Reconstructing Shape from Shading Images under Point Light Source Illumination

Yuji IWAHORI, Hidezumi SUGIE and Naohiro ISHII

Faculty of Engineering,
Nagoya Institute of Technology,
Nagoya, Japan 466

Abstract- A new photometric method is proposed for determining the 3-D shape of the object from multiple shading images under the point light source illumination. When the surface is the perfect diffuser with the uniform reflectance, an algorithm for the determination of 3-D shape with positions is developed by using the method of least squares and basing on the principle of the monocular vision and the inverse square law for illuminance. In the proposed method, the number of the necessary images is four for the general surface, and can be reduced to three for the continuous surface.

I. INTRODUCTION

As a method for estimating 3-D shape from 2-D shading information, Photometric Stereo [1],[2] was proposed by Woodham, and has been studied for the practical applications [3]-[6]. Photometric Stereo can determine the surface gradient of an object from three images illuminated by parallel light beams from three different directions.

However, Photometric Stereo can not determine the 3-D (X,Y,Z) coordinate of the object by itself, since the illumination by parallel light beams does not give the distance information. The process of integrating the gradient (p,q) is required to get the coordinate (X,Y,Z) at a surface element. On the other hand, when the object is illuminated by a point light source, the inverse square law for illuminance may be applied to determine (X,Y,Z) coordinate of the object, because the distance information is included in the law.

In this paper, a new photometric method, named PSIS (Point Source Illuminating Stereo) which can determine the 3-D (X,Y,Z) coordinate, is proposed. PSIS uses multiple images of the object illuminated by a point light source positioned at different (Xs,Ys,Zs) coordinate.

If the surface of the object is the perfect diffuser, the illuminating equation becomes a nonlinear equation with four variables (p,q,Z,C), where (p,q), Z and C are the surface gradient, the distance from the lens plane and the reflectance factor, respectively.

By moving the point light source, we can get a series of illuminating equations, which forms nonlinear simultaneous equations. The coordinate (X,Y,Z) at a surface element can be directly obtained by solving these equations.

Under the conditions that the surface is the perfect diffuser with the uniform reflectance and that at least one local brightest surface element is observed in each image, an algorithm is proposed by using the method of least squares as follows.

Firstly, the brightest surface elements are chosen from all images, and the value of the common reflectance factor C is determined from Z values of these elements. Secondly, the value of Z at each surface element is determined by treating C as a known constant.

In the proposed method, four shading images are needed to determine the unique solution for the general surface (i.e. including the surface discontinuity), and the number of the necessary images can be reduced to three for the continuous surface.

II. ILLUMINATING EQUATION

A. Coordinate System

Fig.1 shows the coordinate system of the perspective projection. The center of the lens is placed at the origin O, and a point light source is located at the coordinate (Xs,Ys,Zs). A surface element P(X,Y,Z) on the object is mapped onto the image pixel p'(x,y) on the film. The symbol b represents the distance between the origin O and the film.

B. Definition of Vectors at Surface Element

We define three vectors at a surface element as follows.

- $\boldsymbol{n}$: unit surface normal vector
- $\boldsymbol{s}$: unit vector from a surface element to a point light source
- $\boldsymbol{v}$: unit vector from a surface element to the origin O

$$\boldsymbol{n} = \frac{(p,q,-1)}{(p^2+q^2+1)^{1/2}} \tag{1}$$

$$\boldsymbol{s} = \frac{(X_s-X, Y_s-Y, Z_s-Z)}{((X_s-X)^2+(Y_s-Y)^2+(Z_s-Z)^2)^{1/2}} \tag{2}$$

$$\boldsymbol{v} = \frac{(-x,-y,-b)}{(x^2+y^2+b^2)^{1/2}} \tag{3}$$

where p and q represent the gradient components:

$$p = -\frac{\partial Z}{\partial X} \quad \text{and} \quad q = -\frac{\partial Z}{\partial Y} \tag{4}$$

The coordinate (X,Y) of the surface element is represented by the distance Z and the coordinate (x,y) of the image pixel as

$$X = x\,\frac{Z}{b} \tag{5}$$

$$Y = y\,\frac{Z}{b} \tag{6}$$

C. Illuminating Equation

The luminous intensity I represents the luminous flux ϕ per unit solid angle emitted by a point light source.

$$I = \frac{d\phi}{d\omega} \tag{7}$$

where $d\omega$ is the solid angle subtended to the small area dA of the surface element P by the point light source. $d\omega$ is represented by

$$d\omega = \frac{dA \cos i}{r^2} \tag{8}$$

where i is the incident angle. The illuminance E_n in the direction of the surface normal $\boldsymbol{n}$ is given by

$$E_n = d\phi/dA = I\,d\omega/dA = I\,\frac{\cos i}{r^2} \tag{9}$$

The luminous exitance M emitted from dA is represented in terms of the surface reflectance R.

CH2898-5/90/0000/0083$01.00 © 1990 IEEE

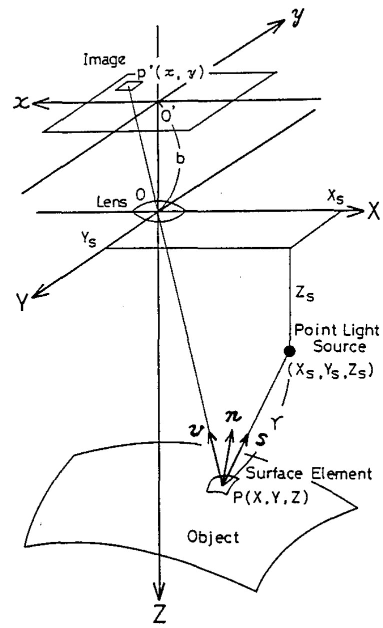

Fig.1. The coordinate system of PSIS.

$$M = R\,E_n = R\,I\,\frac{\cos i}{r^2} \tag{10}$$

Assuming that the reflectance property of the surface is the perfect diffuser of the Lambert's Law,

$$M = \pi L \tag{11}$$

Therefore, the luminance L of the surface element is expressed by

$$L = R\,I\,\frac{\cos i}{\pi\, r^2} \tag{12}$$

$\cos i$ is the scalar product of unit vector $\boldsymbol{n}$ and $\boldsymbol{s}$, then

$$\cos i = \frac{p(Xs-X)+q(Ys-Y)-(Zs-Z)}{r(p^2+q^2+1)^{1/2}} \tag{13}$$

in which r is the distance between the surface element and the point light source. r^2 is represented by

$$r^2 = (Xs-X)^2+(Ys-Y)^2+(Zs-Z)^2 \tag{14}$$

Substituting Eqs.(13) and (14) into Eq.(12) gives

$$L = \frac{R\,I\,(\,p\,l + q\,m - n\,)}{\pi\,(l^2+m^2+n^2)^{3/2}(p^2+q^2+1)^{1/2}} \tag{15}$$

where,

$$l = Xs - x\,\frac{Z}{b} \tag{16}$$

$$m = Ys - y\,\frac{Z}{b} \tag{17}$$

$$n = Zs - Z \tag{18}$$

Let D be the image density on the film, and assuming the Gamma Property to be linear, then

$$D = k\,L \tag{19}$$

where k is a proportional constant.

From Eqs.(15) and (19), the image density D is rewritten by

$$D(x,y) = C\,f(p,q,Z) \tag{20}$$

where,

$$C = k\,\frac{R\,I}{\pi} \tag{21}$$

$$f(p,q,Z) = \frac{p\,l + q\,m - n}{(l^2+m^2+n^2)^{3/2}(p^2+q^2+1)^{1/2}} \tag{22}$$

It should be noted that there are four unknown variables, i.e., (p,q,Z,C) in Eq.(20) and the equation is a nonlinear equation.

III. PRINCIPLE OF PSIS

A. Procedure

The independence of the density D(x,y) can be held by taking images of the object under the different lighting conditions. The surface has the uniform reflectance R, therefore this proposed method uses the constraint that the value of C is constant for all of surface elements. So, the illuminating equations can be solved by the following two steps.

[Step 1] Choose the brightest surface element in each image, and estimate the value of C from Z values obtained by solving the simultaneous illuminating equations of these surface elements.

[Step 2] Determine the values of Z and (p,q) at each surface element by using the value of C obtained in step 1.

B. Property of Brightest Surface Element

The brightest surface element is defined as the surface element on which the value of $\cos i / r^2$ becomes maximum, and exists as one of the local brightest surface elements.

The local brightest surface element has the gradient $\boldsymbol{n}$ given by

$$\cos i = 1,\quad \boldsymbol{n} = \boldsymbol{s} \tag{23}$$

i.e. the surface normal vector directs towards the light source. This can be explained by means of the reductive absurdum as follows.

From the physical constraint, $\cos i$ should be greater than 0 and less than or equal to 1 at the observed surface element. As shown in Fig.2, the surface element (a) is assumed to be a local brightest surface element, and let $\boldsymbol{n} \neq \boldsymbol{s}$ at its surface element. If the local surface is assumed to be a plain, the plain should cross the sphere surface whose center is located at a point light source and on which the local brightest surface element exists. This means that another surface element (b) exists near the local brightest surface element (a) and exists inside the sphere surface.

Then, $(\boldsymbol{n}_a, \boldsymbol{s}_a) < (\boldsymbol{n}_b, \boldsymbol{s}_b)$ and $r_a > r_b$ are obtained. Accordingly, $(\boldsymbol{n}_a, \boldsymbol{s}_a)/r_a^2 < (\boldsymbol{n}_b, \boldsymbol{s}_b)/r_b^2$. Let the corresponding image densities be D_a and D_b respectively, then $D_a < D_b$. This is in conflict with the definition that the surface element (a) is the local brightest surface element. Therefore, $\boldsymbol{n}$ must be equal to $\boldsymbol{s}$ at the local brightest surface element.

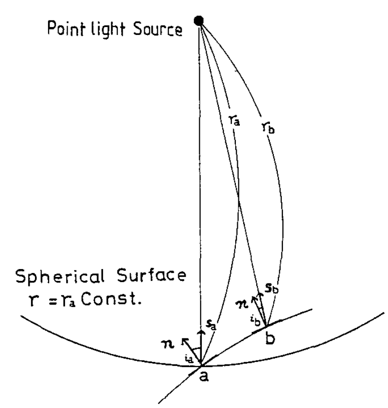

Fig.2. Explanation of the property of brightest surface elements.

C. Step 1

C-1. Simultaneous Equations at Brightest Surface Elements

Let k be the ordinal number of the selected brightest surface element illuminated by the k-th light source, and let j be the ordinal number of the image of the object illuminated by the j-th light source. Letting N be the number of input images, the illuminating equations at the brightest surface elements become

$$D_{kj} = C\, f_j(p_k,q_k,Z_k)$$

$$= C\,\frac{p_k\, l_{kj} + q_k\, m_{kj} - n_{kj}}{(l_{kj}^2+m_{kj}^2+n_{kj}^2)^{3/2}(p_k^2+q_k^2+1)^{1/2}} \qquad (24)$$

(for k=1..N, j=1..N)

where,

$$l_{kj} = Xs_j - x_k\frac{Z_k}{b}$$

$$m_{kj} = Ys_j - y_k\frac{Z_k}{b}$$

$$n_{kj} = Zs_j - Z_k$$

The gradient (p_k,q_k) at the selected surface element is represented by using Eq.(23) as

$$p_k = -\frac{l_{kk}}{n_{kk}} \text{ and } q_k = -\frac{m_{kk}}{n_{kk}} \qquad (25)$$

Substituting Eq.(25) into Eq.(24) gives

$$f_j(p_k,q_k,Z_k) = f_j(Z_k)$$

$$= \frac{l_{kk}\, l_{kj} + m_{kk}\, m_{kj} + n_{kk} n_{kj}}{(l_{kj}^2+m_{kj}^2+n_{kj}^2)^{3/2}(l_{kk}^2+m_{kk}^2+n_{kk}^2)^{1/2}} \qquad (26)$$

By using $f_j(Z_k)$ of Eq.(26), the following simultaneous equations at the brightest surface elements (k=1..N) are obtained

$$D_{kj} = C\, f_j(Z_k) \quad \text{(for j=1..N, k=1..N)} \qquad (27)$$

Let the number of simultaneous equations be n_e and the number of unknown variables be n_v, then $n_e=N^2$ and $n_v=N+1$. In step 1, the values of (N+1) unknown variables, $(Z_k)_{k=1..N}$ and C are estimated by using the method of least squares.

C-2. Objective Function

Let the objective function E_1 be designated as

$$E_1 = \sum_{k=1}^{N}\sum_{j=1}^{N}(\, D_{kj} - C\, f_j(Z_k)\,)^2 \qquad (28)$$

Considering the quadratic expression of C in Eq.(28), and letting $f_j(Z_k)$ be f_{kj}, the objective function E_1 can be rewritten by

$$E_1 = (\Sigma\Sigma f_{kj}^2)(\, C - \frac{\Sigma\Sigma D_{kj} f_{kj}}{\Sigma\Sigma f_{kj}^2}\,)^2 + E_2 \qquad (29)$$

where,

$$E_2 = \Sigma\Sigma D_{kj}^2 - \frac{(\Sigma\Sigma D_{kj} f_{kj})^2}{\Sigma\Sigma f_{kj}^2} \qquad (30)$$

The first term of the right hand side of Eq.(29) equals zero when C satisfies the next relation.

$$C = \frac{\Sigma\Sigma D_{kj} f_{kj}}{\Sigma\Sigma f_{kj}^2} \qquad (31)$$

For a set of the values of $(Z_k)_{k=1..N}$, the value of C that minimizes E_1 can be determined by Eq.(31). Under this condition, E_1 equals E_2. Hence, the searching problem of the minimum value of E_1 becomes equivalent to that of E_2.

C-3. Initial Values of Z_k

N equations of Eq.(27)

$$D_{kj} = C\, f_j(Z_k),\ j=1..N \qquad (32)$$

are held at the brightest surface element k. From two equations of D_{ki} and D_{kj} obtained from the different light source i and j ($i \neq j$), the following high-order equation of Z_k can be obtained as

$$\frac{D_{ki}}{D_{kj}} = \frac{h_{ki}\, r_{kj}^3}{h_{kj}\, r_{ki}^3} \qquad (33)$$

where,

$$h_{ki} = (\, Xs_k - Z_k A_k\,)(\, Xs_i - Z_k A_k\,) + (\, Ys_k - Z_k B_k\,)(\, Ys_i - Z_k B_k\,) + (\, Zs_k - Z_k\,)(\, Zs_i - Z_k\,)$$

$$h_{kj} = (\, Xs_k - Z_k A_k\,)(\, Xs_j - Z_k A_k\,) + (\, Ys_k - Z_k B_k\,)(\, Ys_j - Z_k B_k\,) + (\, Zs_k - Z_k\,)(\, Zs_j - Z_k\,)$$

$$r_{ki} = \{\, (Xs_i - Z_k A_k)^2 + (Ys_i - Z_k B_k)^2 + (Zs_i - Z_k)^2\,\}^{1/2}$$

$$r_{kj} = \{\, (Xs_j - Z_k A_k)^2 + (Ys_j - Z_k B_k)^2 + (Zs_j - Z_k)^2\,\}^{1/2}$$

$$A_k = x_k / b,\quad B_k = y_k / b$$

The number of combinations of the light source i and j is ${}_N C_2$ when N light sources exist. Then,

$$E_{1i}(Z_k) = \sum_{i,j}(\, \frac{D_{ki}}{D_{kj}} - \frac{h_{ki}\, r_{kj}^3}{h_{kj}\, r_{ki}^3}\,)^2 \qquad (34)$$

where Σ means the summation of ${}_N C_2$ combinations. Eq.(33) becomes the following 10th-order equation of Z_k.

$$D_{ki}^2\, r_{ki}^2\, h_{kj}^6 = D_{kj}^2\, r_{kj}^2\, h_{ki}^6 \qquad (35)$$

This means that only one minimum point of $E_{1i}(Z_k)$ exists when the number of light sources (images) is more than three, because more than three light sources produce more than two independent equations of Z_k.

Z_k which minimizes E_{1i} is adopted as the initial value of Z_k which gives the minimum of some minimal points of E_{1i}.

D. Step 2

D-1. Relation between Gradient and Distance at Surface Element from Three Images

In step 2, the values of three variables (p,q,Z) at each surface element are determined under the constraint which C is treated as a known constant. The relation between (p,q) and Z can be obtained from three independent image density D_j (j=1,2,3) as follows.

From two illuminating equations of D_1 and D_2, the operation D_1/D_2 gives

$$D_1\, r_1^3\,(\, p\, l_2 + q\, m_2 - n_2\,) = D_2\, r_2^3\,(\, p\, l_1 + q\, m_1 - n_1\,) \qquad (36)$$

Similarly, from two equations of D_1 and D_3,

$$D_1\, r_1^3\,(\, p\, l_3 + q\, m_3 - n_3\,) = D_3\, r_3^3\,(\, p\, l_1 + q\, m_1 - n_1\,) \qquad (37)$$

These equations are linear equations of (p,q). Then,

$$p = U_1(Z) = \frac{D(Z)\, E(Z) - B(Z)\, F(Z)}{A(Z)\, D(Z) - B(Z)\, C(Z)} \qquad (38)$$

$$q = U_2(Z) = \frac{A(Z)\, F(Z) - C(Z)\, E(Z)}{A(Z)\, D(Z) - B(Z)\, C(Z)} \qquad (39)$$

where,

$$\begin{aligned} A(Z) &= a_1\, l_2 - a_2\, l_1 \\ B(Z) &= a_1\, m_2 - a_2\, m_1 \\ C(Z) &= a_1\, l_3 - a_3\, l_1 \\ D(Z) &= a_1\, m_3 - a_3\, m_1 \\ E(Z) &= a_1\, n_2 - a_2\, n_1 \\ F(Z) &= a_1\, n_3 - a_3\, n_1 \\ a_j &= D_j r_j^3 \quad (j=1,2,3) \end{aligned} \qquad (40)$$

D-2. Determination of Distance at Surface Element from Four Images

(p,q) given by Eqs.(38) and (39) are rewritten by

$$p = U_1(Z) = \frac{V_1(Z)}{V_3(Z)} \qquad (41)$$

$$q = U_2(Z) = \frac{V_2(Z)}{V_3(Z)} \qquad (42)$$

Let C obtained in step 1 be C_e, and let

$$D_j / C_e = d_j \quad (j=1..4) \qquad (43)$$

then,

$$(V_1 l_i + V_2 m_i - V_3 n_i)^2 = d_i^2(V_1^2+ V_2^2+ V_3^2) (l_i^2+ m_i^2+ n_i^2)^3 \quad (44)$$
(for i=1,2,3)

is obtained for D_1 to D_3 by using relations of Eq.(20) to Eq.(22). Similarly, for D_4,

$$(V_1 l_j + V_2 m_j - V_3 n_j)^2 = d_j^2(V_1^2+ V_2^2+ V_3^2) (l_j^2+ m_j^2+ n_j^2)^3 \quad (45)$$
(for j=4)

is obtained. Eq.(44) and Eq.(45) are independent high-order equations of Z each other. Accordingly, an unique solution of Z for each surface element can be obtained when four input images are used.

From the above discussions, Z which minimizes the following objective function $E_3(Z)$ is adopted as the solution of Z which gives the minimum of some minimal points of $E_3(Z)$.

$$E_3(Z) = \Sigma \ (D_j - C_a f_j(U_1(Z),U_2(Z),Z))^2 \quad (46)$$

where Σ means the summation by the point light source ordinal number j (j=1..4). When four images are used, only one minimum point of $E_3(Z)$ exists.

E. Determination of Distance at Surface Element from Three Images

In step 1, more than three input images ($N \geqq 3$) are necessary to obtain unique solutions of Z_K and C_a that minimize Eq.(29). When three input images are used, the assumption that Z values of the neighbouring surface elements are close to each other (i.e. the assumption of the surface continuity) is used in step 2.

In Eq.(46), three terms for the summation are not dependent, then the number of the minimum point of $E_3(Z)$ is not one.

Therefore, Z that minimizes $E_3(Z)$ is orderly estimated at from the brightest surface element towards the surrounding surface elements by using the value of Z_K obtained in step 1 as the initial value. The initial value of Z for minimizing $E_3(Z)$ at the surface element is set to the value of Z obtained at its neighbouring surface element. The searching direction is set to the direction that decreases $E_3(Z)$. Let the initial value be Z and the moving step of Z be dZ, then the sign of dZ that satisfies

$$E_3(Z+dZ) < E_3(Z) \quad (47)$$

is firstly determined. Z which gives the minimal point of $E_3(Z)$ is adopted as the solution of Z at each surface element.

This algorithm of three input images is effective for the continuous surface.

F. Arrangement of Point Light Sources

It is necessary for solving simultaneous equations that the constituent equations are independent each other. Furthermore, it is desirable that each light source gives a great variety of illumination.

Since the luminance of a surface element is determined by the incident angle and the distance from the light source, the light sources must be arranged so that both the incident angles and the distances at the surface elements are much different.

Accordingly, it is required that the distance between light sources should be large.

IV. Experiment

A. Method of Experiment

As an object, a plaster hemisphere with r=45[mm] was adopted, which was put on $Z=Z_0$ plane, where Z_0=695[mm]. The film size was 7x7[mm], b was equal to 25[mm], and the image plane was divided into 64x64 pixels. Fig.3 shows the observation environment.

The diffused spherical light source with 30[mm] diameter was adopted. The distance r is more than ten times of the scale of the light source, then the light source may be regarded as a point light source. Each light source is located so that the brightest surface element in each image exists within the commonly illuminated region in all images.

L_1: (Xs_1,Ys_1,Zs_1)=(32,191,405)
L_2: (Xs_2,Ys_2,Zs_2)=(166,-108,419)
L_3: (Xs_3,Ys_3,Zs_3)=(-84,-130,386)
L_4: (Xs_4,Ys_4,Zs_4)=(180,11,387) [mm]

These values were computed photometrically by the calibration using the method of least squares from the image densities of original 256x256 pixels.

B. Reconstructed Results

Input images with 8-bit quantizations and the commonly illuminated region are shown in Fig.4. In case of four input images, images (a)-(d) are used. On the other hand, in case of three input images, images (a)-(c) are used. The brightest surface element in each image is plotted by a black point in Fig.4.

The obtained objective function $E_{1i}(Z_K)$, $_{K=1..N}$ for N=4 is shown in Fig.5. Z_K which minimizes E_{1i} is determined by getting the values of Z which give the minimal points of E_{1i}.

Furthermore, using the obtained Z_K as the initial values, Powell method [7] is used to search the minimum point of E_2 of Eq.(30).

The objective functions $E_3(Z)$ in step 2 are shown in Fig.6-(a) and (b) for N=4 and N=3 respectively. These are $E_3(Z)$ at the surface elements on the Y-Z plain. In case of four input images, the number of minimum points of $E_3(Z)$ is

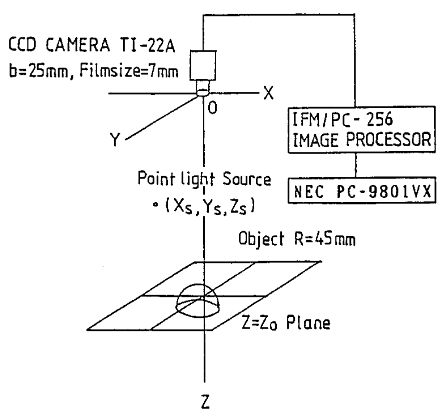

Fig.3. Observation environment.

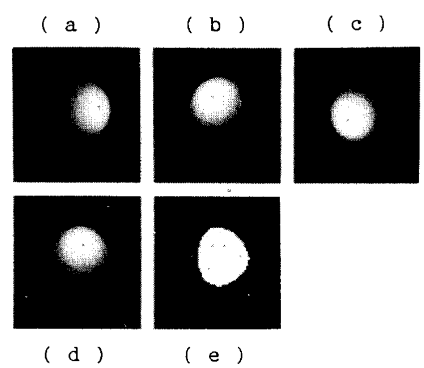

Fig.4. Input images.
(a) - (d) : four input images.
(e): commonly illuminated region in all images.

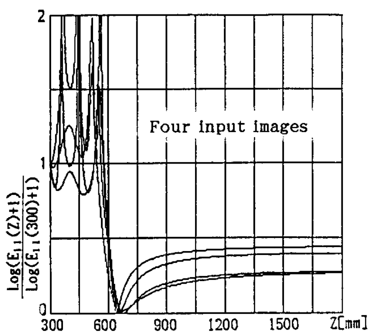

Fig.5. Objective function $E_{11}(Z_K)$.

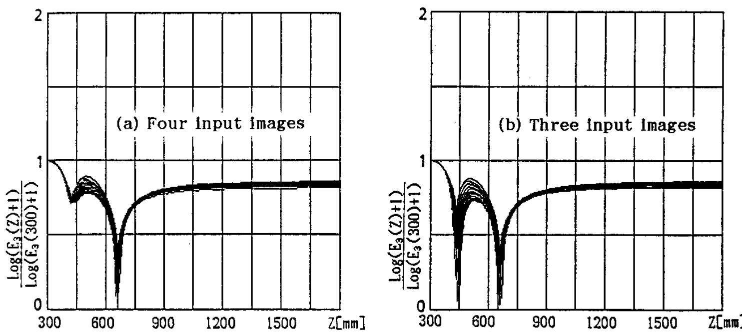

Fig.6. Objective function $E_3(Z)$.

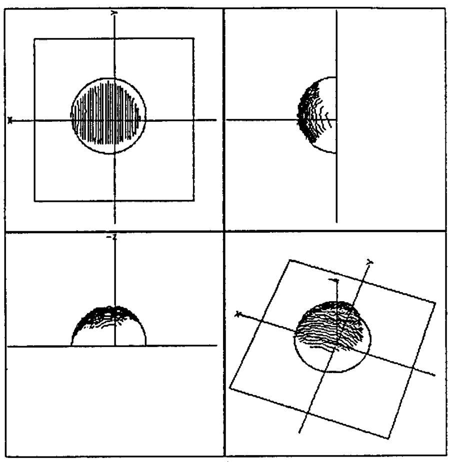

Fig.7. Reconstructed shapes for four input images

only one, while in case of three input images, that is not always only one.

The results of the distance distribution obtained in step 2 are shown in Fig.7, where the coordinate (X,Y) of a surface element is determined from Z by Eqs.(5) and (6). The shape reconstruction of the object is performed by joining (X,Y,Z) values of neighboring surface elements. In Fig.7, (Z_0-Z) is adopted as the measure of the height of the surface element to display reconstructed shapes, and true curves of the object are also shown on the figure for the comparison. Under the condition that the image quantization levels are about maximum 200, the mean absolute errors of the Z-distribution for the reconstruction are within 1[%] both for four input images and three input images. Evaluating errors of the Z-distribution are as follows.

	Four Images	Three Images
Mean absolute errors	2.5 mm	2.7 mm
Mean square errors	9.1 mm^2	10.8mm^2

Results from four images are better than those from three images. The processing times in step 1 are about 3.5 minutes for four images and 1 minute for three images, and those for each surface element in step 2 are about 3 seconds in case of four images and 2 seconds in case of three images. These processing times are measured under the environment of 80286 CPU + 80287 Arithmetic Co-Processor (10MHz clock) hardware and MS-DOS Turbo Pascal system software.

The errors may be caused from some of facts such as that the assumption of the perfect diffused surface is not exactly held and that the gradients at the brightest surface elements are calculated by basing on the limited image sampling resolution. It is considered that the reliability of the estimated value of C is reduced a little by these errors.

V. CONCLUSION

A new photometric method named PSIS is proposed and discussed. The method adopts the principle of the monocular vision and the point light source illumination instead of the parallel light illumination of Photometric Stereo.

Conditions used in this paper are as follows.

(1) the inverse square law for illuminance
(2) the object with the perfectly diffused surface
(3) approximation of the paraxial ray
(4) neglecting the error of the precision of the coordinate of the image pixel

An algorithm based on the method of least squares is developed to determine the distance Z of each surface element of an object from four and three shading images.

Under the above conditions, it is shown that PSIS can reconstruct the shape of an object with positions in 3-D space.

It should be noted that further efforts are required to extend this method for real complex objects, and several extensions of the work presented here may be possible.

REFERENCES

[1] R.J.Woodham, "Photometric Stereo : A reflectance map technique for determining surface orientation from image intensity," Proc. SPIE, vol.155, pp.136-143, 1978

[2] R.J.Woodham, "Photometric method for determining surface orientation from multiple images," Opt. Eng., vol. 19, no.1, pp.139-144, 1980

[3] K.Ikeuchi, "Determining 3D shape from 2D shading information based on the reflectance map technique," Trans. of IECE JAPAN, Vol. J65-D, No.7, pp.842-849, 1982

[4] H.Kitagawa, M.Suzuki, and H.Fujita, "Extraction of surface gradient from shading images," Trans. of IECE JAPAN, Vol.J66-D, No.1, pp.65-72, 1983

[5] H.Hong, T.Kawashima, and Y.Aoki, "A method to reconstruct shape and position of 3-Dimensional objects using photometricstereo system," Trans. of IECE JAPAN, Vol. J69-D, No.3, pp.427-433, 1986

[6] Y.Iwahori, N.Hiratsuka, H.Kamei, and S.Yamaguchi, "Extended photometric stereo for an object with unknown reflectance property," Trans. of IEICE JAPAN, Vol.J71-D, No.1, pp.110-117, 1988

[7] J.Kowalik and M.R.Osborne, "Methods for unconstrained optimization problems", American Elsevier Publishing Company, Inc., Chap.4, 1968