Reduced Form vs. Structural Models of Credit Risk: A Case Study of Three Models

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Abstract

In this paper, we empirically compare two structural models (basic Merton and Vasicek-Kealhofer (VK)) and one reduced-form model (Hull-White (HW)) of credit risk. We propose here that two useful purposes for credit models are default discrimination and relative value analysis. We test the ability of the Merton and VK models to discriminate defaulters from non-defaulters based on default probabilities generated from information in the equity market. We test the ability of the HW model to discriminate defaulters from non-defaulters based on default probabilities generated from information in the bond market. We find the VK and HW models exhibit comparable accuracy ratios on both the full sample and relevant sub-samples and substantially outperform the simple Merton model. We also test the ability of each model to predict spreads in the credit default swap (CDS) market as an indication of each models strength as a relative value analysis tool. We find the VK model tends to do the best across the full sample and relative sub-samples except for cases where an issuer has many bonds in the market. In this case, the HW model tends to do the best. The empirical evidence will assist market participants in determining which model is most useful based on their purpose in hand. On the structural side, a basic Merton model is not good enough; appropriate modifications to the framework make a difference. On the reduced-form side, the quality and quantity of data make a difference; many traded issuers will not be well modeled in this way unless they issue more traded debt.

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Introduction

Complete realism is clearly unattainable, and the question whether a theory is realistic enough can be settled only by seeing whether it yields predictions that are good enough for the purpose in hand or that are better than predictions from alternative theories. (Friedman, 1953)

This insight presented decades ago applies to the current debate regarding structural and reduced-form models. While much of the debate rages about assumptions and theory, relatively little is written about the empirical application of these models. The research results highlighted in this paper improve our understanding of the empirical performance of several widely known credit pricing models. In this way, we can better evaluate whether particular models are good enough for our collective purpose(s) in hand. Let us begin this discussion by considering some of the key theoretical frameworks developed for modeling credit risk.

Credit pricing models changed forever with the insights of Black and Scholes (1973) and Merton (1974). Jones, Mason and Rosenfeld (1984) punctured the promise of these "structural" models of default by showing how these types of models systematically underestimated observed spreads. Their research reflected a sample of firms with simple capital structures observed during the period 1977 to 1981. Ogden (1987) confirmed this result finding that the Merton model under-predicted spreads over U.S. treasuries by an average of 104 basis points. KMV (now Moody's KMV or MKMV) revived the practical applicability of structural models by implementing a modified structural model called the Vasicek-Kealhofer (VK) model (see Crosbie and Bohn (2003), Kealhofer (2003a), Kealhofer (2003b), and Vasicek (1984)). This VK model is combined with an empirical distribution of distance-to-default to generate the commercially available *Expected Default Frequency*™or *EDF*™credit measure. The VK model builds on insights gleaned from modifications to the classical structural model suggested by other researchers. Black and Cox (1976) model the default-point as an absorbing barrier. Geske (1977) treats the liability claims as compound options. In this framework, Geske assumes the firm has the option to issue new equity to service debt. Longstaff and Schwartz (1995) introduce stochastic interest rates into the structural model framework to create a two-factor specification. Leland and Toft (1996) consider the impact of bankruptcy costs and taxes on the structural model output. In their framework, they assume the firm issues a constant amount of debt continuously with fixed maturity and continuous coupon payments. Collin-Dufresne and Goldstein (2001) extend the Longstaff and Schwartz model by introducing a stationary leverage ratio, allowing firms to deviate from their target leverage ratio in the short run, only.

While empirical evidence is still scant, a few empirical researchers have begun to test these model extensions. Lyden and Saraniti (2000) compare the Merton and the Longstaff-Schwartz models and find that both models under-predicted spreads; the assumption of stochastic interest rates did not seem to change the qualitative nature of the finding. Eom, Helwege, and Huang (2003) find evidence contradicting conventional wisdom on the bias of structural model spreads. They find structural models that depart from the Merton framework tend to over-predict spreads for the debt of firms with high volatility or high leverage. For safer bonds, these models, with the exception of Leland-Toft, under-predict spreads.

On the commercial side MKMV offers a version of the VK model applied to valuing corporate securities, which is built on a specification of the default-risk-free rate, the market risk premium, liquidity premium, and expected recovery in the context of a structural model. The VK model framework is used to produce default probabilities defined as EDF credit measures and then extended to produce a full characterization of the value of a credit risky security. This model appears to produce unbiased, robust predictions of corporate bond credit spreads. (see Bohn (2000) and Agrawal, Arora, and Bohn (2004) for more details.) Some important modifications to the typical structural framework include estimation of an implicit corporate-risk-free reference curve instead of using the U.S. treasury curve. Some of the under-prediction found in the standard testing of the Merton model likely results from choosing the wrong benchmark curve in the sense that the spread over U.S. treasuries includes more than compensation for just corporate credit-risk. The assumption here is that the appropriate corporate default risk-free curve is closer to the U.S. swap curve (typical estimates are 10 to 20 basis points less than the U.S. swap curve.) The MKMV implementation of the VK model allows for a time-varying market risk premium, which materially improves the performance of the model. Other important modifications to the framework include the specification of a liquidity premium that may be associated with the firm's access to capital markets and the assumption of a time-varying expected recovery amount. All these modifications contribute to producing a more usable structural model.

The structural model is particularly useful for practitioners in the credit portfolio and credit risk management fields. The intuitive economic interpretation of the model facilitates consistent discussion regarding a variety of credit risk exposures. Corporate transaction analysis is also possible with the structural model. If an analyst wants to understand the impact on credit quality of increased borrowing, share repurchases, or the acquisition of another firm, the structural model naturally lends itself to understanding the transaction's implications. In general, the ability to diagnose the input and output of the structural model in terms of understandable economic variables (e.g. asset volatility as a proxy for business risk, the market's assessment of an enterprise's value, and the market leverage) facilitates better communication among loan originators, credit analysts, and credit portfolio managers.

The other major thread of credit risk modeling research focuses on "reduced-form" models of default. This approach assumes a firms default time is inaccessible or unpredictable and driven by a default intensity that is a function of latent state variables. Jarrow, Lando, and Turnbull (1995), Duffie and Singleton (1999), and Hull and White (2000) present detailed explanations of several well known reduced-form modeling approaches. Many practitioners in the credit trading arena have tended to gravitate toward this modeling approach given its mathematical tractability. Jarrow and Protter (2004) argue further that reduced-form models are more appropriate in an information theoretic context given that we are unlikely to have complete information about the default point and expected recovery. Strictly speaking, most structural models assume complete information.[1] Jarrow and Protter's claim rests on the premise that a modeler only has as much information as the market making the reduced-form approach more realistic. In practice, however, the complete information assumption in structural models is an approximation designed to facilitate a simpler way of capturing the various economic nuances of how a firm operates. The strength or weakness of a model should be evaluated on its usefulness in real world applications. A reduced-form model, while not compromising on the theoretical issue of complete information, suffers from other weaknesses including lack of clear economic rationale for defining the nature of the default process.

Reduced-form models are characterized by flexibility in their functional form. This flexibility is both a strength and a weakness. Given the flexible structure in the functional form for reduced-form models, fitting a narrow collection of credit spreads is straightforward. Unfortunately, this flexibility in functional form may result in a model with strong in-sample fitting properties, but poor out-of-sample predictive ability. Since this type of model represents a generally atheoretic (i.e. less grounded in the economics driving default than in mathematical tractability) characterization of default risk, diagnosing how to improve peformance of these models can be challenging. Difficulties in interpretation of results are particularly acute when modeling large cross-sections of debt instruments– particularly when there is a high degree of heterogeneity in terms of credit quality. Without empirically testing the costs and benefits of any particular modeling approach, it is premature to draw conclusions based on purely theoretical arguments.

The empirical testing of reduced-form models is still nascent. The reason relates back to the lack of theoretical guidance on characterizing the default intensity process. Duffee (1999) found that the parameter estimates using a square-root process of intensity can be fairly unstable. Another reason is the bond data, on which these models are usually calibrated, is typically indicative in nature creating data problems as information slowly leaks into the price, which may produce misleading results.[2] Transaction bond price sources like TRACE may alleviate data problems, but these sources are new and do not provide detailed time-series of data. Other sources of bond data continue to be plagued by missing and mistaken data. A final reason involves the difficulty in empirically separating the merits of the modeling framework and the quality of the underlying data given that bond data are typically used to fit the model as well as test the model. Structural models based on equity price data will not suffer from this difficulty when they are then tested on bond data. The recent availability of credit default swap data provides a new opportunity to understand the power of both the structural and reduced-form modeling frameworks.[3]

The crucial question for academicians and practitioners alike is which modeling approach is better in terms of discriminating defaulters from non-defaulters and identifying relative value?

The objective of this paper is to shed empirical light on this question. We test the performance of a classic Merton model, the VK structural model as implemented by MKMV, and a reduced-form model based on Hull and White (2000) (HW) in separating defaulting and non-defaulting firms in the sample (also known as power-curve testing). We also look at the three models' performance in explaining the levels and cross-sectional variance of credit default swap data. These three models are chosen because they represent three key stages of the development of the literature in credit-risk modeling. The Merton model was the original quantitative structural approach for credit risk modeling. The VK model represents a more realistic and meaningful model for practitioners. (This model was the first commercially-marketed structural model.) The HW model is a reduced-form approach that was developed to address parameter stability problems associated with existing approaches as described in Duffee (1999).

The choice of credit default swap data for testing ensures a neutral ground on which the success of the different models can be evaluated. None of the models are calibrated on the data used for testing. This testing strategy enables us to avoid the pitfalls of testing models on data similar to the data used to fit the models. The structural models are estimated with equity data and the reduced-form model is estimated with bond data. In this way, we conduct a fair, out-of-sample test.[4]

The paper is arranged as follows: Section I describes the basic methodologies of the Merton, VK and HW models. Section II discusses the data and the empirical methodology used in the tests. Section III presents the results. We also elaborate on some of the robustness checks we conducted on our results in this section. Section IV concludes.

I Merton, Vasicek-Kealhofer (VK) and Hull-White (HW) Models

I.i Merton Model

Merton (1974) introduced the original model that led to the outpouring of research on structural models. Merton modeled a firm's asset value as a lognormal process and assumed that the firm would default if the asset value, A, falls below a certain default boundary X. The default was allowed at only one point in time, T. The equity, E, of the firm was modeled as a call option on the underlying assets. The value of the equity was given as:

$$E = A\Phi\left[d_1\right] - X_T \exp\left[-rT\right]\Phi\left[d_2\right] \tag{1}$$

where

$$d_1 = \frac{\log\left[\frac{A}{X}\right] + \left(\mu + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

and Φ represents the cumulative normal distribution function. The debt value, D, is then given by:

$$D = A - E \tag{2}$$

The spread can be computed as:

$$s = -\frac{1}{T}\log\left[\Phi\left[d_2\right] + \frac{A}{X}\exp\left[rT\right]\Phi\left[-d_1\right]\right] \tag{3}$$

where A is the initial asset value of the firm, X is the default barrier for the firm, i.e., if the firm's asset value A is below X at the terminal date T, then the firm is in default. μ is the drift of the asset return, and σ is the volatility of the asset returns.

We included this model in our analysis to start with a simple framework as an initial benchmark. A comparison of the performance of a Merton model with the MKMV implementation of the VK model (which reflects substantial modification to the basic Merton framework) will illuminate the impact of relaxing many of the constraining assumptions in the Merton framework.

I.ii VK Model

MKMV provides a term-structure of physical default risk probabilities using the VK model. This model treats equity as a perpetual down-and-out option on the underlying assets of a firm. This model accommodates five different types of liabilities: short-term liabilities, long-term liabilities, convertible debt, preferred equity and common equity. MKMV uses the option-pricing equations derived in the VK framework to derive a firms market value of assets and its associated asset volatility. The default point term-structure (i.e. the default barrier at different points in time in the future) is determined empirically. MKMV combines market asset value, asset volatility, and the default point term-structure to calculate a Distance-to-default (DD) term-structure. This term-structure is translated to a physical default probability using an empirical mapping between DD and historical default data.

$$DD_T = \frac{log\left[\frac{A}{X_T}\right] + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \tag{4}$$

X_T in VK model has a slightly different interpretation than in Merton model. In the analytical stage of the model, if asset value A falls below X_T at any point in time, then the firm is considered to be in default. In the DD-to-EDF empirical mapping step, VK model estimates a term-structure of this default barrier to come up with a DD term structure that could be mapped to a default-probability term-structure, hence the subscript T for default barrier X. The basic methodology is discussed in Crosbie and Bohn (2003), Kealhofer (2003a), and Vasicek (1984). The model departs from the traditional structural model in many ways. First, it treats the firm as a perpetual entity that is continuously borrowing and retiring debt. Second, by treating different classes of liabilities, it is able to capture richer nuances of the capital structure. Third, it calculates its interim asset volatility by generating asset returns through a de-levering of equity returns. This is different from the popular approaches that compute equity volatility and then de-lever it to compute the asset volatility. Fourth, it generates the final asset volatility by blending the interim empirical asset volatility as computed above together with a modeled volatility estimated on comparable firms. This step helps filter out noise generated in equity data series. The default probability generated by the MKMV implementation of the VK model is called an Expected Default Frequency or EDF credit measure. These modifications address many of the concerns raised by Eom, Helwege and Huang (2003) regarding the tendency of Merton models to over-estimate spreads for riskier bonds and under-estimate spreads for safer bonds. This estimation process also results in term structures of default probabilities that are downward sloping for riskier firms and upward sloping for safer firms. This pattern is consistent with the empirical credit-migration patterns found in the data.[5]

Once the EDF term-structure is obtained, a related cumulative EDF term structure can be calculated up to any term T referred to as $CEDF_T$. This is then converted to a risk-neutral cumulative default probability $CQDF_T$ using the following equation:

$$CQDF_T = N\left[N^{-1}\left[CEDF_T\right] + \lambda \bullet sqrt(R^2) \bullet sqrt(T)\right] \tag{5}$$

where R^2 is the square of correlation between the underlying asset returns and the market index returns, and λ is the market Sharpe ratio.[6]

The spread of a zero-coupon bond is obtained as:

$$s = -\frac{1}{T} log \left[1 - LGD \bullet CQDF_T\right] \tag{6}$$

where LGD stands for the loss given default in a risk-neutral framework. The floating leg of a simple CDS (i.e. a single payment of LGD paid out at the end of the contract with a probability of $CQDF_T$) can also be approximated with this relationship.

I.iii HW Model

Hull and White (2000) provide a methodology for valuing credit default swaps when the payoff is contingent on default by a single reference entity and there is no counterparty default risk. Instead of using a hazard rate for the default probability, this model incorporates a default density concept, which is the unconditional cumulative default probability within one period no matter what happens in other periods. By assuming an expected recovery rate, the model generates default densities recursively based on a set of zero-coupon corporate bond prices and a set of zero-coupon treasury bond prices. Then the default density term-structure is used to calculate the premium of a credit default swap contract. The two sets of zero-coupon bond prices can be bootstrapped from corporate coupon bond prices and treasury coupon bond prices.

They show the credit default swap (CDS) spread s to be:

$$s = \frac{\int_0^T \left[1 - \hat{R}\left(1 + A(t)\right)\right] q(t)v(t)dt}{\int_0^T q(t)\left[u(t) + e(t)\right]dt + \pi u(t)} \tag{7}$$

where:

T: the life of the CDS contract.

q(t): the risk-neutral default probability density at time t.

A(t): the accrued interest on the reference obligation at time t as a percent of face value.

π: the risk-neutral probability of no credit event over the life of the CDS contract.

w: the total payments per year made by the protection buyer.

e(t): the present value of the accrued payment from previous payment date to current date.

u(t): the present value of the payments at time t at rate of $1 on the payment dates.

$\hat{R}$: the expected recovery rate on the reference obligation in a risk-neutral world.

The risk-neutral default probability density is obtained from the bond data using the relationship

$$q_j = \frac{G_j - B_j - \sum_{i=1}^{j-1} q_i \alpha_{ij}}{\alpha_{jj}} \tag{8}$$

where α_{ij} is the present value of the loss on a defaultable bond j relative to an equivalent default-free bond at time t_i. α_{ij} can be described as:

$$\alpha_{ij} = v(t_i)\left[F_j(t_i) - R_j(t_i)C_j(t_i)\right] \tag{9}$$

C_j is the claim made on the j^{th} bond in the event of default at time t_i, while R_j is the recovery rate on that claim. F_j is the risk-free value of an equivalent default-free bond at time t_i, while $v(t_i)$ is the present value of a sure payment of $1 at time t_i.

In this framework, one can infer a risk-neutral default risk density from a cross-section of bonds with various maturities. As long as the bonds measure the inherent credit risk and have the same recovery as used in the CDS, one should be able to recover a fair price for the CDS based on the prices of the obligors traded bonds.

II Data and Empirical Methodology

II.i Data

The data set consists of bond data for each models implementation as described in the preceding section combined with data on actual CDS spreads used to test each models predicted prices. Corporate bond data were provided by EJV, a subsidiary of Reuters. The data include prices quoted daily from 10/02/2000 to 6/30/2004. We selected US dollar-denominated corporate bonds, only. There are 706 firms that have at least two bonds that can be used for bootstrapping. CDS data are from CreditTrade and GFInet, two active CDS brokers. Of the 706 firms, 542 firms have CDS data. Therefore, we restricted our analysis to these 542 firms. The final sample tested represented a reasonable cross-section of firms with traded debt and equity mitigating concerns about biases arising from sample seleciton. While the EJV data are indicative and subject to the price staleness concerns described above, the CDS data are likely to be closer to actual transacted prices. (CDS tend to trade more often than bonds.) The lagging nature of the bond data will be somewhat of a handicap for the HW reduced-form model calibrated on that data. Part of the objective of this study is to present model estimations as they would be done in practice so the nature of the available data is as relevant as the estimation approach.

The Merton default-probabilities and implied spreads are generated using time-series of equity data and financial statements on the sample of firms from COMPUSTAT. The VK daily default probabilities are MKMV EDF credit measures.

The period of study is particularly interesting as it covers two years of recession in many industrialized countries and includes several firms with substantial credit deterioration (e.g. Ahold, Fiat, Ford, Nortel and Sprint.) This time period also includes firms embroiled in major accounting and corporate governance scandals such as Enron and WorldCom. We required each firm to have a minimum of 2 bonds to be included in our data set. The highest number of bonds tracked for any particular issuer in this data set is 24.

II.ii Methodology

For the Merton model, we used the default point as 80% of the overall liabilities of the firm. We chose this number because it performed the best in terms of default predictive power. The concept of default predictive power is discussed in the next section. Regardless, we tested other default point specifications and found that the qualitative nature of the results are not sensitive to how we specify the default point.

For the VK model, we use a market Sharpe Ratio of 0.45 and an LGD of 0.60. MKMV provides a time-series of Sharpe ratios and sector based LGDs which are calibrated from bond data. We specified a constant Sharpe Ratio and a constant LGD so that our test focuses on the main driver of spreads– default probabilities– and reduces the risk of biasing the tests. Using a dynamic time-series of these parameters would have made the results look more favorable for the VK structural approach. The methodology is described in detail in Kealhofer (2003a).

For the reduced-form model, we first derived the corporate zero-rates[7] by bootstrapping. This procedure facilitates calculating zero-coupon yield curves from market data. For each firm, all senior-unsecured straight bonds (i.e. bonds without embedded options, such as those deriving from convertibility and callability) are ranked by their time to maturity from 1 to 6 years. For each 3-month interval, at most one bond is selected. To estimate the zero-rate, a firm must have at least two bonds coming out of the above process.

The bootstrapping procedure is done using a MATLAB financial Toolbox function that takes bond prices, coupon rates, maturities, and coupon frequencies as inputs. The procedure is described in detail in Hull (1999). Using this procedure, we obtain zero-rates corresponding to bond maturities.

Since many firms have 2 to 5 bonds and the zero-rates from bootstrapping are very noisy, a linear interpolation of these zero-rates may lead to unrealistic forward prices. To mitigate this issue, we use a two-degree polynomial function to approximate the zero-curve. We then use the fitted function to generate the zero-rates every 3 months. Corporate zero-coupon bond prices are calculated using the three-month interval zero-rates.

We obtain the treasury zero-rates from 1 month to 30 years from Bloomberg. Treasury zero-coupon bond prices and forward prices are obtained from the risk-free zero curve.

Tables 1(a) through (c) show the descriptive statistics of the bond and CDS data. In Table 1(a), we see that more than 28% of the sample have only 2 bonds usable for bootstrapping. About 4% of the firms in sample have more than 20 bonds that were used for bootstrapping. This limited number of bonds per issuer poses difficulties given that the accuracy of the implied default probability density depends on the number of bonds available. Table 1(b) shows that the time-to-maturity of the bonds is fairly uniformly distributed between 0 and 6 years with the density dropping off at the extremes.

Table 1(c) shows that the underlying issuers span a wide cross-section of financial health, as measured by CDS spreads.

III Results

III.i Default Predictive Power of Models

We first test the ability of the three models to predict defaults. We do this by rank ordering the firms in our sample in their probability of default from highest to lowest. We then eliminate x% of the riskiest firms from our sample and compute the number of actual defaults that were avoided using this simple rule. This number is expressed as a percentage of the total number of defaults, y%. We vary x from 0 to 100 and find a corresponding y for each x.

Ideally, for a sample of size N with D defaults in it, when $x = \frac{D}{N}$ then y should be 100%. This would imply that the default predictive model is perfect, i.e. each firm eliminated would, in fact, be part of the group that actually defaults. The larger the area under the curve of y against x, the better the models power to minimize both Type-I error (holding a position in a firm that later defaults) and Type-II error (avoiding a position in a firm that does not default.) This area is defined as the models accuracy ratio (AR) (See Stein (2002) and Stein (2003) for a more detailed discussion of default model performance evaluation.)

For a random default-risk measure without any predictive power, the x-y graph should be a 45 degree straight line. The more area between the 45 degree line and the power curve, the more accurate the measure.

There are two caveats in interpreting results from this test. First, it is a limited sample: data on bonds and CDS are restricted to many fewer firms than the data available for equities. Of course, a firm that does not issue tradeable bonds may not be as interesting to practitioners; however, the increasing interest in trading bank loans and devising new synthetic credit instruments creates demand for analytics to evaluate instruments for which traded bonds and CDS are not available. Given the large potential for trading credit risk beyond bonds, the applicability of an equity-based structural model to a much more extensive dataset is critical to expanding the coverage of firms and developing market liquidity. The second caveat is that reduced-form models are designed to provide risk-neutral probabilities of default. The order of these might not be the same as that of physical probabilities. For example, a firm with a low physical probability of default but high systematic risk in its asset process might have a higher risk-neutral probability of default compared to a firm with a relatively higher physical probability of default but with no systematic risk in its asset process. As credit investors move toward building portfolios with more optimal return-risk profiles, distinguishing physical default probabilities becomes critical. In this test, we make the strong assumption that the order stays the same.

Figure 1 shows the results of our default prediction test. In the above graph, there are three default-risk measures: default probability from the HW reduced-form model, default probability from a simple Merton structural model, and default probability from the VK model. All three measures are cumulative one-year risk-neutral default probabilities. As we can see, the VK structural model ranks the highest in its ability to predict default with an AR of 0.801. The HW model approach is not too far behind with an accuracy ratio of 0.785. The basic Merton model, however, is far behind with 0.652. This demonstrates that with proper calibration of default models, both equities and bonds can be effective sources for information about impending defaults.

Figure 2 shows the median default probabilities from the three models for every defaulted firm in the dataset before and after default. Month 0 reflects the month in which each company defaulted. Negative numbers reflect the number of months before default and positive numbers reflect the number of months after default. As we can see from the graph, for the limited sample where the defaulted companies did, in fact, have traded bonds, both the VK model and the reduced-form model predict defaults reasonably well. The classic Merton model trails the other two models in terms of signalling distress closer to the actual event of default.

III.ii Levels and Cross-Sectional Variation in CDS Spreads

We next test the ability of the three models to predict the CDS spread levels and explain the cross-sectional variation in CDS spreads. As discussed above, the conventional wisdom is that structural models, in general, underpredict actual credit spreads. If this were indeed the case, then one would expect that the CDS spreads predicted by the Merton model would generally underestimate the observed level of CDS spreads. For a modified structural approach, such as the VK structural model, previous empirical evidence (see Eom, Helwege, and Huang (2003)) suggest one would expect the model-implied CDS spreads to be underpredicted for safer firms and over-predicted for riskier firms. Some of this tendency can be explained by the functional form used for transforming physical default probabilities to risk-neutral default probabilities; the function in the VK framework may be increasing the risk-neutral probabilities too much for high-risk firms. That said, the model structure does not imply a particular bias in either direction.

There is no established pattern in the empirical literature with respect to the accuracy or bias of the spreads predicted by reduced-form models. If bond markets are a fair reflection of the inherent risk of a firm and the corporate bond spreads do, in fact, reflect primarily default risk, then the reduced-form model implied CDS spreads should be an unbiased predictor of CDS spreads. To the extent the reduced-form model is a biased predictor of CDS spreads, the likely cause is factors other than default risk (e.g. liquidity) driving spreads in the corporate bond market.

Figure 2(a) through (c) show the performance of the three models in their ability to predict the

CDS spreads. The graphs show

$$Error = CDS - Model\ CDS$$

as histograms for each model. Consistent with previously published research, we see that the Merton model substantially underpredicts the actual CDS spread, as demonstrated by the right side skew of the frequency chart. Both the reduced-form model and the VK structural model seem to be skewed towards the left side, indicating that these models overpredict CDS spreads. However, the skew is much larger for the reduced-form model, with more than 10% of the sample reflecting overestimation of CDS spreads by more than 200 bps. This compares to 3.5% of the sample where the VK structural model overestimates CDS spreads by more than 200 bps.

The median error in the case of the VK structural model is -33.28 bps, which seems to be consistent with the claim of Eom, Helwege and Huang (2003) that more sophisticated structural models overestimate credit risk. This bias is smaller than the median error of -72.71 bps found for reduced-form models. Consistent with the existing literature, the median error in case of the Merton model is a positive 53.99 bps, implying that the Merton model does underestimate credit-risk, even when measured by CDS spreads. Note also that the VK model generated the smallest median absolute error.

Finally, Figure 3 shows the time-series of the correlation of market CDS spreads with modeled CDS spreads. This correlation reflects the ability of a model to explain the cross-sectional variation of market CDS spreads. If the modeled CDS spreads are the same as the market CDS spreads, then the correlation will be exactly 1. In general, a better correspondence between the levels of modeled with realized market spreads will lead to a higher correlation. As evident from this figure, the VK model performs the best in this regard. Surprisingly, the HW reduced-form model performs the worst. Table 3(a) shows the median of the slope coefficients that result from regressing the market CDS on the modeled CDS cross-sectionally on a daily basis. In general, if the model is an unbiased estimator of the realized spread, then this slope should be 1. While both the VK and Merton models deviate from 1 with median slopes at 0.76 and 1.26 respectively, the HW model's median slope is at 0.07, which seems unusually low. The median R-squareds of these regressions (which should also be the squares of the correlation between the market and modeled spreads) of the HW model, the VK model and the Merton model stand at 0.09, 0.48 and 0.26 respectively, once again demonstrating the strength of the VK structural model in explaining the cross-sectional variation of CDS spreads.[8] These results are particularly striking given that VK-model-based EDF credit measures rely only on equity market input.

III.iii Robustness Tests

In this section, we subject our results to more scrutiny by analyzing various sub-sets of the data to determine whether our results are attributable to the presence of outliers.

One possible explanation for our results may be that healthier firms behave differently than riskier firms. We conduct our tests again on these two subsets of firms separately. Our proxy for a relatively healthy firm is one where the CDS spread is less than 100 bps. As seen in Table 1(c), about 73% of the sample falls in the healthier category. This percentage is in line with our expectations; most of the CDS data are concentrated among larger and less risky firms. Table 2(a) shows that the negative bias of the VK model is typically evident for firms with CDS spreads below 100 bps. For riskier firms, the modeled spreads are largely unbiased with the median error around -7.6 bps. This result is inconsistent with the claim of Eom, Helwege and Huang (2003), that more sophisticated structural models underpredict credit risk at lower levels of risk, and overpredict credit risk at higher levels of credit risk. In comparison, the Merton model consistently underpredicts credit risk, as can be seen by the positive errors for both classes of CDS. The HW reduced-form model overpredicts credit risk for both classes. This overprediction is significantly high and even the 75^{th} percentiles of errors are fairly negative for both classes of CDS.

Table 4(a) highlights the ability of the models to explain the cross-sectional variation of CDS spreads. We find that although on aggregate, the HW model was outperformed by the Merton and VK models, on the subset of data with CDS $<$ 100 bps, the HW model outperforms the Merton model. The VK model outperforms the other two models in all other subsets of the data except the 75th percentile of firms with CDS $<$ 100 bps.

One possible explanation for the HW models difficulties may involve the fact that most issuers have few outstanding bonds. A larger number of bond issues will increase the efficiency of estimating the default density. To test for this possibility, we divided our sample into firms with 2 bonds, firms with 3 to 4 bonds, firms with 5 to 9 bonds, firms with 10 to 15 bonds, and firms with 16 to 24 bonds.[9]

Table 2(b) shows that the negative bias of the VK and HW models is present in all sub-samples of the data. This result indicates that the bias is not being caused by firms with few outstanding bonds. The negative bias of the HW model is larger compared to that of VK model for all data subsets. The Merton model, on the other hand, has a consistently positive bias in all categories, indicating that the model's underestimation of credit risk is fairly consistent across various sub-samples of the data.

Table 4(b) examines the impact of the availability of information, as measured by the number of bonds by an issuer, on the ability of the HW model to explain the cross-sectional variation of the CDS spreads. Interestingly, we find that the HW model outperforms the other two models when there are more than 10 bonds available to calibrate the default probability density. This improvement most likely results from the greater amount of cross-sectional information, in terms of the number of bonds, available to calibrate the default probability density. Similarly, in Table 3(b), we see that the sensitivity of the realized CDS spread to modeled spread, as measured by the median slope of cross-sectional regression of market spreads on model spreads, increases with the number of bonds available.

The ability of both the VK and the Merton model to explain the cross-sectional variation declines among firms with more than 15 bonds issued, as can be observed by the lower R-squareds. This result is surprising given that these models do not use bond information in their calibration. The result is most likely being driven by the fact that firms that issue such large numbers of bonds have other variables impacting their spreads (e.g. interest-rate risk). These firms account for only 5% of the data as can be seen from Table 1(a). That said, in terms of debt outstanding, these firms constitute a larger percentage (but not a majority) of the dollar amount of corporate debt outstanding. For example, on June 2004, firms that had more than 10 bonds used in bootstrapping (after applying all the filters) represented 40% of the total amount outstanding in our sample. Similarly, firms that had more than 15 bonds available for bootstrapping represented about 28% of the amount outstanding. Regardless of the measure, the majority of the firms tested in this research exercise did not have a large number of bonds outstanding.

We have seen that the larger number of bonds available for bootstrapping (all else equal) should improve the performance of the model. Further characterization of the sample of prolific issuers helps us interpret the results on other drivers of model performance. Large firms tend to be the ones that issue more bonds[10]. Table 5(a) shows that large firms account for relatively more of the prolific issuers than small firms. Large issuers also tend to be less risky. Table 5(c) demonstrates this fact by showing the distribution of CDS spreads for the entire sample compared to the distribution of CDS spreads for the largest firms (defined by $log(size) > 11$). The large-firm sample does, in fact, have a higher percentage of firms with lower CDS spreads. We also find that large issuers tend to issue debt of a similar duration as smaller firms. This can be seen from the comparison of the distribution of time to maturity across bonds of large firms and bonds of the overall sample in Table 5(b). Therefore, as a percentage contribution to the overall spread, interest rate risk may dwarf default risk for these types of larger and safer issuers. Corporate bonds issued by small firms tend not to be transacted as much making liquidity risk relatively more important in determining their spreads. These circumstances create countervailing influences on the performance of the HW model using bond data to fit CDS spreads. To the extent that interest rate risk or liquidity risk overpowers default risk as the primary driver of CDS spreads, a HW model calibrated on bond data will not perform as well. Given these circumstances, we cannot predict ex ante how firm size will impact model performance.

We test the impact of size on model performance by further dividing the subsamples based on the number of bonds available for bootstrapping. This procedure is designed to isolate the effect of size from the effect of number of bonds available for bootstrapping (since the two are somewhat positively correlated.) For each subsample, we compute the median size of the firms. We then divide each subsample into two groups: one group where firm size exceeds the subsample median and the other group where firm size is less than the subsample median. Table 6(a) reports the performance of each model in explaining the cross-sectional variation in CDS spreads (as measured by the R-squareds of the regression of CDS spreads on modeled spreads) across the two size categories for each subsample.

The VK model performs fairly consisently across the different subsamples. The HW model performance is a little more varied. As the number of bonds outstanding increase, the HW model performs progressively better. The classic Merton model consistently underperforms the other models in almost every subsample. One interesting pattern is the HW outperformance for large, profilic issuers. We suspect these results reflect the extent to which default risk is not a primary determinant of spreads for large, low-risk, prolific issuers of long-term debt. The performance of the structural models relies on the sensitivity of CDS spreads to changes in the equity-based default probability measures. The standard Merton model reflects primarily equity price movement which may not be a primary determinant of CDS spreads for large firms. The VK model, on the other hand, includes modifications to the specification of the default point, the estimation of asset volatility, and the interaction of firm asset value and the default model in such a way so as to capture more of the determinants of CDS spreads for all firms, regardless of size. As a result, its performance is consistent across subsamples. For the most prolific issuers (i.e. outstanding bonds greater than 16), we should interpret the results with care given the small number in this subsample.

Table 6(b) reports some of the characteristics of the cross-sectional distribution of the CDS spreads in this sample, categorized by firms with different numbers of bond issues available. As we move to firms with a larger number of bonds, the CDS spreads are less diverse, as indicated by the standard deviation and the inter-quartile range ($|p75 - p25|$) of the spreads. There is also a substantial reduction in the number of CDS obervations available on a daily basis (as can be seen by the median number of observations each day). R-squared is not a reliable statistic when the variation in dependent variable (in this case, the CDS spreads) is low, or the number of observations are low. Our sample suffers from both these inadequacies. For example, upon breaking the sample of firms with more than 15 bonds, into two halves according to size (as in Table 6(a)), the median daily number of observations in each half is about 9. This, along with the lowest coefficient of variation $(\frac{StandardDeviation}{Mean})$ leads to an unreliable R-squared measure. The subsamples for firms with fewer numbers of bonds outstanding do not suffer from this particular difficulty. All results should be considered with these characteristics in mind.

In summary, the testing demonstrates that for the vast majority of firms, a structural model, such as the VK model developed at MKMV, calibrated on a time-series of equity data works better in measuring credit risk relative to a HW reduced-form model calibrated on a cross-section of corporate bonds. The VK model substantially outpeforms a simple implementation of the standard Merton model. For practitioners looking across the broad cross-section of traded credit instruments, the data requirements for robust reduced-form modeling and the availability of robust equity-based measures should inform discussions of which modeling approach to use. Moreover, users of reduced-form models to price other credit-risky securities like CDS should bear in mind the potential impact on bond spreads by other forms of risks like interest rate risk and liquidity risk. These effects can be different across size and spread levels, and might therefore distort the performance of these models. A model like VK is relatively more stable in its performance across various categories by size and spreads. The model's strength is partially due to its structural framework that uses equity data, which are less contaminated by other risks, and partially due to its more sophisticated implementation.

IV Conclusion

In this paper, we empirically test the success of three models in their ability to measure credit risk. These models are the Merton model, the Vasicek-Kealhofer (VK) model, and the Hull-White (HW) model. These three models were chosen because they represent three key stages in the development of the theoretical literature in credit risk. These models also represent two main approaches for credit-risk modeling: the structural approach and the reduced-form approach.

This research is the first attempt at testing these types of models on a broad cross-section of credit default swap data. The advantage of these data are that they are not used in the calibration of any of the models and so constitute a true out-of-sample test. Credit default swap spreads have also been accepted by researchers in both academia and the financial industry as efficient measures of credit risk.

We find that, despite the advantages stated by proponents of reduced-form models, a HW reduced-form model largely underperforms a sophisticated structural model like that of the VK model (as implemented by MKMV). Interestingly enough, the HW model outperforms the simple Merton model when a given firm issues a large number of bonds. In these cases of firms issuing more than 10 bonds at any given point in time, the HW model can also outperform (in terms of explaining the cross-sectional variation of CDS spreads) the more sophisticated VK structural model for some subsets of low risk corporate issuers. At this time, the number of such firms is small. In our sample, the HW model was more effective in its ability to explain the cross-sectional variation of the CDS spreads only for the largest 5% of the firms, in terms of the number of issues outstanding on which data were available. Even for this sample, the error in terms of the difference between actual and predicted levels of spreads was much larger for the HW model when compared to the VK model. The VK model consistently outperformed the other two models in terms of default predictive power. The performance of the VK model is more consistent across large and small firms, while the performance of the HW and Merton models worsens considerably across larger firms. The overall results emphasize the importance of empirical evaluation when assessing the strengths and weaknesses of different types of credit risk models.

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Notes

[1]Giesecke and Goldberg (2004) show that it is possible to develop a structural model in which the modeler also has incomplete information about the default point, making the time-to-default inaccessible even in a structural model. Duffie and Lando (2001) propose a hybrid model that assumes accounting information is noisy thereby making the default time inaccessible in the context of a structural model.

[2]Because indicative prices do not always reflect all the information in the market, a researcher may mistakenly conclude that a particular model predicts price changes when, in fact, an actual trade would have reflected an immediate price change eliminating any predictive power of the model. See Duffee (1999) for more discussion on this problem of stale bond prices.

[3]See, for example, Ericsson and Reneby (2004).

[4]The three models take different sources of data as inputs. While the Merton and VK models rely on equity data, HW model is calibrated with bond data. Therefore, any result in this test is a reflection on the framework as well as the quality of data available to the models as input. One of the strengths of a model, especially from a practitioner's point of view, is that it should yield accurate results based on data that are easily available and accessible. A conceptually powerful framework, while intellectually stimulating, can be fairly meaningless if any application of it relies on data that are either unavailable, or are of bad quality.

[5]Note that Helwege and Turner (1999) find evidence of upward sloping term structures of spreads for risky corporate bonds. When we look at the behavior of distance-to-default for firms, the downward sloping term structures of default probabilities for high risk issuers appear more consistent. More research is needed to reconcile these somewhat contradictory findings.

[6]The normal and normal inverse functions act as translators of the default probability estimate without requiring that the default probability estimate,itself, be generated from a normal distribution. In this case, the $CEDF$ was calculated from the empirical distribution estimated at MKMV.

[7]A zero-rate is the implicit interest on a zero-coupon bond of a given maturity.

[8]Median R-squareds of 0.09, 0.48, and 0.26 correspond to median correlations of approximately 30%, 70%, and 51%.

[9]The number of bonds represents the number available for bootstrapping after using all the filters described above– not the actual number of bonds outstanding.

[10]We measure the size of a firm by its book asset value. A large firm is considered to have book assets in excess of about $60 billion (log(60,000) is approximately 11; our data are reported in millions of dollars).

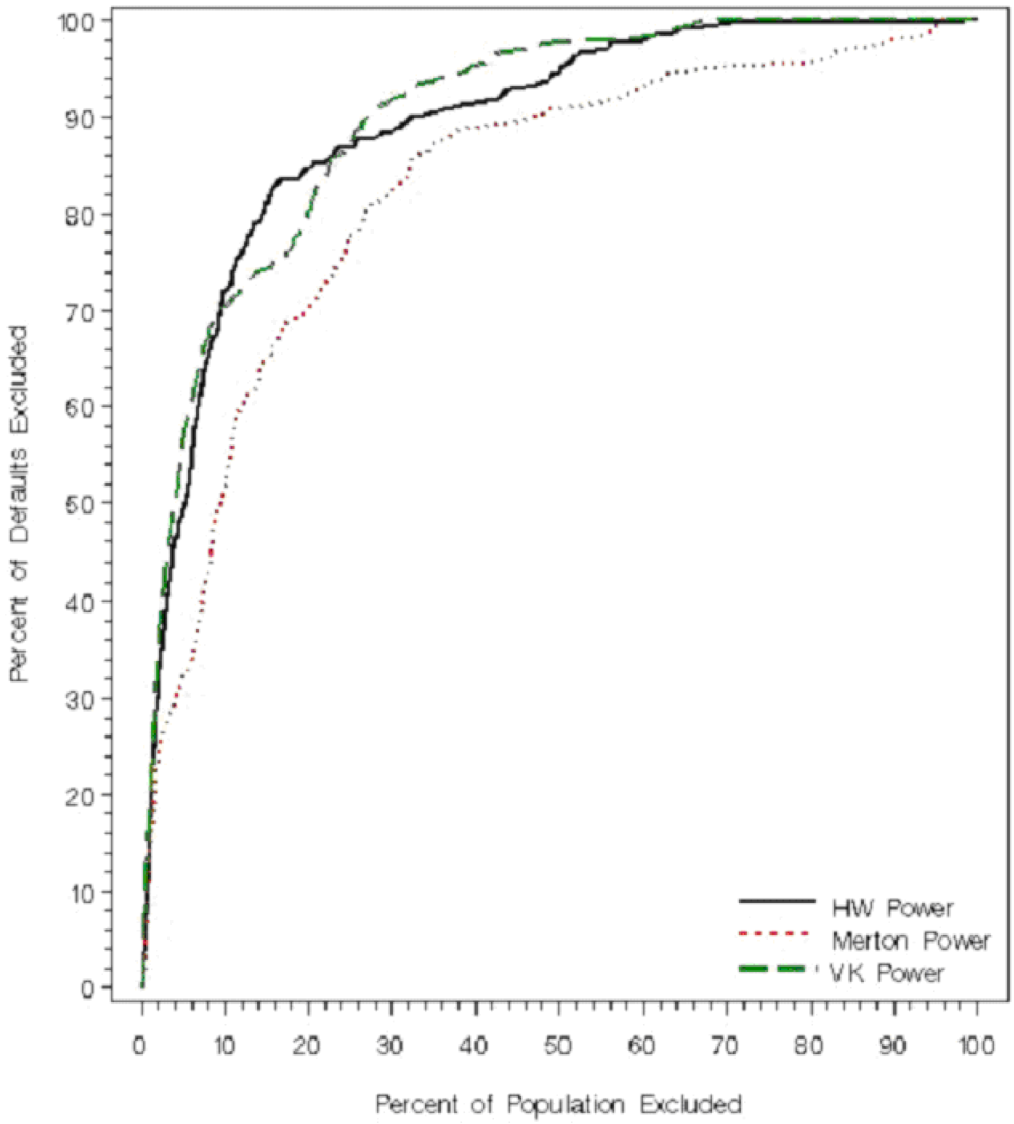

Figure 1(a): A comparison of predictive power across models as given by the accuracy ratio. The accuracy ratios for the VK model, HW model, and Merton models are 0.801, 0.785, and 0.652 respectively.

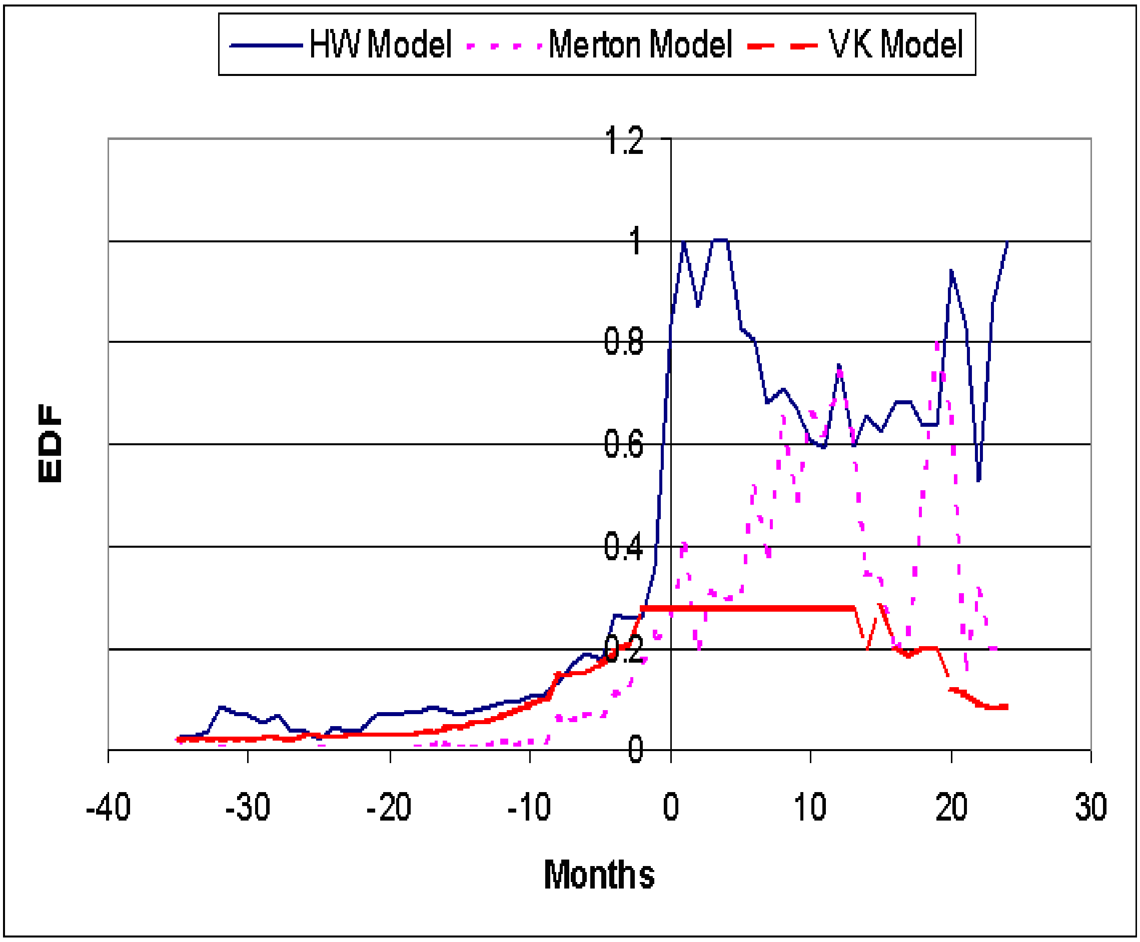

Figure 1(b): Behavior of risk-neutral default probabilities before default for different models. Date 0 represents the date of default.

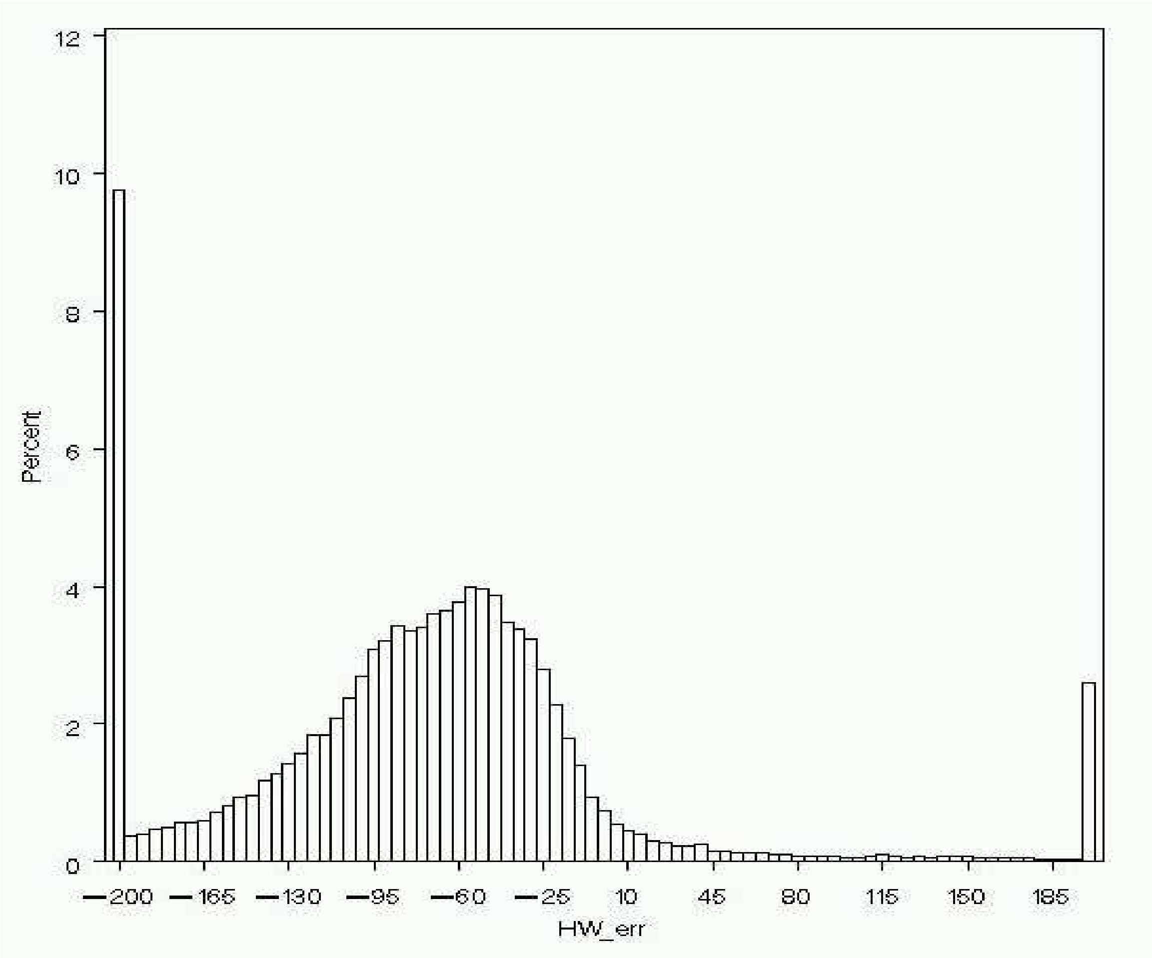

Figure 2(a): Histogram of difference between the market CDS and model CDS prices (as given by $Market\ CDS - Model\ CDS$) for HW reduced-form model.

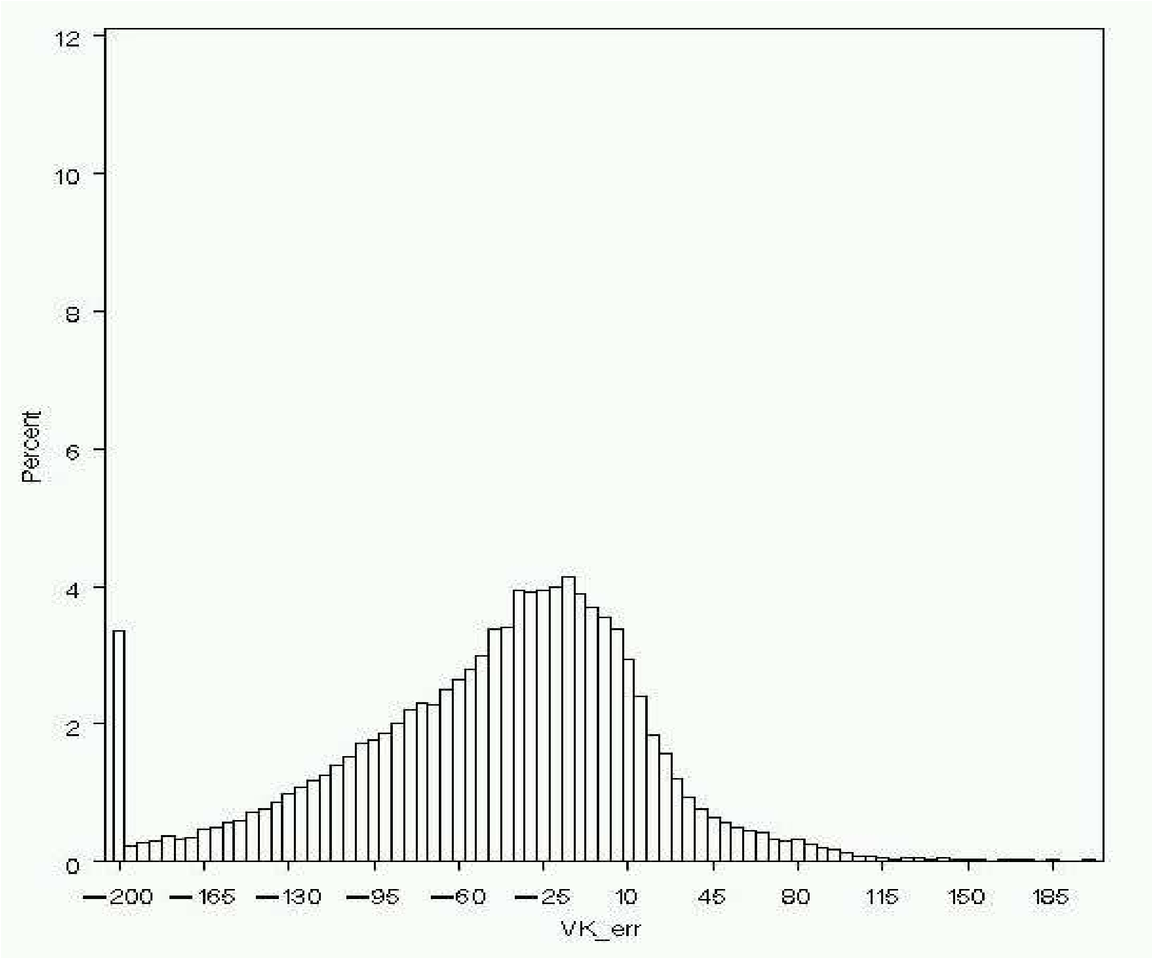

Figure 2(b): Histogram of difference between the market CDS and model CDS prices (as given by $Market\ CDS - Model\ CDS$) for the VK structural model.

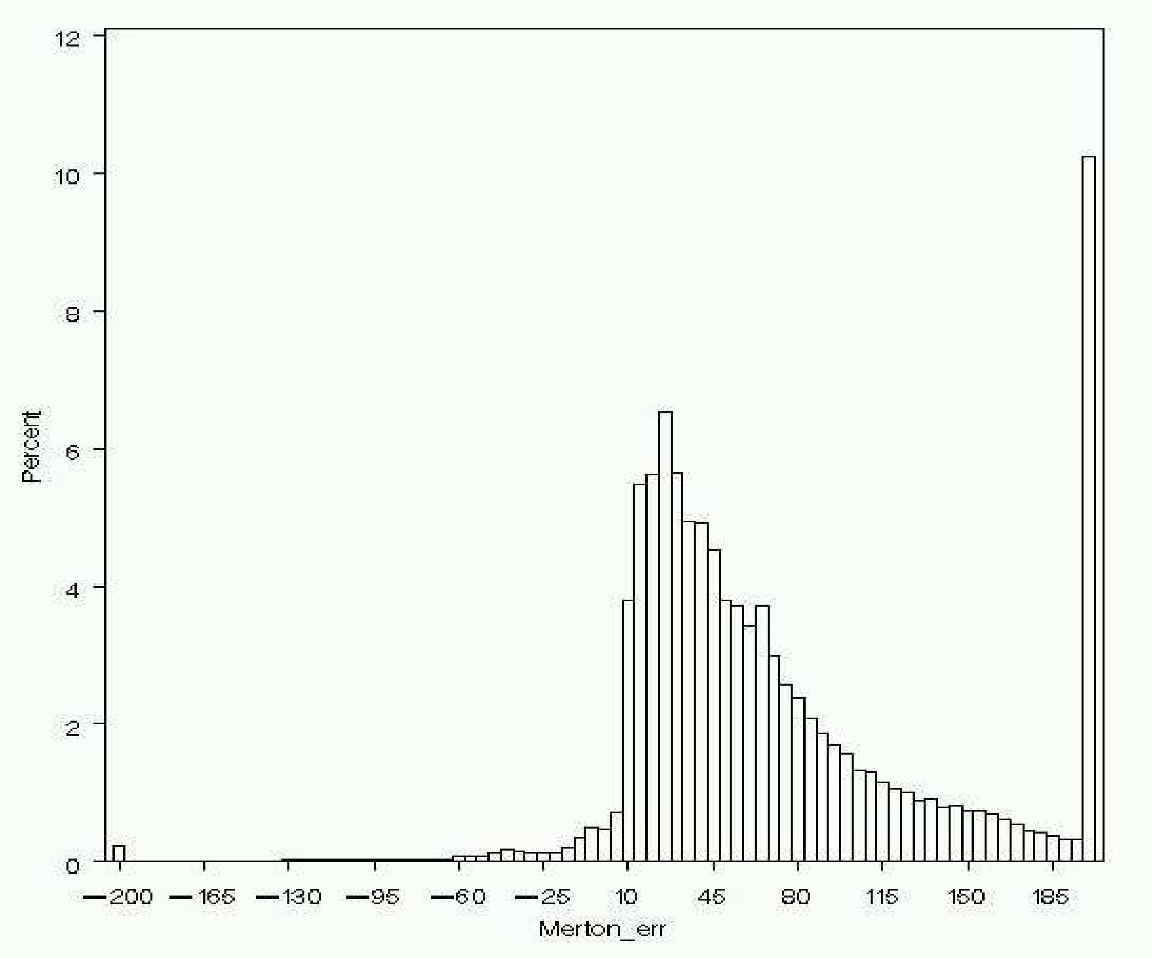

Figure 2(c): Histogram of difference between the market CDS and model CDS prices (as given by $Market\ CDS - Model\ CDS$) for Merton model.

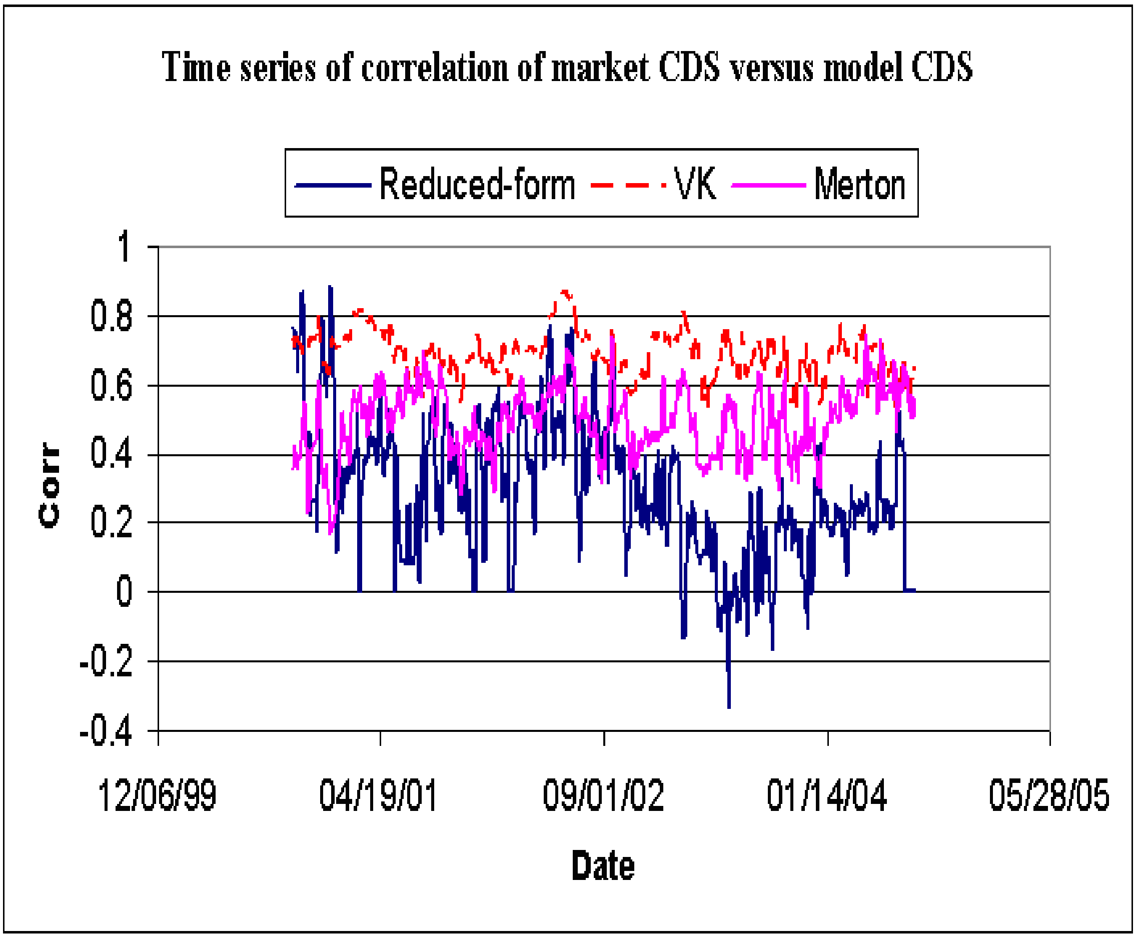

Figure 3: Time-series of correlation between market CDS spreads and model CDS spreads. For each day, the correlation was computed based on cross-sectional data of market and model CDS prices.

Number of Bonds	Percentage of Sample
=2	28.84%
≤ 3	45.83 %
≤ 4	57.32 %
≤ 5	65.35 %
≤ 6	70.96 %
≤ 7	75.74 %
≤ 8	79.38 %
≤ 9	83.31%
≤ 10	85.89 %
≤ 11	87.99 %
≤ 12	89.68 %
≤ 13	91.25 %
≤ 14	92.47 %
≤ 15	93.59 %
≤ 16	94.48 %
≤ 17	95.32 %
≤ 18	96.09%
≤ 19	96.78 %
≤ 20	97.34 %
≤ 21	98.01 %
≤ 22	98.56 %
≤ 23	99.22 %
≤ 24	100 %

Table 1(a): Cumulative percentage distribution of issuers by number of bonds used in bootstrapping for zero-coupon yield curve.

Time to Maturity	Percentage of Sample
$\leq$ 0.25	1.55 %
$\leq$ 0.50	5.99%
$\leq$ 0.75	10.50 %
$\leq$ 1.00	14.93 %
$\leq$ 1.25	19.47 %
$\leq$ 1.50	24.03 %
$\leq$ 1.75	28.54 %
$\leq$ 2.00	33.10 %
$\leq$ 2.25	37.71 %
$\leq$ 2.50	42.16 %
$\leq$ 2.75	46.69 %
$\leq$ 3.00	51.25 %
$\leq$ 3.25	55.69 %
$\leq$ 3.50	59.94 %
$\leq$ 3.75	64.19 %
$\leq$ 4.00	68.43 %
$\leq$ 4.25	72.69 %
$\leq$ 4.50	76.87 %
$\leq$ 4.75	81.09 %
$\leq$ 5.00	85.30 %
$\leq$ 5.25	89.20 %
$\leq$ 5.50	92.35 %
$\leq$ 5.75	95.37 %
$\leq$ 6.00	98.33 %
$\leq$ 6.25	100.00 %

Table 1(b): Cumulative percentage distribution of time-to-maturity of bonds used in bootstrapping for zero-coupon yield curve.

CDS Spread	Percentage of Sample
≤ 25	26.77 %
≤ 50	48.47 %
≤ 75	64.02 %
≤ 100	73.07 %
≤ 125	78.42 %
≤ 150	82.86 %
≤ 175	86.21%
≤ 200	88.33 %
≤ 225	90.17 %
≤ 250	91.49 %
≤ 275	92.82 %
≤ 300	93.81 %
≤ 325	94.69 %
≤ 350	95.50 %
≤ 375	96.16 %
≤ 400	100.00 %

Table 1(c): Cumulative percentage distribution of CDS by range of spreads.

Percentile Difference	Model	Firms with CDS ≤ 100 bps	Firms with CDS > 100 bps	All Firms
p25	HW	-111.70	-135.16	-118.75
	VK	-83.62	-68.60	-80.18
	Merton	21.98	107.30	27.34
p50	HW	-70.73	-77.96	-72.71
	VK	-39.68	-7.57	-33.28
	Merton	37.17	146.11	53.99
p75	HW	-39.02	-33.34	-37.63
	VK	-10.13	62.51	2.01
	Merton	58.24	238.45	102.33

Table 2(a): Difference between the market CDS and model CDS prices (as given by $Market\ CDS - Model\ CDS$) for firms in different CDS buckets

Percentile Difference	Model	Firms with 2 to 4 bonds	Firms with 5 to 9 bonds	Firms with 10 to 15 bonds	Firms with 16 to 24 bonds
p25	HW	-146.62	-120.08	-102.10	-92.29
	VK	-88.81	-68.43	-69.12	-73.79
	Merton	24.50	34.89	29.10	28.32
p50	HW	-80.63	-73.30	-66.99	-63.47
	VK	-37.62	-27.21	-26.63	-34.86
	Merton	51.64	60.40	55.14	48.74
p75	HW	-32.92	-41.15	-37.05	-39.79
	VK	1.80	6.68	3.47	-6.68
	Merton	107.77	98.08	106.46	88.01

Table 2(b): Difference between the market CDS and model CDS prices (as given by $Market\ CDS - Model\ CDS$) for firms with different number of bonds available.

Percentile Slope Coefficient	Model	Firms with CDS $\leq$ 100 bps	Firms with CDS $>$ 100 bps	All Firms
p25	HW	0.05	0.00	0.01
	VK	0.46	0.14	0.58
	Merton	0.43	0.25	0.78
p50	HW	0.33	0.00	0.07
	VK	0.68	0.17	0.76
	Merton	0.82	0.35	1.26
p75	HW	0.50	0.01	0.19
	VK	0.95	0.21	0.94
	Merton	1.47	0.50	1.95

Table 3(a): Slope coefficients resulting from the regression of market CDS on model CDS by firms in different CDS spread buckets. The slope coefficients were computed daily for a three-year period. We report the different quartiles of the time-series distribution of the slope coefficients here.

Percentile Slope Coefficient	Model	Firms with 2 to 4 bonds	Firms with 5 to 9 bonds	Firms with 10 to 15 bonds	Firms with 16 to 24 bonds
p25	HW	0.01	0.20	0.39	0.66
	VK	0.57	0.55	0.34	0.16
	Merton	0.77	0.79	0.56	0.16
p50	HW	0.05	0.36	0.57	0.82
	VK	0.73	0.73	0.56	0.56
	Merton	1.26	1.57	1.29	0.63
p75	HW	0.13	0.53	0.84	0.96
	VK	0.92	1.05	0.92	0.95
	Merton	2.04	2.89	3.36	2.90

Table 3(b): Slope coefficients resulting from the regression of market CDS on model CDS by firms with numbers of bonds available. The slope-coefficients were computed daily for a three-year period. We report the different quartiles of the time-series distribution of the slope coefficients here.

Percentile R-Squared	Model	Firms with CDS ≤ 100 bps	Firms with CDS > 100 bps	All Firms
p25	HW	0.08	0.00	0.02
	VK	0.25	0.20	0.40
	Merton	0.07	0.04	0.16
p50	HW	0.36	0.01	0.09
	VK	0.40	0.25	0.48
	Merton	0.18	0.08	0.26
p75	HW	0.62	0.04	0.21
	VK	0.54	0.30	0.56
	Merton	0.32	0.13	0.36

Table 4(a): Ability of a model to explain the cross-sectional variation of market CDS spreads, as given by the R-squared of the regression of market CDS on model CDS by firms in different CDS buckets. The R-squareds were computed daily for a three-year period. We report different quartiles of the time-series distribution of R-squareds here.

Percentile R-Squared	Model	Firms with 2 to 4 Bonds	Firms with 5 to 9 bonds	Firms with 10 to 15 bonds	Firms with 16 to 24 bonds
p25	HW	0.02	0.17	0.45	0.64
	VK	0.42	0.32	0.27	0.04
	Merton	0.17	0.09	0.09	0.01
p50	HW	0.07	0.38	0.72	0.77
	VK	0.50	0.46	0.44	0.29
	Merton	0.28	0.24	0.29	0.04
p75	HW	0.18	0.61	0.85	0.87
	VK	0.60	0.63	0.63	0.47
	Merton	0.43	0.43	0.50	0.08

Table 4(b): Ability of a model to explain the cross-sectional variation of market CDS spreads, as given by the R-squared of the regression of market CDS on model CDS by firms with different number of bonds available. The R-squared was computed daily for a three-year period. We report different quartiles of the time-series distribution of R-squareds here.

Number of Bonds	Percentage of Sample	Percentage of sample with $log(size) > 11$
=2	28.84%	18.37 %
≤ 3	45.83 %	30.38 %
≤ 4	57.32 %	38.33 %
≤ 5	65.35 %	44.66 %
≤ 6	70.96 %	49.12 %
≤ 7	75.74 %	52.83 %
≤ 8	79.38 %	56.11 %
≤ 9	83.31%	58.43 %
≤ 10	85.89 %	61.17 %
≤ 11	87.99 %	64.45 %
≤ 12	89.68 %	67.72 %
≤ 13	91.25 %	70.73 %
≤ 14	92.47 %	73.24 %
≤ 15	93.59 %	75.97 %
≤ 16	94.48 %	78.56 %
≤ 17	95.32 %	81.32 %
≤ 18	96.09%	84.06 %
≤ 19	96.78 %	86.32 %
≤ 20	97.34 %	88.09 %
≤ 21	98.01 %	90.29 %
≤ 22	98.56 %	93.14 %
≤ 23	99.22 %	96.49 %
≤ 24	100.00 %	100.00 %

Table 5(a): Cumulative percentage distribution of issuers by number of bonds used in bootstrapping for zero-coupon yield curve.

Time to Maturity	Percentage of Sample	Percentage of sample with $log\,(size) > 11$
$\leq$ 0.25	1.55 %	1.22 %
$\leq$ 0.50	5.99%	6.26 %
$\leq$ 0.75	10.50 %	11.38 %
$\leq$ 1.00	14.93 %	16.39 %
$\leq$ 1.25	19.47 %	21.46 %
$\leq$ 1.50	24.03 %	26.35 %
$\leq$ 1.75	28.54 %	31.14 %
$\leq$ 2.00	33.10 %	35.91 %
$\leq$ 2.25	37.71 %	40.73 %
$\leq$ 2.50	42.16 %	45.26 %
$\leq$ 2.75	46.69 %	49.77 %
$\leq$ 3.00	51.25 %	54.33 %
$\leq$ 3.25	55.69 %	58.81 %
$\leq$ 3.50	59.94 %	62.99 %
$\leq$ 3.75	64.19 %	67.10 %
$\leq$ 4.00	68.43 %	71.16 %
$\leq$ 4.25	72.69 %	75.23 %
$\leq$ 4.50	76.87 %	79.18 %
$\leq$ 4.75	81.09 %	83.16 %
$\leq$ 5.00	85.30 %	87.14 %
$\leq$ 5.25	89.20 %	91.12 %
$\leq$ 5.50	92.35 %	93.74 %
$\leq$ 5.75	95.37 %	96.23 %
$\leq$ 6.00	98.33 %	98.59 %
$\leq$ 6.25	100.00 %	100 %

Table 5(b): Cumulative percentage distribution of time-to-maturity of bonds used in bootstrapping for zero-coupon yield curve.

CDS Spread	Percentage of Sample	Percentage of sample with $log\,(size) > 11$
≤ 25	26.77 %	47.72 %
≤ 50	48.47 %	69.87 %
≤ 75	64.02 %	81.53 %
≤ 100	73.07 %	87.46%
≤ 125	78.42 %	90.37 %
≤ 150	82.86 %	93.02 %
≤ 175	86.21%	94.68%
≤ 200	88.33 %	95.72 %
≤ 225	90.17 %	96.43 %
≤ 250	91.49 %	96.85 %
≤ 275	92.82 %	97.41 %
≤ 300	93.81 %	97.97 %
≤ 325	94.69 %	98.40 %
≤ 350	95.50 %	98.68 %
≤ 375	96.16 %	98.86 %
≤ 400	100.00 %	100.00 %

Table 5(c): Cumulative percentage distribution of CDS by range of spreads.

Number of bonds	Model	Below Median Size			Above Median Size		
		p25	p50	p75	p25	p50	p75
2 to 4 bonds	HW	0.09	0.37	0.57	0.01	0.04	0.11
	VK	0.34	0.46	0.57	0.37	0.48	0.62
	Merton	0.11	0.20	0.31	0.24	0.46	0.69
5 to 9 bonds	HW	0.27	0.50	0.76	0.05	0.35	0.67
	VK	0.32	0.52	0.71	0.27	0.42	0.62
	Merton	0.10	0.32	0.63	0.08	0.18	0.42
10 to 15 bonds	HW	0.27	0.64	0.85	0.40	0.73	0.89
	VK	0.31	0.59	0.80	0.16	0.48	0.68
	Merton	0.09	0.46	0.76	0.07	0.31	0.65
16 to 24 bonds	HW	0.46	0.72	0.88	0.64	0.85	0.92
	VK	0.19	0.50	0.70	0.01	0.16	0.35
	Merton	0.02	0.22	0.47	0.10*	0.00	0.01

Table 6(a): Comparison of different models' ability to explain the cross-sectional variation (as measured by the R-squared of the cross-sectional regression of market CDS on model CDS prices) across large and small firms controlling for the number of bonds available for bootstrapping. The R-squareds were computed daily for a three-year period. We report different quartiles of the time-series distribution of R-squareds here. The asterisk indicates that the cross-sectional correlation between market CDS and model CDS price is actually negative, and we see a large number because R-squared is the square of the correlation here.

Percentile	Firms with 2 to 4 Bonds	Firms with 5 to 9 bonds	Firms with 10 to 15 bonds	Firms with 16 to 24 bonds
std. deviation	111.29	99.83	81.20	62.63
$\|p75 - p25\|$	99.77	62.14	80.00	48.71
Median	68.20	70.00	71.39	65.20
Mean	107.74	102.69	95.13	83.22
$\frac{Std.deviation}{Mean}$	1.03	0.97	0.85	0.75
NOBS	88	35	17	21

Table 6(b): Summary statistics of the CDS spreads across firms with different numbers of bond issues. All the statistics were based on daily observations in the period 2000/10 to 2004/06, and the numbers reported here are the medians across the daily time-series of summary statistics.