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## **Reconsidering Cointegration in International Finance: Three Case Studies of Size Distortion in Finite Samples — [Source link](#)**

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Three Case Studies of Size Distortion in Finite Samples**

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# **Reconsidering Cointegration in International Finance: Three Case Studies of Size Distortion in Finite Samples**

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**Keywords: cointegration, exchange rates, stock markets, Monte Carlo**

## **Abstract**

This paper reconsiders several recently published but controversial results about the behaviour of exchange rates. In particular, it explores finite-sample problems in the application of cointegration tests and shows how these may have affected the conclusions of recent research. It also demonstrates how simple simulation methods may be used to check the robustness of cointegration tests in particular applied settings, and provides information on the potential sources of size distortion in these tests.

Three case studies are presented. The first is the literature on cointegration and prediction of nominal spot exchange rates spawned by Baillie and Bollerslev (1989). The second is work on the long-run validity of the monetary model of exchange rate determination, particularly the contributions of MacDonald and Taylor (1993; 1994a). The final case study looks at the evidence presented by Kasa (1992) on common stochastic trends in the international stock market. Our results suggest that Baillie and Bollerslev's results are unaffected by finite-sample problems, but that the opposite is true for the other two case studies.

## **Résumé**

Les auteurs de l'étude se penchent sur les résultats controversés obtenus récemment par certains chercheurs au sujet du comportement des taux de change. Ils essaient notamment d'établir la présence de problèmes d'estimation tenant à la taille limitée de l'échantillon et montrent comment ces problèmes influencent les conclusions des études récentes. Ils montrent également comment de simples méthodes de simulation peuvent servir à évaluer la robustesse des tests de cointégration dans des conditions données et avancent différentes sources possibles de distorsions de niveau.

Les auteurs effectuent trois études de cas. La première s'inspire de la littérature que les recherches de Baillie et Bollerslev (1989) ont suscitée sur la cointégration et la prévision des taux de change nominaux au comptant. La seconde s'appuie sur les travaux relatifs à la validité à long terme du modèle monétaire de détermination du taux de change, notamment ceux de MacDonald et Taylor (1993 et 1994). La troisième et dernière étude passe en revue les résultats présentés par Kasa (1992) relativement à l'existence de tendances stochastiques communes aux marchés boursiers internationaux. Les auteurs concluent que les résultats de Baillie et Bollerslev ne sont pas influencés par des problèmes d'estimation liés à la taille limitée de l'échantillon, mais que l'on ne peut en dire autant des résultats examinés dans les deux autres études de cas.

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## 1.0 Introduction

This paper reconsiders several recently published, but controversial, results in international finance. Specifically, it explores finite-sample problems in the application of cointegration tests and it shows how these may have affected the conclusions of recent research. It also demonstrates how simple simulation methods may be used to check the robustness of cointegration tests in particular applied settings, and provides information on the potential sources of size distortion in these tests. While this methodology is not new, we feel it may be underused.

These results are intended to be of primary interest to those studying international financial markets. However, the results on sources of size distortion may also be of interest to econometricians trying to design more robust empirical methods. In addition, the simulation methods used may be a useful example for applied researchers in other fields who may be concerned about the robustness of their cointegration tests.

We present three case studies, each of which has interesting economic implications.

The first considers whether spot exchange rates for various currencies are cointegrated. If this is true, it implies that the level of current exchange rates can help to forecast future exchange rate changes.

The second considers whether monetary theory provides a useful model of long-run movements in exchange rates. This is not only important for understanding the relative importance of real and monetary shocks in exchange rate determination, but it may also have implications for the desirability of fixed versus flexible exchange rate arrangements.

The third considers whether international stock markets share common trends. If so, then gains to international portfolio diversification may decline as the holding period lengthens.

The next section briefly reviews the maximum-likelihood approach to testing systems for cointegration and surveys the state of the literature on the known weaknesses of these tests. It then describes a simple simulation experiment that can be used to help assess the test's performance in a given setting. The following three sections then present the three case studies mentioned above. Each begins with a brief synopsis of the literature and tries to replicate previously published results. This is followed by simulation experiments to assess whether these results are a reliable basis from which to draw conclusions. Each case study is largely self-contained and may be skipped with little loss of continuity.



## 2.0 Theory and Methodology

The many studies using cointegration tests show that the concept of cointegration has generated considerable interest among economists. Nonetheless, the finite-sample properties of these tests, particularly of tests for the number of cointegrating vectors in a system, has received much less attention. Work to date shows that the usual asymptotic critical values for cointegration tests can cause severe size distortions in systems with a large number of variables. This raises the possibility that previous research testing for cointegration in higher-dimensional systems (for example, with four or more variables) may have concluded that cointegration was present in the data whether this was true or not.

In this section, we briefly review the maximum-likelihood approach to testing for cointegration pioneered by Johansen (1988) and Johansen and Juselius (1990). We then survey various recent investigations of the test's finite-sample performance and consider the available alternatives.

### 2.1 The Maximum-Likelihood Test for Cointegration

While there are many alternative tests for cointegration, Gonzalo (1989) suggests that the maximum-likelihood (ML) system estimation approach performs better than both single-equation and alternative multivariate methods in detecting cointegration.<sup>1</sup> This approach is also among the best known and the most widely applied in empirical work. The starting point of these tests is a VAR specification for the  $n \times 1$  vector of I(1) variables, namely,

$$X_t = \mu + A_1 X_{t-1} + \dots + A_k X_{t-k} + u_t, \quad (\text{EQ 1})$$

where  $u_t$  is assumed to be an independent and identically distributed Gaussian process. Note that we can rewrite (EQ 1) as a Vector Error Correction Model (VECM)

$$\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + u_t, \quad (\text{EQ 2})$$

where

$$\begin{aligned} \Gamma_j &= -(I - A_1 - \dots - A_j) \\ \Pi &= -(I - A_1 - \dots - A_k) \end{aligned} \quad j = 1, \dots, k. \quad (\text{EQ 3})$$

By rewriting (EQ 1) as (EQ 2) we are able to summarize the long-run information in  $X_t$  by the long-run impact matrix,  $\Pi$ , and it is the rank of this matrix that determines the number of cointegrating vectors. Note that under the null hypothesis of  $r$  ( $0 < r < n$ ) cointegrating

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1. See also Watson (1995), especially Section 3.d.

vectors,  $\Pi$  can be factored as  $\Pi = \alpha\beta$ , where  $\alpha$  and  $\beta$  are  $n \times r$  matrices. Therefore under the null we can write the process for  $X_t$  as

$$\Delta X_t = \mu + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \alpha\beta' X_{t-k} + u_t. \quad (\text{EQ 4})$$

Johansen and Juselius (1990) demonstrate that  $\beta$ , the cointegrating vectors, can be estimated as the eigenvectors associated with the  $r$  largest, statistically significant eigenvalues found by solving the problem

$$|\lambda S_{kk} - S_{k0} S_{00}^{-1} S_{0k}| = 0, \quad (\text{EQ 5})$$

where  $S_{00}$  represents the residual-moment matrix from a regression of  $\Delta X_t$  on  $\Delta X_{t-1}, \dots, \Delta X_{t-k+1}$ ,  $S_{kk}$  is the residual-moment matrix from a regression of  $X_{t-k}$  on  $\Delta X_{t-k+1}$ , and  $S_{0k}$  is the cross-moment matrix. These eigenvalues readily permit the formation of likelihood ratios to test the value of  $r$ . Johansen and Juselius propose two tests with differing assumptions about the alternative hypothesis: (i) the Trace statistic tests the restriction  $r \leq q$  ( $q < n$ ) against the unrestricted alternative  $r \leq n$ ; and (ii) the  $\lambda^{max}$  statistic makes the alternative more precise by specifying that only one additional cointegrating vector exists ( $r \leq q + 1$ ). The likelihood-ratio test statistics are formed as

$$\text{Trace} = -T \sum_{i=q+1}^n \ln(1 - \hat{\lambda}_i) \quad (\text{EQ 6})$$

$$\lambda^{max} = -T \ln(1 - \hat{\lambda}_{q+1}). \quad (\text{EQ 7})$$

The asymptotic critical values are non-standard and are tabulated in Osterwald-Lenum (1992). They depend on whether one includes a constant in (EQ 1), whether one imposes the restriction that any cointegrating vectors must also annihilate drift, and whether drift is present when it is allowed for. The test without a constant was originally proposed by Johansen (1988), the others by Johansen and Juselius (1990).

## 2.2 Finite Sample Performance

The critical values for the ML tests described above are based on their asymptotic distribution under the null hypothesis of interest. Recently, a number of authors have begun to examine how reliable this asymptotic approximation is in finite samples, and some have suggested modifications to the ML test.<sup>2</sup>

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2. In addition to the papers we discuss below, see the literature survey in Ho and Sørensen (1994). Gonzalo and Lee (1995) give examples of several “pitfalls” — cases where the size of the ML test for cointegration approaches 1 asymptotically.

Toda (1994; 1995) shows that the ML test statistics are invariant to a number of data transformations, so their finite sample performance can be completely characterized by a canonical data-generating process (DGP), thereby reducing the size of the parameter space to be explored in simulation experiments. In the case where there is no cointegration, the canonical DGP depends only on the sample size and on the rate of deterministic drift, and tests appear to be well-sized for samples of 100 observations. However, Toda restricts his attention to the case of a correctly specified bivariate system with independent and identically distributed normal disturbances and no short-run dynamics (i.e.,  $k=1$ .)

Gregory (1994) compares the finite sample performance of a variety of tests for cointegration, exploring DGPs resulting from linear quadratic (i.e., partial-adjustment) models. He finds that

The [maximal eigenvalue and trace] tests have a tendency to over-reject when the null is true and the rejection frequency rises in  $k$  [the number of variables in the system.] The overrejection occurs even at  $T=200$  for  $k=3$  or  $4$ .... The size results are somewhat better for the maximum eigenvalue test LR1 than for the trace test LR, which uses all of the eigenvalues. Despite this overrejection, the size-adjusted power is quite good.<sup>3</sup>

Gonzalo and Pitarakis (1994) address the problem of size distortion in systems where  $k$  becomes large. They propose a Linear Combination Test (LCT) statistic, calculated as

$$LCT = T \cdot \left( \sum_{i=r+1}^N \tau_i - \ln(1 - \lambda_i) \right), \quad (\text{EQ 8})$$

which seems to have much more reliable size properties in finite samples. They also consider alternatives to the usual sequential procedure for determining the number of cointegrating vectors in a system. Like Toda, however, they do not consider the effects of lag length on test size.

Reinsel and Ahn (1988) propose a modification of (EQ 6) to correct for size distortion in finite samples caused by either the number of lags included in the VECM or the number of variables in the system. Their modified trace statistic is

$$RA = -(T - Np) \cdot \sum_{i=r+1}^N \ln(1 - \lambda_i), \quad (\text{EQ 9})$$

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3. Gregory (1994, 353).

and it has the same asymptotic distribution as (EQ 6).<sup>4</sup>

Cheung and Lai (1993) show that the ML test's bias towards finding excessive evidence of cointegration increases with the number of lags and the number of variables in the system, and decreases with the sample size. They also suggest the use of scaled critical values (as opposed to the scaled test statistics of Reinsel and Ahn 1988) to make test size more constant. Unfortunately, Cheung and Lai's scaling factors are based solely on simulations with mutually orthogonal random walks. Given that they find the test to be very sensitive to misspecification of the dynamics, it is not clear how precise their corrections would be for more general processes.

The results above show that for some DGPs and sample sizes, ML tests for cointegration may be subject to size distortion. The importance of this problem will depend on the specific application. Edison, Gagnon and Melick (1994) examine the size and power of ML cointegration tests in the context of testing purchasing power parity in the post-Bretton Woods period, and find important size distortions. Hendry (1995) examines the ML tests in the context of Canadian money demand and reaches similar conclusions. Ho and Sørensen (1994) review Durlauf's (1989) evidence on cointegration in value-added across U.S. industrial sectors and suggest that neither asymptotic critical values nor the finite-sample corrections suggested by Reinsel and Ahn (1992) or by Cheung and Lai (1993) are reliable in very short samples (40-50 observations.) Edison and Melick (1995) find evidence of significant size distortions in examining evidence on real interest rate parity. In arriving at these conclusions, the above studies use a simulation methodology similar to that which we use below.<sup>5</sup>

### 2.3 A Simulation Approach

For most of the applications that we consider below, we will want to determine the degree of size distortion caused by using the asymptotic critical values. That is, we want to determine how frequently we will conclude that there is significant evidence of

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4. In preparing this paper, we calculated but chose not to report the PGp statistic,

$$PGp = \frac{1}{2} \cdot \left( (T - Np + p) \cdot \sum_{i=r+1}^N \lambda_i - \left( T - Np + \frac{N}{2} \right) \cdot \sum_{i=r+1}^N \ln(1 - \lambda_i) \right),$$

which was suggested in an earlier draft of Gonzalo and Pitarakis (1994). Its results were always very similar to those of the RA statistic and gave identical conclusions. The PGp statistic was abandoned by its authors in a revised version of their manuscript because they felt that it failed to adequately control for size distortion due to lag lengths in finite samples.

5. Our simulation methodology was designed in consultation with Scott Hendry before we were aware of its application elsewhere.

cointegration (say, at the 5 per cent significance level) when there is in fact no cointegration in the true DGP.

To do so, we will simply simulate data from a given DGP under the null hypothesis of no cointegration. Running cointegration tests on this artificial data then allows us to estimate the frequency with which we would falsely reject the null hypothesis of no cointegration. Of course, these rejection rates may be specific to the DGP that we select, and in general it is always possible to find a DGP that will cause severe size distortion for a given cointegration test. To produce meaningful results, we need to use a “reasonable” DGP, where reasonable means that the data might well have been generated by such a process.

The DGP we use below is based on the general VECM given by (EQ2). In this case, the assumption of no cointegration implies that  $\Pi = 0$ , so this reduces to a  $k-1$  order VAR in first differences. We choose the order of the VAR to be identical to that used in the VECM to test for cointegration and estimate the parameters of the VAR by ordinary least squares (OLS). Since this DGP will be encompassed by the VECM, the cointegration test will always be well specified. This means that our DGP may understate the degree of size distortion that could be expected from a VECM that misspecifies the DGP. We will provide an example of this in one of the case studies. We also tried estimating a VECM, then using only the estimated coefficients of the first-differenced variables to simulate the data, thus ignoring the  $\Pi$  matrix of cointegration relationship. These results led to the same conclusions as the method we use below and so are not reported.

We will usually simulate these first-differenced VAR systems under the assumption that the residuals were independently and identically distributed with a multivariate normal distribution whose variance-covariance matrix is the estimated variance-covariance matrix of the VAR residuals. We investigated the possibility of relaxing the normality assumption with the use of bootstrap methods. These experiments lead to the same conclusions as the assumption of normality.<sup>6</sup>

We construct 1000 artificial data sets for each experiment. This may seem small when compared with the 50,000 or more simulations that are often performed in published Monte Carlo experiments. However, we feel that this number is appropriate, bearing in mind that the purpose of this paper differs significantly from much of this published literature. If our purpose were to produce a table of critical values that could be broadly

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6. Maddala (1993) argues that the usual techniques for bootstrapping are not valid for unit-root or cointegrated systems and that there is no consensus on whether some of the proposed alternatives are reliable. Nonetheless, standard bootstrap methods have been used in other published studies; see Evans and Lewis (1995) or Cushman et al. (1996).

applied (such as the original tabulations of Dickey-Fuller t-statistics), then it would be reasonable to try to make these estimates as precise as possible. Instead, our purpose is to determine whether a set of asymptotic critical values leads to reliable inferences for a very specific DGP. We therefore do not require the highest degree of precision; if an asymptotic 5 per cent critical value gives a finite sample test size of, say 3-7 per cent, this should still be a useful guide for inference. We can therefore reliably answer the latter question with far fewer simulations than those required to precisely estimate critical values.

### 3.0 Cointegration among Spot Exchange Rates

Baillie and Bollerslev (1989) (hereinafter BB) use daily data on seven spot exchange rates (against the U.S. dollar) from March 1, 1980 to January 28, 1985 (1245 observations) with the Johansen (1988) test for cointegration.<sup>7</sup> Using the asymptotic critical values, they find evidence of a single cointegrating vector that is significant at the 1 per cent level. This implies that spot exchange rate movements must be at least partly predictable, which is a violation of weak-form market efficiency.<sup>8</sup>

This finding prompted a number of studies that examine variations of BB's original test. Sephton and Laursen (1991) show that BB's results are sensitive to the precise sample period used. Diebold, Gardeazabal and Yilmaz (1994) show that the cointegration model has no predictive power for exchange rate movements in an out-of-sample experiment, and that the evidence of cointegration vanishes if the Johansen and Juselius (1990) tests are used instead of the Johansen (1988) test. In response, Baillie and Bollerslev (1994a) presented evidence suggesting that these exchange rates displayed a slightly different form of long-run relationship, which they termed fractional cointegration. Ho and Sørensen (1994) present evidence that in this case the test statistics converge to their asymptotic distributions quite slowly. While they do not find that BB's original results suffer from finite-sample problems, they suggest that this slow convergence might account for some (but not all) of the fragility reported by Sephton and Laursen (1991).

#### 3.1 An Attempt at Replication

The first step in analysing BB's results is to try to reproduce them.<sup>9</sup> Table 1 shows that while we not able to replicate their results precisely, our trace statistics are quite close to BB's and produce the same conclusions when using asymptotic critical values. Furthermore, we reach the same conclusion if we instead use the maximum-eigenvalue statistic, or Reinsel and Ahn's (1988) scaled version of the trace statistic. Given the large number of observations available and the small number of lags used to construct the test, it is not surprising to find that the Reinsel-Ahn (RA) statistic is very close to the usual trace

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7. The seven currencies are the British pound, the German mark, the French franc, the Italian lira, the Swiss franc, the Japanese yen and the Canadian dollar. BB present similar results for the seven corresponding 30-day forward exchanges, but we will restrict our attention to their results for spot rates.

8. See Baffes (1994) for a more complete discussion of market efficiency in this context.

9. The authors would like to thank Richard Baillie for providing the data used in Baillie and Bollerslev (1989).

statistic. Similarly, according to Cheung and Lai (1993), we should expect virtually no size distortion in samples of this size.<sup>10</sup>

**TABLE 1. Baillie and Bollerslev Test Results: One Lag (Johansen 1988)**

Number of cointegrating vectors	Maximum eigenvalue			Trace		
	Godbout & van Norden	5% Critical Value <sup>a</sup>	Godbout & van Norden	Baillie & Bollerslev	5% Critical Value <sup>a</sup>	Reinsel & Ahn
0	46.98	41.51	117.27	119.70	109.99	115.95
1	28.28	36.36	70.28	76.29	82.49	69.49
2	22.78	30.04	42.00	45.59	59.46	41.53
3	9.18	23.80	19.22	21.20	39.89	19.01
4	6.76	17.89	10.04	11.01	24.31	9.93
5	2.02	11.44	3.28	4.35	12.53	3.24
6	1.26	3.84	1.26	0.686	3.84	1.25

a. Taken from Osterwald-Lenum (1991, Table 0). Baillie and Bollerslev report slightly different critical values.

## 3.2 Simulations

The set of simulations shown in Table 2 was prepared exactly as described in Section 3.2. The logs of the spot exchange rates were first-differenced and then used to estimate a first-order VAR. The VAR was then used to simulate 1000 data sets of the same length as the original data, using multivariate-normal mean-zero errors. Johansen (1988) test statistics (using a single lag) were then calculated for each data set. The resulting test statistics were then compared with their 95 per cent asymptotic critical values from Osterwald-Lenum (1991), and the number of significant cointegrating vectors detected was tabulated.<sup>11</sup>

10. Cheung and Lai's equations (10) and (11) give the ratio of the finite-sample critical value to its asymptotic value. In this case the 5 per cent critical value for the Trace statistic is given by

$$\frac{CR_{TJ}}{CR_{\infty}} = 0.10902 + 0.89150 \cdot \frac{T}{T - nk},$$

where  $T = 1244$ ,  $n = 7$ ,  $k = 2$ .

11. We considered, but rejected, the idea of presenting percentiles of each of the  $n$  test statistics produced by each type of test. As we will see in subsequent case studies, it is sometimes critical to determine the number of cointegrating vectors found. One can conclude that  $r$  cointegrating vectors are present only if *all* of the first  $r$  test statistics exceed their critical values. Since percentiles of each of the test statistics would ignore this interdependence, we felt that the tabulations we present would be more meaningful in most circumstances.



Table 2 shows, in percentages, the fraction of all trials in which a given number of cointegrating vectors were found. Since the data were not cointegrated by construction, a properly sized test would find zero cointegrating vectors in 95 per cent of the trials. This is very close to what was found in simulation. There was no significant evidence of cointegration in 90-95 per cent of the trials, regardless of the test statistic used.

**TABLE 2. Baillie and Bollerslev: Cointegration Test Results for Artificial Data**  
**No intercept, no drift (Johansen 1988)**  
**1244 observations, 7 variables, VAR(1), estimated VCV, VECM(1) 1000 trials**

Number of cointegrating vectors	Maximum eigenvalue	Trace	Reinsel & Ahn
0	91.9	92.6	93.6
1	7.8	6.4	5.6
2	0.2	0.7	0.5
$\geq 3$	0.2	0.3	0.3

We then performed additional simulation experiments to check the robustness of the cointegration tests to three other factors. First, to allow for the possibility that the errors may have a non-Gaussian distribution, we tried simulating the data-generating VAR using errors bootstrapped from the VAR residuals. Second, to allow the errors to be non-independently and identically distributed, we also simulated the VAR using GARCH(1,1) errors, where the parameters of the GARCH process were estimated to fit the VAR residuals. Finally, we also simulated the VAR after setting to zero all off-diagonal elements of the variance-covariance matrix of the residuals. All these variants produced results very similar to those reported above. We therefore conclude that the ML cointegration tests seem to suffer little from these finite-sample problems in this application, and that the asymptotic critical values are a useful basis on which to perform inference.

## 4.0 Cointegration and the Monetary Model of Exchange Rates

MacDonald and Taylor (1993; 1994a; 1994b) used ML cointegration tests to assess the monetary model of exchange rate behaviour. They found strong evidence of cointegration in their seven-variable system and therefore concluded that the monetary model gave useful information about the long-run behaviour of exchange rates. However, their results contrast with those reported by other authors using Johansen tests, such as Gardeazabal and Regúlez (1992), Sarantis (1994) and Cushman et al. (1996). In this section, we examine MacDonald and Taylor's results in more detail.

### 4.1 MacDonald and Taylor (1993)

In their study of the U.S. dollar/German mark exchange rate, MacDonald and Taylor (1993) (hereinafter MT93) use the Johansen test to determine the number of cointegrating vectors in their system. This system consisted of seven variables: the spot exchange rate and, for each country, the money supply (M1), the short-term interest rate and industrial production. The data were monthly observations over the period January 1976 to December 1990 (giving 179 observations after taking first differences.) Their VECM used eight lags of every variable in each equation, with the lag length chosen to eliminate autocorrelation in the system's residuals.<sup>12</sup> MT93 reported evidence of one cointegrating vector when using the maximum-eigenvalue test and three cointegrating vectors when using the trace statistic.

Of course, MT93 present other evidence to support their claim of the adequacy of the monetary model. However, we believe that some of their other results may be open to other interpretations if the evidence of cointegration is questionable. For example, while MT93 also test restrictions on the coefficients of the cointegrating vector, it seems likely that such tests will be unreliable if the cointegration tests upon which they are based are invalid.<sup>13</sup> MT93 also present some evidence of the out-of-sample forecasting performance of their model, comparing it to a random-walk model. However, while they show that their model produces better forecasts, they do not determine whether this difference is statistically significant. In the case of multi-period forecasts, they do not show whether their model's ability to forecast is due to Granger-causality from exchange rates to their

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12. This implies that every equation in the system would estimate coefficients on seven variables in levels plus eight lags times seven variables in differences plus one constant to produce 64 parameters on 180-8 lags minus 1 for differences to produce 171 observations.

13. Elliot (1993) shows that such tests may themselves be subject to severe size distortion in large and small samples when variables contain a root that is close to, but not equal to, 1.

other variables, or vice versa. We therefore feel that it is important to re-examine their evidence of cointegration.

## 4.2 An Attempt at Replication

In this section, we first attempt to replicate as closely as possible MT93's cointegration results. Using data from the same sources,<sup>14</sup> we estimate the same VECM system using  $k=9$  and the Johansen and Juselius (1990) test with no restrictions on intercept or drift.<sup>15</sup>

**TABLE 3. MacDonald and Taylor (1993): Results for Germany  
IFS data, VECM(8), 180 observations, 7 variables**

Number of cointegrating vectors	Maximum eigenvalue			Trace			
	G & vN	MT93	5% critical value <sup>a</sup>	G & vN	MT93	5% critical value	Reinsel & Ahn
0	44.89	43.99	45.28	160.93	159.87	124.24	110.87
1	41.31	40.87	39.37	116.05	115.87	94.15	79.94
2	27.35	35.81	33.46	74.74	77.10	68.52	51.49
3	20.53	21.02	27.07	47.39	39.19	47.21	32.65
4	14.48	11.84	20.97	26.86	18.17	29.68	18.51
5	9.54	4.90	14.07	12.39	6.33	15.41	8.53
6	2.85	1.43	3.76	2.85	1.43	3.76	1.96

a. Taken from Osterwald-Lenum (Dec. 1991).

Table 3 reports our results. There is no significant evidence of cointegration according to the maximum-eigenvalue test but evidence of four cointegrating vectors using the trace test. The latter result is similar to the results reported by MT93, who found evidence of

14. We were not able to precisely replicate their original data. MT93 list the IMF's *International Financial Statistics* (IFS) as their only data source. However, the U.S. M1 series is not available from IFS at a monthly frequency. Instead, we used the monthly M1 series published in the *Federal Reserve Bulletin*, Table H.6. In addition, MT93 deseasonalize their data, but do not report the method used. We used the *constant* method available in FAME, which forces the seasonal component to be identical in every year. Finally, data in IFS are subject to revision from time to time, which might also cause our data to differ from theirs. Despite these three factors, we feel that our data sets should be close to identical, and that any differences are almost certainly irrelevant to the results that we discuss below.

15. This case corresponds to case 1 in Osterwald-Lenum (1992). We noted what appears to be a discrepancy between the critical values reported by MT93 and the construction of their test statistics. The former seem to imply the restriction that none of the variables in the system is subject to drift, a restriction that MT93 do not mention imposing in their construction of the test statistics. Since variables such as money supply and industrial production clearly drift upwards over time, we do not impose the no-drift assumption. Note that the critical values we use are almost without exception lower than those used by MT93, so that their evidence of cointegration would be slightly more significant with our critical values.

one and three cointegrating vectors using the maximum-eigenvalue and trace tests respectively.<sup>16</sup> We attribute the difference to minor differences in the data used (see footnote 14.) However, the Reinsel and Ahn test finds no evidence of cointegration significant at the 5 per cent level, suggesting that size distortion due to the large number of variables and lags may play a significant role in this case.

**TABLE 4. Single Equation Cointegration Tests  
(Exchange rate as dependent variable)**

Test <sup>a</sup>	Statistic	5 per cent Critical Value (n=5)	P-value
$P_u$	7.152	57.79	>0.05
$P_z$	228.436	241.33	>0.05
$Z_\alpha$	-8.651	-41.94	>0.05
$Z_t$	-2.037	-4.71	>0.05
H(0,3) (CCR):	147.873	7.815	0.000
H(0,3) (FM):	152.763	7.815	0.000

a. H() is from Park (1992), and the rest are from Phillips and Ouliaris (1990). All estimate the long-run covariance matrix non-parametrically using data-dependent truncation lag length selection, a Parzen kernel and AR(1) prewhitening. QS kernels without prewhitening gave similar results.

Table 4 presents additional tests for cointegration based on single-equation methods. Critical values are only available for systems with up to five explanatory variables for the  $P$  and  $Z$  statistics. However, since correct critical values would be still further from zero, we can conclude that none of these tests finds significant evidence of cointegration with the exchange rate. In addition, the H tests (which have  $\chi^2(3)$  critical values) strongly reject the null hypothesis of cointegration with the exchange rate.

In summary, single-equation methods give no significant evidence of cointegration and give significant evidence against the null hypothesis of cointegration. Using ML tests, the maximum-eigenvalue tests show no significant evidence of cointegration. The trace test provides strong evidence of cointegration, but (as we show below) this vanishes when we attempt to correct for the size distortion caused by the number of variables and lags in the estimated system.

16. We determine the number of cointegrating vectors by applying the Johansen tests sequentially, starting with the null hypothesis of no cointegration. This implies that, even though our maximum eigenvalue statistic in Table 3 appears to reject the null hypothesis of one cointegrating vector in favour of the alternative of two cointegrating vectors, we conclude that it finds zero cointegrating vectors since it is unable to reject that null in favour of the alternative of one cointegrating vector. In theory, this sequential method must give *less* evidence of size distortion than would using the largest number of cointegrating vectors for which we reject any of the various null hypotheses. (For example, the latter would imply that we find two cointegrating vectors in the above example.) In practice, the difference between these two methods was negligible.

### 4.3 Simulation Methodology

The simulation results presented in the next section were prepared exactly as described in Section 3.2. All series were first-differenced and then used to estimate an eighth-order VAR. The VAR was then used to simulate 1000 data sets of the same length as the original data, using multivariate-normal mean-zero errors. ML cointegration test statistics were then calculated for each data set and compared with their 95 per cent asymptotic critical values from Osterwald-Lenum (1991).

Note that this methodology assumes that the data series are all  $I(1)$ . We found some evidence that money supplies for both nations might be  $I(2)$ . In this case, the first-differenced VAR could still generate data that is cointegrated  $CI(2,1)$ , in the notation of Engle and Granger (1987). As long as the exchange rate is  $I(1)$ , the monetary model makes no prediction about the presence or absence of higher-order cointegration relationships. Therefore, we have no direct interest in testing for the presence or absence of higher-order cointegration, and we ignore this possibility in our simulations. Because the Johansen-Juselius test used by MT93 is valid only in the case where all series in  $X$  are  $I(1)$ , this is another possible source of size distortion in MT93's results.

### 4.4 Simulation Results

The Monte Carlo test results in Table 5 show the frequency (in per cent) of the number of cointegrating vectors that we find using asymptotic 95 per cent critical values. For the two tests reported by MT93, we find that we reach the correct conclusion less than 2 per cent of the time, finding instead up to seven cointegrating vectors. Size distortion was less severe for the RA test, but still led to the correct conclusion less than 40 per cent of the time.

We also used the Monte Carlo experiment to generate approximate corrected 5 per cent critical values for MT's application of the Johansen test. These are shown in Table 6 for the maximum-eigenvalue and trace tests, along with their asymptotic critical values. For both tests, the finite-sample critical values are almost double their asymptotic values. Comparing the test statistics for the true data (Table 3) with the finite-sample critical values or those suggested by Cheung and Lai (1993), we find no evidence of cointegration that is significant at the 5 per cent level.

We found the evidence of size distortion to be unexpectedly strong, particularly given the degree of distortion present in the RA test. We therefore conducted a series of additional experiments to isolate the factors contributing to this result. Hopefully, this will not only

make our results more understandable, but it may shed light on which other published results are likely to be affected by serious size distortion.

**TABLE 5. Cointegration Test Results for Artificial (Non-Cointegrated) Data: Frequency (in per cent) 180 observations, seven variables, VAR(8), estimated VCV, VECM(8)**

Number of cointegrating vectors	Maximum eigenvalue	Trace	Reinsel & Ahn
0	1.90	0.20	36.10
1	21.30	2.90	40.90
2	39.00	19.20	18.10
3	24.10	34.80	3.90
4	9.40	25.20	0.30
5	1.50	11.10	0.00
6	0.00	0.00	0.00
7	2.80	6.60	0.70

**TABLE 6. Monte Carlo 95 per cent Critical Values for MT93: Null Hypothesis of No Cointegration**

Maximum-eigenvalue test				Trace test			
Test statistic	95% critical values			Test statistic	95% critical values		
	Asymp-totic	Cheung & Lai	Monte Carlo		Asymp-totic	Cheung & Lai	Monte Carlo
44.89	46.45	67.3	80.0	160.93	131.70	188.9	242

First, we examined the role of sample length by generating additional data sets with 1000 observations each (versus 180 in MT93) from the same DGP as before. The results in Table 2 show considerable reduction in size distortion for the maximum-eigenvalue and trace statistics, but little change in the RA statistics. Despite this, the degree of size distortion remains very large even in a sample of 1000 observations; the null hypothesis of no cointegration is rejected over 60 per cent per cent of the time for every test statistic. It seems that the finite sample distribution of these test statistics converge to their asymptotic values only very slowly.<sup>17</sup>

Note that this result contrasts with the results from our simulations for the Baillie and Bollerslev (1989) paper, where in a similar sample size there was little or no size distortion. One difference between these simulations is that those in Table 2 use many more lagged first differences in the estimated VECM, a factor known to worsen the size

17. To confirm that they would eventually converge, we ran a small number of replications that showed that the tests had approximately the correct size in samples of 10,000 observations.

**TABLE 7. Cointegration Test Results for Artificial (Non-Cointegrated) Data:  
Frequency (in per cent) 1000 observations, seven variables, VAR(8), estimated VCV, VECM(8)**

Number of cointegrating vectors	Maximum eigenvalue	Trace	Reinsel & Ahn
0	31.0	22.1	35.9
1	50.4	48.6	45.5
2	16.7	23.1	15.6
3	1.9	5.0	2.6
≥ 4	0.0	1.0	0.4

distortion. Although the RA statistic should compensate for this, it still performs poorly in the large sample. Another possible difference is the variance-covariance matrix of the matrix of the residuals, which differs from one data set to another.<sup>18</sup>

The next step was to determine the effects of the variance-covariance matrix of the residuals, by setting it equal to the identity matrix for the simulated data. The results in Table 8 show that this worsens the degree of size distortion compared with the original results in Table 5. We now find at least two cointegrating vectors for every one of our 1000 trials, and the frequency with which we find seven cointegrating vectors also increases.

**TABLE 8. Cointegration Test Results for Artificial (Non-Cointegrated) Data:  
Frequency (in per cent) 180 observations, seven variables, VAR(8), VCV=I, VECM(8)**

Number of cointegrating vectors	Maximum eigenvalue	Trace	Reinsel & Ahn
0	0.00	0.00	0.00
1	0.00	0.00	0.00
2	16.60	1.80	57.90
3	41.20	22.50	32.20
4	27.30	37.80	7.20
5	9.20	22.60	1.40
6	0.00	0.00	0.00
7	5.70	15.30	1.30

We then simplified the DGP used in Table 8 to seven random walks while keeping the number of lagged first differences used in the Johansen test at eight. The results in Table 9 show a marked improvement in test size. The RA statistic now appears to be correctly

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18. Gonzalo and Lee (1995) show that if this matrix is singular or near-singular, then the ML cointegration tests will find spurious cointegration with probability approaching 1 asymptotically.

sized, while the size of the maximum-eigenvalue and trace tests falls from 100 per cent to 68.8 per cent and 93.6 per cent respectively. Presumably, all the remaining distortion stems from the use of too many lagged differences in a short sample. Our final simulation (Table 10) reduces the number of lagged differences from eight to one. This eliminates most (but not all) of the size distortion in the maximum eigenvalue and trace tests, which now reach the correct conclusion over 80 per cent of the time. Interestingly, the performance of the RA test deteriorates slightly relative to the previous case, although it remains a more reliable test than the trace test on which it is based.

**TABLE 9. Cointegration Test Results for Artificial (Non-Cointegrated) Data:  
Frequency (in per cent) 180 observations, seven variables, VAR(0), VCV=I, VECM(8)**

Number of cointegrating vectors	Maximum eigenvalue	Trace	Reinsel & Ahn
0	31.20	6.40	94.60
1	43.60	28.70	4.90
2	20.50	36.20	0.40
3	4.00	19.40	0.10
≥ 4	0.70	9.30	0.00

**TABLE 10. Cointegration Test Results for Artificial (Non-Cointegrated) Data:  
Frequency (in per cent) 180 observations, seven variables, VAR(0), VCV=I, VECM(1)**

Number of cointegrating vectors	Maximum eigenvalue	Trace	Reinsel & Ahn
0	87.9	83.4	91.1
1	11.5	14.1	8.0
2	0.6	2.1	0.90
≥ 3	0.0	0.4	0.0

The remaining distortion is presumably due to a combination of the limited sample size and the high dimension of the system. While the distortion is small compared with that found in Table 5, it is still high enough to cause concern. For example the 5 per cent asymptotic critical value for the trace test for cointegration gives an actual size of about 15 per cent. It seems that even in this very simple case, the use of asymptotic critical values may require significantly larger samples when dealing with large-dimension systems.



## 4.5 An Extension

As noted at the beginning of this section, MacDonald and Taylor present similar cointegration evidence supporting the monetary model of exchange rates for other currencies as well. It would be useful to know whether the results we presented above for the German mark generalize to these other cases. Therefore, we also examined the results for the British pound presented in MacDonald and Taylor (1994a) (hereinafter MT94).

Table 11 presents the results of our attempt to replicate their results, again using data from the IMF's *International Financial Statistics* (IFS). Once again, while we were unable to precisely reproduce their test statistics, our test results were similarly supportive of cointegration. Using the 5 per cent significance level, we found evidence of six cointegrating vectors using either the maximum-eigenvalue or the trace statistic, whereas MT94 found three in both cases. However, the RA statistic gives evidence of only one cointegrating vector.

**TABLE 11. McDonald and Taylor (1994a): United Kingdom  
VECM(12), 215 observations, seven variables, Johansen and Juselius (1990)**

Number of cointegrating vectors	Maximum eigenvalue			Trace			
	G & vN	MT93	5% critical value <sup>a</sup>	G & vN	MT93	5% critical value	Reinsel & Ahn
0	74.31	74.43	46.45	251.26	221.87	131.70	138.62
1	45.48	50.12	40.30	176.95	147.44	102.14	97.63
2	41.60	48.13	34.40	131.48	97.31	76.07	72.54
3	38.30	23.83	28.14	89.87	49.18	53.12	49.58
4	27.57	16.97	22.00	51.57	25.35	34.91	28.45
5	17.19	8.37	15.67	24.00	8.38	19.96	13.24
6	6.81	0.00	9.24	6.81	0.00	9.24	3.76

a. Taken from Osterwald-Lenum case 1\*(Dec. 1991).

We then performed the simulation experiment as before, running the same cointegration tests on artificial data generated from a VAR in first differences. The results are shown in Table 12. Once again, we find extensive evidence of size distortion for the maximum-eigenvalue and trace tests, with a nominal 5 per cent critical value giving a true test size in excess of 98 per cent. Surprisingly, however, the RA statistic has low size distortion, in contrast to the results found in Table 5 with the German data.

**TABLE 12. MT94 Simulation Results for ML cointegration tests**  
**No intercept, no drift: OL case 1 (drift)**  
**215 observations, seven variables, VAR(12), estimated VCV**

Number of cointegrating vectors	Statistic		
	Maximum eigenvalue	Trace	Reinsel & Ahn
0	1.8	0.0	88.4
1	15.7	0.3	9.8
2	33.7	6.9	1.5
3	28.2	23.5	0.2
4	15.3	34.8	0.1
5	3.9	21.3	0.0
6	1.0	5.8	0.0
7	0.0	0.0	0.0

Finite-sample 5 per cent critical values for the null hypothesis of no cointegration were estimated at 85 for the maximum-eigenvalue test and at 237 for the trace test. Both are slightly less than double the asymptotic critical values. However, the 5 per cent critical value for the RA statistic was estimated at 131, which is effectively identical to its asymptotic value. Unlike the results for the German data, our RA statistics (but not the maximum-eigenvalue or trace statistic) exceed their 5 per cent finite-sample critical values. This implies that, after correcting for the severe size distortion, there may still be some support for the monetary model of exchange rates in the British data.

## 5.0 Cointegration and the International Stock Market

Kasa (1992) studies the equity markets in five major industrialized countries (Canada, England, Germany, Japan and the United States). After concluding that the upward trend in each national stock market is due to a stochastic trend, Kasa raises the following question: If prices in each individual country's stock market follow a random walk with a drift component, are these random-walk components different or do they arise from the response of each country to a single, common world-growth factor?

Although Kasa concentrates on detecting and estimating common stochastic trends, he notes that if stock markets share a common trend, long-term gains to international diversification may be overstated. He also investigates whether the cointegration structure of dividend payments is the same as corresponding stock prices. Indeed, since the price of equity is the discounted cash flows (dividends), the unit-root and cointegration properties of stock prices should derive from the unit-root and cointegration properties of their dividend payments.<sup>19</sup> Thus, a single stochastic trend in the stock prices of these five markets should be reflected in a single stochastic trend in their dividend payments. Kasa finds two stochastic trends in the dividend payments. However, using the GNP as a proxy, he finds evidence of a single common trend, as in the stock market price series.<sup>20</sup>

In order to detect the number of common stochastic trends shared by the five equity markets, Kasa uses ML cointegration tests; if there are  $r$  cointegration relationships, there will be  $n-r$  common stochastic trends. Kasa observes that tests with a small number of lagged first differences reveal little evidence of cointegration, while larger numbers of lags provide much stronger evidence in favour of cointegration. He performs the test on monthly and quarterly data for one lag to fourteen lags, but only reports the results for one and fourteen lags, noting that results are intermediate for intermediate lag length.

The stock prices and dividends are the Capital International indices constructed by Morgan Stanley. The price data are value-weighted indices computed from end-of-month price observations on a very large sample of firms in each market. The series were converted to real U.S.\$ using end-of-month exchange rates and the U.S. CPI, and transformed in logarithms. The time period is January 1974 through August 1990, and monthly and quarterly frequencies are used.

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19. This assumes that discount rates follow stationary stochastic processes and bubbles are not present.

20. Kasa uses a proxy because he has a reservation about the series: it is in the form of a 12-month moving average, and it is not clear how this might distort the results in small sample.

Since we have Kasa's data base, we are able to perfectly replicate his results, which are shown in Table 13 for the monthly data. According to the maximum-eigenvalue test, there is evidence of one cointegrating vector for both the VECM(1) and VECM(14). Using the trace test, we find no evidence of cointegration for a VECM(1) lag and one cointegrating vector for a VECM(14). We also computed the RA statistic, which finds no significant evidence of cointegration in either case.

**TABLE 13. Kasa: Stock markets. Monthly Data 1974M1-1990M8  
199 observations, 5 variables, OL1.1\* Johansen and Juselius (1990)**

Number of cointegrating vectors	Maximum eigenvalue			Trace			Reinsel & Ahn
	1 lag	14 lags	5% critical value <sup>a</sup>	1 lag	14 lags	5% critical Value	14 lags
0	36.60	58.75	33.26	63.62	103.3	69.98	61.40
1	15.11	21.08	27.34	27.03	44.52	48.42	26.47
2	8.05	10.79	21.28	11.92	23.45	31.26	13.94
3	3.63	9.05	14.59	3.87	12.65	17.84	7.52
4	0.24	3.60	8.08	0.24	3.60	8.08	2.14

a. Taken from Osterwald-Lenum case 1.1\*(Dec. 1991).

The simulation experiment is performed the same way as before, running the same cointegration tests on artificial data generated from a VAR in first differences. For the VECM(1), as shown in Table 14, we do not find evidence of size distortion for maximum-eigenvalue, trace tests and the RA statistic.

**TABLE 14. Kasa: Simulation Results for ML Cointegration Tests  
OL case 1.1\* (drift)  
199 observations, five variables, VAR(1), estimated VCV, VECM(1)**

Number of cointegrating vectors	Statistic		
	Maximum eigenvalue	Trace	Reinsel & Ahn
0	92.2	91.5	95.3
1	7.7	7.7	4.6
2	0.1	0.7	0.1
3	0.0	0.1	0.0
4	0.0	0.0	0.0

However, as in the MacDonald and Taylor case, performing the test with more lags has a dramatic effect, as shown in Table 15 and 16 for a VAR(1) and a VAR(14) respectively. There is now strong evidence of size distortion for the maximum-eigenvalue and trace test, albeit no size distortion for the RA test. Adding 1000 observations (Table 17) significantly improves the results: the eigenvalue and trace tests now reach the correct conclusion over 90 per cent of the time.

Kasa also reports results for quarterly data. With quarterly stock indices, Kasa uses nine lagged first differences, which means that each equation in his VECM estimates 51 parameters on 55 observations.<sup>21</sup> With so few degrees of freedom, it would not be surprising if the test statistics deviated from their asymptotic distribution. This was confirmed by our simulations for this case (not reported), which produced the same conclusions as reported above for monthly data.

Kasa also tests quarterly series on dividends and on GNP for cointegration. These data cover the same time period as the stock market indices, but Kasa now states that only seven lagged first differences are needed to correct for the short-run dynamics. This means that each equation in the VECM is now estimating 41 parameters on 57 usable observations. Given the results from our simulations of the stock price data, we would expect to find similarly strong evidence of size distortion in this case, although we have not attempted to verify this by simulation. As Kasa notes, the absence of bubbles generally implies that we would expect the same number of cointegrating relationships between the fundamentals as we find between the stock prices.

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21. Five variables in levels plus nine lags times five variables in differences plus one constant equals 51 parameters; 66 quarterly observations minus nine lags minus one for first differences equals 55 usable observations.

**TABLE 15. Kasa: Simulation Results for ML Cointegration Tests  
OL case 1.1\* (drift)  
199 observations, five variables, VAR(1), estimated VCV, VECM(14)**

Number of cointegrating vectors	Statistic		
	Maximum eigenvalue	Trace	Reinsel & Ahn
0	24.3	11.4	96.5
1	44.8	33.0	3.4
2	25.1	40.3	0.1
3	5.1	12.7	0.0
4	0.7	2.2	0.0
5	0.0	0.4	0.0

**TABLE 16. Kasa: Simulation Results for ML Cointegration Tests  
OL case 1.1\* (drift)  
199 observations, five variables, VAR(14), estimated VCV, VECM(14)**

Number of cointegrating vectors	Statistic		
	Maximum eigenvalue	Trace	Reinsel & Ahn
0	24.2	8.1	95.1
1	43.0	32.6	4.6
2	24.8	39.6	0.3
3	6.6	16.9	0.0
4	1.4	2.3	0.0

**TABLE 17. Kasa: Simulation Results for ML Cointegration Tests  
OL case 1.1\* (drift)  
1199 observations, five variables, VAR(14), estimated VCV, VECM(14)**

Number of cointegrating vectors	Statistic		
	Maximum eigenvalue	Trace	Reinsel & Ahn
0	90.8	91.1	96.1
1	8.8	8.0	3.5
2	0.3	0.6	0.2
3	0.1	0.2	0.1
4	0.0	0.1	0.1

## 6.0 Conclusions and Directions for Research

We have used a simple method for checking the validity of conclusions drawn using some popular cointegration tests. By simulating the data under the null hypothesis of no cointegration, we are able to assess whether asymptotic critical values are a reliable guide for applied research. Furthermore, given the number of simulations required to check the approximate distribution of the test statistics, these calculations can easily be performed on modern computers.

Our results confirm that great care must be taken in using maximum-likelihood tests for cointegration in high-dimensional systems. In the examples we considered, the combination of multiple lags in the VECM and a high-dimensional system seemed to produce considerable size distortion, sometimes even in samples of 1000 observations. The use of the correction factors suggested by Reinsel and Ahn (1988) reduced these distortions, but the results were still not a reliable guide for applied research in this situation. Besides correcting for the dimension of the system and the number of lags used to construct the ML test statistics, it appears that a reliable test will also have to take into account the covariance matrix of the residuals of the VECM.

Our three case studies of high-dimensional systems produced evidence suggesting that some important results concerning exchange rate behaviour may simply reflect finite-sample size distortion. In this category would fall MacDonald and Taylor's (1993; 1994a; 1994b) results documenting long-run support for the monetary model of exchange rates. Their conclusions are subject to very severe size distortion. Once we corrected for this, there was no significant evidence of cointegration remaining for Germany, although some evidence remained for Britain. The same is true for Kasa (1992), who finds evidence of cointegration among international stock markets and suggests that long-term gains from international diversification may be overstated. In contrast, we found no evidence of important size distortions in our examination of Baillie and Bollerslev's (1989) analysis of cointegration in a system of spot exchange rates.

The high degree of size distortion that we appear to have found suggests that it would be interesting to re-examine other recent reports of cointegration in the exchange rate and other literatures. One interesting candidate may be the work on the long-run determinants of real exchange rates, such as Amano and van Norden (1995).

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