Recursive Diffusion Layers for Block Ciphers and Hash Functions

Mahdi Sajadieh, Mohammad Dakhilalian, Hamid Mala and
<u>Pouyan Sepehrdad</u>

Isfahan University of Technology, Isfahan, Iran
Isfahan University, Isfahan, Iran
EPFL, Lausanne, Switzerland

Happy Persian New Year!

Outline

- Lightweight Algorithms and Diffusion Layers
- Designing A Recursive Diffusion Layer
- Designing A Diffusion Layer with One Linear Function
- Designing A Diffusion Layer with Two Linear Functions
- Conclusion

Lightweight Algorithms and Diffusion Layers

- Most block ciphers: A round consists of confusion and diffusion layers.
- The confusion layer: often uses small S-boxes.
- The diffusion layer: plays an efficacious role in providing resistance against DC and LC.
- Diffusion layers must
  - have large branch numbers.
  - be efficient, both the layer and its reverse.

Lightweight Algorithms and Diffusion Layers

- Lightweight block ciphers:
  - New ciphers appear everyday in the literature.
  - Compete over the throughput and GE.
  - Not providing the same level of security (LC, DC, AC).
  - Does the comparison make sense?

- Designing lightweight and efficient diffusion layers:
  - Efficient and perfect recursive Feistel-like diffusion layers.
  - We design one without any finite fields operations.
  - Only have “XOR”s and “Shift” or “Rotations”.
  - LED and PHOTON: a nice and efficient MDS diffusion layer.

Designing A Recursive Diffusion Layer

$$D : \begin{cases} y_0 = x_0 \oplus F_0(x_1, x_2, \ldots, x_{s-1}) \\ y_1 = x_1 \oplus F_1(x_2, x_3, \ldots, x_{s-1}, y_0) \\ \vdots \\ y_{s-1} = x_{s-1} \oplus F_{s-1}(y_0, y_1, \ldots, y_{s-2}) \end{cases}$$

- Maximal branch number
- Length of the input words be changeable
- Have a very simple inverse
- An efficient linear functions F.

Designing A Recursive Diffusion Layer

$$D : \begin{cases} y_0 = x_0 \oplus F_0(x_1, x_2, \ldots, x_{s-1}) \\ y_1 = x_1 \oplus F_1(x_2, x_3, \ldots, x_{s-1}, y_0) \\ \vdots \\ y_{s-1} = x_{s-1} \oplus F_{s-1}(y_0, y_1, \ldots, y_{s-2}) \end{cases}$$

$$D^{-1} : \begin{cases} x_{s-1} = y_{s-1} \oplus F_{s-1}(y_0, y_1, \ldots, y_{s-2}) \\ x_{s-2} = y_{s-2} \oplus F_{s-2}(x_{s-1}, y_0, \ldots, y_{s-3}) \\ \vdots \\ x_0 = y_0 \oplus F_0(x_1, x_2, \ldots, x_{s-1}) \end{cases}$$

Some Instances of Recursive Diffusion Layers

- Feistel with a linear F

$$D : \begin{cases} y_0 = x_0 \oplus L(x_1) \\ y_1 = x_1 \oplus L(y_0) \end{cases}$$

- Salsa20 (non linear)

$$D : \begin{cases} y_1 = x_1 \oplus ((x_0 + x_3) <<< 7) \\ y_2 = x_2 \oplus ((x_0 + y_1) <<< 9) \\ y_3 = x_3 \oplus ((y_1 + y_2) <<< 13) \\ y_0 = x_0 \oplus ((y_2 + y_3) <<< 18) \end{cases}$$

- PHOTON matrix

$$\mathbf{B} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 4 \end{pmatrix} \Rightarrow \mathbf{B}^4 = \begin{pmatrix} 1 & 2 & 1 & 4 \\ 4 & 9 & 6 & 17 \\ 17 & 38 & 24 & 66 \\ 66 & 149 & 100 & 11 \end{pmatrix}$$

The Proposed Regular s n-bit Words Diffusion Layer

Input : s n-bit words $x_0, \ldots, x_{s-1}$
Output : s n-bit words $y_0, \ldots, x_{s-1}$
for $i = 0$ to $s - 1$ **do**
  $y_i = x_i$
end for
for $i = 0$ to $s - 1$ **do**
  $y_i = \bigoplus_{j=0}^{s-1} \alpha_{[(j-i) \mod s]} y_j \oplus L\left(\bigoplus_{j=0}^{s-1} \beta_{[(j-i) \mod s]} y_j\right)$
end for

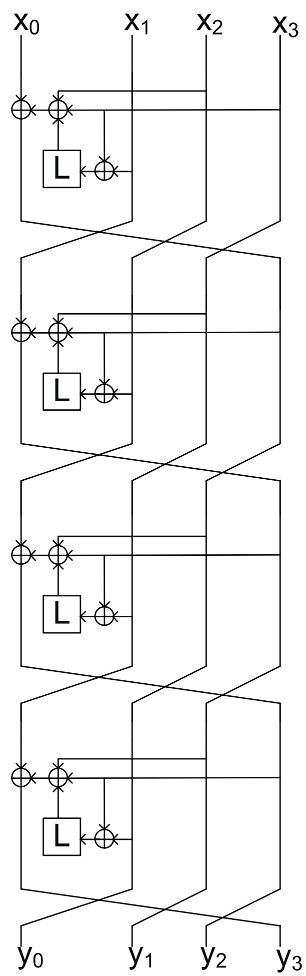
- In the main pseudo code: $F_i(x_1, x_2, \ldots, x_{s-1}) = F_0(x_1, x_2, \ldots, x_{s-1})$

$$F_i(x_1, x_2, \ldots, x_{s-1}) = \bigoplus_{j=1}^{s-1} \alpha_j x_j \oplus L\left(\bigoplus_{j=1}^{s-1} \beta_j x_j\right)$$

A Regular 4 × 4 In/Out Diffusion Layer with Perfect Diffusion

$$D:\begin{cases} y_0 = x_0 \oplus x_2 \oplus x_3 \oplus L(x_1 \oplus x_3) \\ y_1 = x_1 \oplus x_3 \oplus y_0 \oplus L(x_2 \oplus y_0) \\ y_2 = x_2 \oplus y_0 \oplus y_1 \oplus L(x_3 \oplus y_1) \\ y_3 = x_3 \oplus y_1 \oplus y_2 \oplus L(y_0 \oplus y_2) \end{cases} \quad \Rightarrow \quad D^{-1}:\begin{cases} x_3 = y_3 \oplus y_2 \oplus y_1 \oplus L(y_0 \oplus y_2) \\ x_2 = y_2 \oplus y_1 \oplus y_0 \oplus L(x_3 \oplus y_1) \\ x_1 = y_1 \oplus y_0 \oplus x_3 \oplus L(x_2 \oplus y_0) \\ x_0 = y_0 \oplus x_3 \oplus x_2 \oplus L(x_1 \oplus x_3) \end{cases}$$

A Regular 4 × 4 In/Out Diffusion Layer with Perfect Diffusion



Conditions on L: Maximal Branch Number

- Outputs based on inputs

$$D : \begin{cases} y_0 = x_0 \oplus L(x_1) \oplus x_2 \oplus x_3 \oplus L(x_3) \\ y_1 = x_0 \oplus L(x_0) \oplus x_1 \oplus L(x_1) \oplus L^2(x_1) \oplus x_2 \oplus L^2(x_3) \\ y_2 = L^2(x_0) \oplus x_1 \oplus L(x_1) \oplus L^3(x_1) \oplus x_2 \oplus L(x_2) \oplus x_3 \oplus L^2(x_3) \oplus L^3(x_3) \\ y_3 = x_0 \oplus L^2(x_0) \oplus L^3(x_0) \oplus L(x_1) \oplus L^2(x_1) \oplus L^3(x_1) \oplus L^4(x_1) \\ \qquad \oplus L(x_2) \oplus L^2(x_2) \oplus L^2(x_3) \oplus L^4(x_3) \end{cases}$$

- The linear functions must be invertible for maximal branch number:

$$\begin{cases} L(x) \\ x \oplus L(x) \\ x \oplus L^3(x) \\ x \oplus L^7(x) \end{cases}$$

$L(x) = (x \oplus (x \& 2$ [illegible]

Some Linear Functions

- Large number of linear functions satisfying the conditions, some are:

word size	Some linear functions L
4	$L(x) = (x \oplus x \lll 3) \oplus 1$
8	$L(x) = (x \oplus (x \& 0x2) \lll 1) \oplus 1$
16	$L(x) = (x \oplus x \lll 15) \oplus 1$
32	$L(x) = (x \oplus x \lll 31) \oplus 15$ or $L(x) = (x \oplus 24) \oplus (x \& 0xFF)$
64	$L(x) = (x \oplus x \lll 63) \oplus 1$ or $L(x) = (x \oplus 8) \oplus (x \& 0xFFFF)$

- Without any circular shift:

word size	Sample linear functions L
32	$L(x) = (x \ll 3) \oplus (x \gg 1)$
64	$L(x) = (x \ll 15) \oplus (x \gg 1)$

Replacement of Some Diffusion Layers

- MDS_H of Hierocrypt
  - Performance two times better
- Binary matrix of MMB
  - Branch number of the MMB diffusion layer increases to 5.
    - prevents the attacks [SAC'09] presented on this block cipher.
  - Performance is decreased by 10%.
- PHOTON
  - If $L(x)=2x$ in $GF(2^4)$, the matrix is:

$$B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 3 \end{pmatrix} \Rightarrow B^4 = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 3 & 7 & 1 & 4 \\ 4 & 11 & 3 & 13 \\ 13 & 30 & 6 & 20 \end{pmatrix}$$

All Other Regular Diffusion Layers

For $s > 4$, no diffusion layer was found with only one linear function.

All Other Regular Diffusion Layers

s	Diffusion Layer	Function that must be invertible
2	$D: \begin{cases} y_0 = x_0 \oplus L(x_1) \\ y_1 = x_1 \oplus L(y_0) \end{cases}$	$L(x)$ and $x \oplus L(x)$
3	$D: \begin{cases} y_0 = x_0 \oplus L(x_1 \oplus x_2) \\ y_1 = x_1 \oplus L(x_2 \oplus y_0) \\ y_2 = x_2 \oplus L(y_0 \oplus y_1) \end{cases}$	$L(x)$, $x \oplus L(x)$ and $x \oplus L^3(x)$
3	$D: \begin{cases} y_0 = x_0 \oplus x_1 \oplus L(x_1 \oplus x_2) \\ y_1 = x_1 \oplus x_2 \oplus L(x_2 \oplus y_0) \\ y_2 = x_2 \oplus y_0 \oplus L(y_0 \oplus y_1) \end{cases}$	$L(x)$, $x \oplus L(x)$, $x \oplus L^3(x)$ and $x \oplus L^7(x)$
3	$D: \begin{cases} y_0 = x_0 \oplus x_2 \oplus L(x_1 \oplus x_2) \\ y_1 = x_1 \oplus y_0 \oplus L(x_2 \oplus y_0) \\ y_2 = x_2 \oplus y_1 \oplus L(y_0 \oplus y_1) \end{cases}$	$L(x)$, $x \oplus L(x)$, $x \oplus L^3(x)$ and $x \oplus L^7(x)$
3	$D: \begin{cases} y_0 = x_0 \oplus x_1 \oplus x_2 \oplus L(x_1 \oplus x_2) \\ y_1 = x_1 \oplus x_2 \oplus y_0 \oplus L(x_2 \oplus y_0) \\ y_2 = x_2 \oplus y_0 \oplus y_1 \oplus L(y_0 \oplus y_1) \end{cases}$	$L(x)$, $x \oplus L(x)$, and $x \oplus L^3(x)$
4	$D: \begin{cases} y_0 = x_0 \oplus x_2 \oplus x_3 \oplus L(x_1 \oplus x_3) \\ y_1 = x_1 \oplus x_3 \oplus y_0 \oplus L(x_2 \oplus y_0) \\ y_2 = x_2 \oplus y_0 \oplus y_1 \oplus L(x_3 \oplus y_1) \\ y_3 = x_3 \oplus y_1 \oplus y_2 \oplus L(y_0 \oplus y_2) \end{cases}$	$L(x)$, $x \oplus L(x)$, $x \oplus L^3(x)$ and $x \oplus L^7(x)$
4	$D: \begin{cases} y_0 = x_0 \oplus x_1 \oplus x_2 \oplus L(x_1 \oplus x_3) \\ y_1 = x_1 \oplus x_2 \oplus x_3 \oplus L(x_2 \oplus y_0) \\ y_2 = x_2 \oplus x_3 \oplus y_0 \oplus L(x_3 \oplus y_1) \\ y_3 = x_3 \oplus y_0 \oplus y_1 \oplus L(y_0 \oplus y_2) \end{cases}$	$L(x)$, $x \oplus L(x)$, $x \oplus L^3(x)$ and $x \oplus L^7(x)$
4	$D: \begin{cases} y_0 = x_0 \oplus x_2 \oplus L(x_1 \oplus x_2 \oplus x_3) \\ y_1 = x_1 \oplus x_3 \oplus L(x_2 \oplus x_3 \oplus y_0) \\ y_2 = x_2 \oplus y_0 \oplus L(x_3 \oplus y_0 \oplus y_1) \\ y_3 = x_3 \oplus y_1 \oplus L(y_0 \oplus y_1 \oplus y_2) \end{cases}$	$L(x)$, $x \oplus L(x)$, $x \oplus L^3(x)$, $x \oplus L^7(x)$ and $x \oplus L^{15}(x)$
4	$D: \begin{cases} y_0 = x_0 \oplus x_1 \oplus x_3 \oplus L(x_1 \oplus x_2 \oplus x_3) \\ y_1 = x_1 \oplus x_2 \oplus y_0 \oplus L(x_2 \oplus x_3 \oplus y_0) \\ y_2 = x_2 \oplus x_3 \oplus \oplus y_1 \oplus L(x_3 \oplus y_0 \oplus y_1) \\ y_3 = x_3 \oplus y_0 \oplus y_2 \oplus L(y_0 \oplus y_1 \oplus y_2) \end{cases}$	$L(x)$, $x \oplus L(x)$, $x \oplus L^3(x)$, $x \oplus L^7(x)$ and $x \oplus L^{15}(x)$

Non-regular Recursive Diffusion Layers

- In the non-regular diffusion layers: F_i's are different.
  - Use only one linear function.

Input : s n-bit words $x_0, \ldots, x_{s-1}$
Output : s n-bit words $y_0, \ldots, x_{s-1}$
for $i = 0$ to $s-1$ **do**
    $y_i = x_i$
end for
for $i = 0$ to $s-1$ **do**
    $$y_i = y_i \oplus \left(\bigoplus_{j=0, j \neq i}^{s-1} A_{i,j} y_j \right) \oplus L \left(\bigoplus_{j=0, j \neq i}^{s-1} B_{i,j} y_j \right)$$
end for

- The space for a complete search is 2^{2s^2} for an s input/output diffusion layer.

Non-regular Recursive Diffusion Layers

- After a complete search
  - For s = 3, the one with the least number of XORs:

$$\mathbf{D}:\begin{cases} y_0 = x_0 \oplus x_1 \oplus x_2 \\ y_1 = x_1 \oplus x_2 \oplus L(y_0 \oplus x_2) \\ y_2 = x_2 \oplus y_0 \oplus y_1 \end{cases}$$

  - For s = 4, the one with the least number of XORs:

$$\mathbf{D}:\begin{cases} y_0 = x_0 \oplus x_1 \oplus x_2 \oplus L(x_3) \\ y_1 = x_1 \oplus x_3 \oplus y_0 \oplus L(x_2 \oplus y_0) \\ y_2 = x_2 \oplus x_3 \oplus y_0 \oplus L(x_3 \oplus y_1) \\ y_3 = x_3 \oplus y_1 \oplus y_2 \oplus L(y_0) \end{cases}$$

- For s > 4, a complete search is too costly.

Regular Recursive Diffusion Layers with Two Linear Functions

1: Input : s n-bit words $x_0, \ldots, x_{s-1}$
2: Output : s n-bit words $y_0, \ldots, x_{s-1}$
3: **for** $i = 0$ to $s-1$ **do**
4: $\quad y_i = x_i$
5: **end for**
6: **for** $i = 0$ to $s-1$ **do**
7: $$y_i = \bigoplus_{j=0}^{s-1} \alpha_{[(j-i) \mod s]} y_j \oplus L_1\left(\bigoplus_{j=0}^{s-1} \beta_{[(j-i) \mod s]} y_j\right) \oplus L_2\left(\bigoplus_{j=0}^{s-1} \gamma_{[(j-i) \mod s]} y_j\right)$$
8: **end for**

- If L_1 and L_2 do not have any relation, the analysis is hard.

$$L_2(x) = L_1^2(x) \quad \text{or} \quad L_2(x) = L_1^{-1}(x)$$

Regular Diffusion Layer for s > 4

s	y_0 in a perfect diffusion Layer
5	$y_0 = x_0 \oplus x_2 \oplus x_3 \oplus L(x_4) \oplus L^2(x_1)$
5	$y_0 = L^{-1}(x_4) \oplus x_0 \oplus x_2 \oplus L(x_1 \oplus x_3 \oplus x_4)$
6	$y_0 = x_0 \oplus x_5 \oplus L(x_3 \oplus x_5) \oplus L^2(x_1 \oplus x_2 \oplus x_4)$
6	$y_0 = L^{-1}(x_2 \oplus x_5) \oplus x_0 \oplus x_3 \oplus L(x_1 \oplus x_3 \oplus x_4 \oplus x_5)$
7	$y_0 = x_0 \oplus x_2 \oplus x_4 \oplus L(x_3) \oplus L^2(x_1 \oplus x_2 \oplus x_4 \oplus x_5 \oplus x_6)$
7	$y_0 = L^{-1}(x_1 \oplus x_3 \oplus x_6) \oplus x_0 \oplus x_6 \oplus L(x_1 \oplus x_2 \oplus x_4 \oplus x_5)$
8	$y_0 = x_0 \oplus x_1 \oplus x_3 \oplus x_4 \oplus L(x_2 \oplus x_3 \oplus x_5) \oplus L^2(x_1 \oplus x_5 \oplus x_6 \oplus x_7)$
8	$y_0 = L^{-1}(x_3 \oplus x_4 \oplus x_7) \oplus x_0 \oplus x_1 \oplus x_2 \oplus x_4 \oplus L(x_1 \oplus x_5 \oplus x_6 \oplus x_7)$

- The linear functions which must be invertible:

$$
\begin{array}{lll}
L(x) & I \oplus L(x) & I \oplus L^3(x) \\
I \oplus L^7(x) & I \oplus L^{15}(x) & I \oplus L^{31}(x) \\
I \oplus L^{63}(x) & I \oplus L^{127}(x) & I \oplus L^{255}(x) \\
I \oplus L^{511}(x) & I \oplus L^{1023}(x) & I \oplus L^{2047}
\end{array}
$$

Conclusion

- We introduced some efficient and perfect recursive diffusion layers.
- Found all regular *s* input/output recursive diffusion layers with $s < 8$ and one linear function.
- Found all non-regular *s* input/output recursive diffusion layers with
- $s < 5$ and one linear function.
- Found some efficient regular *s* input/output recursive diffusion layers with $s < 9$ and two linear function.
- A good candidate for designing new block ciphers or hash functions.


Questions?