

RECONSTRUCTIVE SCHEMES FOR VARIATIONAL ITERATION METHOD WITHIN YANG-LAPLACE TRANSFORM WITH APPLICATION TO FRACTAL HEAT CONDUCTION PROBLEM

by

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A reconstructive scheme for variational iteration method using the Yang-Laplace transform is proposed and developed with the Yang-Laplace transform. The identification of fractal Lagrange multiplier is investigated by the Yang-Laplace transform. The method is exemplified by a fractal heat conduction equation with local fractional derivative. The results developed are valid for a compact solution domain with high accuracy.

Key words: *fractal heat conduction equation, local fractional variational iteration method, Yang-Laplace transform, local fractional derivative*

Introduction

Heat transfer problems can be mathematically modelled by ordinary and partial differential equations [1], integral equations [2], and integro-differential equations [3]. Commonly heat conduction problems are solved either analytically or analytical and numerically applying various methods, among them: finite difference techniques (FDT) [4], regression analysis (RA) [5], Adomian decomposition method (ADM) [6], homotopy analysis method (HAM) [7], differential transformation method (DTM) [8], boundary element method (BEM) [9], the heat-balance integral method (HBIM) [10-12], *etc.*

The variational iteration method (VIM) was first proposed by He [13] and was successfully applied to deal with heat conduction equations [14-17]. In 2010, the fractional variational iteration method (FVIM) via modified Riemann-Liouville derivative was conceived [17]. On the other hand, the local fractional variational iteration method via local fractional calculus [18-20] was developed in [21] and successfully applied to differential equations on Cantor sets [22-24]. In this context, recently, a method combining the variational iteration method and Laplace transform method was suggested [25, 26] and a modification via fractional calculus and Laplace transform was conceived by Wu [27].

Fractal heat conduction problems describe heat transport in inhomogeneous materials as fibrous materials, textiles, coal deposits, and other discontinuous media where homogenizations are unacceptable because this approach neglects important physical characteristics of the transport processes. The non-smoothness raises problems and avoids application of the classical calculus of integer and fractional order. To avoid this problem, the local fractional

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calculus [20, 21] was successfully applied to ordinary and partial differential equations with either fractal media or with fractal boundary conditions. Solutions of such local problems have been developed the local fractional Fourier series method [28], Yang-Fourier and Yang-Laplace transforms [29], local fractional variational iteration method [21-24], etc.

As a stand of portion of development of solutions to fractal heat conduction problems, the present communication refers to a coupling method combining the features of the variational iteration method and the Yang-Laplace transform [29] local fractional derivatives.

Reconstructive processes

Let us consider the following local fractional partial differential equations:

$$L_\alpha u - R_\alpha u = g(x) \quad (1)$$

where $u = u(x)$, L_α is the linear local fractional operator, R_α is the linear local fractional operator of order less than L_α , and $g(x)$ is a source term of the non-differential function.

According to the rule of local fractional variational iteration method, the correction local fractional functional for eq. (1) is constructed as [21-24]:

$$u_{n+1}(x) = u_n(x) + {}_0I_x^{(\alpha)} \left\{ \frac{\lambda(t)^\alpha}{\Gamma(1+\alpha)} [L_\alpha u_n(t) + R_\alpha \tilde{u}_n(t) - g(t)] \right\} \quad (2)$$

In eq. (2) the local fractional integral operator is defined as [18-20]:

$${}_aI_b^{(\alpha)} f(x) = \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t) (dt)^\alpha = \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \rightarrow 0} \sum_{j=0}^{N-1} f(t_j) (\Delta t_j)^\alpha \quad (3a)$$

and the partition of the interval $[a, b]$, is:

$$\Delta t_j = t_{j+1} - t_j, \quad \Delta t = \max\{\Delta t_1, \Delta t_2, \Delta t_3, \dots\} \quad \text{and} \quad j=0, \dots, N-1, \quad t_0 = a, t_N = b \quad (3b)$$

In eq. (2) $\lambda(t)^\alpha/\Gamma(1+\alpha)$ is a fractal Lagrange multiplier.

When $\delta^\alpha \tilde{u}_n$ is considered as a restricted local fractional variation [20], i. e. $\delta^\alpha \tilde{u}_n = 0$, we obtain the following fractal Lagrange multiplier:

$$\frac{\lambda(t)^\alpha}{\Gamma(1+\alpha)} = -\frac{(x-t)^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} \quad (4)$$

where L_α in eq. (1) are $k\alpha$ times local fractional partial differential equations.

Therefore, eq. (2) becomes:

$$u_{n+1}(x) = u_n(x) - {}_0I_x^{(\alpha)} \left\{ \frac{(x-t)^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} [L_\alpha u_n(t) + R_\alpha \tilde{u}_n(t) - g(t)] \right\} \quad (5)$$

For initial value problems of eq. (1), we can start with:

$$u_0(x) = u(0) + \frac{x^\alpha}{\Gamma(1+\alpha)} u^{(\alpha)}(0) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} u^{(2\alpha)}(0) + \dots + \frac{x^{k\alpha}}{\Gamma(1+k\alpha)} u^{(k\alpha)}(0) \quad (6)$$

where local fractional derivative is given by [18-20]:

$$D_x^{(\alpha)} f(x_0) = f^{(\alpha)}(x_0) = \left. \frac{d^\alpha f(x)}{dx^\alpha} \right|_{x=x_0} = \lim_{x \rightarrow x_0} \frac{\Delta^\alpha [f(x) - f(x_0)]}{(x-x_0)^\alpha} \quad (7a)$$

with $\Delta^\alpha [f(x) - f(x_0)] \cong \Gamma(1+\alpha)[f(x) - f(x_0)]$, and local fractional derivative of high order is denoted as [20]:

$$f^{(k\alpha)}(x) = \overbrace{D_x^{(\alpha)} \dots D_x^{(\alpha)}}^{k \text{ times}} f(x) \quad (7b)$$

The Yang-Laplace transform is defined as [18, 19, 29]:

$$\tilde{L}_\alpha \{f(x)\} = f_s^{\tilde{L},\alpha}(s) = \frac{1}{\Gamma(1+\alpha)} \int_0^\infty E_\alpha(-s^\alpha x^\alpha) f(x) (dx)^\alpha, \quad 0 < \alpha \leq 1 \quad (8a)$$

and its inverse formula is [18, 19, 29]:

$$f(x) = \tilde{L}_\alpha^{-1} \{f_s^{\tilde{L},\alpha}(s)\} = \frac{1}{(2\pi)^\alpha} \int_{\beta-i\infty}^{\beta+i\infty} E_\alpha(s^\alpha x^\alpha) f_s^{\tilde{L},\alpha}(s) (ds)^\alpha \quad (8b)$$

where $f(x)$ is local fractional continuous, $s^\alpha = \beta^\alpha + i^\alpha \alpha^\alpha$ and $\text{Re}(s^\alpha) = \beta^\alpha$.

Thus, following eqs. (2) and (5), we obtain a new local fractional functional in the form:

$$u_{n+1}(x) = u_n(x) + {}_0I_x^{(\alpha)} \left\{ \frac{\lambda(x-t)^\alpha}{\Gamma(1+\alpha)} [L_\alpha u_n(t) + R_\alpha \tilde{u}_n(t) - g(t)] \right\} \quad (9)$$

We now take Yang-Laplace transform of eq. (2), namely:

$$\tilde{L}_\alpha \{u_{n+1}(x)\} = \tilde{L}_\alpha \{u_n(x)\} + \tilde{L}_\alpha \left\{ {}_0I_x^{(\alpha)} \left[\frac{\lambda(x-t)^\alpha}{\Gamma(1+\alpha)} [L_\alpha u_n(t) + R_\alpha \tilde{u}_n(t) - g(t)] \right] \right\} \quad (10a)$$

or

$$\tilde{L}_\alpha \{u_{n+1}(x)\} = \tilde{L}_\alpha \{u_n(x)\} + \tilde{L}_\alpha \left\{ \frac{\lambda(x-t)^\alpha}{\Gamma(1+\alpha)} \right\} L_\alpha \{[L_\alpha u_n(x) + R_\alpha \tilde{u}_n(x) - g(x)]\} \quad (10b)$$

Here, we first take the variation, which is given by:

$$\begin{aligned} & \delta^\alpha \{ \tilde{L}_\alpha \{u_n(x)\} \} = \\ = & \delta^\alpha \{ \tilde{L}_\alpha \{u_{n+1}(x)\} \} + \delta^\alpha \left\{ \tilde{L}_\alpha \left[\frac{\lambda(x-t)^\alpha}{\Gamma(1+\alpha)} \right] \tilde{L}_\alpha \{[L_\alpha u_n(x) + R_\alpha \tilde{u}_n(x) - g(x)]\} \right\} \end{aligned} \quad (11a)$$

or

$$\delta^\alpha \{ \tilde{L}_\alpha \{u_{n+1}(x)\} \} = \delta^\alpha \{ \tilde{L}_\alpha \{u_{n+1}(x)\} \} + \delta^\alpha \left\{ \tilde{L}_\alpha \left[\frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} \right] \tilde{L}_\alpha \{L_\alpha u_n(x)\} \right\} \quad (11b)$$

By using computation of (11b), we get:

$$\delta^\alpha \{ \tilde{L}_\alpha \{u_{n+1}(x)\} \} = \delta^\alpha \{ \tilde{L}_\alpha \{u(x)\} \} + \tilde{L}_\alpha \left\{ \frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} \right\} \{ \delta^\alpha \tilde{L}_\alpha \{L_\alpha u_n(x)\} \} \quad (12a)$$

where there is a following relation [18, 19]:

$$\tilde{L}_\alpha \{u_n^{(k\alpha)}(x)\} = s^{k\alpha} \tilde{L}_\alpha \{u_n(x)\} - s^{(k-1)\alpha} u_n(0) - s^{(k-2)\alpha} u_n^{(\alpha)}(0) - \dots - u_n^{[(k-1)\alpha]}(0) \quad (12b)$$

Hence, from (12a) we get:

$$1 + \tilde{L}_\alpha \left\{ \frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} \right\} s^{k\alpha} = 0 \quad (13a)$$

or

$$\tilde{L}_\alpha \left\{ \frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} \right\} = -\frac{1}{s^{k\alpha}} \quad (13b)$$

Taking the inverse version of the Yang-Fourier transform, we have:

$$\frac{\lambda(x)^\alpha}{\Gamma(1+\alpha)} = \tilde{L}_\alpha^{-1} \left\{ -\frac{1}{s^{k\alpha}} \right\} = -\frac{(x)^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]}, \quad k \in N \quad (14)$$

In view of eq. (14), we obtain:

$$\tilde{L}_\alpha \{u_{n+1}(x)\} = \tilde{L}_\alpha \{u_n(x)\} - \tilde{L}_\alpha \left\{ {}_0 I_x^{(\alpha)} \left[\frac{(x-t)^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} [L_\alpha u_n(t) + R_\alpha \tilde{u}_n(t) - g(t)] \right] \right\} \quad (15a)$$

Therefore, we have the following iteration algorithm:

$$\tilde{L}_\alpha \{u_{n+1}(x)\} = \tilde{L}_\alpha \{u_n(x)\} - \tilde{L}_\alpha \left\{ \frac{x^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} \right\} \tilde{L}_\alpha \{[L_\alpha u_n(x) + R_\alpha u_n(x) - g(t)]\} \quad (15b)$$

From eq. (12b) we get:

$$\delta^\alpha \left\{ \frac{s^{(k-1)\alpha} u_n(0) + s^{(k-2)\alpha} u_n^{(\alpha)}(0) + \dots + u_n^{[(k-1)\alpha]}(0)}{s^{k\alpha}} \right\} \quad (16a)$$

Hence, from eq. (16a) the initial value is determined by:

$$u_0(x) = \tilde{L}_\alpha^{-1} \left\{ \frac{s^{(k-1)\alpha} u(0) + s^{(k-2)\alpha} u^{(\alpha)}(0) + \dots + u_n^{[(k-1)\alpha]}(0)}{s^{k\alpha}} \right\} = \\ = u(0) + \frac{x^\alpha}{\Gamma(1+\alpha)} u^{(\alpha)}(0) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} u^{(2\alpha)}(0) + \dots + \frac{x^{k\alpha}}{\Gamma(1+k\alpha)} u^{(k\alpha)}(0) \quad (16b)$$

Consequently, the Yang-Laplace transform for identification of fractal Lagrange multiplier is shown in eqs. (15a, b) which leads to the local fractional series solution:

$$u = \lim_{n \rightarrow \infty} \tilde{L}_\alpha^{-1} \{ \tilde{L}_\alpha u_n \} \quad (17)$$

Application to fractal heat-conduction problem

In order to illustrate the proposed method, we give the fractal heat-conduction problem. As it was demonstrated earlier [20, 21], the heat equation on Cantor sets (local fractional heat equation) can be expressed as:

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} = \frac{\partial^{2\alpha} u(x, t)}{\partial x^{2\alpha}} \quad (18a)$$

where $u(x, t)$ is a fractal heat flux.

The initial value is presented as [21]:

$$\frac{\partial^\alpha u(0, t)}{\partial x^\alpha} = E_\alpha(t^\alpha), \quad u(0, t) = 0 \quad (18b,c)$$

By using eq. (15a) we structure the iterative relation as:

$$\tilde{L}_\alpha \{u_{n+1}(x, t)\} = \tilde{L}_\alpha \{u_n(x, t)\} - \tilde{L}_\alpha \left\{ \frac{x^\alpha}{\Gamma(1+\alpha)} \right\} \tilde{L}_\alpha \left\{ \left[\frac{\partial^{2\alpha} u_n(x, t)}{\partial x^{2\alpha}} - \frac{\partial^\alpha u(x, t)}{\partial t^\alpha} \right] \right\} = \\ = \tilde{L}_\alpha \{u_n(x, t)\} - \frac{1}{s^{2\alpha}} \left\{ \left[s^{2\alpha} \tilde{L}_\alpha \{u_n(x, t)\} - s^\alpha u_n(0, t) - u_n^{(\alpha)}(0, t) - \frac{\partial^\alpha \tilde{L}_\alpha \{u_n(x, t)\}}{\partial t^\alpha} \right] \right\} = \\ = \frac{1}{s^\alpha} u_n(0, t) + \frac{u_n^{(\alpha)}(0, t)}{s^{2\alpha}} + \frac{1}{s^{2\alpha}} \frac{\partial^\alpha \tilde{L}_\alpha \{u_n(x, t)\}}{\partial t^\alpha} \quad (19a)$$

In view of eqs. (17 b, c), the initial value reads:

$$u_0(x, t) = \frac{x^\alpha E_\alpha(t^\alpha)}{\Gamma(1 + \alpha)} \quad (19b)$$

and its Yang-Laplace transform is:

$$\tilde{L}_\alpha \{u_0(x, t)\} = \frac{E_\alpha(t^\alpha)}{s^{2\alpha}} \quad (19c)$$

where [19]

$$\tilde{L}_\alpha \left\{ \frac{x^{k\alpha}}{\Gamma(1 + k\alpha)} \right\} = \frac{1}{s^{(k+1)\alpha}} \quad (19d)$$

Hence, we get the first approximation, namely:

$$\tilde{L}_\alpha \{u_1(x, t)\} = \frac{1}{s^\alpha} u_0(0, t) + \frac{u_0^{(\alpha)}(0, t)}{s^{2\alpha}} + \frac{1}{s^{2\alpha}} \frac{\partial^\alpha \tilde{L}_\alpha \{u_0(x, t)\}}{\partial t^\alpha} = E_\alpha(t^\alpha) \left(\frac{1}{s^{2\alpha}} + \frac{1}{s^{4\alpha}} \right) \quad (20a)$$

Thus,

$$u_1(x, t) = \tilde{L}_\alpha^{-1} \left\{ E_\alpha(t^\alpha) \left(\frac{1}{s^{2\alpha}} + \frac{1}{s^{4\alpha}} \right) \right\} = E_\alpha(t^\alpha) \left(\frac{x^\alpha}{\Gamma(1 + \alpha)} + \frac{x^{3\alpha}}{\Gamma(1 + 3\alpha)} \right) \quad (20b)$$

The second approximation reads:

$$\tilde{L}_\alpha \{u_2(x, t)\} = \frac{1}{s^\alpha} u_1(0, t) + \frac{u_1^{(\alpha)}(0, t)}{s^{2\alpha}} + \frac{1}{s^{2\alpha}} \frac{\partial^\alpha \tilde{L}_\alpha \{u_1(x, t)\}}{\partial t^\alpha} = E_\alpha(t^\alpha) \left(\frac{1}{s^{2\alpha}} + \frac{1}{s^{4\alpha}} + \frac{1}{s^{6\alpha}} \right) \quad (21a)$$

Therefore, we get:

$$u_1(x, t) = \tilde{L}_\alpha^{-1} \left\{ E_\alpha(t^\alpha) \left(\frac{1}{s^{2\alpha}} + \frac{1}{s^{4\alpha}} + \frac{1}{s^{6\alpha}} \right) \right\} = E_\alpha(t^\alpha) \left[\frac{x^\alpha}{\Gamma(1 + \alpha)} + \frac{x^{3\alpha}}{\Gamma(1 + 3\alpha)} + \frac{x^{5\alpha}}{\Gamma(1 + 5\alpha)} \right] \quad (21b)$$

The other approximations are written as:

$$\begin{aligned} \tilde{L}_\alpha \{u_3(x, t)\} &= \frac{1}{s^\alpha} u_2(0, t) + \frac{u_2^{(\alpha)}(0, t)}{s^{2\alpha}} + \frac{1}{s^{2\alpha}} \frac{\partial^\alpha \tilde{L}_\alpha \{u_2(x, t)\}}{\partial t^\alpha} = \\ &= E_\alpha(t^\alpha) \left(\frac{1}{s^{2\alpha}} + \frac{1}{s^{4\alpha}} + \frac{1}{s^{6\alpha}} + \frac{1}{s^{8\alpha}} \right) \\ &\vdots \end{aligned} \quad (22)$$

Consequently, the local fractional series solution is:

$$\begin{aligned} u &= \lim_{n \rightarrow \infty} \tilde{L}_\alpha^{-1} \{ \tilde{L}_\alpha u_n \} = \lim_{n \rightarrow \infty} \tilde{L}_\alpha^{-1} \left\{ E_\alpha(t^\alpha) \left(\sum_{i=1}^n \frac{1}{s^{2i\alpha}} \right) \right\} = \\ &= \lim_{n \rightarrow \infty} \left[E_\alpha(t^\alpha) \sum_{k=0}^n \frac{x^{(2k+1)\alpha}}{\Gamma[1 + (2k+1)\alpha]} \right] = E_\alpha(t^\alpha) \sinh_\alpha(x^\alpha) \end{aligned} \quad (23)$$

The same final result was developed by the local fractional variational method [21].

Conclusions

A local fractional continuous solution to fractal heat conduction problems was developed is obtained by a new approach coupling process the variational iteration method and the Yang-Laplace transform. The method is straightforward and well exemplified by a solution of fractal heat conduction with fractal Neumann condition.

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Nomenclature

\tilde{I}_α	– Yang-Laplace transform of $f(x)$	$u(x,t)$	– the temperature function, [K]
\tilde{I}_α^{-1}	– inverse version of Yang-Laplace transform of $f_\omega^F(\omega)$	x	– space co-ordinate, [m]
t	– time, [s]	α	– time fractal dimensional order, [–]

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