Reconstruction of Medical Images by Perspective Shape-from-Shading

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Abstract

Shape-from-Shading (SfS) is a fundamental problem in Computer Vision; it is based upon the image irradiance equation. Recently, the authors proposed to solve the image irradiance equation under the assumption of perspective projection rather than the common orthographic one. The solution was a modification of the Fast Marching method of Kimmel and Sethian. This paper presents an application of this novel perspective algorithm to reconstruction of medical images. We focus on gastrointestinal endoscopy and compare the two versions of the Fast Marching method (orthographic vs. perspective). The examples and comparison show that, unlike orthographic SfS, perspective SfS is robust and can be utilized for real-life applications.

1. Introduction

Shape-from-Shading (SfS) is a fundamental problem in Computer Vision. The common way to obtain shape information is to solve the image irradiance equation, which relates the reflectance map to image intensity. As this task is nontrivial, most of the works in the field employ simplifying assumptions, and in particular the assumption that projection of scene points during a photographic process is orthographic ([2], [15], [6], [1], [12] to name just few; see [14] for a survey). This resulted in low stability of reconstruction algorithms.

Later algorithms which did assume the perspective model have either been too restrictive or have not addressed the general problem (e.g., [8], [4], [13], [9]). Recently, two works ([10] and [5]) have developed the perspective image irradiance equation simultaneously.

Despite the large amount of literature on the subject, SfS is still considered too unstable for real-life tasks by many researchers. The goal of this paper is to show that the recent incorporation of the perspective model into SfS yields robust and efficient algorithms that can be employed for real-life applications. To achieve this goal, we present a real-life application of the perspective algorithm of [10], and compare it with a state-of-the-art orthographic method, the Fast Marching of [3]. The application is reconstruction of medical images. More specifically, our application deals with gastrointestinal endoscopic imagery.

This paper is organized as follows. We first review the image irradiance equation under the perspective projection model and the perspective SfS algorithm of [10] (Sect. 2). Section 3 presents the application. Section 4 introduces perspective reconstruction of medical images of gastrointestinal endoscopy and compares it with orthographic reconstruction. Finally, Sect. 5 draws conclusions.

2. The Perspective Image Irradiance Equation

We first present the perspective image irradiance equation (see [10] for more details). We employ the following notation and assumptions. $\hat{z}(x, y)$ denotes the depth function in a real-world Cartesian coordinate system (origin at camera plane). If the real-world coordinate $(x, y, \widehat{z}(x, y))$ is projected onto image point (u, v), then its depth is denoted $z(u, v)$. By definition, $z(u, v) = \hat{z}(x, y)$. f is the focal length. The scene object is Lambertian, and is illuminated by a point light source at infinity whose direction is $\vec{L} = (p_s, q_s, -1)$. $\vec{N}(x, y)$ is the surface normal.

2.1. The Equation in Image Coordinates

The perspective image irradiance equation is given by:

$$I(u, v) = \vec{L} \cdot \vec{N}(x, y) \qquad (1)$$

where: $x = -\frac{u \cdot \widehat{z}(x,y)}{f}$, $y = -\frac{v \cdot \widehat{z}(x,y)}{f}$. In [10] we show that these equations can be combined to form the *perspective image irradiance equation*:

$$I(u, v) = \frac{(u - fp_s)z_u + (v - fq_s)z_v + z}{||\vec{L}||\sqrt{(uz_u + vz_v + z)^2 + f^2(z_u^2 + z_v^2)}} \qquad (2)$$

If one denotes $p \stackrel{def}{=} \frac{z_u}{z} = \frac{\partial \ln z}{\partial u}$ and $q \stackrel{def}{=} \frac{z_v}{z} = \frac{\partial \ln z}{\partial v}$ and substitutes into Eq. 2, the equation becomes:

$$I(u, v) = \frac{(u - fp_s)p + (v - fq_s)q + 1}{||\vec{L}||\sqrt{(up + vq + 1)^2 + f^2(p^2 + q^2)}} \qquad (3)$$

Eq. 3 depends on the derivatives of $\ln(z(u,v))$, but not on $\ln(z(u,v))$ itself, which implies that the perspective image irradiance equation (Eq. 2) is invariant to scale changes of $z(u,v)$ (unlike the orthographic equation, which is invariant to translation of $z(u,v) = \hat{z}(x,y)$). That is, the perspective intensity functions of $c \cdot z(u,v)$ and $z(u,v)$ are identical (where c is constant). This property is more similar to that of real cameras than invariance to translation.

2.2. Perspective Fast Marching

The perspective SfS algorithm was suggested in [11], and is based on the Fast Marching method of [3].

The original algorithm [3] solves the orthographic image irradiance equation $I(x,y) = \vec{L} \cdot \vec{N}(x,y)$, which can be transformed into: $\tilde{p}^2 + \tilde{q}^2 = \tilde{F}^2(x,y)$, where $\tilde{p} \stackrel{\text{def}}{=} z_u = z_x$, $\tilde{q} \stackrel{\text{def}}{=} z_v = z_y$ and $\tilde{F} = \sqrt{(I(x,y))^{-2} - 1}$. Note, that $\tilde{F}(x,y)$ is independent of $\tilde{p}$ and $\tilde{q}$. Similarly, the perspective image irradiance equation (Eq. 3), can be transformed into:

$$p^2 A_1 + q^2 B_1 = F \tag{4}$$

where A_1 and B_1 are non-negative and independent of p or q (see [11]). However, F depends on both p and q.

In order to solve an Eikonal equation, we assume that we have estimated $p(u,v)$ and $q(u,v)$ by some process. We use these estimations to derive $F(u,v) = F(p(u,v), q(u,v), u, v)$ and substitute it into the right-hand side of Eq. 4. This results in an Eikonal equation, which we solve for p and q of the left-hand side. We may then use the solution to improve our initial estimation of p and q. This results in the following iterative scheme:

$$p_{n+1}^2 A_1 + q_{n+1}^2 B_1 = F(p_n, q_n) \tag{5}$$

where p_n and q_n are the values of p and q at the $n^{\underline{\text{th}}}$ iteration. We initialize this perspective process by the orthographic Fast Marching method [3].

For each iteration we convert the numerical approximation of [3] to the perspective case:

$$\begin{aligned} p_{ij} &\approx \max\{D_{ij}^{-u} z, -D_{ij}^{+u} z, 0\} \\ q_{ij} &\approx \max\{D_{ij}^{-v} z, -D_{ij}^{+v} z, 0\} \end{aligned}$$

where $D_{ij}^{-u} z = \frac{z_{ij} - z_{i-1,j}}{\Delta u}$ is the standard backward derivative and $D_{ij}^{+u} z = \frac{z_{i+1,j} - z_{ij}}{\Delta u}$, the standard forward derivative in the u-direction ($z_{ij} \stackrel{\text{def}}{=} z(i\Delta u, j\Delta v)$; $p_{ij} \stackrel{\text{def}}{=} p(i\Delta u, j\Delta v)$; $q_{ij} \stackrel{\text{def}}{=} q(i\Delta u, j\Delta v)$). $D_{ij}^{-v} z$ and $D_{ij}^{+v} z$ are defined in a similar manner for the v-direction. We then substitute the numerical approximation into Eq. 5, and solve the discrete equation for z_{ij}. The resultant solution depends on the depth of neighboring pixels ($z_{i-1,j}$, $z_{i+1,j}$, $z_{i,j-1}$, $z_{i,j+1}$). Nevertheless, z_{ij} can be computed in one pass, because information always flows from small to large values (see [7]), and our reconstruction begins from points of minimal depth.

3. The Application

We applied the perspective Fast Marching [11] to medical images taken by endoscopy of the gastrointestinal channel. In addition, we juxtaposed it with the orthographic Fast Marching method [3], so the orthographic results are provided as well. Images of different parts of the gastrointestinal tract were chosen to demonstrate the application.

As our theoretic model requires the knowledge of the light source direction and focal length, while in practice these data were unavailable, a human viewer had to produce [very rough] estimations of light source directions from the images themselves. The focal length was set to the same arbitrary constant in all experiments. The points of local minimal depth were also located manually.

In our comparison we checked 5 iterations of the perspective Fast Marching for each example (in addition to the orthographic initialization). We found out that all perspective iterations yielded visually-identical images, which implies the suggested algorithm converges very fast. We therefore introduce only the first iteration in this paper.

4. Experimental Results

Fig. 1 presents reconstructions by the perspective Fast Marching method (after 1 perspective iteration). Reconstruction by the orthographic method is also supplied. Each row presents an example from a different part of the gastrointestinal pathway (or from the larynx).

The top row[1] of Fig. 1 presents reconstruction of a cavity in the larynx. The cavity is clearly reconstructed. Pay special attention to the smaller cavity which is also reconstructed (at the right hand side of the cropped image).

The second row[2] shows an enormous tumor (leiomyoma) of the stomach. The cropped image is of the lower part of the leiomyoma, at it base. The strong noise is due to the texture of the tumor. Perspective SfS reconstructs the shape despite the noise.

The next row[1] exhibits the descending duodenum. The cropped version contains three plicae circulares (folds typical of the small intestine). The perspective method recovered the folds, and even the concavity of the duodenal wall.

The fourth row[2] is of the ileocecal valve, the opening between the large and small intestines. The face of the valve was reconstructed; the reconstruction is displayed from a

1 Image from www.gastrolab.net, courtesy of The Wasa Workgroup on Intestinal Disorders, GASTROLAB, Vasa, Finland.

2 Image from www.gastrointestinalatlas.com, courtesy of the Department of Gastroenterology, Hospital Centro de Emergencias, Jerusalem Medical Center, San Salvador, El-Salvador.

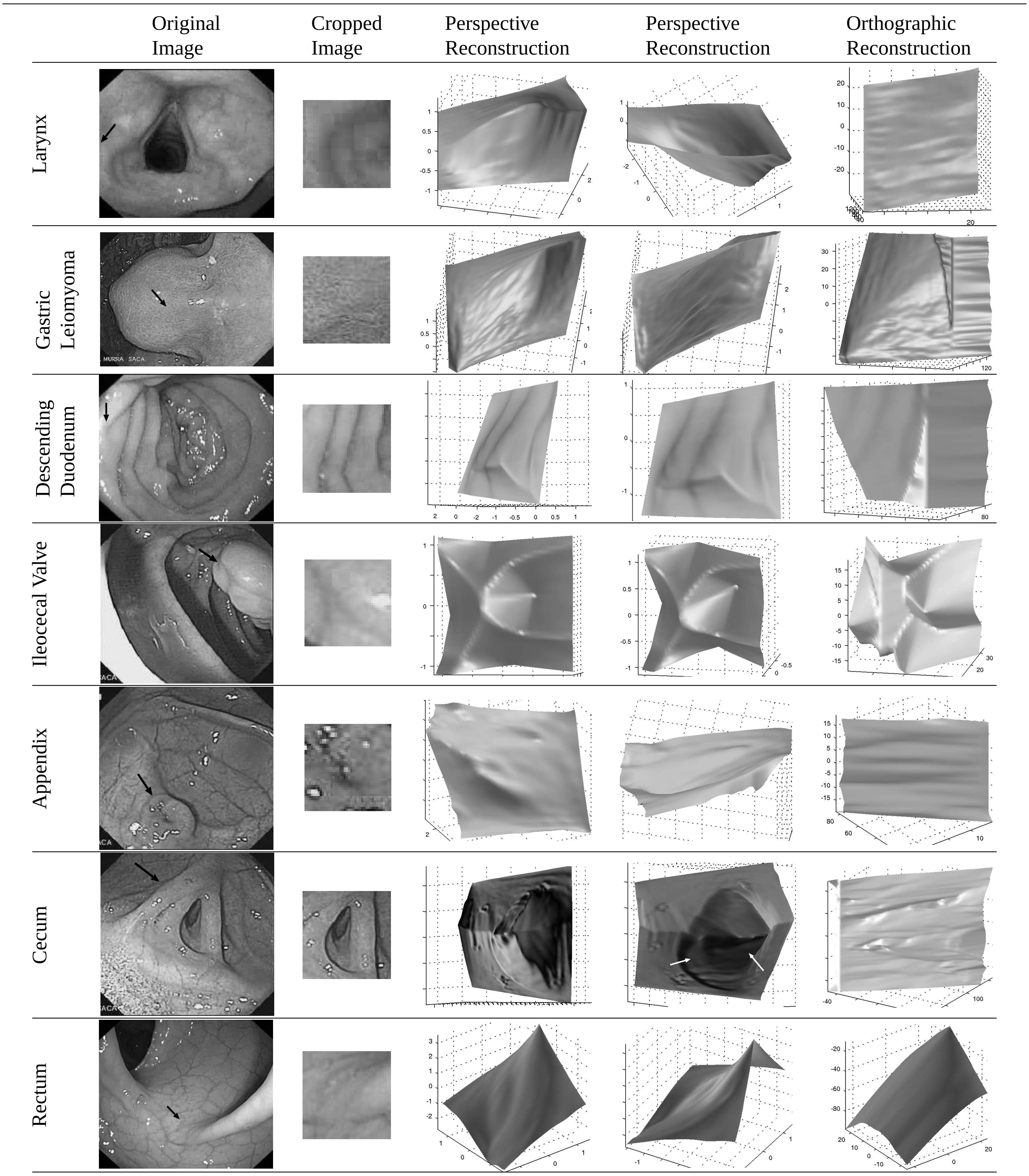

Figure 1. Perspective (and orthographic) reconstruction of endoscopic images by the Fast Marching method. Each row refers to a different organ (as indicated to the left of each row). The arrow on the original image points to the upper-left corner of the cropped image. Perspective reconstruction is presented from two points of view.

frontal viewpoint. The three part of the valve were reconstructed (the top one is split into two, due to a change in convexity of this part).

The next row[2] shows the reconstruction of an inverted appendix. The appendiceal orifice is faithfully reconstructed, despite the low quality of the cropped image.

The following example[2] is of the cecum with appendiceal orifice; view of the crow's foot appearance of the haustral folds. The main cavity was recovered. It is even possible to note the two folds inside the cavity (indicated by white arrows in the right image). The right hand side of the rightmost fold was recovered inaccurately, probably due to the in-out ambiguity (surface duality). Note, however, the accurate reconstruction of the leftmost fold, outside the main cavity.

The root of a rectal plica is reconstructed on the bottom row[1]. The plica can be distinguished from the walls of the rectum in the perspective reconstruction (but not in the orthographic one).

In general, all orthographic reconstructions either did not reconstruct the cavity (Larynx, Appendix and Cecum), or failed to recover the correct shape (Leiomyoma, Duodenum, Rectum) or had extra structures not present in the original image (Leiomyoma, Duodenum, Ileocecal valve).

Even though both algorithms use the same numerical methodology, the perspective Fast Marching method appears to outrank the orthographic one. This suggests that the assumption of a perspective rather than an orthographic image irradiance equation yields an important increase in both robustness and accuracy of the reconstruction.

While many orthographic algorithms rival the best numerical way to solve the classic equation, the novel perspective algorithm under study adopts its numerical scheme from Kimmel and Sethian [3] and thus avoids competition in the numerical arena. Instead, it demonstrates that the *perspective equation* may be better suited for the task of SfS in real-life scenes.

5. Conclusions

This paper presented an application of a novel perspective SfS algorithm to reconstruction of medical images. The algorithm is the perspective Fast Marching presented in [11]. The images under study were taken by endoscopy from different parts of the gastrointestinal channel. The perspective algorithm was compared with the orthographic Fast Marching [3] and was shown to outmatch it. In addition, the application demonstrated that perspective Fast Marching is an efficient and robust algorithm adequate for real-life tasks. This indicates that Shape-from-Shading under the perspective projection model can be utilized for real-life scenes.

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References

[1] M. Bichsel and A. P. Pentland. A simple algorithm for Shape from Shading. In *Computer Vision and Pattern Recognition*, pages 459–465, 1992.

[2] B. K. P. Horn. Image intensity understanding. *Artificial Intelligence*, 8(2):201–231, Apr. 1977.

[3] R. Kimmel and J. A. Sethian. Optimal algorithm for Shape from Shading and path planning. *Journal of Mathematical Imaging and Vision*, 14(3):237–244, 2001.

[4] K. M. Lee and C.-C. J. Kuo. Shape from Shading with a generalized reflectance map model. *Computer Vision and Image Understanding*, 67(2):143–160, Aug. 1997.

[5] E. Prados and O. Faugeras. Perspective Shape from Shading and viscosity solutions. In *Proc. of the Intl. Conference on Computer Vision*, volume 2, pages 826–831, October 2003.

[6] A. Robles-Kelly and E. R. Hancock. Model acquisition using Shape-from-Shading. In F. J. Perales and E. R. Hancock, editors, *Intl. Workshop on Articulated Motion and Deformable Objects*, pages 43–55, Palma de Mallorca, Nov. 2002.

[7] E. Rouy and A. Tourin. A viscosity solutions approach to Shape-from-Shading. *SIAM Journal of Numerical Analysis*, 29(3):867–884, June 1992.

[8] D. Samaras and D. Metaxas. Coupled lighting direction and shape estimation from single images. *Proceedings of the Intl. Conference on Computer Vision*, 2:868–874, 1999.

[9] I. Seong, S. Hideo, and O. Shinji. A divide-and-conquer strategy in Shape from Shading problem. In *Conf. on Computer Vision and Pattern Recognition*, pages 413–419, 1997.

[10] A. Tankus, N. Sochen, and Y. Yeshurun. A new perspective [on] Shape-from-Shading. In *Proceedings of the* $9^{\underline{th}}$ *IEEE International Conference on Computer Vision*, volume II, pages 862–869, Nice, France, Oct. 2003.

[11] A. Tankus, N. Sochen, and Y. Yeshurun. Perspective Shape-from-Shading by Fast Marching. In *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, Washington, DC, Jun.–Jul. 2004.

[12] P.-S. Tsai and M. Shah. Shape from Shading using linear approximation. *Image and Vision Computing*, 12(8):487–498, Oct. 1994.

[13] S. Yamany, A. Farag, D. Tazman, and A. Farman. A robust 3D reconstruction system for human jaw modeling. *SPIE-Int. Soc. Opt. Eng., 1999, USA.*, 3640:125–135, 1999.

[14] R. Zhang, P.-S. Tsai, J. E. Cryer, and M. Shah. Shape from Shading: A survey. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 21(8):690–705, August 1999.

[15] Q. Zheng and R. Chellappa. Estimation of illuminant direction, albedo, and shape from shading. *Trans. on Pattern Analysis and Machine Intelligence*, 13(7):680–702, 1991.