Faculty of Informatics - Papers (Archive)

Faculty of Engineering and Information Sciences

Recovering the absolute phase maps of two fringe patterns with selected frequencies

Yi Ding
University of Wollongong

Jiangtao Xi
University of Wollongong, jiangtao@uow.edu.au

Yanguang Yu
University of Wollongong, yanguang@uow.edu.au

Joe F. Chicharo
University of Wollongong, chicharo@uow.edu.au

Recommended Citation

Recovering the absolute phase maps of two fringe patterns with selected frequencies

Abstract

Phase unwrapping is an important and challenging issue in fringe pattern profilometry. In this Letter we propose an approach to recover absolute phase maps of two fringe patterns with selected frequencies. Compared to existing temporal multiple frequency algorithms, the two frequencies in our proposed algorithm can be high enough and thus enable efficient and accurate recovery of absolute phase maps. Experiment results are presented to confirm the effectiveness of the proposed technique.

Disciplines

Physical Sciences and Mathematics

Publication Details

Recovering the absolute phase maps of two fringe patterns with selected frequencies

Yi Ding,[1,2] Jiangtao Xi,[1,*] Yanguang Yu,[1] and Joe Chicharo[1]

[1]*School of Electrical, Computer and Telecommunications Engineering, University of Wollongong,
Northfields Avenue, Wollongong, New South Wales 2522, Australia*

[2]*Department of Electronic and Information Engineering, Huazhong University of Science and Technology,
1037 Luoyu Road, Wuhan, Hubei 430074, China*

Corresponding author: jiangtao@uow.edu.au

Phase unwrapping is an important and challenging issue in fringe pattern profilometry. In this Letter we propose an approach to recover absolute phase maps of two fringe patterns with selected frequencies. Compared to existing temporal multiple frequency algorithms, the two frequencies in our proposed algorithm can be high enough and thus enable efficient and accurate recovery of absolute phase maps. Experiment results are presented to confirm the effectiveness of the proposed technique.   © 2011 Optical Society of America

OCIS codes: 120.5050, 100.2650, 120.2830.

Fringe projection profilometry (FPP) is one of the most promising approaches for noncontact three-dimensional (3D) shape measurement. A problem associated with all existing phase measurement techniques in FPP is phase unwrapping, which aims to recover the absolute phase maps from those wrapped into the interval $(-\pi, \pi)$. Although various phase unwrapping methods have been proposed [1,2], their efficiency and accuracy remains an issue, particularly in situations where there is noise present or when sharp changes or discontinuities are present in object surfaces [3]. The accuracy of period coding phase unwrapping is highly dependent on the features of patterns themselves [4].

The problem of absolute phase recovery also exists in traditional interferometry. In an effort to overcome this problem, Wyant [5] proposed a method based on two-frequency interferometry. This method employed two interferometric patterns to generate an equivalent phase map with its frequency much lower (a wavelength longer) than each of the individual patterns. The equivalent phase map can be used as a reference to recover the absolute phase of the two original patterns [6,7].

To achieve reliable and accurate phase unwrapping for FPP, a class of approaches called temporal phase unwrapping was also proposed based on the utilization of multiple fringe patterns with different frequencies. In [8,9], two image patterns were employed, one of which had a very low spatial frequency with its absolute phase value falling within $(-\pi, \pi)$. This low frequency phase map was used as a reference to recover the absolute map of the other fringe pattern. However, the gap between the frequencies of the two image patterns must be smaller than a certain value. In order to unwrap a high frequency phase map, multiple image patterns with their frequencies filling the gaps should be added. Zhang [10,11] studied the selection of the multiple frequencies, showing that absolute phase maps can be recovered if the frequency increase between two adjacent patterns was 2 times. The same problem was studied by Saldner and Huntley [3,12], showing that, in order to unwrap a phase map of frequency f, $\log_2 f + 1$ fringe patterns are required. Hence, the task of reducing the number of image patterns for unwrapping a high frequency phase map in FPP remains an open challenge.

This Letter presents a novel temporal phase unwrapping technique that is able to recover the absolute phase maps of two fringe patterns with selected frequencies. Compared to the two-frequency techniques in [6,7], the proposed approach is able to directly recover the absolute phases of the two fringe patterns, without referring to or formulation of the equivalent phase map, thus leading to a much simpler implementation. The two frequencies in the proposed approach can be high enough to achieve the desired resolution for 3D shape measurement using FPP.

Let us consider a FPP system, with which two image patterns are projected onto the object surface. We employ normalized spatial frequencies f_1 and f_2 to describe the two patterns, which are positive integer numbers representing the total number of fringes on the respective patterns. The fringe on the frequency f_1 is $d_1(x, y)$; the fringe on the frequency f_2 is $d_2(x, y)$. The fringe patterns are characterized by vertical strips (i.e., in the y direction) whose intensity varies in a sinusoidal manner horizontally (i.e., in the x direction). The reflected images from the object surface can be obtained as follows:

$$\begin{cases} d_1(x,y) = A(x,y) + B(x,y)\cos[\Phi_1(x)] \\ d_2(x,y) = A(x,y) + B(x,y)\cos[\Phi_2(x)] \end{cases}, \qquad (1)$$

where (x, y) is the pixel number index in either the horizontal or the vertical direction, $A(x, y)$ is the background illumination, $B(x, y)$ is the projected fringe amplitude, and $\Phi_1(x)$ and $\Phi_2(x)$ are the absolute phase maps, which are required for accurate reconstruction of the 3D surface shape. The ranges of the two phase maps should be

$$-f_1\pi < \Phi_1(x) < f_1\pi, \qquad -f_2\pi < \Phi_2(x) < f_2\pi. \quad (2)$$

The wrapped phases $\phi_1(x)$ and $\phi_2(x)$ obtained by phase detection range from $-\pi$ to π. Retrieving the absolute phase maps $\Phi_1(x)$ or $\Phi_2(x)$ from the wrapped ones, $\phi_1(x)$ and $\phi_2(x)$, we have

$$\begin{cases} \Phi_1(x) = 2\pi m_1(x) + \phi_1(x) \\ \Phi_2(x) = 2\pi m_2(x) + \phi_2(x) \end{cases}. \qquad (3)$$

In order to determine the two integers $m_1(x)$ and $m_2(x)$, let us employ the following relationship [11]:

$$f_2\Phi_1(x) = f_1\Phi_2(x). \qquad (4)$$

Combining Eqs. (3) and (4) yields

$$\frac{f_2\phi_1(x) - f_1\phi_2(x)}{2\pi} = m_2(x)f_1 - m_1(x)f_2. \qquad (5)$$

Equation (5) reveals an interesting property that might be employed to determine $m_1(x)$ and $m_2(x)$: The right-hand side is an integer and so the left-hand side must also be the same integer. Because the left-hand side can be obtained from the wrapped phase maps, we should be able to determine these two integers based on the value of the left-hand side, if there exists a unique mapping between all the possible values of the right-hand side to $m_1(x)$ and $m_2(x)$. In order to explore such a possibility, we have the following analysis. As $-\pi < \phi_1(x)$ and $\phi_2(x) < \pi$, from Eqs. (2) and (3) we have

$$m_1(x) = \begin{cases} \lfloor f_1/2 \rfloor & [f_1 - (f_1 \bmod 2 + 1)]\pi \le \Phi_1(x) < f_1\pi \\ \cdots & \cdots \\ 1 & \pi \le \Phi_1(x) < 3\pi \\ 0 & -\pi < \Phi_1(x) < \pi \\ -1 & -3\pi < \Phi_1(x) \le -\pi \\ \cdots & \cdots \\ -\lfloor f_1/2 \rfloor & -f_1\pi < \Phi_1(x) \le -[f_1 - (f_1 \bmod 2 + 1)]\pi \end{cases}, \qquad (6)$$

$$m_2(x) = \begin{cases} \lfloor f_2/2 \rfloor & [f_2 - (f_2 \bmod 2 + 1)]\pi \le \Phi_2(x) < f_2\pi \\ \cdots & \cdots \\ 1 & \pi \le \Phi_2(x) < 3\pi \\ 0 & -\pi < \Phi_2(x) < \pi \\ -1 & -3\pi < \Phi_2(x) \le -\pi \\ \cdots & \cdots \\ -\lfloor f_2/2 \rfloor & -f_2\pi < \Phi_2(x) \le -[f_2 - (f_2 \bmod 2 + 1)]\pi \end{cases}, \qquad (7)$$

where $[x]$ denotes the largest integer not greater than x.

Since both $\Phi_1(x)$ and $\Phi_2(x)$ are not directly available, the above relationships cannot be used to determine $m_1(x)$ and $m_2(x)$. In order to work out a way to determine $m_1(x)$ and $m_2(x)$, let us assume that we have another image pattern $d_0(x, y)$ containing a single fringe, that is, the spatial frequency is $f_0 = 1$. In this case, the acquired phase $\phi_0(x)$ is the same as the absolute phase $\Phi_0(x)$ and phase unwrapping is not needed. Let us also assume that $\Phi_0(x)$ increases monotonically from $-\pi$ to π with respect to x. Taking $\Phi_0(x)$ as the reference, we have

$$\Phi_1(x) = f_1\Phi_0(x), \qquad \Phi_2(x) = f_2\Phi_0(x). \qquad (8)$$

Hence, Eqs. (6) and (7) can be rewritten in the form of Eqs. (9) and (10), respectively. It is evident that these two equations provide a unique mapping from $\Phi_0(x)$ to $m_1(x)$ and $m_2(x)$:

$$m_1(x) = \begin{cases} \lfloor f_1/2 \rfloor & [f_1 - (f_1 \bmod 2 + 1)]\pi \le f_1\Phi_0(x) < f_1\pi \\ \cdots & \cdots \\ 1 & \pi \le f_1\Phi_0(x) < 3\pi \\ 0 & -\pi < f_1\Phi_0(x) < \pi \\ -1 & -3\pi < f_1\Phi_0(x) \le -\pi \\ \cdots & \cdots \\ -\lfloor f_1/2 \rfloor & -f_1\pi < f_1\Phi_0(x) \le -[f_1 - (f_1 \bmod 2 + 1)]\pi \end{cases}, \qquad (9)$$

$$m_2(x) = \begin{cases} \lfloor f_2/2 \rfloor & [f_2 - (f_2 \bmod 2 + 1)]\pi \le f_2\Phi_0(x) < f_2\pi \\ \cdots & \cdots \\ 1 & \pi \le f_2\Phi_0(x) < 3\pi \\ 0 & -\pi < f_2\Phi_0(x) < \pi \\ -1 & -3\pi < f_2\Phi_0(x) \le -\pi \\ \cdots & \cdots \\ -\lfloor f_2/2 \rfloor & -f_2\pi < f_2\Phi_0(x) \le -[f_2 - (f_2 \bmod 2 + 1)]\pi \end{cases}. \qquad (10)$$

These two equations imply that, when $m_2(x)f_1 - m_1(x)f_2$ is given and if we are able to uniquely determine the range of $\Phi_0(x)$, we can use Eqs. (9) and (10) to determine $m_1(x)$ and $m_2(x)$. In order to confirm this idea, let us choose $f_1 = 5$ and $f_2 = 8$ as an example, and from Eqs. (9) and (10) we can derive the following relationship in Table 1.

It is seen that the first column in Table 1 covers the whole range $-\pi < \Phi_0(x) < \pi$, and the third column also gives all possible values of $m_2(x)f_1 - m_1(x)f_2$. As the elements of $m_2(x)f_1 - m_1(x)f_2$ are all different, we can give the corresponding relationship from $m_2(x)f_1 - m_1(x)f_2$ to $m_1(x)$ and $m_2(x)$ in Table 2 by rearranging Table 1.

From Table 2, we can see that the left half of the table is inversely symmetrical to the right half. That is, each of the entries on the left half has a mirror entry on the right half with an opposite sign. Therefore, only half of the entries are required. Similar relationships can be found

Table 1. Mapping from $\Phi_0(x)$ to $m_2(x)f_1 - m_1(x)f_2$

$\Phi_0(x)$	$m_1(x), m_2(x)$	$m_2(x)f_1 - m_1(x)f_2$
$7\pi/8 \le \Phi_0(x) < \pi$	2, 4	4
$5\pi/8 \le \Phi_0(x) < 7\pi/8$	2, 3	-1
$3\pi/5 \le \Phi_0(x) < 5\pi/8$	2, 2	-6
$3\pi/8 \le \Phi_0(x) < 3\pi/5$	1, 2	2
$\pi/5 \le \Phi_0(x) < 3\pi/8$	1, 1	-3
$\pi/8 \le \Phi_0(x) < \pi/5$	0, 1	5
$-\pi/8 < \Phi_0(x) < \pi/8$	0, 0	0
$-\pi/5 \le \Phi_0(x) \le -\pi/8$	0, -1	-5
$-3\pi/8 < \Phi_0(x) \le -\pi/5$	-1, -1	3
$-3\pi/5 < \Phi_0(x) \le -3\pi/8$	-1, -2	-2
$-5\pi/8 < \Phi_0(x) \le -3\pi/5$	-2, -2	6
$-7\pi/8 < \Phi_0(x) \le -5\pi/8$	-2, -3	1
$-\pi < \Phi_0(x) \le -7\pi/8$	-2, -4	-4

Table 2.   Mapping from $m_2(x)f_1 - m_1(x)f_2$ to $m_1(x)$, $m_2(x)$

$m_2(x)f_1 - m_1(x)f_2$	$m_1(x)$, $m_2(x)$	$m_2(x)f_1 - m_1(x)f_2$	$m_1(x)$, $m_2(x)$
6	−2, −2	−1	2, 3
5	0, 1	−2	−1, −2
4	2, 4	−3	1, 1
3	−1, −1	−4	−2, −4
2	1, 2	−5	0, −1
1	−2, −3	−6	2, 2
0	0, 0	—	—

Table 3.   Mapping from $m_2(x)f_1 - m_1(x)f_2$ to $m_1(x)$, $m_2(x)$

$m_2(x)f_1 - m_1(x)f_2$	$m_1(x)$, $m_2(x)$	$m_2(x)f_1 - m_1(x)f_2$	$m_1(x)$, $m_2(x)$
14	2, 4	6	−6, −8
13	−5, −6	5	−1, −1
12	0, 1	4	4, 6
11	5, 8	3	−3, −4
10	−2, −2	2	2, 3
9	3, 5	1	−5, −7
8	−4, −5	0	0, 0
7	1, 2	—	—

for other frequency pairs, such as $f_1 = 12$ and $f_2 = 17$; results are given in Table 3.

With the above results we can reconstruct the absolute phase maps of two fringe patterns by the following steps.

1.   Select two frequencies (f_1, f_2) and construct a table similar to Table 2, making sure the table provides a unique mapping from $m_2(x)f_1 - m_1(x)f_2$ to $m_1(x)$ and $m_2(x)$.

2.   Project two fringe patterns onto the object and acquire the two phase maps $\phi_1(x)$ and $\phi_2(x)$ by a phase detection algorithm.

3.   Calculate $[f_2\phi_1(x) - f_1\phi_2(x)]/2\pi$ by rounding its value to the closest integer, denoted as M. Using the lookup table derived in Step 1, find the row (or entry) whose value of $m_2(x)f_1 - m_1(x)f_2$ is the closest to M. Record the corresponding $m_1(x)$ and $m_2(x)$ in the same row.

4.   Using $m_1(x)$ and $m_2(x)$ obtained in Step 3, reconstruct the absolute phase maps by Eq. (3).

Experiments are carried out to verify the proposed approach. We project two fringe patterns with frequencies 5 and 8 onto a plaster hand model object, as depicted in Figs. 1(a) and 1(b). The resolution of these images is 1392×1038. The wrapped phase maps of the two fringes are shown in Figs. 1(c) and 1(d). Using the proposed approach, we successfully recovered the absolute phase maps of the two fringes in Figs. 1(e) and 1(f).

The proposed approach is also valid for many other frequency pairs, such as (130, 9), (100, 9), (72, 25), and (32, 45). Note that the gaps between the two adjacent entries of $m_2(x)f_1 - m_1(x)f_2$ in the table (i.e., Table 3)
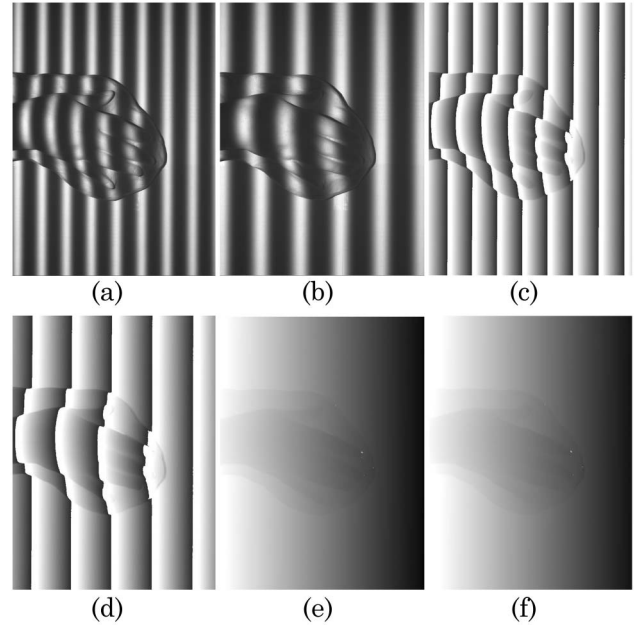

Fig. 1.   (a) Fringe patterns on the object at $f_2 = 8$. (b) Fringe pattern on the object at $f_1 = 5$. (c) Wrapped phase map on the object at $f_2 = 8$. (d) Wrapped phase map at $f_1 = 5$. (e) Recovered absolute phase at $f_2 = 8$. (f) Recovered absolute phase at $f_1 = 5$.

determine the antinoise capability of the proposed technique. The larger the gaps, the more reliable the proposed approach. The relationship between frequency selection and the noise performance will be studied in our future work.

In summary, we have proposed a new approach to recover absolute phase maps with only two fringe patterns. Essentially, we use Eq. (3) in concert with a lookup table to determine the absolute phase maps. This operation is obviously simpler than the approaches proposed in [6,7], making it suitable for time critical 3D object acquisition applications.

References
1. S. Zhang, X. Li, and S. T. Yau, Appl. Opt. **46**, 50 (2007).
2. S. Li, W. Chen, and X. Su, Appl. Opt. **47**, 3369 (2008).
3. J. M. Huntley and H. O. Saldner, J. Opt. Soc. Am. A **14**, 3188 (1997).
4. K. Liu, Y. Wang, D. L. Lau, Q. Hao, and L. G. Hassebrook, Opt. Express **18**, 5229 (2010).
5. J. C. Wyant, Appl. Opt. **10**, 2113 (1971).
6. P. J. de Groot, Appl. Opt. **33**, 5948 (1994).
7. K. Houairi and F. Cassaing, J. Opt. Soc. Am. A **26**, 2503 (2009).
8. H. Zhao, W. Chen, and Y. Tan, Appl. Opt. **33**, 4497 (1994).
9. J. Li, L. G. Hassebrook, and C. Guan, J. Opt. Soc. Am. A **20**, 106 (2003).
10. S. Zhang, Proc. SPIE **7432**, 74320N (2009).
11. S. Zhang, Opt. Eng. **48**, 105601 (2009).
12. H. O. Saldner and J. M. Huntley, Appl. Opt. **36**, 2770 (1997).