been chosen. So we use these model-independent $H(z)$ data. The χ^2 for $H(z)$ is

$$\chi^2_H(H_0) = \sum_{i=1}^{i=19} \frac{(H_{th}(z_i) - H_{obs}(z_i))^2}{\sigma^2_{H_i}}. \tag{9}$$

The total χ^2 is $\chi^2_{tot}(H_0) = \chi^2_{SN}(H_0) + \chi^2_H(H_0)$. Then we minimize the total χ^2_{tot} to get the best-fit parameter H_0 of the $R_\mathrm{h} = ct$ universe.

The best-fit Hubble constant is $H_0 = 70.01 \pm 0.40\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ **at** 1σ confidence level with $\chi^2_{min} = 573.13$ from SNe Ia. After including the 19 Hubble parameter data, the best-fit Hubble parameter is $H_0 = 69.83 \pm 0.40\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ at 1σ confidence level with $\chi^2_{min} = 604.03$. Recently, the Planck team derived the Hubble constant $H_0 = 67.3 \pm 1.2\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ in the ΛCDM model, which is consistent with our result.

3.2 Testing the $R_\mathrm{h} = ct$ universe with old objects

The old objects are usually used to test cosmological models, especial the old high redshift objects [47]. In previous literature, many cosmological models cannot pass the cosmic age test. We use the nine extremely old GCs in M31 and the old quasar APM 08279+5255 to test the $R_\mathrm{h} = ct$ universe. Any object at any redshift z must be younger than the age of the universe at z, i.e., $t_{obj}(z) < t_{cos}(z)$, where $t_{obj}(z)$ is the age of a object at redshift z, and $t_{cos}(z)$ is the age of the universe at redshift z. The age of a flat universe is given by [17]

$$t_{cos}(z) = \int_z^\infty \frac{d\tilde{z}}{(1+\tilde{z})H(\tilde{z})}. \tag{10}$$

From Eq. (5), the age of the $R_\mathrm{h} = ct$ universe at redshift z is

$$t_{cos}(z) = \frac{1}{H_0(1+z)}. \tag{11}$$

We use the best-fit value of Hubble constant $H_0 = 70.01 \pm 0.40\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ from SNe Ia data to calculate the age of the universe. For this result, the age of the local $R_\mathrm{h} = ct$ universe $t_0 = 13.97 \pm 0.08$ Gyr. For the best-fit value of Hubble constant $H_0 = 69.83 \pm 0.40\mathrm{km\ s^{-1}\ Mpc^{-1}}$ from SNe Ia and Hubble parameter data, the age of local universe is $t_0 = 14.01 \pm 0.08$ Gyr. We choose the second one. In Fig. 1, the blue line shows the evolution of cosmic age at different redshifts, and the red lines are for the 1σ dispersion. For a given diagonal line, the area below this diagonal line corresponds to a larger cosmic age. From Fig. 1, we find that the $R_\mathrm{h} = ct$ universe accommodates the old quasar APM 08279+5255 at more than 3σ confidence level. In Fig. 2, the blue line shows the best-fit line of the age of local universe, and the red lines are for the 1σ dispersion. From Fig. 2, we find that five GCs (B239, B144D, B260, B383, B495) can be accommodated by the $R_\mathrm{h} = ct$ universe at 1σ confidence level but the other four GCs (B129, B024, B297D, B050) cannot. But the age estimates of some GCs are controversial. For example, the metallicities of B129, B024, B297D, and

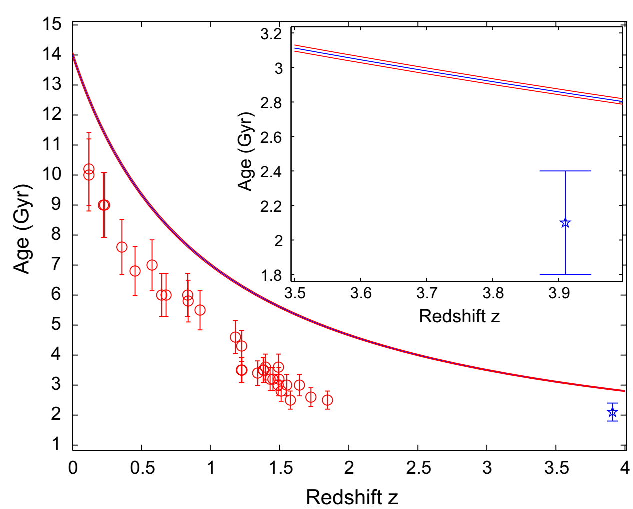


Fig. 1 The *blue line* shows the evolution of cosmic age in the $R_\mathrm{h} = ct$ universe using the best-fit value from SNe Ia and Hubble parameter; the *red lines* are the 1σ deviation. The *star* is the old quasar APM 08279+5255. We can find the quasar is below the *lines*, which means the old quasar APM 08279+5255 is younger than the age of the $R_\mathrm{h} = ct$ universe. The *open circles* are old galaxies data with 1σ error taken from [50]. The *inset* shows the dispersion and data clearly

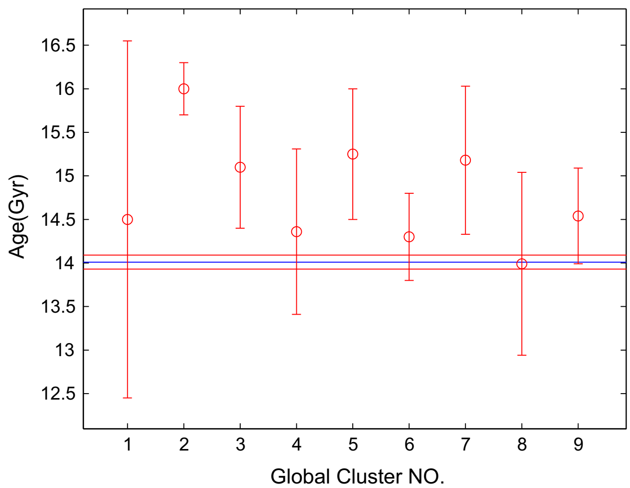


Fig. 2 The *blue line* shows the cosmic age in the $R_h = ct$ using the best-fit value from SNe Ia and the Hubble parameter, the *red lines* are for the 1σ deviation. The *red circles* are the nine extremely old GCs and this also gives the error of their age

B050 measured by [24,25] are higher than those of [48]. The values are significantly different. So the GCs ages derived by [24,25] may be larger than the true ages. Due to the uncertainty of the age determination, we can claim that the $R_h = ct$ universe can marginally solve the cosmic age problem.

4 Testing other models

In order to compare with the $R_h = ct$ universe, we also investigate some other models. The theoretical luminosity distance is

$$d_L = c(1+z)\int_0^z \frac{d\tilde{z}}{H(\tilde{z})}, \tag{12}$$

where $H(z)$ is the Hubble parameter. Then we can use Eq. (7) to get the distance modulus. But the SNe Ia data should be re-optimized for each model except the ΛCDM model, which needs lots of work. So, like previous literature, we just use the SNe Ia data based on the ΛCDM model. The 19 model-independent Hubble parameters chosen by [36] are also used.

4.1 ΛCDM model

The Hubble parameter in the flat ΛCDM model is

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}. \tag{13}$$

Using the same method as that used in the $R_h = ct$ model, we find that the best-fit Hubble constant value is $H_0 = 69.93 \pm 0.50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and the best-fit Ω_m value is $\Omega_m = 0.28 \pm 0.02$. Panel (a) of Fig. 3 shows the constraints on the h–Ω_m plane at 1σ, 2σ, and 3σ confidence levels. The blue line and the two red lines represent the age of that old quasar APM 08279+5255 and 1σ error, respectively. From Fig. 3 we can see that the ΛCDM model cannot accommodate the old quasar APM 08279+5255. From Eq. (10), we can find that the age of the local universe, which means $z = 0$, is $t_0 = 13.71^{+0.30}_{-0.28}$ Gyr. From Fig. 4, which is similar to Fig. 2, we can also find that there are only five GCs (B239, B144D, B260, B383, B495) that can be accommodated by the ΛCDM universe at 1σ confidence level.

4.2 Interacting dark energy model

In [12], they introduce three interacting dark energy models. We take the first one called IΛCDM as an example. For a flat universe, the Hubble parameter in this model is

$$H(z) = H_0\sqrt{\frac{\Omega_m}{1-\alpha}(1+z)^{3(1-\alpha)} + \left(1 - \frac{\Omega_m}{1-\alpha}\right)}, \tag{14}$$

where the α is a parameter denoting the strength of interaction. The best-fit values are $H_0 = 69.95 \pm 0.50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_m = 0.28 \pm 0.03$ and $\alpha = -0.01$. From Panel (b) of Fig. 3, we can see that the IΛCDM cannot accommodate the old quasar APM 08279+5255. From Eq. (10), we can find that the age of the local universe is $t_0 = 13.62^{+0.31}_{-0.27}$Gyr. From Fig. 4, we can also find that there are only four GCs (B239, B144D, B260, B383) that can be accommodated by the IΛCDM universe at 1σ confidence level.

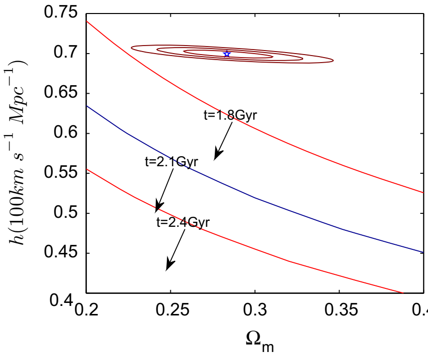
4.3 Generalized Chaplygin gas model

For the generalized Chaplygin gas (GCG) model, one has [17,18]:

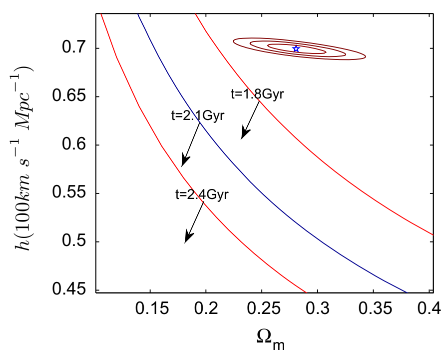
$$H(z) = H_0\sqrt{\Omega_b(1+z)^3 + (1-\Omega_b)[A_s + (1-A_s)(1+z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}}}, \tag{15}$$

where Ω_b is the energy density of baryon matter, and A_s and α are model parameters. The best-fit parameters are $H_0 = 70.07 \pm 0.35\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $A_s = 0.78 \pm 0.05$, and $\alpha = 0.17 \pm 0.38$. From panel (c) of Fig. 3, we can see that the age of the old quasar APM 08279+5255 is in tension (over the 2σ confidence level) with the age of universe for GCG model. A similar result is also found by [18]. From Eq. (10), we can find that the age of the local universe is $t_0 = 13.73^{+0.38}_{-0.62}$ Gyr. From Fig. 4, we can also find that there are only five GCs (B239, B144D, B260, B383, B495) that can be accommodated by the GCG model at 1σ confidence level.

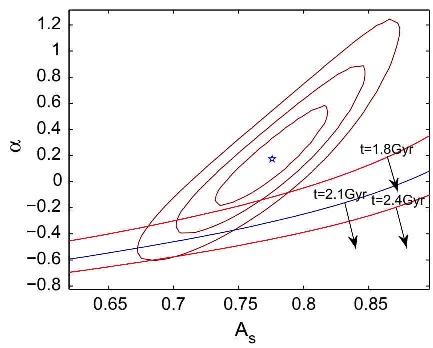
Fig. 3 Contour plot for the ΛCDM model, IΛCDM model, the GCG model, and the holographic dark energy model, respectively. The *ellipses* represent confidence intervals from 1σ to 3σ and the *blue star* means the optimal value. The *blue line* represents that the age of universe at $z = 3.91$ is 2.1 Gyr and the two *red lines* represent the 1σ error ± 0.3 Gyr. The *arrowhead* points to the allowed region



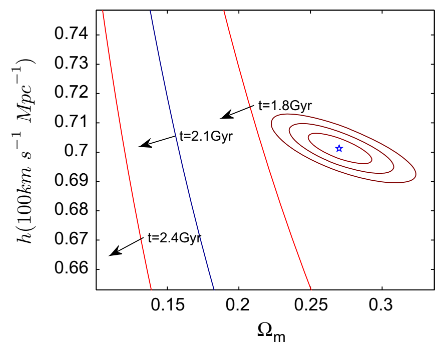
(a) ΛCDM model.



(b) IΛCDM model.

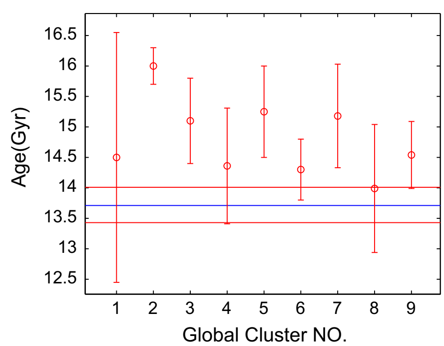


(c) The GCG model.

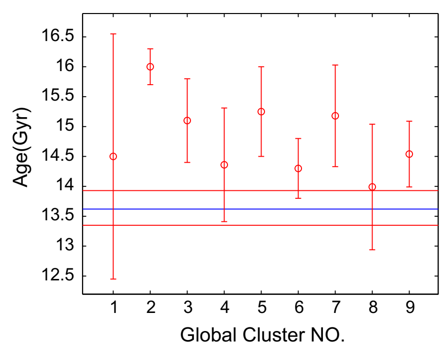


(d) The holographic dark energy model.

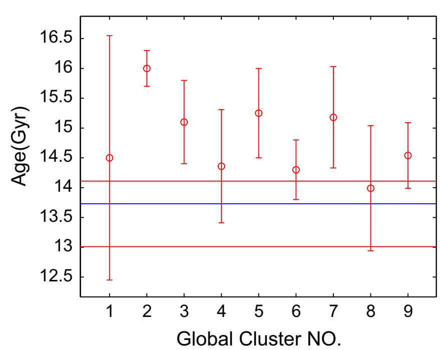
Fig. 4 Similar to Fig. 2 but for the Λ CDM model, the IΛCDM model, the generalized Chaplygin gas model, and the holographic dark energy model, respectively



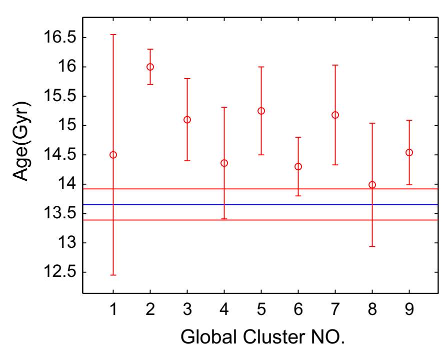
(a) ΛCDM model.



(b) IΛCDM model.



(c) The GCG model.



(d) The holographic dark energy model.

4.4 Holographic dark energy model

We will test the holographic dark energy model in this section. The Hubble parameter in this model is [49]

$$H(z) = H_0\sqrt{\frac{\Omega_{m0}(1+z)^3}{1-\Omega_\Lambda}}, \tag{16}$$

where $\Omega_{\rm m0}$ is the matter density at present and Ω_Λ is the energy density of the dark energy at redshift z, which can be calculated by

$$\ln\Omega_\Lambda - \frac{d}{2+d}\ln(1-\sqrt{\Omega_\Lambda}) + \frac{d}{2-d}\ln(1+\sqrt{\Omega_\Lambda}), \\ - \frac{8}{4-d^2}\ln(d+2\sqrt{\Omega_\Lambda}) = -\ln(1+z) + y_0, \tag{17}$$

where d is a free parameter and y_0 is a constant which can be calculated by Eq. (17) with $z = 0$ and $\Omega_\Lambda = 1 - \Omega_{\rm m0}$. The best-fit parameters are $H_0 = 70.13 \pm 0.51\,{\rm km\,s^{-1}\,Mpc^{-1}}$, $\Omega_{\rm m0} = 0.27 \pm 0.02$ and $d = 0.81 \pm 0.05$. From panel (d) of Fig. 3, we can see that the holographic dark energy model cannot accommodate the old quasar APM 08279+5255. From Eq. (10), we can find that the age of the local universe is $t_0 = 13.65^{+0.27}_{-0.26}$ Gyr. From Fig. 4, we can also find that there are only four GCs (B239, B144D, B260, B383) that can be accommodated by the holographic dark energy model at 1σ confidence level.

5 Conclusions

In this paper, we test the cosmic age problem in several cosmological models by using nine extremely old GCs in M31 and the old quasar APM 08279+5255. We find that the best-fit value of the Hubble constant in the $R_{\rm h} = ct$ universe is $H_0 = 70.01 \pm 0.40\,{\rm km\,s^{-1}Mpc^{-1}}$ at 1σ confidence level by using the SNe Ia data. In this case, the age of the local $R_{\rm h} = ct$ universe $t_0 = 13.97 \pm 0.08$ Gyr. If we fit the $R_{\rm h} = ct$ universe with the SNe Ia and $H(z)$ data, the Hubble constant is $H_0 = 69.83 \pm 0.40\,{\rm km\,s^{-1}\,Mpc^{-1}}$ at the 1σ confidence level. The age of the local universe is $t_0 = 14.01 \pm 0.08$ Gyr. From Fig. 1, we find that the $R_{\rm h} = ct$ universe can accommodate the old quasar APM 08279+5255 at more than 3σ confidence level. From Fig. 2, we find that there are five GCs (B239, B144D, B260, B383, B495) that can be accommodated by the $R_{\rm h} = ct$ universe at 1σ confidence level. But the age estimates of some GCs are controversial. For example, the metallicities of B129, B024, B297D, and B050 measured by [24,25] and [48] are significantly different. So the derived ages are different. Due to the uncertainty of the age determination, we can claim that the $R_{\rm h} = ct$ universe can marginally solve the cosmic age problem.

Using the same method, we also test some other cosmological models, such as ΛCDM, the interacting dark energy model, the generalized Chaplygin gas model, and the holographic dark energy model. In Sect. 4, we show that these models cannot accommodate all nine old GCs in M31. Meanwhile, for the old quasar APM 08279+5255 at $z = 3.91$, the $R_{\rm h} = ct$ model can accommodate it at more than 3σ confidence level. But these models cannot accommodate it. The generalized Chaplygin gas model is in tension (over the 2σ confidence level) with the age of APM 08279+5255. So the $R_{\rm h} = ct$ universe can marginally solve the cosmic age problem, especially at high redshift.

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