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RED GIANTS IN OLD OPEN CLUSTERS

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SUMMARY

A detailed comparison is made between the predictions of stellar evolution calculations and the observations of red giants in old open clusters. Colour–magnitude diagrams are presented for several such clusters, in which relative proper motions have been used to eliminate non-members. The published observations of other old clusters are summarized.

A prominent feature of all these clusters is a ' clump ' of red giants, centred near $M_v = +1$, $B-V = +1{\cdot}0$, which is tentatively identified with the horizontal branch found in globular clusters. Comparison with various theoretical calculations, mostly by I. Iben and J. Faulkner, supports this identification and indicates that the stars have a helium-burning core surrounded by a hydrogen-burning shell.

The near constancy of the absolute magnitude of the clump is then interpreted as observational evidence for the occurrence of electron degeneracy in the cores of low mass red giants. The use of the clump for crude determinations of cluster distances and reddening, and as evidence of the internal composition of the stars, is advocated. Finally, it is suggested that a large number of field red giants may be in the helium-burning phase of evolution, some of which may be used to investigate mass loss and internal mixing in evolving stars.

1. INTRODUCTION

Star clusters are widely used in observational checks on theories of stellar evolution, since they are samples of stars with similar properties. It appears that the members of a cluster are distinguished principally by their different masses, while such parameters as initial composition and age vary little from star to star. Up to now, attention has largely been focused on the globular clusters since they give the largest samples of stars, and are reasonably free from field star interference. Consequently a large proportion of the published evolutionary calculations have been for the low mass, low metal abundance giant stars in such clusters.

In contrast to the globular clusters, there are few open clusters with even as many as ten red giants, and most are seriously contaminated by field stars. In spite of these difficulties, several studies have been made recently in which the observations of open clusters of all ages have been compared with Population I evolutionary models, principally those of Iben (see Iben 1967c). To try to overcome the problem of small numbers, the authors have either combined the observations for large numbers of clusters (Barbaro, Dallaporta & Nobili 1967; Lindoff 1968b, c; Barbaro Dallaporta & Fabris 1969), or used a few relatively rich and well-observed clusters (Schlesinger 1969; Sandage & Eggen 1969). However, all of these studies have been affected by uncertainties in cluster membership, and the need for proper motion studies of the older clusters in particular was emphasized by Schlesinger (1969). These older clusters are generally the richest clusters, and consequently offer the best data for detailed studies of red giant evolution.

In an attempt to extend the useful observational data from the globular clusters to the higher mass stars of 'normal' composition in younger clusters, relative proper motions have been determined by the author and co-workers in several old open clusters, and used to obtain nearly pure samples of cluster members. In the present paper, these data will be used as the basis for a discussion of the red giant branches of old open clusters, and for a detailed comparison with the theoretical evolution of Population I stars with masses between about 1 $M_\odot$ (one solar mass) and 3 $M_\odot$.

2. CLUSTER AGES

The age of an open cluster can be estimated from the colour–magnitude (C–M) diagram in various ways; probably the simplest is to use the position of the main sequence turn-off. Throughout this paper, the unreddened colour of this turn-off, i.e. of the bluest main sequence stars, $(B-V)_{o,t}$, will be used as a crude age parameter, since it can be readily determined for the majority of open clusters. From a theoretical point of view, the absolute magnitude of the turn-off is probably a more reliable age criterion, but it can only be determined for a few of the best observed clusters, and then only if the appropriate zero-age main sequence is known. The special technique developed by Lindoff (1968b) is not very accurate for the old rich clusters.

Since the turn-off colour is also a function of chemical composition, the ages determined from it will only be rough estimates. In fact, a serious difficulty arises for the very old clusters, since stars with low metal abundance have less line blanketing than do normal stars, and therefore appear bluer in $B-V$. This effect means that the relation between turn-off colour and age is not single-valued. Thus the observed turn-off colours for the globular clusters are bluer than those of some much younger open clusters. Arp (1962) deduced that his sample of intermediate-age clusters had slightly low abundances relative to 'nearby stars', while Spinrad & Taylor (1969) have recently suggested that the oldest open clusters are super metal rich. However, Eggen & Sandage (1969) found several old clusters to have very nearly solar abundances, with no evidence for a systematic trend with age, or for individual abundances differing from the solar value by more than a factor of two. Therefore, the observed turn-off colour should still be adequate for a crude classification of star clusters, and for the present purposes all the open clusters will be assumed to have about the same metal abundance. Detailed discussions of the whole problem of open cluster abundances, and the relation to main sequence model stars, have been given recently by Demarque (1968) and by Sandage & Eggen (1969).

An open cluster will be called 'old' if its turn-off colour is redder than about 0·4 in $B-V$, and 'intermediate-age' if $0{\cdot}0<(B-V)_{o,t}<0{\cdot}4$. An extensive series of Population I evolutionary calculations has been published by Iben (see Iben 1967c, for a summary and further references). On the basis of these models, intermediate-age clusters have ages between $3{\cdot}10^8$ and $3{\cdot}10^9$ years and red giants of masses between about 2·5 $M_\odot$ and 1·5 $M_\odot$, while old clusters are from $3{\cdot}10^9$ to 10^{10} years old, and have red giant masses in the range 1·5 $M_\odot$ to 1 $M_\odot$.

The ages of individual clusters are estimated as follows. The best fitting line is drawn through the observed main sequence stars in a C–M diagram; this assumes that the width of the observed main sequence is due mainly to observational scatter, and that the intrinsic width is small, as it is found to be in the few photoelectrically observed clusters. A correction for interstellar reddening is applied, generally found from the two-colour diagram, assuming that the abundances of the stars are not too different from that of Hyades stars. Variations in chemical abundance between the open clusters seem unlikely to lead to errors as large as $0^{m}\cdot1$ in such reddening estimates; Sandage & Eggen (1969) applied blanketing corrections of only $0^{m}\cdot03$ to the oldest open clusters. The age versus $(B-V)_{o,t}$ relation is calibrated using Iben's models. The forms of the isochrones (equal age loci) in the C–M diagram are found by interpolation between the published evolutionary tracks; the somewhat complicated procedure has been described elsewhere (Cannon 1968) and yields results very close to those found by Sandage & Eggen (1969). The Johnson (1966) bolometric corrections (b.c.'s) and calibration of the surface temperature scale are used throughout.

3. THE SELECTION OF CLUSTERS

An extensive search of the literature has been made, in order to find those clusters which appear to have $(B-V)_{o,t} \gtrsim 0\cdot0$ mag., or ages $\gtrsim 3\cdot10^{8}$ years. There is a tendency for these clusters to include the richest (i.e. most populous) open clusters, presumably on account of their greater dynamical stability and also, perhaps, their detectability. The delineation of the principal features of the C–M diagrams of these clusters is therefore in many cases straightforward. However, in order to study the details of the C–M diagrams, some technique is necessary to eliminate field stars. This elimination can be done using relative proper motions, which have recently been determined for several intermediate-age and old clusters, by the author and others, at the Royal Greenwich Observatory.

The clusters thus investigated are used as a basis in discussing the schematics of the C–M diagrams for clusters of various ages. Full details of the methods and results are being published elsewhere; here only a summary will be given. For most of the clusters, proper motions were determined using plates taken on the 60″ reflector at Mount Wilson, and for several, photographic photometry was obtained using the 26″ refractor at the Royal Greenwich Observatory.

4. THE INTERMEDIATE-AGE CLUSTERS

Three clusters of this type, NGC 752, NGC 2158 and NGC 7789, were compared by Arp (1962). Proper motions were determined for NGC 752 by Ebbighausen (1939), photoelectric photometry by Johnson (1953) and by Eggen (1963), and photographic photometry by Rohlfs & Vanysek (1961). New proper motions and photographic photometry have been determined by the author (1968), extending the limiting magnitude to $B = 17$. The resultant C–M diagram is shown in Fig. 1, where the photometry used is a combination from the sources quoted above. Filled circles represent stars found to be members in the author's proper motion study. Open circles represent additional stars brighter than $B = 12$ which were found to be probable members (Classes 1, 2 and 3) by Ebbighausen (1939). Very good agreement was found for those stars in common to the two proper motion studies. The filled circles represent a complete sample of probable members down to about $B = 17$ within an area of about $\frac{1}{2}°$ by $\frac{1}{2}°$, while the open circles should be complete to about $B = 12$ within a circle of diameter 50′. Eggen & Sandage (1964) find a reddening $E(B-V) = 0\cdot07$ in NGC 752.

The appearance of main sequence stars below $V = 12$, contrary to the preliminary finding of Eggen (1963), enables a more reliable estimate of the cluster

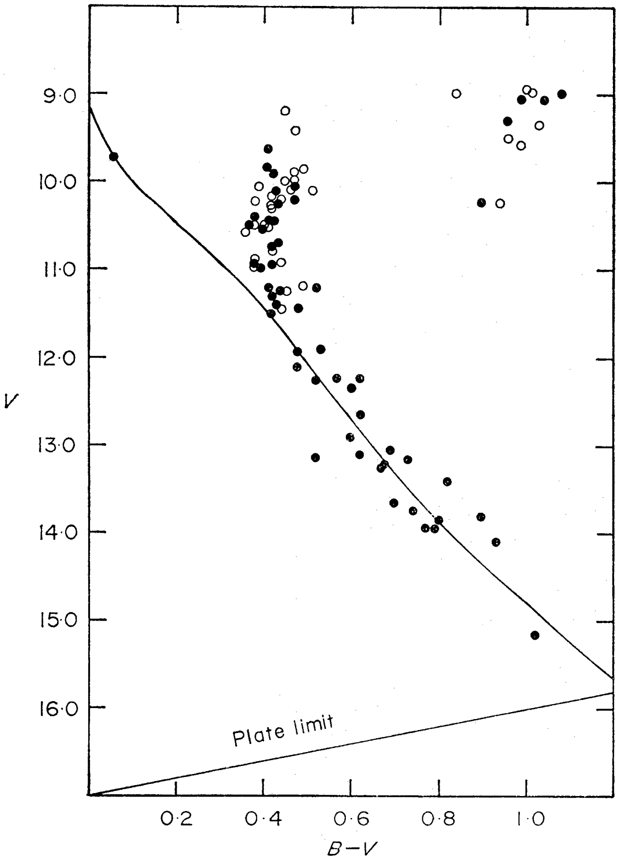

FIG. 1. *A C–M diagram of NGC 752. Filled circles represent a complete sample of cluster members to magnitude B = 17, while open circles represent bright members over a larger area. See text for full explanation and sources of photometry.*

distance to be established, although the resulting true modulus $(m\text{–}M)_0$ of $7^{\mathrm{m}}{\cdot}9 \pm 0^{\mathrm{m}}{\cdot}1$ (applying the same blanketing correction as Eggen & Sandage (1964)) is very close to the figure of $7^{\mathrm{m}}{\cdot}75$ originally adopted by Eggen. It is interesting to note the virtual absence of cluster members between $B = 15$ and the proper motion plate limit at $B = 17$. In such an old, sparse, cluster this may either be evidence for the preferential dynamical escape of low mass stars, or it may be that such stars were never formed.

Proper motions have also been measured for NGC 7789, another of Arp's original group of clusters, and for the similar cluster NGC 6939. Both of these clusters are very much richer than NGC 752, but are also more distant so that the relative proper motions provide a less sensitive criterion of membership. Fig. 2 is a C–M diagram for the central region of NGC 7789, using photometry by Burbidge & Sandage (1958), where preliminary proper motion results by the author have been used to eliminate some certain field stars. Burbidge and Sandage found a reddening of $E(B\text{–}V) = 0{\cdot}28$ in this cluster. Fig. 3 shows the C–M diagram for NGC 6939, using the photographic photometry by Cannon & Lloyd (1969) in conjunction with the relative proper motions by Cannon & Purcell (1970). Filled circles represent an almost pure sample of cluster members. Open circles are fainter stars whose proper motions were not determined; about 25 per cent of such stars in the neighbourhood of the main sequence, and all of the faint redder stars, are estimated to be field stars

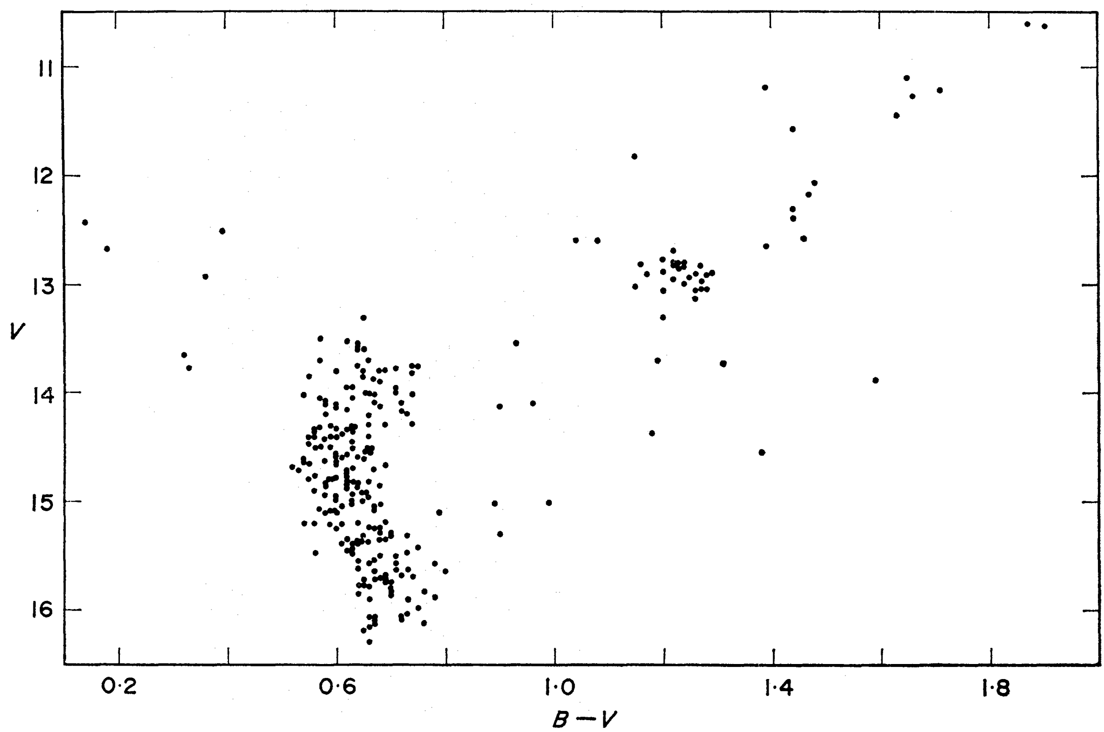

FIG. 2. *A C–M diagram for NGC* 7789, *using photometry by Burbidge & Sandage* (1958). *Those stars are plotted which lie within* 5·5′ *of the cluster centre and have proper motions compatible with cluster membership.*

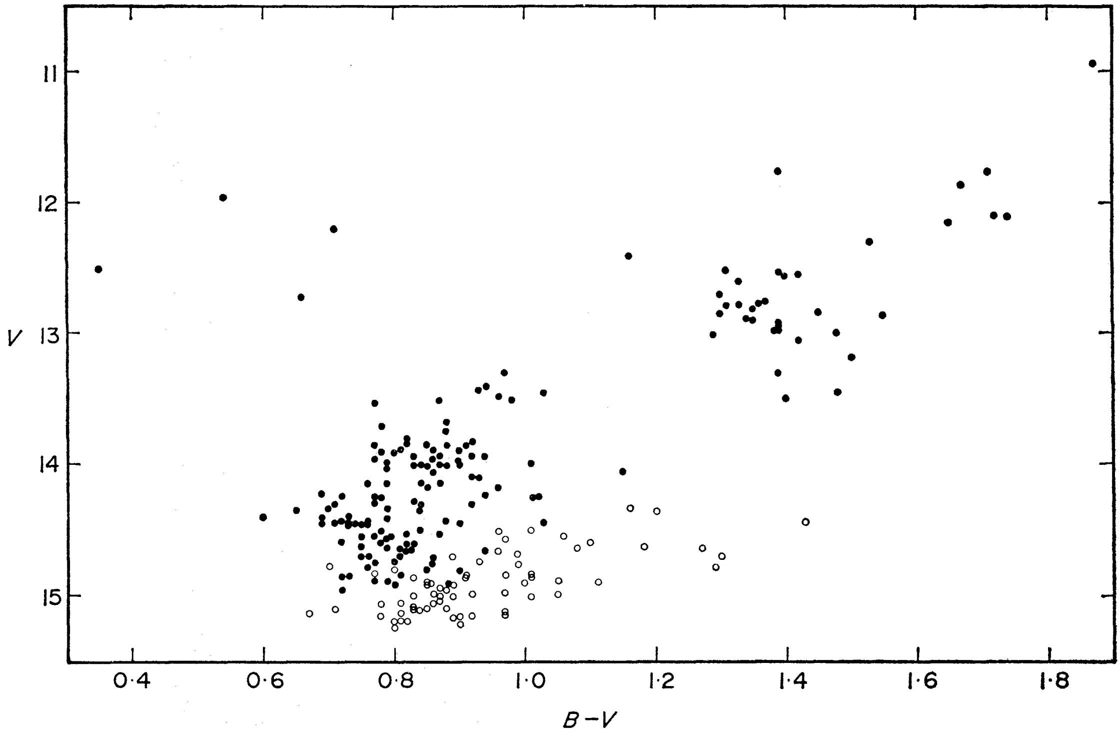

FIG. 3. *A C–M diagram for NGC* 6939, *taken from Cannon & Lloyd* (1969). *Filled circles are stars with proper motions compatible with cluster membership; open circles are fainter stars of unknown proper motion.*

on the basis of spatial distribution. NGC 6939 appears to be heavily reddened, and Chincarini (1963) found $E(B-V) \approx 0{\cdot}5$.

Certain common features of these three C–M diagrams can be discerned. The unreddened turn-off colour is near $(B-V)_0 = 0{\cdot}3$ in each case, and each cluster has a similar red giant branch in which the dominant feature is a strong clump of stars near $(B-V)_0 = 1{\cdot}0$, with a thin line of brighter, redder, stars (missing in NGC 752, presumably on account of the small total membership). The cluster membership of stars in the Hertzsprung Gap, between the top of the main sequence and the clump of red giants, is debatable, but if such stars do exist they must be very few in number. Another common feature of these C–M diagrams is the presence of ' blue straggler ' stars, lying near the main sequence but above the break-point defined by the majority of the stars. Such stars appear to occur in the majority of old and intermediate-age clusters, although there seems to be little similarity from cluster to cluster, either in number or in position in the C–M diagram.

One respect in which the clusters appear to differ is in the presence or absence of a small gap within and near the top of the main sequence. Such a gap is shown clearly in Fig. 3 for NGC 6939, at about $V = 14{\cdot}1$. However, there is no indication of a similar gap in NGC 7789, while in NGC 752, Eggen (1963) noted a gap at about $V = 10{\cdot}5$, but its reality is difficult to establish with such a small sample of stars. Recent work, notably by Aizenman, Demarque & Miller (1969), has shown

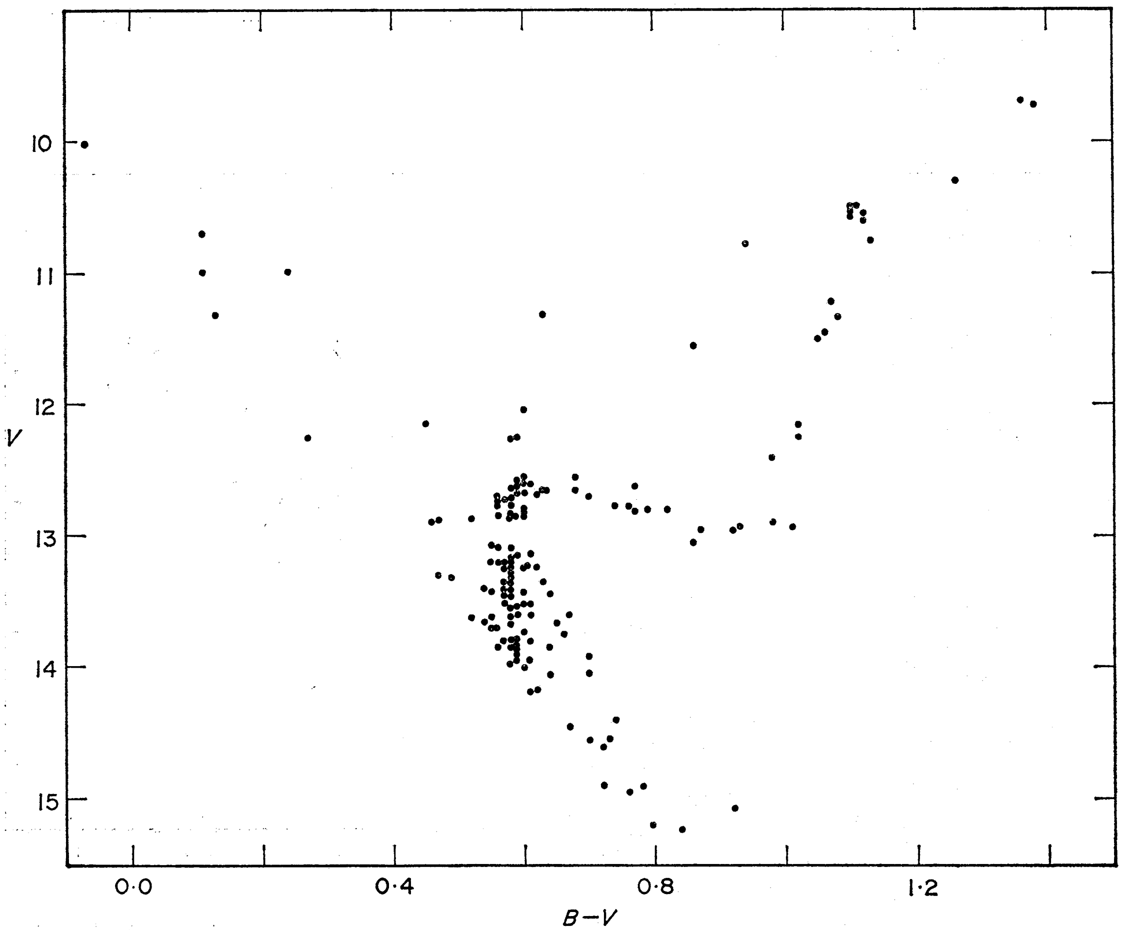

FIG. 4. *A C–M diagram for probable members of M67, using photoelectric photometry by Eggen & Sandage* (1964).

that the existence of such gaps is sensitively dependent both on the mass and on the chemical composition of the stars concerned; thus the differences between the three clusters studied here, of apparently similar age, may indicate differences of chemical composition.

5. THE OLD CLUSTERS

The two best observed old clusters are M67 and NGC 188. A photoelectric C–M diagram for M67 is shown in Fig. 4, taken from Eggen & Sandage (1964). Proper motions by Murray, Corben & Allchorn (1965) and radial velocities by Pesch (1967) were used to eliminate field stars in this case. The reddening is given as $E(B-V) = 0{\cdot}06$ by Sandage & Eggen (1969). A recent C–M diagram for the nucleus of NGC 188 has been produced by Eggen & Sandage (1969), but the total number of red giants found is rather few for the present study.

The main feature distinguishing the C–M diagram of an old cluster from that of an intermediate-age cluster is the continuous, fairly uniform, line of subgiants and giants starting from the top of the main sequence. There is no Hertzsprung Gap. However, a small, inconspicuous, clump of stars does exist on the M67 giant branch, at about $(B-V) = 1{\cdot}1$, $V = 10{\cdot}5$ (Fig. 4). NGC 188 also shows such a concentration of stars, at $V \approx 12{\cdot}5$, which is confirmed in proper motion work over an extended area by the author (1968), and most other old clusters show a similar feature. Although the significance of this clump may be doubted in any one cluster, where only a few stars are involved, its appearance in a similar position in the C–M diagrams of many old clusters is not consistent with its being an accidental statistical clumping. Regarding other features of the C–M diagrams of old clusters, both M67 and NGC 188 appear to contain a considerable number of blue stragglers, and both have gaps near the top of the main sequence.

6. OBSERVATIONS OF OTHER CLUSTERS AND GROUPS

An attempt has been made to obtain a complete list of all open clusters which are known or believed to have unreddened turn-off colours redder than $(B-V)_0 = 0{\cdot}0$.

The results of an extensive literature search, up to November 1969, are given in Table I. Earlier compilations by Johnson *et al.* (1961), Lindoff (1968b) and Barbaro *et al.* (1969) were particularly useful in this search.

The clusters are listed in order of Right Ascension, the first column giving the NGC number or other designation. The next two columns give the reddening, $E(B-V)$, and the true distance modulus, $(m-M)_0$, generally quoted from the literature. In most cases both of these quantities have been determined by reference to the Hyades. The fourth column gives the unreddened turn-off colour, $(B-V)_{0,t}$, while the fifth gives the corresponding age in units of 10^8 years. These quantities were evaluated as described in Section 2, and the ages should only be taken as rough guides for the initial classification of the clusters. The remaining columns describe the properties of the red giant clump. This was generally straightforward for the richer clusters, but in poor clusters could not be unambiguously distinguished when a high proportion of field stars was present, or when the photometry was of low accuracy, so that the results given are then of necessity rather subjective. However, there do not appear to be any clusters which could have shown a clump but which certainly do not. The position of the clump was estimated by visual inspection of the

TABLE I

Clusters with turn-off redder than $(B-V)_0 = 0{\cdot}0$

NGC	Name	$E(B-V)$	$(m-M)_0$	$(B-V)_{0,t}$	Age (10^8 years)	$\overline{M_v}$	$\overline{(B-V)_0}$	ΔV	$\Delta(B-V)$	References
188		0·09	10·85	0·59	80	1·1	1·1	2·7	0·5	Eggen & Sandage 1969
559		0·45	11·1	0·20	7	1·4	1·05	—	0·85	Lindoff 1968a
752		0·04	7·9	0·33	12	1·1	1·0	0·8	0·6	Fig. 1, this paper
1245		0·28	11·8	0·16	6	1·4:	0·9	0·0	0·8	Hoag *et al.* 1961
1342		0·28	8·7	0·05	4	—	—	—	—	Hoag *et al.* 1961
—	Hyades	0·00	3·0	0·12	5	0·7	1·0	0·5	0·9	Johnson & Knuckles 1955
1817*		—	—	—	—	—	—	—	—	Cuffey 1938
1907		0·38	10·7	0·07	4	0·7:	0·9	0·0	0·9	Hoag *et al.* 1961; Lavdovskii 1965
2158		0·43	13·4	0·27	9	0·4	0·9	1·3	0·6	Arp & Cuffey 1962
2194*		—	—	—	—	—	—	—	—	Cuffey 1943
2215		0·10	10·0	0·10	4	—	—	—	—	Becker 1960
2266*		—	—	—	—	—	—	—	—	Cuffey 1938
2324		0·11	12·4	0·15	5	0·7:	1·0	0·0	0·8	Hoag *et al.* 1961
2360		0·07	10·3	0·30	9	0·7	0·9	0·4	0·6	Eggen 1968
2420		0·01	11·9	0·39	20	0·7	1·0	1·8	0·6	West 1967b; Cannon 1970
2423		0·13	9·7	0·2	7	0·9	0·9	0·0	0·7	Smyth & Nandy 1962
2477		0·25	10·0	0·25	8	1·6	1·0	0·0	0·7	Eggen & Stoy 1961
2506		0·10	11·7	0·40	20	1·1	0·9	1·3	0·5	Purgathofer 1964

TABLE I (*continued*)

2632	Praesepe	0·00	6·0	0·15	5	0·5	1·0	0·0	0·8	Johnson 1952
2682	M67	0·06	9·6	0·51	50	0·8	1·0	2·0	0·5	Eggen & Sandage 1964
3496		0·5	10·3	0·1	4	1·2	1·1	0·0	1·0	Sher 1965
3680		0·04	9·5	0·44	30	1·3	1·1	1·0	0·7	Eggen 1969a
—	Coma	0·00	4·5	0·05	4	—	—	—	—	Johnson & Knuckles 1955
5822		0·19	9·3	0·00	4	0·7	0·8	0·4	0·7	Brück *et al.* 1968
5823		0·18	9·2	0·4:	20:	—	—	—	—	Brück *et al.* 1968
IC4651†		—	—	—	—	—	—	—	—	Eggen 1969b
6633		0·17	7·5	0·05	4	0·5	0·9	0·0	0·9	Hiltner *et al.* 1958
6791		0·22	13·5	0·58	70	0·6	1·1	2·5	0·5	Kinman 1965
6811*		—	—	—	—	—	—	—	—	Becker 1947
6819*		—	—	—	—	—	—	—	—	Barkhatova *et al.* 1963
6866		0·14	10·4	0·06	4	—	—	—	—	Hoag *et al.* 1961
6882/5		0·08	8·9	0·36	16	—	—	—	—	Hoag *et al.* 1961; Lavdovskii 1965
6939		0·5	10·5	0·25	8	0·8	0·8	1·0	0·6	Fig. 3, this paper
6940		0·3:	9·5	0·15:	5:	0·5:	0·8:	0·0:	0·7:	Larsson-Leander 1964
7062*		—	—	—	—	—	—	—	—	Fenkart 1965
7142		0·46	11·4:	0·39:	20:	0·9:	1·0:	—	0·7:	van den Bergh 1962
7789		0·28	11·4	0·32	11	0·7	1·0	0·8	0·7	Fig. 2, this paper

* These clusters may be old from the characteristics of the C–M diagram, but cannot be used quantitatively either because the photometry is not on the *UBV* system, or because of excessive confusion due to field stars.

† IC4651 is undoubtedly old, but final results are not yet available.

C–M diagrams, and corrected using the reddening and distance modulus given in columns 2 and 3. Columns 6 and 7 give the resultant mean absolute magnitude, $\overline{M_v}$, and colour $\overline{(B-V)_0}$, of the clump stars. Columns 8 and 9 give the position of the clump relative to the top of the main sequence; this is independent of the reddening and distance modulus assumed, and is used later in a rough check on the results. ΔV is defined to be the difference between the apparent magnitude of the brightest main sequence stars and that of the clump; $\Delta(B-V)$ is the difference between the colour of the clump and the turn-off colour. The final column gives references to the C–M diagrams used.

Several of the moving groups of stars investigated by Eggen have C–M diagrams similar to that of M67, and can therefore be included with the old clusters. Data on these are given in Table II, which gives the same quantities as Table I, except that $E(B-V)$ and $(m-M)_0$ do not apply.

TABLE II

Old stellar groups

Name	$(B-V)_{0,t}$	Age (10^8 years)	$\overline{M_v}$	$\overline{(B-V)_0}$	ΔV	$\Delta(B-V)$	References
Wolf 630	0·5	50	1·0:	1·1	2·0:	0·6	Eggen 1969c
ζ Her	0·5	50	1·2	1·0	1·8	0·5	Eggen 1958
γ Leo	0·55	60	—	—	—	—	Eggen 1959
61 Cyg	0·5	50	0·8	1·1	2·0	0·6	Eggen 1969c
η Cep	0·55	60	0·2	1·1:	2·8	0·50:	Eggen 1964a
σ Pup	0·52	50	1·0	1·0:	3·4	0·4:	Eggen 1964b
ϵ Indi	0·5	50	—	—	—	—	Eggen 1958

For comparison purposes, data have also been put together for a few younger clusters, i.e. with $(B-V)_{0,t} \lesssim 0·0$. Even in the youngest such clusters, h and χ Persei, most of the red giants fall within a small region of the C–M diagram, so that the quantities $\overline{M_v}$ and $\overline{(B-V)_0}$ can still be given meaningful values. Table III lists the relevant quantities for these younger clusters, the column headings having the same meanings as in Tables I and II.

TABLE III

Some clusters with turn-off colour $(B-V)_0 \leqslant 0·0$

NGC	Name	$E(B-V)$	$(m-M)_0$	$(B-V)_{0,t}$	Age (10^8 years)	$\overline{M_v}$	$\overline{(B-V)_0}$	References
869+884	$h+\chi$ Persei	0·56	11·8	−0·30	~0·1	−5·0	1·9	Wildey 1964
6664		0·60	10·8	−0·20	0·3:	−2·2	1·3:	Arp 1958
3114		0·04	9·7	−0·10	1·5:	−1·7	1·2:	Lynga 1962
6494		0·38	9·1	−0·06	2	−0·6	1·0	Hoag *et al.* 1961
2287	M41	0·00	9·1	−0·05	2	−1·3	1·1:	Hoag *et al.* 1961
6705	M11	0·40	11·2	−0·05	2	−0·7	1·1:	Johnson *et al.* 1956
2099	M37	0·27	10·8	0·00	3	−0·3	1·0	West 1967a

7. PROPERTIES OF THE RED GIANT CLUMP

While for many of the clusters and groups listed in Tables I, II and III the identification of a clump of red giants may be regarded as uncertain and subjective, there can be no doubt of its reality in the richest intermediate-age clusters, such as

NGC 7789 (see Fig. 2). The C–M diagram of such a cluster shows some suggestive similarities to that of 47 Tucanae (Tifft 1963), one of the most metal rich globular clusters. While the much greater age ($\sim 10^{10}$ years) and different chemical composition of 47 Tucanae lead to a much lower turn-off point and to a very different shape for the giant branch, the short, red, horizontal branch of this cluster is very reminiscent of the clump in NGC 7789 and other open clusters. The remainder of this paper will be an attempt to justify this tentative identification of the open cluster clump with the globular cluster horizontal branch, a possibility first noted by Faulkner (1966) for the case of M67, and by Barbaro *et al.* (1967) for some younger clusters.

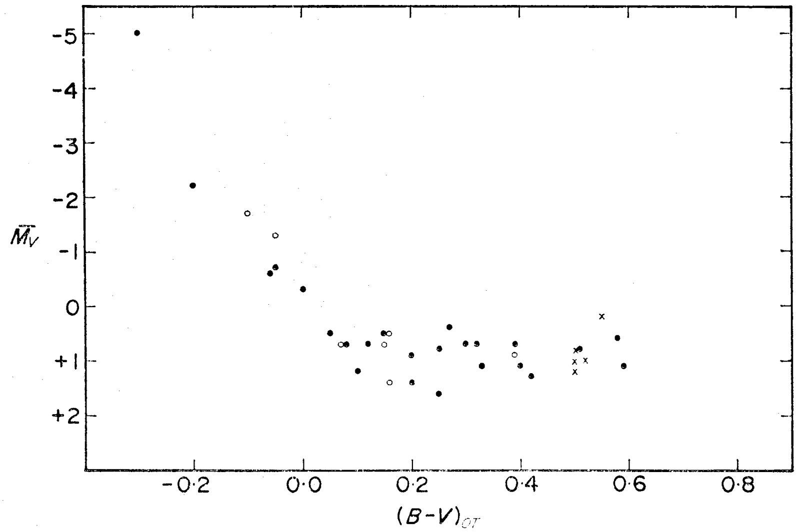

FIG. 5. *The mean absolute magnitude of the red giant clump, $\overline{M}_v$, as a function of cluster turn-off colour. Filled circles are for clusters with a well-defined, well-observed clump, open circles represent less certain results, while crosses are for the Eggen groups.*

The most striking feature of the clump data for the older clusters given in Tables I and II is the near constancy of the absolute magnitude, $\overline{M}_v$, of the clump. This is illustrated in Fig. 5, where for each cluster the value of $\overline{M}_v$ is plotted versus the turn-off colour, $(B-V)_{o,t}$. Filled circles represent clusters with a reasonably well-defined clump, open circles those for which either the position or the existence of the clump is uncertain, while the Eggen groups are represented by crosses. The most obvious feature of this diagram is the sudden change of slope at about $(B-V)_{o,t} = +0{\cdot}05$, corresponding to a cluster age of about $4{\cdot}10^8$ years, and to red giant stars of masses about $2{\cdot}3\ M_\odot$. For younger clusters, $\overline{M}_v$ is a rapidly increasing function of cluster age, but for all clusters with ages $\geqslant 4{\cdot}10^8$ years, up to the maximum observed at about 10^{10} years, $\overline{M}_v$ varies by only about 1 magnitude, with no systematic trend. The observational errors may easily be half a magnitude in some cases; further uncertainties may be introduced by variations in chemical composition leading to changes in the position of the unevolved main sequence used to estimate the cluster distances. Nevertheless, some of the scatter seen in Fig. 5 does seem to be intrinsic; the values of $\overline{M}_v$ for the best-observed clusters, taken from a recent paper by Sandage & Eggen (1969), range from $+0{\cdot}8$ for NGC 7789 to $+1{\cdot}6$ for NGC 2477.

The mean value of $\overline{M}_v$ for clusters with $(B-V)_{o,t} \geqslant 0{\cdot}05$ is $+0^{\mathrm{m}}{\cdot}9$ with a precision estimated from the scatter in Fig. 5 of $\pm 0^{m}\cdot 1$. Inspection of Tables I, II and III shows that the colour of the clump is also very constant for all but the youngest clusters, at about $\overline{(B-V)_0} = +1\cdot 0$.

It is possible to check the validity of these results in an approximate manner without any knowledge of the reddening, absorption, or distances of the clusters. As the age of a cluster increases, the turn-off colour becomes redder and the brightest remaining main sequence star becomes fainter, in such a way that the shape of the upper main sequence remains approximately constant but is shifted downwards parallel to the zero-age main sequence. Therefore if the clump of red giants remains at constant luminosity and colour, the position of the top of the main sequence relative to the clump should move roughly parallel to the zero-age main sequence. The quantities ΔV and $\Delta(B-V)$ which thus crudely specify the relative position of the upper main sequence are given in Tables I and II and shown graphically in Fig. 6. The symbols used have the same meanings as in Fig. 5, and the points do scatter about a locus approximately parallel to the main sequence, which is indicated by the broken line in Fig. 6, for a cluster with a clump at $\overline{M_v} = +0\cdot 9$, $\overline{(B-V)_0} = +1\cdot 0$. A recent composite C–M diagram by Sandage & Eggen (1969) shows that the upper ends of cluster main sequences in fact vary considerably in shape, so that no very tight correlation would be expected in Fig. 6 even if the clump did occupy a constant position and there were no observational uncertainties.

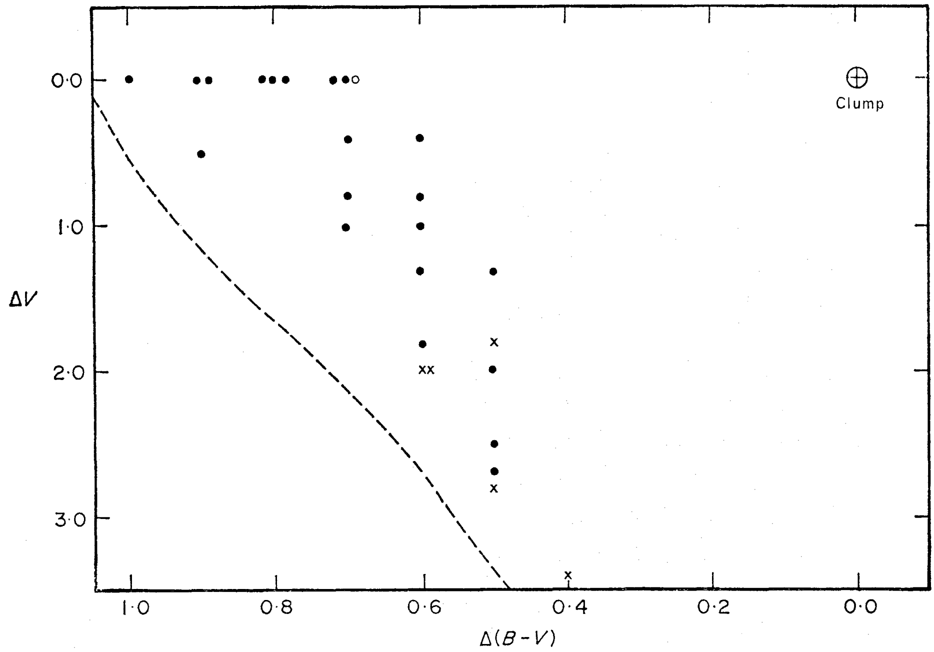

FIG. 6. *The position in the C–M diagram of the top of the main sequence relative to the clump, using the same symbols as in Fig. 5. The approximate position of the main sequence is also indicated.*

8. COMPARISON OF OBSERVATIONS WITH THEORY

There are two major uncertainties in comparing the observations of red giants with theoretical models. In the first place, the envelopes of red giants are almost entirely convective and their structure at least in the outer parts depends critically on the theory of convection employed. Secondly, the bolometric corrections and temperature scales, which are needed to relate the theoretical quantities L (the luminosity in solar units) and T_e (the effective temperature) to the corresponding observables, V and B–V, are less certain for these very cool stars than for main sequence stars. Demarque (1967) and Hofmeister (1967) have investigated the effect of changing the mixing length in the simplified convection theory usually adopted. Even main sequence colours may be uncertain by $0^m{\cdot}2$ in B–V.

Iben (1967c) concludes that calculated values of log T_e for red giants are uncertain by $\pm 0{\cdot}1$ as a result of the over-simplified convection theory. For red giants of spectral type K, which comprise the majority of those in old open clusters, this means an uncertainty of up to $0^m{\cdot}5$ in predicted values of B–V. It appears that the colours of red giant stars must also be very dependent on metal abundance (cf. Iben 1965a). In other words, the colours of theoretical red giant models mean very little.

The luminosities on the other hand do not seem to be much affected by changes in mixing length but the predictions of observable magnitudes *are* affected since the bolometric corrections are strongly dependent on temperature; a change of 0·1 in log T_e can produce a change of up to $0^m{\cdot}75$ in the b.c. for a K giant, and hence a change of $0^m{\cdot}75$ in M_v for a given bolometric magnitude M_{bol} (Johnson 1966).

Consequently it seems at present more profitable to transform from the observed C–M diagram to the theoretical $\log L - \log T_e$ diagram. This has been done for a schematic representation of the NGC 7789 C–M diagram shown in Fig. 2, using Johnson's (1966) transformations; the result is shown by the shaded areas in Fig. 7. Also shown in Fig. 7 are some of Iben's evolutionary tracks (1967a, b), with the appropriate mass of star written against each track. Following Iben (1967c), a small correction has been subtracted from values of $\log T_e > 3{\cdot}75$, to bring the theoretical main sequence into coincidence with that observed. This is necessary since line opacity effects were omitted in the models. The zero-age main sequence (ZAMS) is taken from Johnson (1964).

The only track which Iben has carried up to the stage of the helium flash is that for the $2{\cdot}25\ M_\odot$ star. This star spends only about 10 per cent of its life prior to the helium flash as a red giant. Thus the isochrones for hydrogen-burning red giants are similar to the evolutionary tracks of individual stars. In the case of the observations shown in Fig. 7, the red giants must have masses of between 1·6 and $1{\cdot}7\ M_\odot$, since by interpolation the brightest stars on the evolved main sequence have about $1{\cdot}6\ M_\odot$. The track for $1{\cdot}6\ M_\odot$ cannot lie much above that shown for $1{\cdot}5\ M_\odot$ (compare the track for $2{\cdot}25\ M_\odot$), and thus the evolutionary tracks calculated by Iben are too cool, and consequently appear to be too faint by about 0·8 in log L, or two magnitudes in M_{bol}, at least for $\log T_e \lesssim 3{\cdot}7$.

This is at first sight surprising, since Iben (1967a) obtained good agreement between his tracks and observations of the slightly lower mass stars in M67 and NGC 188. Part of the reason appears to be that he used different b.c.'s and temperature scales, derived ultimately from some early results of Kuiper (1938). It seems likely that a small change in the convection parameters could bring the models into better agreement with the present observations. Since the bolometric corrections are also still insecure, the discrepancy seen in Fig. 7 is perhaps not too serious. However, if the theoretical luminosities really are too low, then the calculated red giant lifetimes will probably be too long.

The predicted red giant isochrone for NGC 7789 must follow a $1{\cdot}6\ M_\odot$ track initially, and thus will run from the top of the main sequence to a relative minimum

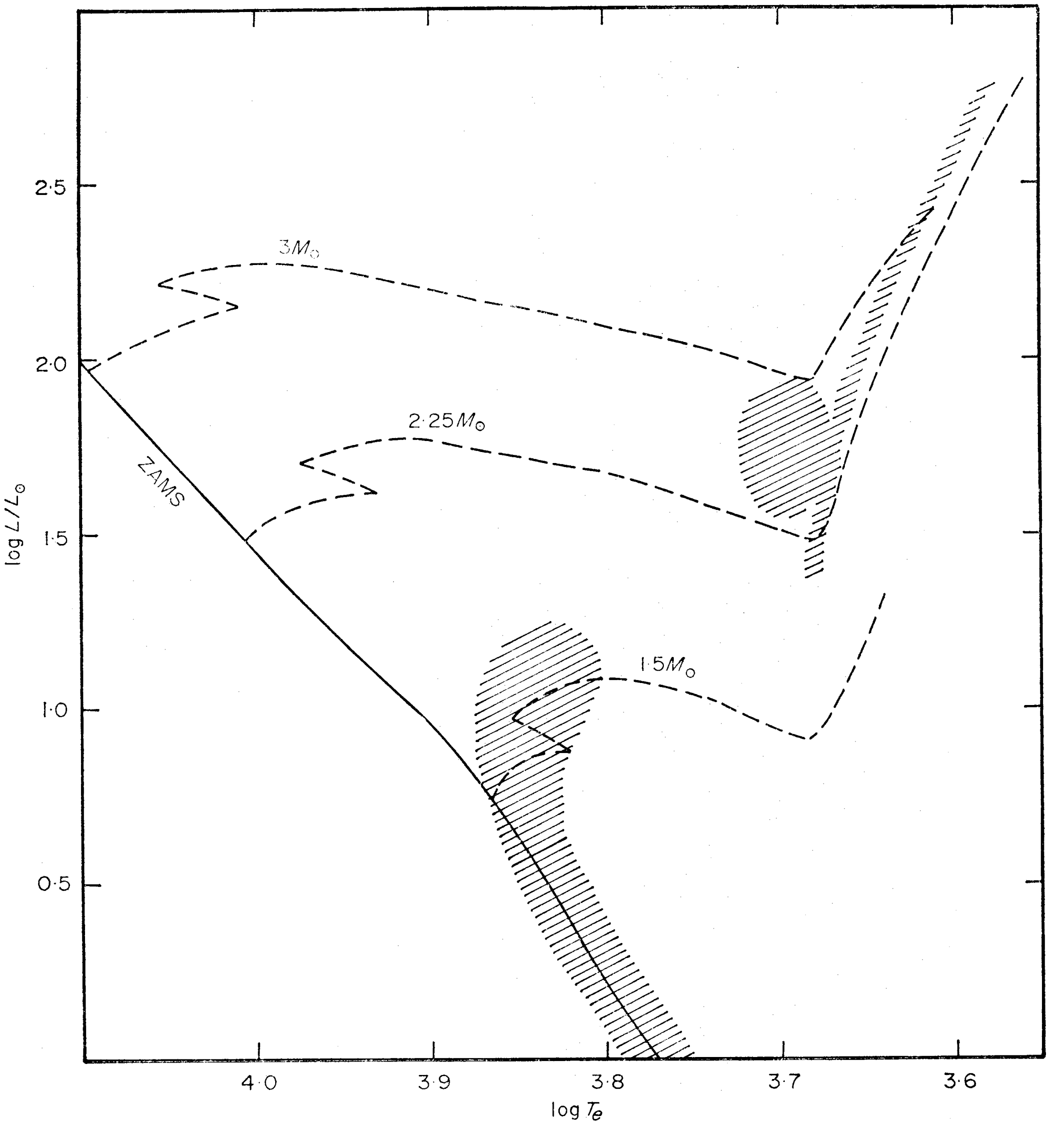

FIG. 7. *Comparison of observations with theory in the log $L - $log T_e diagram. The shaded area is a schematic representation of the NGC* 7789 *C–M diagram shown in Fig.* 2. *The broken lines are some theoretical evolutionary tracks by Iben* (1967*a, b*), *with the appropriate star mass written against each track, while the solid line is the zero-age main sequence.*

at log $T_e \approx 3{\cdot}7$, log $L \approx 1{\cdot}0$, and then turn steeply upwards to pass near the observed clump of red giants. Presumably, by analogy with the 2·25 $M_\odot$ model, it will run from there up the observed upper red giant sequence until the helium flash occurs, which is the limit of the available calculations. One point stands out immediately in Iben's evolutionary models for stars from 1 $M_\odot$ to 2·25 $M_\odot$ (1967a, b): there is no prediction of the clump of red giants. The theoretical evolution proceeds smoothly with time prior to the helium flash, and while there is some variation in the rate of evolution up the giant branch, with the stars of mass 1·5 $M_\odot$ or less showing a tendency to accelerate as they ascend the branch, this rate does not seem to vary by a factor of more than two. Therefore, the variations in number density on the giant branch also should not exceed a factor of two. However, if stars are counted, for example in NGC 7789 (Fig. 2), the density in the neighbourhood of the clump is about ten times greater than elsewhere on the giant branch. A second point is that the proper motion work described previously for NGC 6939 and NGC 7789 demonstrated conclusively that the majority of the stars lying between the main sequence and the clump of red giants, and many of the red stars lying below the clump, are field stars. Thus there is no observational evidence concerning the shape of evolutionary tracks in the Hertzsprung Gap, but the evolution must be sufficiently rapid for very few stars to be observed.

9. EMPIRICAL RATES OF EVOLUTION

It is possible to use the number of stars observed in a given evolutionary stage in a C–M diagram to obtain an estimate of the time which individual stars spend in that stage, a principle first extensively employed by Sandage (1957). In order to do this, it is necessary to assume an initial distribution with luminosity, or luminosity function, of the main sequence progenitors of the stars being examined. Star counts were made in half magnitude intervals down the main sequence for each of the clusters shown in Figs 1–4, and the numbers of stars compared with the numbers in the red giant branch. For this purpose the red giant branch was divided into three sections: the region between the evolved main sequence and the Hayashi limit (Hayashi & Hoshi 1961), which is the Hertzsprung Gap in intermediate-age clusters and the horizontal section of the giant branch in old clusters; the continuous line of red giants running up the Hayashi limit to the tip of the red giant branch, but excluding any clump; and finally, the clump itself. Clearly such divisions are somewhat subjective, as is the choice of the upper limit of the main sequence, but since both the statistical fluctuations in the observations and the present uncertainties in the theory give rise to uncertainties of factors of at least two, the exact way in which the star counts are made is not too important.

TABLE IV

Star counts in some clusters

		NGC 752		NGC 6939		NGC 7789		M67	
		obs	norm	obs	norm	obs	norm	obs	norm
Main sequence	1st ½ mag.	7	47	38	44	35	30	35	29
	2nd ½ mag.	7	47	42	48	57	49	59	50
	3rd ½ mag.	8	53	45:	52:	59	51	60	50
	4th ½ mag.	2	13	—	—	55	47	59	50
Hertszprung Gap		0	0	$\leqslant 3$	$\leqslant 3$	$\leqslant 3$	$\leqslant 3$	15	13
Red giant sequence		1?	7:	15	17	19	16	19	16
Red giant clump		4	27	20	23	41	35	5	4

The results of these star counts are given in Table IV, where the left-hand column of figures for each cluster gives the actual counts, while the right-hand column gives those counts normalized to 100 stars in the interval between $0^{\mathrm{m}}\cdot 5$ and $1^{\mathrm{m}}\cdot 5$ below the top of the main sequence. The counts were made from Figs 1, 2 and 3 for the first three clusters, while for M67 the C–M diagram presented by Murray *et al.* (1965) was used, since that shown in Fig. 4 does not represent a complete sample of stars. In NGC 752, the counts were restricted to the filled circles in Fig. 1, again for completeness.

The main sequence luminosity functions given in Table IV are in all cases surprisingly flat, after the first half magnitude, and it is this feature which permits the simple normalization of the data.

Some thinning out near the top of the main sequence is expected on account of the increasing rate of evolution, and also if any small gap is present as in NGC 6939 and M67. It should therefore be reasonable to assume that the initial luminosity function was flat over the region of interest; as can be seen from Table IV, it is only necessary to extrapolate the observed function by about half a magnitude to obtain the progenitors of the present red giants.

Suppose that N stars are observed in an advanced stage of evolution in a cluster, and that they came from a section of main sequence having N_0 stars per magnitude at age zero. These N stars must have come from a section of initial main sequence of mean magnitude m and length Δm, where $\Delta m = N/N_0$ magnitudes. The length of time, Δt years, which stars spend in the given stage is then equal to the difference in total time taken by stars of initial magnitudes $m - \Delta m/2$ and $m + \Delta m/2$ to reach the stage. A crude estimate of these times can be made using simple homology arguments (see for example Stein 1966), which predict that $L \propto M^5$ for main sequence stars of luminosity L and mass M between $1 \cdot 25\ M_\odot$ and $2\ M_\odot$, a good approximation for the models by Iben. For homologous models, the main sequence lifetimes are given by $t \propto M/L = L^{-0 \cdot 8}$. One therefore obtains the relations

$$\Delta t/t = -0 \cdot 8 \Delta L/L = -0 \cdot 8 \Delta \log_e L,$$

where t is the age of the cluster, ΔL is the range in luminosity and L the mean initial luminosity of stars in the specified evolutionary stage, and $\Delta \log_e L$ is the corresponding interval on the main sequence. Since $\Delta \log_{2 \cdot 5} L = -\Delta m$, the final result is

$$\Delta t/t = 0 \cdot 74 \Delta m = 0 \cdot 74 . N/N_0. \qquad (1)$$

Although the later red giant stages of evolution are no longer even approximately homologous, the total time spent in such stages is sufficiently small that the breakdown of the assumption is not important. It is only necessary that the red giant rates of evolution should be slowly varying functions of mass over the small range of masses observed in any one cluster (see below). The error in $\Delta t/t$ caused by the over-simplified form of (1) seems to be about 10 per cent, judging by the accuracy with which Iben's models can be fitted, so that observational uncertainty in N and N_0 will normally be the limiting factor in estimates of $\Delta t/t$.

TABLE V

Empirical lifetimes (years)

	NGC 6939	NGC 7789	M67
Assumed age of cluster	10^9	10^9	$5 . 10^9$
Hertzsprung Gap	$2 . 10^7$	$2 . 10^7$	$5 . 10^8$
Red giant sequence	$1 \cdot 3 . 10^8$	$1 \cdot 2 . 10^8$	$6 . 10^8$
Red giant clump	$1 \cdot 7 . 10^8$	$2 \cdot 6 . 10^8$	$1 \cdot 5 . 10^8$

Applying equation (1) to the data in Table IV gives the lifetimes shown in Table V, where NGC 752 has been omitted on account of the small number of stars. The cluster ages which were used (see Section 2), i.e. the values of t, are given in the first line of Table V, while $N_0 = 100$ for the normalized data in Table IV. The age of NGC 7789 is uncertain because the theoretical isochrone does not agree well with that observed; there is no observational evidence for a gap near the top of the main sequence, and no consequent sudden change of $\sim 0 \cdot 03$ in $\log T_e$. This may simply be due to a small difference in chemical composition (Aizenman, Demarque

& Miller 1969). The fitting of 1·7 $M_\odot$ red giants, based on Fig. 7, in fact requires an age of about $1{\cdot}4 . 10^9$ years. Also, the true turn-off colour of NGC 6939 is observationally very uncertain. The age of 10^9 years assigned to both of these clusters is only a crude estimate, and any future revision of their ages will simply lead to proportionate changes in the lifetimes given in Table V. The total lifetime for all of the red giant phases can be compared with the estimates given by Lindoff (1968c) for ' mean ' clusters of comparable age. Lindoff also derived his lifetimes from star counts, but in a different way to that used here, and with the extra uncertainty due to contamination by field stars. For cluster ages of 4·1, 2·1 and $0{\cdot}545 . 10^9$ years, Lindoff (1968c, Table 3) obtained total red giant lifetimes of 9, 4 and $0{\cdot}8 . 10^8$ years respectively. The agreement with the figures in Table V is good, well within the uncertainties of either estimate.

The homology arguments can also be used to estimate the spread in mass of the red giants. If $L \propto M^5$ and $t \propto M/L$, then $t \propto M^{-4}$, so that $\Delta t/t = 4\Delta M/M$, where ΔM and M are the range of mass and mean mass respectively of the stars. Substitution in equation (1) gives

$$\Delta M/M = 0{\cdot}18 . N/N_0. \qquad (2)$$

For NGC 7789, assuming that the stars just leaving the main sequence have 1·6 $M_\odot$, the spread must be about 0·16 $M_\odot$, so that the red giants must have masses of about 1·7 $M_\odot$, if there is no mass loss. In M67, with Iben's (1967a) value of about 1·25 $M_\odot$ for the red giant mass, equation (2) applied to the counts of Table IV suggests a smaller figure of about 0·07 $M_\odot$ for the total mass spread between the subgiants and giants.

In these estimates of lifetimes and mass spreads, it is tacitly assumed that all of the red giant sequence and the clump represent stages in the evolution of every star. A possible evolutionary sequence for the red giant stages can now be established.

10. HYDROGEN-BURNING RED GIANTS

The observations of NGC 7789 (and of NGC 6939) put an upper limit of about three on the number of stars in the Hertzsprung Gap, so that the time taken for a 1·6 $M_\odot$ star to cross the gap must be $\lesssim 2{\cdot}10^7$ years. It is not easy to estimate the corresponding theoretical rates of evolution of red giants, starting from the few available calculated tracks, since the evolution is not homologous. However, Iben (1967a, b) finds that a 2·25 $M_\odot$ star requires about 10^7 years to cross the Hertzsprung Gap, while a 1·5 $M_\odot$ star requires nearly 10^8 years, which would suggest a slightly higher mass than 1·6 $M_\odot$ for stars currently becoming giants in NGC 7789. For M67, the time taken to traverse the horizontal part of the giant branch is given in Table V as about $5{\cdot}10^8$ years, which has to be compared with the time of $2 . 10^8$ years found by Iben (1967a) for a star of 1·25 $M_\odot$.

According to the theoretical calculations, the next stage of evolution is a steady increase in luminosity and decrease in effective temperature as the star, now with an almost entirely convective envelope, moves up the boundary of the Hayashi forbidden region, until central temperatures and pressures rise sufficiently to start helium burning in the core. This is probably a violent event since the helium core is degenerate for masses below 2·25 $M_\odot$, and the subsequent evolution is not yet known. It is natural to identify this phase of evolution with the long sequence of red giants observed in clusters such as NGC 7789. At first sight this identification is not easy using the tracks shown in Fig. 7, but as explained previously, the extreme sensitivity of the surface temperatures of red giant models to the parameters used to describe convection, and the uncertainties involved in transforming the obstructions, may mean that the discrepancies in the position and shape of the tracks are not significant.

A more crucial test is whether or not the theoretical lifetimes are compatible with the observed numbers of such stars. Unfortunately, Iben (1967b) only carried his computations through to the helium flash in the case of the 2·25 $M_\odot$ star, but the lifetimes of lower mass red giants can be very crudely estimated from the existing calculations. The 1·5$M_\odot$ model appears, from the growth of the helium core, to have completed somewhat less than half of its hydrogen-burning red giant evolution in $1{\cdot}6 . 10^8$ years. Assuming the fairly constant rate of evolution in the upper giant branch which is suggested by the more massive 2·25 $M_\odot$ model (Iben 1967b) and by the observations (Fig. 2), the total lifetime in this stage must be about $3 . 10^8$ years. For the slightly more massive stars in NGC 7789, with between 1·6 $M_\odot$ and 1·7 $M_\odot$, interpolation between the models suggests a hydrogen-burning red giant phase of about $2 . 10^8$ years. For the corresponding stars in M67, if the masses are about 1·25 $M_\odot$, this phase should last about $4 . 10^8$ years.

These theoretical lifetimes in the early red giant phase are to be compared with those derived empirically and given in Table V, the corresponding times being $1{\cdot}2 . 10^8$ and $6 . 10^8$ years for NGC 7789 and M67 respectively. The agreement is very satisfactory, since both theoretical and empirical estimates are uncertain by factors of about two. There is certainly no sign of the gross deficiency in the observed numbers of red giants which Lindoff (1968c) found for younger clusters with ages $\lesssim 3 \times 10^8$ years. The empirical lifetimes are of course upper estimates in so far as it has been assumed that few of the giants, apart from the clump stars, are in phases more advanced than hydrogen burning. In conclusion, the identification of the long sequence of bright red giants with simple hydrogen shell-burning models seems to be very plausible.

II. RED GIANTS BURNING BOTH HYDROGEN AND HELIUM

The identification of the long line of red giants leaves the strong clump unexplained. If all stars pass through both red giant regions during their lifetimes, the clump must represent a long period during which a star's luminosity and colour are nearly constant. There are in fact some possible phases of roughly constant luminosity during the first hydrogen-burning red giant stage, occurring when the outward-moving shell source reaches discontinuities in the hydrogen abundance. Two such discontinuities can occur, one marking the maximum outward extent of the convective core during main sequence evolution, and the other, the maximum inward extent of the convective envelope during shell burning. Both are present in Iben's 2·25 $M_\odot$ model (1967b), but together cause the luminosity to stay constant for only about $3 . 10^6$ years, less than 10 per cent of the total red giant lifetime. The point has been discussed at length for the Population II low mass giants in M15, again by Iben (1968a), but the longest standstill found is still less than 20 per cent of the time spent on the upper part of the red giant branch, and a much smaller proportion of the total red giant lifetime. In contrast, the clump of giants in the 10^9 years old cluster NGC 7789, which are presumably intermediate in mass between the cases considered by Iben, represents about two thirds of the total red giant lifetime. Therefore it does not seem reasonable to identify the clump with such minor features in the evolution, although some explicit models for stars with the appropriate mass and composition would be very valuable to confirm this conclusion, particularly for the old clusters such as M67 which have a relatively weak clump.

All of the observations concerning the clump have a simple explanation if these stars are in a post-helium flash stage of evolution, with a helium-burning core and a hydrogen-burning shell source, similar to the models suggested by Faulkner (1966) to explain the horizontal branch observed in globular clusters. Iben (1967b) has found that a large part of the pressure support must be supplied by degenerate electrons in the cores of hydrogen-burning red giants with masses $\leqslant 2{\cdot}25\ M_\odot$. In ordinary matter, the strongly temperature-dependent nuclear reactions are stable since a local increase in energy production leads to expansion, cooling, and a subsequent decrease in energy production. Degenerate matter cannot expand in this way, so that the onset of a nuclear process must lead to an instability which cannot be controlled until the degeneracy has been lifted. For this reason it is not yet possible to carry evolutionary calculations through to normal helium burning without very great effort. Even if the computations are carried out, it is not clear that the correct results can be obtained assuming smooth evolution of spherically symmetrical stars (Eggleton 1966). By making these and various other simplifying assumptions, Härm & Schwarzschild (1964) were able to follow the evolution of a few models through the helium flash. One of the questions which they tried to answer was whether or not convection would spread sufficiently to cause mixing of the hydrogen-rich envelope with the helium core, thus seriously affecting the subsequent evolution of the star. They found that such mixing did not occur, but did not regard their calculations as conclusive. Faulkner (1966) assumed that no mixing occurred, and constructed models in the immediate post-helium flash stage with non-degenerate helium-burning cores, hydrogen-burning shells, and a sharp composition discontinuity at the interface. To construct these models he had to assume a mass for the helium core, and used the argument that the helium flash should occur when the mass of the degenerate core reaches a certain value, dependent on the chemical composition but virtually independent of the total mass of the star. This argument is based on the demonstration by Schwarzschild (1958) that conditions at the edge of the helium core are almost independent of the total mass of the star, while it is precisely these conditions which determine the onset of helium burning. The helium core behaves rather like an independent star within the red giant envelope.

This argument has recently been used by Eggleton (1968) in an investigation of the development of the helium flash in red giant stars of different compositions. For a Population I composition, with hydrogen abundance $X = 0{\cdot}65$ and heavy element abundance $Z = 0{\cdot}01$ by weight, Eggleton finds that the flash occurs when the core mass reaches $0{\cdot}45\ M_\odot$. This should apply for total star masses between about 0·9 and 2 $M_\odot$. The critical core mass does not appear to depend very strongly on metal abundance, and varies by only a few per cent over the full range of likely Population I compositions. A critical core mass of about $0{\cdot}45\ M_\odot$ is also predicted by Iben (1968b) although there are considerable uncertainties in the nuclear reaction rates and opacities used in the models.

Some series of models of double energy source stars with constant core mass but varying total mass have been constructed by Faulkner (1966). Those with $Z = 0{\cdot}02$

should be directly applicable as continuations of Iben's evolutionary tracks. Faulkner used two values of X, $X = 0{\cdot}90$ and $X = 0{\cdot}65$, the latter being close to Iben's value of $X = 0{\cdot}702$, and a range of total masses from $0{\cdot}60\,M_\odot$ to $1{\cdot}25\,M_\odot$. The most striking feature of the results is that for a given helium abundance and core mass, the luminosity and temperature do not vary significantly with total mass. This rule is only violated when the mass in the envelope is $\lesssim 0{\cdot}2\,M_\odot$. The constancy of effective temperature is a consequence of the high metal abundance, since with $Z = 0{\cdot}02$ the star envelopes are almost wholly convective so that the star must lie near the boundary of the Hayashi forbidden region. The constancy of luminosity is straightforward for the stars of low helium abundance ($X = 0{\cdot}9$), since for these stars most of the energy is generated in the helium core (Faulkner & Iben 1966) which behaves like a more or less independent star. It is not so easy to understand for the stars with $X = 0{\cdot}65$, in which most of the energy is produced in the hydrogen shell, but the range in luminosity found by Faulkner in this case is still only about $0^{\mathrm{m}}{\cdot}25$. A similar constancy in luminosity was found by Giannone (1967), who constructed sequences of stars with more massive cores.

Before these models can be accepted as appropriate to the observed clump of red giants, it is necessary to investigate their subsequent evolution. Evolutionary tracks have been computed by Faulkner & Iben (1967), but only for compositions with $Z = 2.10^{-4}$ and $Z = 2.10^{-5}$. For most of these tracks the total change in luminosity is about $0^{\mathrm{m}}{\cdot}25$, while the greatest change was about $0^{\mathrm{m}}{\cdot}75$. There is no reason why metal abundance should seriously affect the sizes of evolutionary luminosity changes, which depend mainly on the evolution of the helium core, and so the changes for models with $Z = 0{\cdot}02$ would probably be less than one magnitude. The range of luminosities observed in NGC 7789, one of the few clusters rich enough to give a meaningful result, is about $0^{\mathrm{m}}{\cdot}5$. Regarding changes in effective temperature, Faulkner (1966) has suggested that for Population I metal abundances, horizontal branch stars will always remain near the Hayashi limit, and thus maintain constant temperature. Stars of this type therefore evolve without much change in either luminosity or colour, and should lie in a clump in the C–M diagram. The absolute magnitude of this clump should be approximately the same in all old Population I clusters.

The predicted and observed values of the clump absolute magnitude must now be compared. For a star with $X = 0{\cdot}65$, $Z = 0{\cdot}02$, core mass $0{\cdot}45\,M_\odot$, and total mass $1{\cdot}25\,M_\odot$, interpolation between Faulkner's (1966) models for core masses $0{\cdot}4$ and $0{\cdot}5\,M_\odot$ yields a predicted value of $M_{\mathrm{bol}} = +0^{\mathrm{m}}{\cdot}3$. In the subsequent work (Faulkner & Iben 1966) the effects of electron degeneracy were included and produced increases in luminosity of between $0^{\mathrm{m}}{\cdot}1$ and $0^{\mathrm{m}}{\cdot}2$. The bolometric correction for giants with the observed colour of $B\text{–}V = +1$ is $+0^{\mathrm{m}}{\cdot}3$ (Johnson 1966), so that the predicted 'horizontal branch' has $M_v \approx +0{\cdot}5$. The present uncertainties in the initial core mass and in the helium abundance, lead to an uncertainty of at least $0^{\mathrm{m}}{\cdot}5$ in this predicted value of M_v, so that the good agreement with the average observed figure of $+0^{\mathrm{m}}{\cdot}9$ (Section 7) is to some extent fortuitous. The predicted colours for these stars are very uncertain, as always for red giants. A better procedure is to try to determine the Hayashi limit observationally. The relevant limit must be approximately delineated by the stars ascending the red giant branch prior to the helium flash, and in all the clusters this track is observed to pass through or very near to the clump, in agreement with the model predictions.

As a final check on the validity of these double-source models for the clump stars, the predicted and observed lifetimes can be compared. For each of the clusters discussed earlier, a clump lifetime of about 2.10^8 years was found (Table V), with an uncertainty of about a factor two. For their various calculations, Faulkner & Iben (1966) found ages in the range 0·7 to $1{\cdot}2.10^8$ years, so that although none of their calculations was for the composition of interest here, the rough agreement of the ages is very encouraging. The time which stars spend in the double-source phase is essentially determined by evolution within the helium core, and is probably not very dependent on the chemical composition, or on the total mass of the star. This approximate constancy of horizontal branch lifetime for stars of different total mass gives an immediate explanation for the near-disappearance of the clump in the oldest clusters such as NGC 188. For stars of 1 $M_\odot$, evolution up the first part of the red giant branch covered in the calculations by Iben (1967a) took 5.10^8 years, and the helium core was still not near the critical mass for the helium flash. Therefore if only twenty red giants are observed in such a cluster, three or four should be in the clump, which will consequently be very poorly defined.

It remains to examine the theoretical predictions for stars with masses $\gtrsim 2{\cdot}25\ M_\odot$. For such stars, electron degeneracy never becomes significant and the evolutionary calculations can be followed through the stages of helium burning without difficulty. The argument that the helium burning must begin when the core reaches a fixed critical mass no longer applies, and the calculations by Iben show that the hydrogen-burning red giant phase is very short. Consequently, for masses $\gtrsim 3\ M_\odot$, helium burning begins in a core which is approximately the same size as the Schönberg–Chandrasekhar limiting core, about 12 per cent of the total mass of the star (Schönberg & Chandrasekhar 1942). Such stars are found to have luminosities during helium burning which increase monotonically with the mass of the star. One slight complication is that for masses near 3 $M_\odot$, the initial mass of the helium-burning core is still near 0·45 $M_\odot$, so that at first sight such stars might be expected to have the same luminosity as the clump discussed above. Since the core is never degenerate, these more massive stars commence helium burning quietly, and begin to evolve down towards the position of the clump. However, this evolution is sufficiently slow that the core mass exceeds 0·45 $M_\odot$ while the luminosity is still decreasing, and the stars never reach the position of the clump. A star of 3 $M_\odot$ (Iben 1965b) reaches a minimum luminosity of $M_{\text{bol}} = -0{\cdot}5$, or $M_v = -0{\cdot}2$, during core helium burning. Since the evolution is slowest near this minimum luminosity, a clump of red giants is still predicted for clusters containing such stars, although the clump should become less well defined as the mass of the red giants increases, that is as the age of the cluster decreases. The red giant branch in the younger clusters has recently been discussed in more detail by Schlesinger (1969), who also pointed out that a large proportion of the red giants in Hyades-like clusters must be helium-burning.

The theory therefore predicts a clump of red giants in the C–M diagram for all except possibly the youngest Population I clusters. This clump should be at constant luminosity when the helium cores are initially degenerate, and should become brighter and less well defined for younger clusters. In all of these respects the observations support the theory well. The change of slope of the clump luminosity versus turn-off colour relation shown in Fig. 5, occurring at a turn-off colour corresponding to red giants of mass about 2·3 $M_\odot$, can then be interpreted as direct observational support for the prediction by Iben (1967b) that electron degeneracy sets in for stars of mass $\leqslant 2{\cdot}25\ M_\odot$.

One objection to the identification of the red giant clump with the horizontal branch is that several authors have already suggested that the ' blue straggler ' stars, seen in Figs 1–4 and mentioned in Sections 4 and 5, are the Population I analogues of the horizontal branch. This interpretation is most plausible in the case of M67 (Eggen & Sandage 1964; Sargent 1968). However, comparison of the observations in several clusters favours a main sequence interpretation for at least the majority of the blue stragglers (Cannon 1968), possibly as a result of mass exhange in close binary systems (McCrea 1964). Also, the number of blue stragglers is generally much too small for the expected horizontal-branch lifetimes. Nevertheless it may be that a few clump-type stars occur at relatively blue colours, either as a consequence of mixing during the helium flash, or as a result of considerable mass loss at some stage.

12. RED GIANTS IN THE GENERAL FIELD

The demonstration that all old clusters contain a clump of red giants at virtually the same magnitude represents a refinement and extension of the ' funnel effect ' first noted by Sandage (1957). He superimposed the C–M diagrams of a few clusters and discovered that the fainter red giants tended to concentrate at about $M_v = +0{\cdot}5$. Sandage used this empirical result to explain the observed luminosity function of stars of spectral types K0–K2 in the solar neighbourhood. A re-examination of this luminosity function should be worthwhile in view of the much more precise knowledge of evolution now available. Since the clump at $M_v \approx +1$ is a notable feature of the C–M diagram of any cluster more than $3 . 10^8$ years old, it should appear in the C–M diagram for any selection of red giants having ages between $3 . 10^8$ and 10^{10} years, i.e. born during the first 97 per cent of the life of the Galaxy. Presumably it should therefore be a prominent feature of the C–M diagram of stars in the solar neighbourhood.

An early H–R diagram for stars in the solar neighbourhood given by Adams *et al.* (1935) shows a strong concentration of stars at around $M_v = +0{\cdot}5$, spectral type K0 III, although this concentration is over-emphasized by the reduction procedure (Blaauw 1963). More recently, the C–M diagram given by Wilson (1959) shows a concentration of stars at about $M_v = +1$, $B\text{–}V = +1{\cdot}0$. The stars in both of these diagrams were presumably selected partly on the basis of apparent magnitudes, and therefore the volume of space sampled will be an increasing function of luminosity. Corrections for this volume effect were included by Halliday (1955) who gave luminosity functions for stars of spectral types G8–9 and K0–1. The G8–9 stars, corresponding to $B\text{–}V = +1{\cdot}0$, show a very sharp maximum at $M_v = +0{\cdot}8$, a decrease to near zero at $M_v = +2$, and a rapid increase again for fainter stars. The number of stars with $M_v = +3$ is already comparable to the number at the $M_v = +0{\cdot}8$ peak. For K0–1 stars, the luminosity function is qualitatively similar but with a less-marked peak which is about $0^{\mathrm{m}}{\cdot}5$ fainter.

The absolute magnitudes of the peaks in the luminosity functions found by Halliday are in excellent agreement with the mean luminosity of the clump found in Section 7. The observations of fainter stars are not sufficiently complete for a detailed analysis, but comparison with old clusters leads to the prediction that a second maximum will occur at about $M_v = +4$, the magnitude at $B\text{–}V = +1{\cdot}0$ of the horizontal section of the subgiant branch in the oldest open clusters such as NGC 188, and a third maximum at about $M_v = +6$ where main sequence dwarfs of type K0 are found.

Further possible support for the clump comes from the star counts in the nuclear bulge of our Galaxy, as reported by van den Bergh (1968), which indicate that the major contribution among the brighter stars comes from stars with $M_v \approx 0$ and $B-V \approx +1 \cdot 0$.

13. CONCLUSIONS

Many of the steps in the theoretical arguments presented here are uncertain, and will only be put on a sound basis when detailed advanced evolutionary calculations are available for low mass Population I stars. However, the agreement between theory and observation seems sufficiently good to make such calculations worthwhile, particularly in an attempt to improve understanding of the horizontal branch.

Observationally, accurate modern photoelectric observations in a few clusters would be useful to investigate the variation in absolute magnitude of the clump from cluster to cluster, and to find if there is any structure within the clump, which might be expected from some of the models of evolving horizontal branch stars.

The approximate constancy of the position of the clump in the C–M diagram indicates that it could be used to determine roughly the reddening and distance of the older clusters which are too distant for the observation of the unevolved main sequence, while its relative strength would give an indication of cluster age. For those clusters which can be well observed it should give a sensitive check on the internal composition of the stars.

It is believed that a large proportion of the giants with $B-V \approx +1 \cdot 0$ in the general field are probably helium burning horizontal branch-type stars, and as such should repay careful investigation. Spectroscopic evidence should help to determine the nature of the helium flash, and in particular whether any significant mixing occurs, while any estimates of their masses could help to determine what mass loss, if any, occurs during the flash.

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