Reduced acoustic cloaks based on temperature gradients

Victor M. García-Chocano, Daniel Torrent, and José Sánchez-Dehesa[a)]
Wave Phenomena Group, Departamento de Ingeniería Electrónica, Universitat Politècnica de València, Camino de Vera s.n. (Edificio 7F), E-46022 Valencia, Spain

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This letter presents the design of a reduced acoustic cloak that uses a temperature gradient in order to obtain sound speeds larger than in air. The cloak consists of a circular acoustic crystal made of ten concentric layers of rigid cylinders whose surfaces are heated or cooled in order to get the temperature gradient needed for cloaking behavior. The total pressure field produced by the scattering of sound waves impinging this complex structure is computed and it is shown how acoustic waves are bent in a way similar to that predicted for perfect cloaking devices. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4747197]

Cloaking is perhaps one of the more fascinating phenomena proposed in the acoustic realm by the transformation-based solutions.[1] Following the photonic cloaking predicted for electromagnetic (EM) waves, acoustic cloaking is designed to shield any object from external sound and, moreover, has also the property of restoring the wavefront of the impinging wave.[2,3] As in the case for EM waves, the physical realization of acoustic cloaks depends on the ability of engineering artificial structures named acoustic metamaterials or metafluids with the required parameters.

Acoustic cloaks based on metafluids with isotropic inertia and anisotropic stiffness have been also proposed by Norris,[4] but we are here interested with the realization of two-dimensional (2D) cloaking shells having inertia with cylindrical anisotropic (ρ_r, ρ_θ) and isotropic modulus (B). Their dependence on the distance r to the shell axis is[2]

$$\rho_r(r) = \frac{r}{r - R_a}\rho_0; \;\; \rho_\theta(r) = \frac{r - R_a}{r}\rho_0, \tag{1}$$

$$B(r) = \left(\frac{R_b - R_a}{R_b}\right)^2 \frac{r}{r - R_a} B_0, \tag{2}$$

ρ_0 and B_0 being the static parameters of the fluid background and $R_b - R_a$ being the shell thickness.

Anisotropy inertia is not a property of natural fluids, but it can be artificially obtained in one dimension (1D) by using a multilayer of two acoustic materials[5] and in 2D by using non-isotropic lattices of sonic scatterers embedded in a fluid of a gas background.[6] Moreover, the realization of cylindrical cloaks based on layered metafluids was proposed using alternating layers of two types of isotropic fluids: one with high density (ρ_+) and another with low density (ρ_-) whose expressions are[7,8]

$$\rho_\pm(r) = \rho_r(r) \pm \sqrt{\rho_r^2(r) - \rho_r(r)\rho_\theta(r)}, \tag{3}$$

where ρ_r and ρ_θ are given by Eq. (1). The main issue for the practical realization of this layered structure is the intrinsically instability of any mixture of fluids with different densities.

Acoustic cloaks with stable structures have been proposed in the framework of acoustic metafluids by Torrent and Sánchez-Dehesa[8] and by Pendry and Li,[9] who described how to engineer the parameters given in Eq. (3) using arrays of solid scatterers embedded in a fluid background. However, the acoustic cloaks experimentally demonstrated are based on different physical mechanisms. In particular, the one reported by Zhang and coworkers[10] consisted of an underwater corrugated waveguide while the directional cloak demonstrated by García-Chocano and coworkers[11] consisted of 120 cylindrical rods whose positions in air were obtained by an optimization algorithm. Very recently, a cloaking structure for elastic waves in thin plates has been constructed and characterized[12] following an earlier proposal by Farhat *et al.*[13]

In terms of sound propagation, it is important to remark that acoustic cloaks are materials in which sound propagates anisotropically with phase speed components given by

$$c_r = \sqrt{\frac{B(r)}{\rho_r(r)}} = \left(\frac{R_b - R_a}{R_b}\right)c_0, \tag{4}$$

$$c_\theta = \sqrt{\frac{B(r)}{\rho_\theta(r)}} = \left(\frac{R_b - R_a}{R_b}\right)\left(\frac{r}{r - R_a}\right)c_0, \tag{5}$$

where c_0 is the sound speed of the fluid background. Note that, inside the cloak, the angular speed of sound, c_θ, is always larger than c_0 and increases for distances $r \rightarrow R_a$, which is the condition allowing the bending of waves around the cloaked object.

The recent demonstration of inertial mass anisotropy of airborne sound propagating in 2D corrugated waveguides[14–16] also confirmed that phase velocity of sound inside a waveguide cannot be made larger than that of air. In other words, there is a basic physical principle showing that sound waves propagating along waveguides enclosed by rigid corrugated surfaces will never travel at velocities larger than that of the fluid background. It can be said that this is the main challenge to be solved when engineering an acoustic cloak for sound waves; it is required to find any physical mechanism making possible that sound waves travel at velocities larger than that

[a)]Author to whom correspondence should be addressed. Electronic mail: jsdehesa@upvnet.upv.es.

of the background, which could be air, water, or any other fluid.

This letter proposes using temperature as the physical mechanism allowing the design of cloaks for airborne sound in which the bending of sound waves is possible thanks to the fact that the sound speed can achieve values larger than that of the surrounded air. In fact, bending of ultrasonic waves by temperature gradients caused by cooling or heating a tube has been already demonstrated.[17,18]

The problem of engineering a fluid composite with alternating regions of high and low densities like those proposed in Eq. (3) can be solved by generating a temperature gradient with alternating layers of low and high temperature gradients in air since it is well known that the air density depends on temperature T.[19] The realization of this temperature gradient can be performed by fixing a set of discrete points at selected temperatures, which define the boundary conditions for the Laplace's equation satisfied by the temperature; i.e., $\nabla^2 T = 0$.

The proposal reported here considers cylinders acoustically rigid to create this temperature function. It is assumed that the surface of the cylinders can be fixed at any temperature and, therefore, a cluster of identical cylinders surrounding the cloaking object can be heated or cooled according to their positions to derive the required temperature gradient. Note that the density of the effective medium will depend on the temperature as well as on the filling fraction f of the cluster of cylinders. This approach allows in principle designing any mass density profile while the effective bulk modulus $B_{\rm eff}$, which is temperature independent, will be always a constant that only depends on f as follows:[20,21]

$$B_{\rm eff} = \frac{1}{1-f} B_0, \tag{6}$$

and therefore $B_{\rm eff} > B_0$ for any $f \neq 0$. Since the modulus of perfect cloak is a function that can take values lower than B_0 [see Eq. (2)], we concluded that perfect cloaks are not realizable by just using the temperature approach as described above.

However, the temperature gradient approach can be employed for the realization of a *reduced* cloak in similar grounds than that proposed and realized for EM waves.[22] In other words, a cloaking shell is here designed in such a way that the components of the tensorial phase speed are identical to that in Eq. (4) but with the following simplified acoustic parameters:

$$\hat{\rho}_r(r) = \alpha \left(\frac{R_b}{R_b - R_a} \right)^2 \rho_0, \tag{7}$$

$$\hat{\rho}_\theta(r) = \alpha \left(\frac{R_b}{R_b - R_a} \right)^2 \left(\frac{r - R_a}{r} \right)^2 \rho_0, \tag{8}$$

$$\hat{B}(r) = \alpha B_0, \tag{9}$$

where α is an arbitrary parameter that takes values $\alpha \geq 1$. Note that the only penalty for using the reduced set of material properties described above is a nonzero reflectance.

It must be stressed that reduced cloaks can be made for any possible value α. On the one hand, a large value of α implies a high value of f, which is a good condition to create a temperature gradient (i.e., there is less empty space between cylinders) but is bad regarding the impedance mismatch produced between cloak and background. On the other hand, a value of α near one produces a good impedance matching with the surrounding background but is worse regarding the temperature gradient. The choice in this work is $\alpha = 2.4$, leading to $f \approx 0.3$, obtained from the comparison between Eqs. (6) and (9):

$$f = 1 - \frac{1}{\alpha}. \tag{10}$$

Once f is fixed, the temperature of cylinders at the different concentric layers can be determined from the fact that the effective isotropic inertia of a cluster embedded in air at T is[21]

$$\rho_{\rm eff}(T) = \frac{1+f}{1-f} \rho_0(T), \tag{11}$$

where $\rho_0(T)$ is the static density of air, which can be calculated as[19]

$$\rho_0(T) = 1.29 \frac{273}{T} \frac{P_0}{0.76} = \tilde{\rho}_0 \frac{T_0}{T}, \tag{12}$$

where T is the absolute temperature in degrees Kelvin and P_0 is the barometric pressure in meters of mercury. The last term is a simplified version in which $\tilde{\rho}_0 \equiv 1.29\,{\rm kg/m^3}$, $T_0 \equiv 273\,{\rm K}$ and it has been considered that the cloak will be made in an ambient with $P_0 = 0.76$ m of Hg.

Now, the temperature at the different layers, $T\pm$, can be derived from

$$\frac{T_\pm(r)}{T_0} = \frac{1+f}{1-f} \frac{\rho_0}{\rho_\pm(r)}, \tag{13}$$

where $\rho_\pm(r)$ are the mass densities from Eq. (3). This equation is employed to fix the temperature of the cylinders belonging to the layers making the cloak.

The cylinders are arranged in a series of concentric layers surrounding the cloaked object. This distribution is named circular acoustic crystal (CAC) since it is similar to that of a circular photonic crystal.[23] The position of the mth cylinder in the ℓth layer is given by

$$\hat{\boldsymbol{r}}_{\ell m} = a(\ell + \ell_0) \cos \frac{2m\pi}{6(\ell+\ell_0)} \hat{\boldsymbol{x}} + a(\ell + \ell_0) \sin \frac{2m\pi}{6(\ell+\ell_0)} \hat{\boldsymbol{y}}, \tag{14}$$

where a is the radial distance between concentric layers, ℓ_0 is an integer defining the cloak inner radius ($R_a = a\ell_0$), and m is an integer taking values $m = 1, 2, \ldots, 6(\ell + \ell_0)$. The number of concentric layers N defining the shell is determined from the condition $R_b = aN + R_a$. Therefore, for a cloaking shell with given inner and outer radii, R_a and R_b, respectively, the number N of the layers completely defines the cylinders' position in the CAC. Note that a also defines the angular distance between cylinders in the same layer since cylinders are placed in the circles of radius $|\hat{\boldsymbol{r}}_{lm}| = a(\ell + \ell_0)$ and the angular distance between them is $\Delta\varphi_{\ell m} = 2\pi 6(\ell + \ell_0)$; thus, the arc length between two nearest-neighbor cylinders is $d = |\hat{\mathbf{r}}_{\ell m}|\Delta\varphi_{\ell m} = 2\pi a/6$.

The total number of cylinders M in a CAC cluster can be easily calculated by considering that the number of cylinders in the ℓ-layer is $6(\ell + \ell_0)$ and for the case of N layers,

$$M = \sum_{\ell=1}^{N} 6(\ell + \ell_0) = 3N(N+1) + 6N\ell_0. \quad (15)$$

In this work, we have considered a shell with $R_b = 3R_a$ that is made of $N = 20$ concentric layers. Consequently, $\ell_0 = 10$, $a = R_a/10$, and $M = 2460$ cylinders. However, since we need that the temperature from $r = R_b$ to ∞ be equal to that of the background, we need to add an additional layer of cylinders fixed at this temperature; otherwise, the temperature would decrease slowly depending on the temperature of the last layer. This additional layer makes that the total number of cylinders in the cloak be $M = 2646$.

The continuous line in Figure 1 represents the required temperature profile for a 20-layer reduced cloak with $R_b = 3R_a$ while the dot symbols depict the temperature obtained by heating and cooling the cylinders in the cluster. The last solution has been found by solving Laplace's equation by COMSOL Multiphysics. Note that there is a perfect agreement between the required and the obtained temperature function. The inset shows the resulting inhomogeneous temperature in logarithmic scale. It is evident that a layered temperature profile has been created and, consequently, a layered mass density profile is also expected.

In order to check the cloaking performance of our structure, we have made full wave simulations using COMSOL Multiphysics in which we have combined the inhomogeneous mass density created by the temperature gradient and the scattering properties of the cluster of cylinders. Thus, for a given solution $T = T(x,y)$ obtained by solving Laplace's equation, a background density and a bulk modulus with the form $\rho(x,y) = \tilde{\rho}_0 \frac{T_0}{T(x,y)}$ and $B(x,y) = B_0$, respectively, are derived. Afterwards, the scattering of sound waves impinging the bare object is computed and it is compared with the fields scattered by the object with the reduced cloak as well as with the perfect cloak.[8] We have studied the backscattered field as the parameter characterizing the performance of the cloak.

Figure 2 shows the backscattered fields produced by a bare rigid cylinder in air, by the cylinder surrounded with the reduced cloak designed above and by a 20-layer thick perfect cloak proposed in Ref. 8. We can observe that the reduced cloak is able of reducing the backscattered field produced by the bare cylinder in some frequency bands, where its performance is comparable to that of the ideal cloak with the same number of layers. Since the performance of a layered cloaking shell is a function of the number of layers, as it was demonstrated in Ref. 8, a large number of layers should be employed to improve the cloaking performance of the studied cloaks. However, we consider that the small number of layers here analyzed is enough as a proof of concept and, moreover, the calculated reduction in the backscattered field can be enough for a practical demonstration.

Figure 3 shows the real part of the total pressure obtained when a plane wave with wavelength $\lambda = 3R_a$ arriving from the left impinges the object with the reduced cloaking shell. Left panels show the numerical simulations for the layered reduced cloak, while right panels show the simulations for the system of rigid cylinders with the designed temperatures. Plots are shown at three different times in order to observe the evolution of the wavefront. It is clear that the field distributions are quite similar, showing that the layered shell can be properly created by temperature gradients. Also we observe how the wavefront surrounds the object and is partially reconstructed at the outer side of the cloak. Note that some scattered field appears due to the fact that a

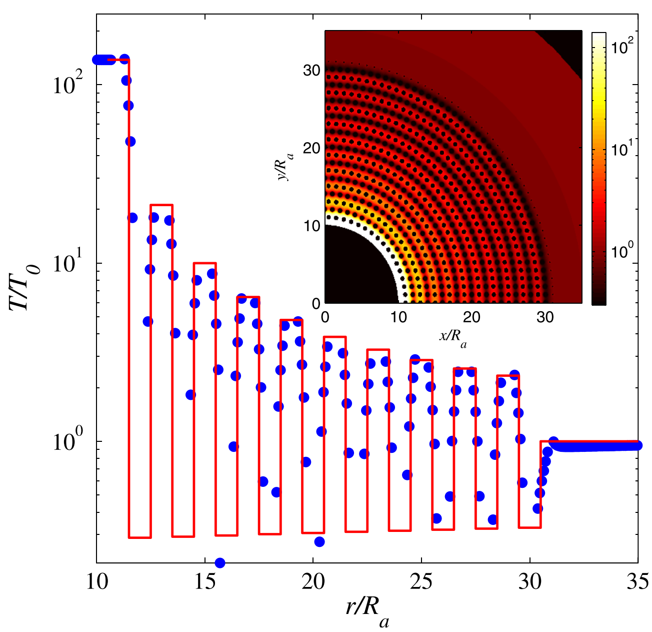

FIG. 1. Temperature profile of the reduced acoustic cloak (continuous line) compared with the profile obtained by heating or cooling the cylinders belonging to the concentric layers forming the cloak (dots). The inset shows the two dimensional temperature distribution.

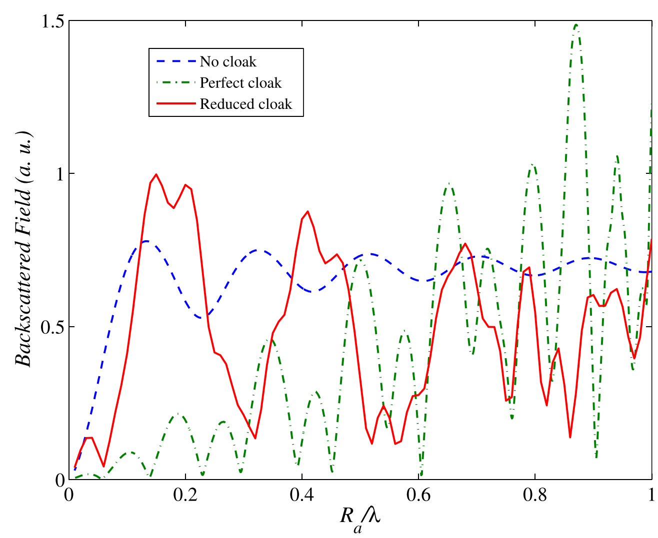

FIG. 2. Frequency dependence of the backscattered field produced by a rigid cylinder with radius R_a (dashed line) and its comparison with the fields produced by the cylinder surrounded with a perfect cloak made of a layered shell with 20 layers [6] (dotted line) and by the 20 layers thick reduced cloak proposed here (continuous line).

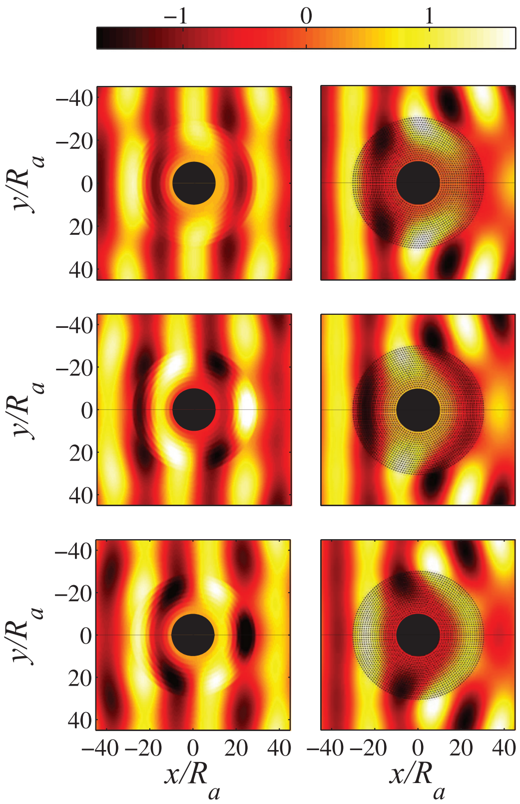

FIG. 3. Snapshots of the total pressure pattern (real part) produced by the scattering of a plane sound wave, with wavelength $\lambda = 3R_a$, impinging a reduced cloak made of layered metafluids (left panels) and the corresponding cloak made a cluster of heated cylinders (right panels). Simulations are shown for three consecutive times: $t = 0$ (upper panel), $\pi/2$ (middle panel), and π (lower panel).

reduced cloak has an unavoidable impedance mismatch with the surrounded fluid, an effect not existing for perfect cloaks. Also, the small number, N, of layers in the cloak affects its performance, which can be improved by increasing N.

In summary, we have designed a type of reduced acoustic cloak based on tailoring a temperature gradient. A layered dynamical mass density is designed here by heating or cooling the cylinders placed in concentric circles surrounding the cloaked object. The field backscattered by this reduced cloak is computed by full wave simulations and compared with that of a perfect cloak with the same number of layers. It has been shown that the reduced cloak reduces the backscattered field of the bare object as the perfect cloak does but only in certain frequency bands. A better performance is expected by increasing the number of concentric layers. Although the reported cloak is just a theoretical proposal that could be considered as hardly realizable, it contains a physical mechanism (the temperature) providing a sound speed larger than that of the surrounding background. We expect that our study will motivate further research work proposing alternative mechanisms giving more feasible cloaking structures, or other acoustic devices with inhomogeneous acoustic parameters that could be tailored using temperature.

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