

Reduced-Order Modeling: Cooperative Research and Development at the NASA Langley Research Center

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Cooperative research and development activities at the NASA Langley Research Center (LaRC) involving reduced-order modeling (ROM) techniques are presented. Emphasis is given to reduced-order methods and analyses based on Volterra series representations, although some recent results using Proper Orthogonal Decomposition (POD) are discussed as well. Results are reported for a variety of computational and experimental nonlinear systems to provide clear examples of the use of reduced-order models, particularly within the field of computational aeroelasticity. The need for and the relative performance (speed, accuracy and robustness) of reduced-order modeling strategies is documented. The development of unsteady aerodynamic state-space models directly from CFD analyses is presented in addition to analytical and experimental identifications of Volterra kernels. Finally, future directions for this research activity are summarized.

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Introduction

THE inclusion of CFD-based analyses into disciplines such as aeroelasticity, aeroservoelasticity, and optimization is currently not routine due to the high computational costs of the CFD analyses. One solution to this problem is the development of reduced-order models (ROMs). Reduced-order models capture the essence of the dynamical system under investigation, resulting in a significantly less complex model. The reduced complexity of the ROM translates into improved computational efficiency. The ROM can then be used for subsequent analyses at reduced computational costs. It should be stated that the ROMs discussed within this context are a generalization of the traditional ROMs that involve the reduction of the dimensions of a given matrix. So, for example, the lift response of a three-dimensional aeroelastic CFD system undergoing plunging motions can be characterized, as will be shown, using a single impulse response function that relates lift due to plunge. In this case, the single impulse response function is the ROM. Therefore, a ROM can also be a functional condensation of the original system in addition to the traditional interpretations with respect to matrices.

Early mathematical models of unsteady aerodynamic response capitalized on the efficiency and power of superposition of scaled and time shifted fundamental responses, or convolution. Classical models of two-dimensional airfoils in incompressible flow¹ include Wagner's function (response to a unit step variation in angle of attack), Kussner's function (response to a sharp-edged gust), Theodorsen's function (frequency response to sinusoidal pitching motion), and Sear's function (frequency response to a sinusoidal gust). As geometric complexity increased from airfoils to wings to complete configurations, the analytical derivation of response functions was no longer practical and the numerical computation of linear unsteady aerodynamic responses in the frequency domain became the method of choice.² And, when geometry-and/or flow-induced nonlinearities aerodynamic effects became significant, the nonlinear equations were computed via time integration. The direct time-integration approach for solving aeroelastic problems via the coupling of the nonlinear aerodynamic equations with the linear structural equations has yielded a very powerful simulation capability with two primary challenges. The first challenge is the computational cost of this simulation, which increases with the fidelity of the nonlinear aerodynamic equations to be solved. The

second challenge is that simulations cannot be used effectively for preliminary aeroservoelastic design. Design by simulation inevitably becomes design by trial-and-error, an impractical approach. The development of ROMs is targeted precisely at addressing these two challenges. While a ROM provides increased computational efficiency over the original more complex system, it is the mathematical model that is extracted from the original system that enables the interconnection with other disciplines. This simplified reduced-order mathematical model transforms a highly complex "black box" into a system with distinct physical and mathematical properties suitable for design analyses. Although the first challenge can be improved via parallel processing and advanced algorithms, the direct time-domain approach does not address the second challenge.

Attempts to address the problem of high computational cost include the development of transonic indicial responses.³⁻⁵ Transonic indicial (step) responses are responses due to a step excitation of a particular input, such as angle of attack, about a transonic (or nonlinear) steady state condition. More recently, Marzocca et al⁶ have analytically derived indicial functions for three-dimensional wings in compressible flow. Neural networks have also been used to develop nonlinear models of unsteady aerodynamics⁷ and nonlinear models of maneuvers (using an experimental database).⁵ Neural networks and Volterra series have some similarities, since each involves the characterization of a system via an input-output mapping.^{7,8} In particular, there is a direct relationship between the weights of a neural network and the kernels of a Volterra series representation for a particular system.⁹

A major difference between Volterra series and neural networks, however, is in the training effort. Neural networks can require a substantial training effort⁵ while Volterra series require neither a training period nor curve fitting for model construction. Also, Volterra kernels provide a direct means for physical interpretation of a system's response characteristics in the time and frequency domains. However, potential disadvantages of the Volterra theory method include input amplitude limitations related to convergence issues and the need for higher order kernels.¹⁰

Another approach for reducing the computational cost of CFD analysis is to restrict attention to linearized dynamics. The response of the linearized system about a nonlinear steady-state condition can be obtained via several methods. Some of these methods include the model order reduction of state-space models using various techniques^{11,12} One method for building a linearized, low-order, frequency-domain model from

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CFD analysis is to apply the exponential (Gaussian) pulse input.¹³ This method is used to excite the aeroelastic system, one mode at a time, using a broadband Gaussian pulse. The time-domain responses are transformed into the frequency domain to obtain the frequency-domain generalized aerodynamic force (GAF) influence coefficient matrix. These linearized GAFs can then be used in standard linear aeroelastic analyses.¹⁴ Raveh et al¹⁵ applied this method while replacing the Gaussian pulse with step and impulse inputs. Recently, these time-domain, linearized impulse and step responses have been transformed directly into state-space form for use in other disciplines such as controls or optimization.¹⁶ Guendal and Cesnik¹⁷ applied the Aerodynamic Impulse Response (AIR) technique, based on the Volterra theory, to the PMARC aerodynamic panel code. The PMARC/AIR code was applied to a simplified High Altitude Long Endurance (HALE) aircraft for rapid linear and nonlinear aeroelastic analysis of the vehicle. Yet another method, different from the Volterra-based ROM approach, is the Proper Orthogonal Decomposition (POD) technique. The POD is a method that is used extensively at several research organizations for the development of reduced-order models. A thorough review of POD research activities can be found in Beran and Silva.¹⁸ A brief overview of the POD method and a representative result are included in this paper for completeness. In addition, a review of the issues involved in the development of reduced-order models for fluid-structure interaction problems is provided by Dowell and Hall.¹⁹ A topic of recent interest is the potential development of hybrid POD/Volterra methods. These hybrid techniques would combine the spatial resolution possible with POD methods with the low dimensionality and computational efficiency of Volterra methods. This is a very appealing concept that merits serious investigation.

A valuable and important characteristic of the Volterra theory of nonlinear systems is that the theory is well defined in the time and frequency domains for continuous- and discrete-time systems. In particular, this theory has found wide application in the field of nonlinear discrete-time systems²⁰ and nonlinear digital filters for telecommunications and image processing.²¹ However, application of nonlinear system theories, including Volterra theory, to modeling nonlinear unsteady aerodynamic responses has not been extensive. One approach for modeling unsteady transonic aerodynamic responses is Ueda and Dowell's²² application of describing functions, which is a harmonic balance technique involving one harmonic. Tobak and Pearson²³ apply the continuous-time Volterra concept of functionals to indicial (step)

aerodynamic responses to compute nonlinear stability derivatives. Jenkins²⁴ also investigates the determination of nonlinear aerodynamic indicial responses and nonlinear stability derivatives using similar functional concepts. Stalford et al²⁵ develop Volterra models for simulating the behavior of a simplified nonlinear stall/post-stall aircraft model and the limit cycle oscillations of a simplified wing-rock model. In particular, they establish a straightforward analytical procedure for deriving the Volterra kernels from known nonlinear functions.

A particular response from a CFD code may provide information regarding the nonlinear aeroelastic behavior of a complex configuration due to a particular input at a particular flight condition. It does not, however, provide general information regarding the behavior of the configuration to a variation of the input, or the flight condition, or both. As a result, repeated use of the CFD code is necessary as input parameters and flight conditions are varied. A primary feature of the Volterra approach is the ability to characterize a linearized or nonlinear system using a small number of CFD-code analyses. Once characterized (via step or impulse responses of various orders), the functions can be implemented in a computationally-efficient convolution scheme for prediction of responses to arbitrary inputs without the costly repeated use of the CFD code of interest. Characterization of the nonlinear response to an arbitrary input via the Volterra theory requires identification of the nonlinear Volterra kernels for a specified configuration and flight condition. Clearly, development and application of Volterra-based ROM concepts depend on the identification of the associated kernels for the problem of interest.

The problem of Volterra kernel identification is addressed by many investigators, including Rugh,²⁶ Clancy and Rugh,²⁷ Schetzen,²⁸ and more recently by Boyd, Tang, and Chua.²⁹ There are several ways of identifying Volterra kernels in the time and frequency domains that can be applied to continuous- or discrete-time systems. Tromp and Jenkins³⁰ use indicial (step) responses from a Navier-Stokes CFD code and a Laplace domain scheme to identify the first-order kernel of a pitch-oscillating airfoil. Rodriguez³¹ generates realizations of state-affine systems, which are related to discrete-time Volterra kernels, for aeroelastic analyses. Assuming high-frequency response, Silva³² introduces the concept of discrete-time, aerodynamic impulse responses, or kernels, for a rectangular wing under linear (subsonic) and nonlinear (transonic) conditions. Silva³³ improves upon these results by extending the methodology to arbitrary input frequencies, resulting in the first identification of discrete-time impulse responses of

an aerodynamic system. It should be noted that owing to separation of the downwash input terms, Silva's first approach had limited applicability for the identification of nonlinear Volterra kernels³³ a situation which has been resolved recently.¹⁰

In his dissertation, Silva¹⁰ discusses the fundamental differences between traditional, continuous-time theories and modern discrete-time formulations that allow the identification of discrete-time kernels. The discrete-time methods are then applied to various nonlinear systems including a nonlinear Riccati circuit, the viscous Burger's equation, an aeroelastic wing in transonic flow using a transonic small-disturbance code, and a supercritical airfoil undergoing large plunge motions at transonic conditions using a Navier-Stokes flow solver with the Spalart-Allmaras turbulence model. Although the majority of the research mentioned thus far involving aeroelastic Volterra kernel identification has been of a computational/analytical nature, the experimental identification of Volterra kernels for a nonlinear aeroelastic system has been performed. Kurdila et al³⁴ applied an efficient wavelet-based algorithm to the extraction of the nonlinear Volterra kernels of an aeroelastic system exhibiting limit cycle oscillations (LCO). There is increased interest in the development of these experimental techniques for use in various experimental settings. The identification of LCO during flight flutter tests is a case in point.

One of the goals of the present paper is to provide enough information to motivate readers to consult the references. The methods and results presented are not intended to be complete and all inclusive. Another goal of the present paper is to familiarize the reader with NASA Langley Research Center's (LaRC's) vision for ROM research, unifying several of the methods and results presented. It should be stated that some of the research discussed in the paper is funded by NASA while the other activities are purely of a cooperative nature. Development of ROMs using POD methods is currently funded by the Air Force Research Laboratory (AFRL) although a cooperative effort between the NASA LaRC. The paper begins with a brief description of the methods discussed: Volterra theory of nonlinear systems, state-space models from aerodynamic impulse responses, and POD. This is followed by presentation of various computational, analytical, and experimental results, including recent results based on the application of the POD method. The paper concludes with recommendations for future research.

Theoretical Background

The research and development of reduced-order models (ROMs) for applications in nonlinear aeroservoelasticity at the NASA Langley Research

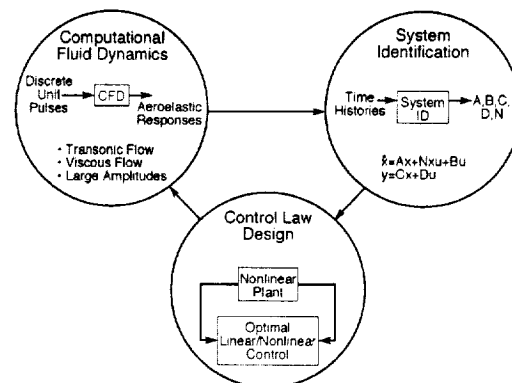


Fig. 1 Schematic representation of the Reduced-Order Modeling research program at the NASA Langley Research Center.

Center can be categorized into three disciplines: computational fluid dynamics (CFD), system identification, and control law design. A schematic representation of the components of this research program is presented as Figure 1.

In addition to representing the vision for ROM research at the NASA LaRC, Figure 1 also serves as an organizational outline for this paper. The CFD portion consists of the development and application of techniques implemented into various CFD codes for the extraction of Volterra kernels. As will be discussed, the Volterra kernels represent linear and nonlinear functional ROMs for these CFD models. These functional ROMs can be used within a digital convolution setting to provide increased computational efficiency over the original CFD codes. These functional ROMs also can be transformed into the more traditional state-space models, suitable for use in modern control theory and optimization. The transformation of the Volterra kernels into state-space form is performed within the System Identification category. In addition, the analytical derivation and the experimental identification of aeroelastic kernels will be discussed under this heading. The Control Law Design portion of Figure 1 is still in its early stages and is left as a topic for future publications. Research involving POD could easily fit within either the CFD or System Identification topics but is treated separately with various important and valuable references for description of details.

Prior to discussion of the various results under the categories just described, a brief description of the various theoretical concepts follows.

Volterra Theory

We begin by reviewing key features of the Volterra theory, as applied to time-invariant, nonlinear, continuous- and discrete-time systems. The literature on Volterra theory is significant, including several texts.^{26,35-37} This section will concentrate on time-domain Volterra formulations.

consistent with the implied application to time-domain, computational aeroelasticity methods. Details regarding the foundations and applications of the frequency-domain Volterra theory can be found in several references.^{26,29,36-39} Marzocca et al.^{38,39} present a thorough discussion of the frequency-domain Volterra theory with respect to nonlinear aeroelastic phenomena. This research is discussed in a subsequent section.

We first consider time-invariant, nonlinear, continuous-time, systems followed by the application of Volterra theory to discrete-time systems (e.g., systems arising in CFD). Of interest is the response of the system about an initial state $\mathbf{w}(0) = \mathbf{W}_0$ due to an arbitrary input $u(t)$ (we take u as a real, scalar input, such as pitch angle of an airfoil) for $t \geq 0$. As applied to these systems, Volterra theory yields the response

$$\begin{aligned} \mathbf{w}(t) = & \mathbf{h}_0 + \int_0^t \mathbf{h}_1(t - \tau)u(\tau)d\tau \\ & + \int_0^t \int_0^t \mathbf{h}_2(t - \tau_1, t - \tau_2)u(\tau_1)u(\tau_2)d\tau_1d\tau_2 + \\ & \sum_{n=3}^N \int_0^t \dots \int_0^t \mathbf{h}_n(t - \tau_1, \dots, t - \tau_n)u(\tau_1) \dots u(\tau_n)d\tau_1 \dots d\tau_n. \end{aligned} \quad (1)$$

The Volterra series in expression (1) contains three classes of terms. The first is the steady-state term satisfying the initial condition, $\mathbf{h}_0 = \mathbf{W}_0$. Next is the first response term, $\int_0^t \mathbf{h}_1(t - \tau)u(\tau)d\tau$, where \mathbf{h}_1 is known as the first-order kernel (or the linear/linearized unit impulse response). This term represents the convolution of the first-order kernel with the system input for times between 0 and t . Lastly are the higher order terms involving the second-order kernel, \mathbf{h}_2 , through the n^{th} -order kernel, \mathbf{h}_n . The existence of these terms is an indication that the system is nonlinear.⁴⁰

The convergence of the Volterra series is dependent on input magnitude and the degree of system nonlinearity. Boyd⁴¹ shows that the convergence of the Volterra series cannot be guaranteed when the maximum value of the input exceeds a critical value, which is system dependent. Of course, the issue of convergence is important, since the Volterra series must be truncated for analysis of practical systems. Silva^{10,40} and Raveh et al.¹⁵ consider a weakly nonlinear formulation, where it is assumed that the Volterra series can be accurately truncated beyond the second-order term:

$$\begin{aligned} \mathbf{w}(t) = & \mathbf{h}_0 + \int_0^t \mathbf{h}_1(t - \tau)u(\tau)d\tau \\ & + \int_0^t \int_0^t \mathbf{h}_2(t - \tau_1, t - \tau_2)u(\tau_1)u(\tau_2)d\tau_1d\tau_2. \end{aligned} \quad (2)$$

For linear systems, only the first-order kernel is non-trivial, and there are no limitations on input amplitude.

Silva¹⁰ derives the first- and second-order kernels, which are presented here in final form in terms of various response functions:

$$\mathbf{h}_1(t) = 2\mathbf{w}_1(t1) - \frac{1}{2}\mathbf{w}_2(t1), \quad (3)$$

$$\mathbf{h}_2(t1, t2) = \frac{1}{2}(\mathbf{w}_1(t1, t2) - \mathbf{w}_1(t1) - \mathbf{w}_1(t2)). \quad (4)$$

In (3), $\mathbf{w}_1(t1)$ is the time response of the system to a unit impulse applied at time 0 and $\mathbf{w}_2(t1)$ is the time response of the system to an impulse of twice unit magnitude at time 0. If the system is linear, then $\mathbf{w}_2 = 2\mathbf{w}_0$ and $\mathbf{h}_1 = \mathbf{w}_0$. If the system is nonlinear, then this identification of the first-order kernel captures an amplitude-dependent nonlinear effect. The identification of the second-order kernel is more demanding, since it is dependent on two parameters. Assuming $t2 > t1$ in (4), $\mathbf{w}_1(t2)$ is the response of the system to an impulse at time $t2$.

Time is discretized with a set of time steps of equivalent size. Discrete time increments are indexed from 0 (time 0) to n (time t), and the evaluation of \mathbf{w} at time n is denoted by $\mathbf{w}[n]$. The convolution in discrete time is

$$\mathbf{w}[n] = \mathbf{h}_0 + \sum_{k=0}^N \mathbf{h}_1[n - k]u[k] \quad (5)$$

$$+ \sum_{k_1=0}^N \sum_{k_2=0}^N \mathbf{h}_2[n - k_1, n - k_2]u[k_1]u[k_2]. \quad (6)$$

where N is the total time record of interest.

It should be noted that an important conceptual breakthrough in the development and application of the discrete-time Volterra theory as a ROM technique is the distinction between a continuous-time unit impulse response and a discrete-time unit impulse response.^{10,40} The continuous-time unit impulse response is an abstract function typically defined as a function with an amplitude that reaches infinity while its width approaches zero but its integral is unity. This function is difficult, if not impossible, to apply in practical applications (i.e., discrete-time problems). The discrete-time unit impulse response (known as a unit sample response), on the other hand, is specifically designed for discrete-time (i.e., numerical) applications. This function is defined as having a value of unity at one point in time and zero everywhere else. This is clearly a simpler function to implement in a numerical setting. The proof of this and details regarding the very powerful unit sample response can be found in any modern text on digital signal processing.⁴²

The identification of linearized and nonlinear Volterra kernels is an essential step in the development of ROMs based on Volterra theory, but it is not the final step. Ultimately, these functional kernels can be transformed into linearized and nonlinear (bilinear) state-space systems that can be easily implemented into other disciplines such as controls and optimization.^{10,26} Recently, linearized state-space models of an unsteady aerodynamic system have been developed while research into the development of nonlinear state-space models continues.¹⁶

The frequency-domain version of the Volterra theory is simply the Fourier transform of the series shown in (1). Therefore, the Fourier transform of the first-order kernel (for a linear system) is the frequency response function of the system. Higher-order kernels are Fourier transformed into higher-order frequency response functions, discussed in most of the references already mentioned. The primary benefit of these higher-order frequency response functions is that they provide information regarding the interaction of frequencies due to a nonlinear process. For example, bispectra (the frequency-domain version of the time-domain second-order kernel) have been used in the study of grid-generated turbulence to identify the nonlinear exchange of energy from one frequency to another. Linear concepts, by definition, cannot provide this type of information. In addition, some very interesting and fundamental applications using the frequency-domain Volterra theory,^{38,39} and experimental applications of Volterra methods^{43,44} are providing new “windows” on the world of nonlinear aeroelasticity.

Discrete-Time State-Space Models

The basic formulation used in the development of state-space models of the unsteady aerodynamic system using the impulse responses (Volterra kernels) obtained directly from a CFD code is described in this subsection. The ability to generate state-space models of systems using impulse responses was a primary motivation for the development of aerodynamic impulse response functions.¹⁰ Although the method and results presented for this activity are linearized results, the long-term goal is the development of nonlinear state-space models directly from the nonlinear Volterra kernels. The linearized results presented here are a starting point for the nonlinear state-space model development activity. Details can be found in the reference by Silva and Raveh.¹⁶

A finite-dimensional, discrete-time, linear, time-invariant dynamical system has the state-variable equations

$$\dot{x}[n+1] = Ax[n] + Bu[n] \quad (7)$$

$$y[n] = Cx[n] + Du[n] \quad (8)$$

where x is an m -dimensional state vector, u an p -dimensional control input, and y a r -dimensional output or measurement vector with n being the discrete time index. The transition matrix, A , characterizes the dynamics of the system. The goal of system realization is to generate constant matrices (A , B , C , and D) such that the output responses of a given system due to a particular set of inputs is reproduced by the discrete-time state-space system described above.

For the system of (7) and (8), the time-domain values of the systems unit sample response are also known as the Markov parameters and are defined as

$$Y[n] = CA^{n-1}B \quad (9)$$

with B an m by p matrix and C a q by m matrix. System realization techniques provide the constant matrices A , B , C and D using $Y(n)$.

The Eigensystem Realization Algorithm (ERA) algorithm^{45,46} begins by defining the generalized Hankel matrix consisting of the Markov parameters for all input/output combinations. The algorithm then uses the singular value decomposition (SVD) to compute the A , B , and C matrices. Often, the direct feedthrough matrix, D , is nonzero whenever the initial values of the Markov parameters are nonzero.

The ERA algorithm has been used successfully for the identification of several experimental structural dynamic systems. Although the algorithm also has been used to extract damping and frequency information from CFD-generated aeroelastic transients (no published references), this research represents the first time that the ERA algorithm is applied to the development of unsteady aerodynamic state-space models using aerodynamic pulse responses (Markov parameters). Additional details regarding the ERA algorithm and its numerous applications are discussed in the references provided.

Proper Orthogonal Decomposition (POD)

POD is a linear method for establishing an optimal basis, or modal decomposition, of an ensemble of continuous or discrete functions. The variables and parameters defined in this subsection are not related to variables from any previous subsection.

Beran¹⁸ provides an excellent summary of the POD method and its origins. Detailed derivations of the POD and its properties are available elsewhere^{47,48} and not repeated herein. In this discussion of POD, M basis vectors are used to represent deviations of $\mathbf{w}(t)$ from a base solution, \mathbf{W}_0 . These basis vectors are written as $\{\mathbf{e}^1, \mathbf{e}^2, \dots, \mathbf{e}^M\}$, and are referred to by many names, including POD vectors,⁴⁹ empirical eigenfunctions⁴⁷ or, simply,

modes.^{47,50} For the sake of brevity, we shall use the term “modes” to denote the POD basis vectors. Each mode is normalized such that $\mathbf{e}^i \mathbf{e}^i = 1$ ($i = 1, \dots, M$), and computed in a manner to be described shortly. The modal decomposition of \mathbf{w} using M modes, or \mathbf{w}_M , is given by

$$\mathbf{w}_M = \mathbf{W}_0 + \sum_{i=1}^M \hat{w}_i \mathbf{e}^i = \mathbf{W}_0 + \Phi \hat{\mathbf{w}}, \quad (10)$$

where Φ is an $N \times M$ matrix containing the ordered set of modes, $\Phi = [\mathbf{e}^1, \mathbf{e}^2, \dots, \mathbf{e}^M]$ and $\hat{\mathbf{w}}$ is an M -dimensional vector of modal amplitudes, $\hat{\mathbf{w}} = [\hat{w}_1, \hat{w}_2, \dots, \hat{w}_M]$. As a time-varying function, \mathbf{w} is approximated by $\mathbf{W}_0 + \Phi \hat{\mathbf{w}}(t)$.

As stated by Holmes et al.,⁴⁷ “Linearity is the source of the [POD] method’s strengths as well as its limitations ...” The method is linear owing to the independence of the modes from the modal amplitudes, thereby allowing for the straightforward construction of reduced-order equation sets from the full equation sets following mode computation.

The POD modes are constructed by first computing samples, or snapshots, of system behavior (solutions at different instants in time for dynamic problems, or equilibrium solutions at different parameter values for static problems) and storing these samples in a snapshot matrix, \mathbf{S} . This sampling process is often referred to as POD “training.” For now, we assume that M snapshots are collected and column-wise collocated into the $N \times M$ snapshot matrix: $\mathbf{S} = [\mathbf{w}^1, \mathbf{w}^2, \dots, \mathbf{w}^M]$. The basis provided by the POD, known as the Karhunen-Loeve^{51,52} (or K-L) basis, has been shown to minimize the error of the approximation of functions using these basis functions.^{47-49,53,54}

In practice, fewer than M modes are retained to simulate system behavior. These are selected based on the size of the modal eigenvectors. Simply put, the K-L basis for a subspace of dimension $M_r < M$ is obtained by retaining the modes associated with the M_r largest eigenvalues computed.

The techniques described in Beran and Silva¹⁸ provide different means for obtaining reduced-order equation sets governing $\mathbf{w}(t)$ in the POD subspace. There are several methods for accomplishing this including the Galerkin projection, “subspace” projection (for linear and nonlinear systems), and collocation. These methods are discussed in detail by Beran and Silva¹⁸ and will not be repeated here. In addition, recent results by Kim and Bussolletti⁵⁵ and Kim⁵⁶ are indicative of the efficiency and suitability of these techniques. However, due to the importance and value of POD techniques, a sample result is provided in a subsequent section in order to familiarize the reader with the appropriate references concerning POD research.

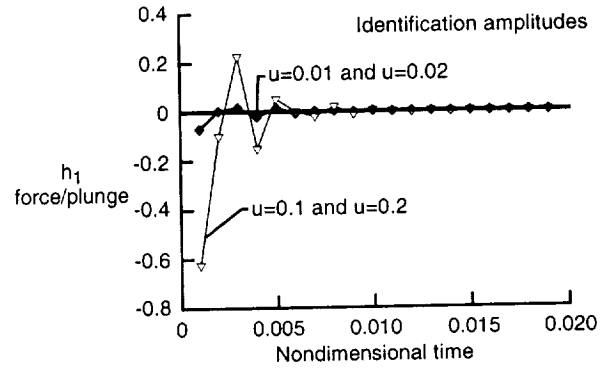


Fig. 2 Volterra kernels for CFD analysis of RAE airfoil: First-order kernel and effect of identification amplitudes.

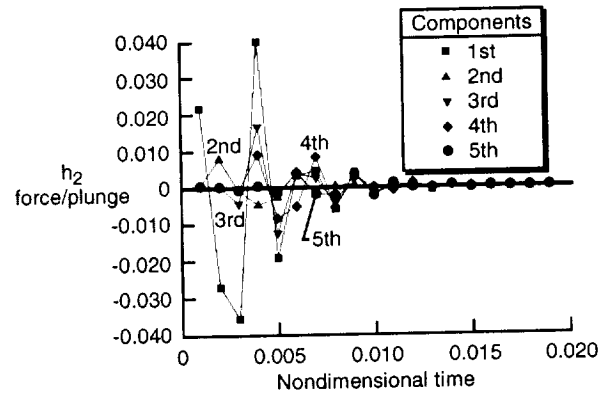


Fig. 3 Volterra kernels for CFD analysis of RAE airfoil: First five components of the second-order kernel for plunge amplitude of 0.1.

Results

The results discussed in this section address the top two disciplines (circles) of Figure 1. The first subsection discusses the approximation of CFD results using ROMs and the second subsection discusses results from system identification studies.

Approximating CFD Results with ROMs

This section describes some results obtained in the identification of CFD-based (time-domain) Volterra kernels. The first result presented is from Silva¹⁰ for a supercritical airfoil at a transonic Mach number.

First- and second-order kernels for the Navier-Stokes solution (with Spalart-Allmaras turbulence model) of an RAE airfoil undergoing plunging motions at a transonic Mach number using the CFL3D code⁵⁷ are presented in Figures 2 and 3. Shown in Figure 2 are two sets of first-order kernels due to two different sets of excitation amplitudes. Recall that the first-order kernel is computed using (3) and is the result of two pulse responses, one at a particular amplitude and the second at double the first amplitude. One of the first-order kernels shown in Figure 2 was computed using excitation plunge amplitudes of $w = 0.01$ and $w = 0.02$, where w is a fraction

of the chord of the airfoil. The other first-order kernel was computed using excitation plunge amplitudes of $w = 0.1$ and $w = 0.2$. It is clear that the two kernels are not linearly related, demonstrating how the first-order kernel can capture amplitude-dependent nonlinear effects.

Shown in Figure 3 are five components of the second-order kernel for this case. The second-order kernel is more complicated because it is a two-dimensional function of time. The second-order kernel is presented as a family of functions in Figure 3 for simplicity. The important point to be made is that this kernel is readily generated and its relatively smaller values (compared with the first-order kernel) and its rapid convergence indicate a small (but not negligible) level of nonlinearity present. This information may be used to determine if the first-order kernel is sufficient to capture the dominant nonlinear effects. This point is demonstrated in Figure 4.

Figure 4 is a comparison of plunge responses for three different plunge amplitudes for the same configuration. Specifically, three comparisons are made between the full CFD solution due to a sinusoidal plunging motion (labelled in the figure as "actual nonlinear") and that obtained using the first-order kernel from Figure 2 due to the larger excitation amplitudes. As can be seen, the plunge response obtained using the Volterra first-order kernel compares almost identically with the response obtained from the full CFD solution for the two smaller amplitude responses. In fact, the two curves for these responses, are almost not discernable in Figure 4. The comparison for the largest amplitude response is very good as well, with a slight but noticeable difference between the two results. The nonlinearity of the large-amplitude plunge responses is confirmed by linearly scaling the smallest amplitude (i.e., linear) response which, as shown in Figure 4, cannot capture the amplitude-dependent nonlinear plunge dynamics seen at the larger amplitude.

The turnaround time ("wallclock") for the full CFD solution was on the order of a day whereas the Volterra first-order solution was computed on a workstation in 30 seconds using digital convolution. The initial cost of computing the first-order kernel was trivial as well due to the rapid convergence of the pulse responses. In fact, since each pulse (unit and double amplitudes) goes to zero in less than 100 time steps, the responses were generated using a debugging mode option available on the supercomputer system used. Using this option, computations requiring less than 300 time steps are executed immediately, intended for debugging purposes. As a result, each pulse was computed within five minutes, resulting in a first-order kernel that was computed in about ten minutes. Of

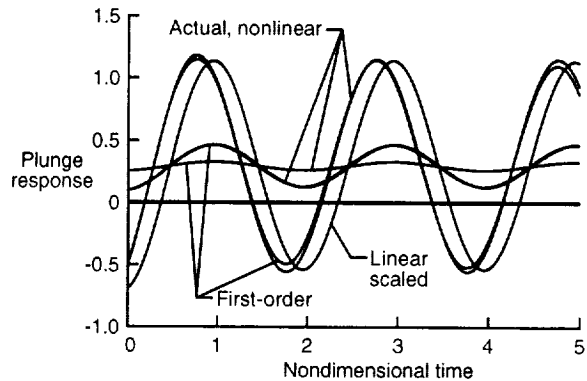


Fig. 4 Comparison of actual nonlinear and first-order Volterra responses for three different plunge motions and a linear response for the largest motion.

course, once the kernel has been computed, it can be used to predict the response to any arbitrary input (steady, any and all frequencies, random) of arbitrary length via digital convolution on a workstation. Using this method, there is no need for the repeated, and costly, execution of the CFD code for different inputs.

Raveh, Levy and Karpel have recently applied the Volterra-based ROM approach to analysis of the AGARD 445.6 wing.¹⁵ They simulate the flow field around the wing using the EZNSS Euler/Navier-Stokes code.⁵⁸ This code provides a choice between two implicit algorithms, the Beam-Warming algorithm⁵⁹ and the partially flux-vector splitting algorithm of Steger et al.⁶⁰ Grid generation and inter-grid connectivity are handled using the Chimera approach.⁶¹ The code was enhanced with an elastic capability to compute trimmed maneuvers of elastic aircraft.⁵⁸ For the CFD computations, the flow field around the wing was evaluated on a C-H type grid, with 193 points in the chordwise direction along the wing and its wake, 65 grid points in the spanwise direction, and 41 grid points along the normal direction.

A process of mode-by-mode excitation, discussed previously, was performed for this wing using four elastic modes at a Mach number of 0.96. The mode-by-mode excitation technique provides the unsteady time-domain generalized aerodynamic forces (GAFs) in all four modes due to an excitation of one of the modes. In this fashion, a matrix of four-by-four functions (sixteen total) is developed. Two sets of excitation inputs were used: the discrete-time pulse input and the discrete-time step input. The cost of computing these functions is minimal due to the rapid convergence of these functions and it consists of only four code executions. Once these functions were defined, several full CFD solutions were generated that were due to various sinusoidal inputs at various frequencies. Shown in Figure 5 is just one of

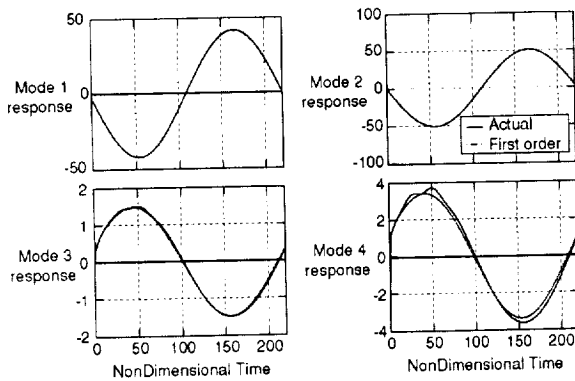


Fig. 5 Comparison of actual and first-order Volterra responses to sinusoidal excitation at 5 Hz (Mach 0.96).

these results for a 5 Hz input frequency, comparing the result obtained from the full CFD solution to that obtained via convolution of the step or pulse responses with a 5 Hz sinusoid. As can be seen, the comparison is exact to plotting accuracy for most of the responses. The full CFD solution, consisting of 8000 iterations required approximately 24 hours on an SGI Origin 2000 computer with 4 CPUs. By comparison, the Volterra-based ROM response shown required about a minute. Even including the upfront cost of computing the pulse (or step responses), the computational cost savings are significant. More importantly, the same pulse (or step) functions can now be used to predict the response of the aeroelastic system to any arbitrary input of any length.

As a validation, a comparison of this approach to another result available in the literature is presented as Figure 7. Shown in Figure 7 is a comparison of linear and nonlinear GAFs for the first two modes of the AGARD 445.6 Aeroelastic Wing at a Mach number of 0.96. Nonlinear GAFs refers to the GAFs computed using the Volterra pulse-response technique about a nonlinear steady-state value by exciting one mode at a time and obtaining the resultant responses in the other modes. The CFD results are compared with those using the ZAERO code for a purely linear case. Frequency-domain values were obtained by performing a convolution of several frequencies of interest with the computed CFD-based pulse responses. As can be seen, the comparison is reasonable and shows the small (but not negligible) nonlinear aerodynamic effects identified using the Volterra pulse-response technique.

Additional computational applications of the Volterra-based ROM technique include the application to a High Altitude Long Endurance (HALE) wing using a panel method for the aerodynamics.¹⁷ The simplified wing is referred to as the Simple High Aspect Ratio (SHAR) Wing. The particular panel method selected for this research was the PMARC aerodynamic panel code, a code devel-

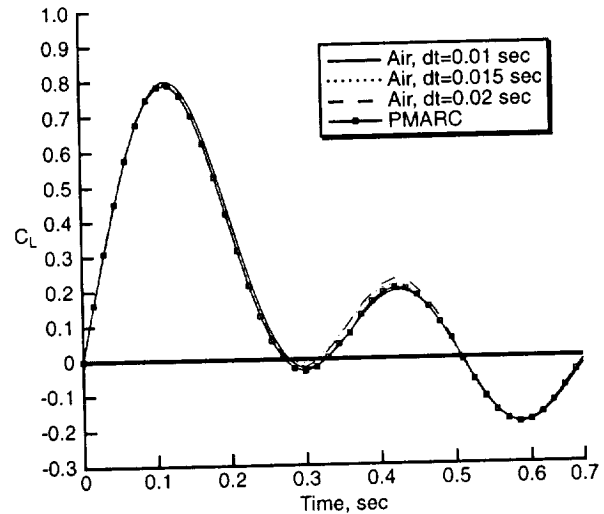


Fig. 6 Pitch responses of the SHAR Wing to a prescribed pitch command.

oped in the late 1980s at NASA Ames Research Center.⁶² This code is a low-order panel method, meaning that the source and doublet singularities are considered to be constant on each panel. PMARC is one of the most advanced panel methods available, featuring an advanced time-stepping relaxed wake, internal flow capabilities, jet plume modeling, and rudimentary unsteady capabilities.

The implementation of the Volterra-based ROM technique into the PMARC code is referred to as the Aerodynamic Impulse Response (AIR) technique. A particularly useful feature of PMARC/AIR is the ability to predict the response to an arbitrary input. For the purpose of testing this feature, the following Fourier sine series with three components was used:

$$\theta[t] = 3^\circ \sin(2\pi t) + 3^\circ \sin(4\pi t) + 3^\circ \sin(6\pi t) \quad (11)$$

The quantity theta [t] is a signal with three frequencies: 1 Hz, 2 Hz, and 3 Hz. The pitch response of the SHAR wing is shown in Figure 6. The response was calculated with the AIR method using impulse responses generated with three time steps, $\Delta t = 0.01$ seconds, 0.015 seconds, and 0.02 seconds, and with the full solution from the PMARC code at $\Delta t = 0.015$ seconds. All three time step choices produce a fairly accurate prediction of the response. These results indicate that the method is a useful one for predicting the response to arbitrary inputs, especially those that may not be as easy to implement within the PMARC code. This is a significant improvement over the capabilities of the original PMARC code.

System Identification

In this section, some of the results classified under the system identification portion of Figure 1 are discussed. These results include the tranfor-

mation of Volterra kernels into state-space form, and analytical and experimental identification of Volterra kernels for various systems. The state-space results follow the method described in an earlier subsection. The analytical identification of Volterra kernels consists of two parts: 1) the derivation of compressible aerodynamic indicial functions for subsonic, supersonic, and hypersonic Mach number regimes and 2) the derivation of frequency-domain Volterra kernels for aeroelastic systems with structural nonlinearities. It should be pointed out that the relationship between indicial functions and Volterra kernels is the same as the relationship between a step and impulse responses: one is the derivative of the other. The experimental identification of Volterra kernels consists of the experimental extraction of second-order Volterra kernels for an aeroelastic system undergoing limit cycle oscillations (LCO). Results from these categories are now discussed.

In an earlier subsection, the generation of time-domain GAFs using a CFD code was presented. Validation of these functions was performed in the time and frequency domains. Time-domain validation consisted of a comparison between the full CFD solution and the solution obtained via convolution for forced harmonic responses at several frequencies. Frequency-domain validation was performed by Fourier transforming the time-domain GAFs to the frequency domain and comparing these with results available in the literature for that configuration. Because traditional flutter analyses are performed in the frequency domain using frequency domain GAFs, the standard approach for using linearized GAFs from a CFD code is to transform the time-domain GAFs into the frequency domain and use the standard flutter analysis routines. Then, if a time-domain model of the GAFs is needed for ASE analyses (i.e., state-space form), then rational function approximations (RFAs) are employed to transform the frequency-domain GAFs back into the time domain. But rather than transforming the time-domain GAFs into the frequency domain only to transform them back into the time domain, discrete-time, state-space systems can be created using the Volterra kernels (time-domain GAFs) directly.¹⁶

Using the ERA method previously discussed, a 32nd-order state-space model of the AGARD 445.6 Aeroelastic Wing was generated. This state-space model consists of four inputs and four outputs, one for each of the four structural modes. Then, in order to validate the accuracy of the state-space model, the frequency content of the state-space model is compared to the frequency content of the Volterra kernels extracted from the CFD code. Presented in Figure 8 is a comparison of the frequency response for the CFD solution (kernels)

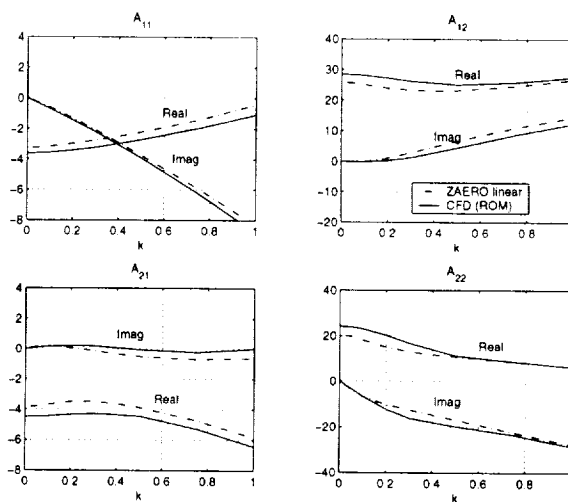


Fig. 7 Comparison of linear and nonlinear GAFs for the first two wing modes.

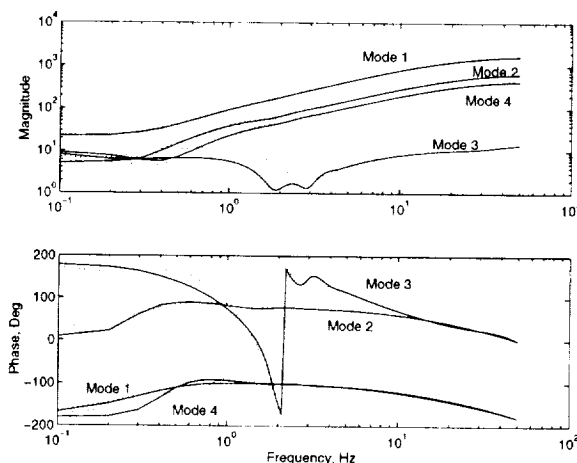


Fig. 8 Comparison of frequency responses for AGARD 445.6 Aeroelastic Wing. CFD (Solid Line), State-Space (Dotted Line).

versus the frequency response for the state-space system for the AGARD 445.6 Aeroelastic Wing. Presented in the figure are the responses (output) for all four modes due to an input in the first mode. The frequency responses of the pulses computed directly from the CFD code (solid lines) compare very well with the frequency responses obtained from 32nd-order state-space system generated to model this system. The responses due to inputs in the second, third, and fourth modes are just as good as those shown in Figure 8, but are not presented here for brevity. The important point to be made is that time-domain kernels, extracted from a CFD code, can now be transformed directly into state-space form, amenable for use in modern control theory and optimization.

In terms of analytically-derived kernels (indicial functions), Marzocca et al⁶ have developed a unified approach, based upon the use of aerodynamic indicial functions, that enables the determination of the aerodynamic lift and moment

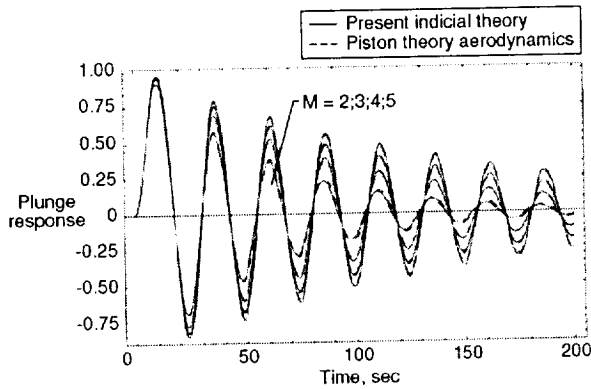


Fig. 9 Comparison of aeroelastic responses due to an external blast load.

in subsonic, supersonic, and hypersonic compressible flight speed regimes for three-dimensional wings. These indicial functions are the compressible, three-dimensional extensions of the classical incompressible, two-dimensional indicial functions, such as Wagner's function. As a result, these indicial aerodynamic functions can now be used to perform rapid evaluations of preliminary designs over a wide range of Mach number regimes and for three-dimensional wings.

A sample result from this research for a swept wing in the supersonic and hypersonic flight speed regimes is presented as Figure 9. Shown in Figure 9 is a comparison of the aeroelastic responses due to a blast load using the method of Marzocca et al⁶ and piston theory for various Mach numbers. As can be seen, the comparisons are excellent, validating this methodology for this speed range. Similar comparisons for subsonic speed regimes can be found in the stated reference. A fundamental contribution of this research is that it defines the relationship between aerodynamic indicial responses and aerodynamic Volterra kernels. This functional relationship can then be used to derive nonlinear aeroelastic Volterra kernels.

Analytical derivation of frequency-domain, nonlinear aeroelastic Volterra kernels is presented by Marzocca et al.^{38,39} Aeroelastic systems with one (plunge) and two (plunge and pitch) degrees of freedom with linear aerodynamic equations but with nonlinear structural parameters were defined. The nonlinear structural parameters consisted of quadratic and cubic stiffness and damping terms. Analytical derivation of the Higher-Order Frequency Response Functions (HO-FRF), which is another term for frequency-domain Volterra kernels, was performed on these nonlinear aeroelastic systems. Evaluation of the HO-FRFs was performed by comparing the responses obtained using these HO-FRFs with the responses obtained via direct simulation of the systems.

Presented in Figure 10 are the results of a convergence study involving the first three kernels for

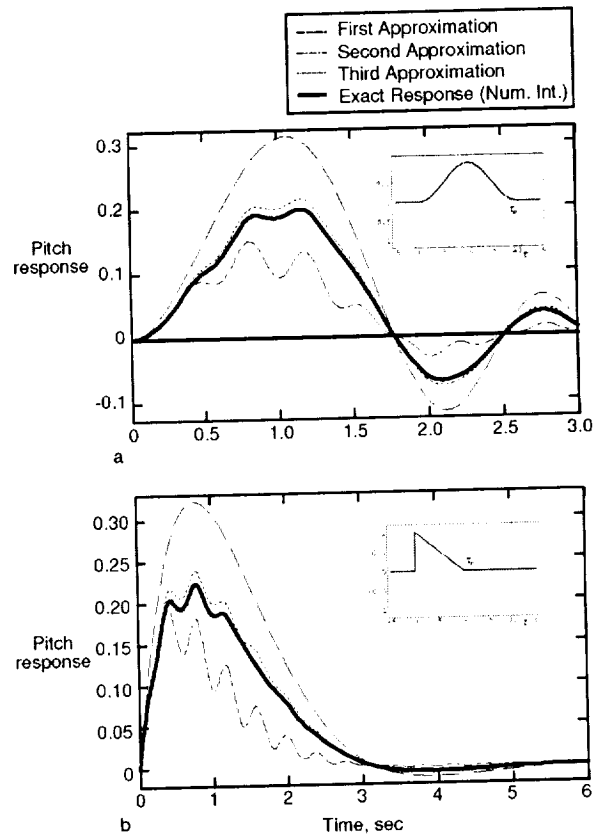


Fig. 10 Convergence study involving the first three kernels and comparison with the exact nonlinear aeroelastic response to a 1). 1-cosine gust and a 2). triangular external blast load, as shown in insets.

the one-degree-of-freedom system and comparison with the "exact" nonlinear aeroelastic response to a (a) 1-cosine gust pulse and a (b) triangular blast load as shown in the insets of the figure. These time-domain results were obtained by computing the inverse Fourier transform of the derived HO-FRFs. This demonstrates the rapid and accurate convergence of the Volterra kernel approach for these types of problems.

For the two-degree-of-freedom system, a three-dimensional plot of the second-order HO-FRF is presented as Figure 11, where the theoretical symmetry about the diagonal elements of these Volterra kernels is verified. The importance of these results is that these analytically-derived functions provide a closed-form nonlinear solution of simple nonlinear aeroelastic systems which can be used for the validation of computational techniques. In addition, enhanced understanding of these HO-FRFs can provide significant insight into the nonlinear dynamics of a system that would not be visible via standard linear (or linearized) processes.

A discussion of the experimental results, as performed by Kurdila et al⁴³ and Prazenica et al⁴⁴ now follows. First of all, it is important to mention that, historically, nonlinear Volterra series have

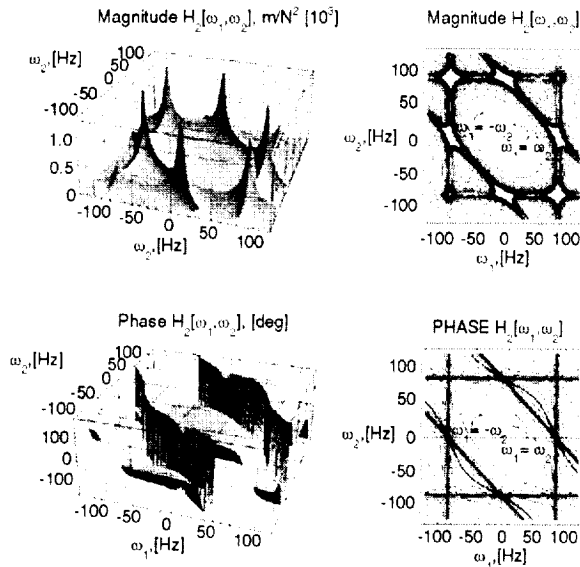


Fig. 11 Three-dimensional and contour plots of second-order aeroelastic kernel.

not seen widespread use in system synthesis precisely because of the high dimensionality of the higher order, nonlinear terms. This is true from experimental, computational, and analytical perspectives. However, recent work by researchers in multiresolution analysis of the Volterra kernels has shown that the dimensionality of the higher order terms can be significantly reduced. This reduction is due to the fact that wavelet and multiresolution analysis have shown considerable promise for the compression of signals, images, and, in particular, some integral operators. Numerical studies of the theoretical derivations are carried out using experimental pitch and plunge response data from the Texas A & M University's (TAMU) Nonlinear Aeroelastic Testbed (NAT).

The TAMU-NAT, as used for this study, was comprised of a NACA0012 airfoil with a flap mounted on a specially-designed carriage capable of undergoing large amplitude pitch and plunge motions. Aside from the potential nonlinear aerodynamic effects introduced by the stall of the airfoil at large angles of attack, this experimental setup results in an aeroelastic system with, primarily, nonlinear structural effects. The nonlinearity in the system is introduced by the appropriate variation of the various parameters of the pitch-and-plunge carriage such as cams and springs.

A wavelet-based kernel identification algorithm was developed to extract Volterra kernels from experimental data. This algorithm was then applied to the experiment involving the TAMU-NAT. The information used by the wavelet-based kernel identification algorithm consisted of the flap deflection (input) and pitch angle (output) as functions of time. Variation of the flow velocity of the TAMU-NAT ranged between 23 and 24 m/s,

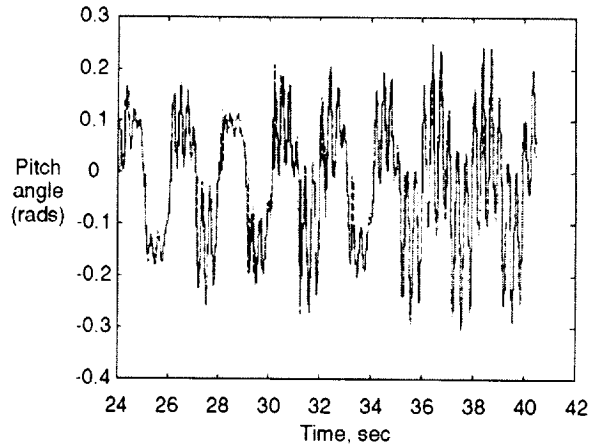


Fig. 12 Comparison of predicted output from model identification and experimental output.

which classifies this system as a time-varying or non-stationary system. Strictly speaking, this classification is not consistent with the theoretical developments for stationary nonlinear systems. However, this flow velocity variation was seen to be small enough to classify the system as "nearly stationary". In addition, windowed sampling of the data was performed in order to further reduce the non-stationarity of the system.

Presented in Figure 12 is a comparison of the predicted output from the model identification and the experimental output for the airfoil exhibiting limit cycle oscillations (LCO). As can be seen, the two results are indistinguishable, verifying the accuracy of the identified model. In addition, Figures 13 and 14 are second-order kernels identified for this system at different samplings (temporal windows) during the experiment. A noticeable difference exists between the second-order kernel of Figure 13 and that of Figure 14, which is indicative of a modification of the nonlinear system. Clearly, some event took place during the test that altered, in some sense, the nonlinear system. This change is also visible in Figure 12 as a noticeable change with time in the nature of the time series of the output. This change in the nonlinear dynamics of the system is not due to flow velocity variations but the exact cause is not yet known. These results demonstrate and validate two fundamental properties of Volterra kernels: 1) the existence of the higher-order kernels is an indication of a nonlinear process or system, and 2) alterations of the nonlinear dynamics of a system are reflected as alterations of these kernels. Finally, the identification of the second-order kernels was performed using as few as 14 wavelets, significantly reducing the computational burden as compared to more traditional Volterra kernel identification techniques.

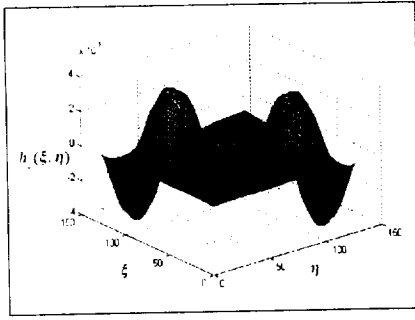


Fig. 13 Volterra second-order kernel for initial portion of output data.

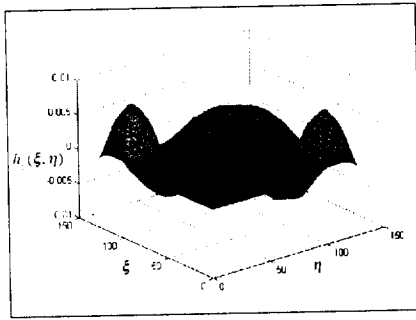


Fig. 14 Volterra second-order kernel for latter portion of output data.

POD Results

To assess the applicability of POD-based ROMs to differential equations that exhibit limit-cycle oscillation (LCO), Beran et al.⁶³ computed solutions with the subspace projection technique of a tubular reactor, known to experience LCO.^{64,65} The governing equations are

$$\frac{\partial w_1}{\partial t} = Lw_1 - w_1Q(w_2), \quad (12)$$

$$\frac{\partial w_2}{\partial t} = Lw_2 - \beta_1(w_2 - \beta_3) + \beta_2w_1Q(w_2), \quad (13)$$

$$L \equiv \frac{1}{Pe} \frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial x}, \quad Q(w_2) = \mu \exp\left(\Gamma - \frac{\Gamma}{w_2}\right),$$

where Pe , β_1 , β_2 , β_3 , Γ , and μ are specified parameters. Equations (12) and (13) describe convection, diffusion and reaction (CDR) within the reactor, and are referred to as the CDR equations. The variables w_1 and w_2 represent concentration and temperature, respectively, and the parameter μ (the Damkohler number) determines the ability of the CDR equations to sustain LCO. The spatial domain is normalized; boundary conditions are applied at $x = 0$ and $x = 1$. Following spatial discretization of the equations and specification of suitable initial conditions, which are

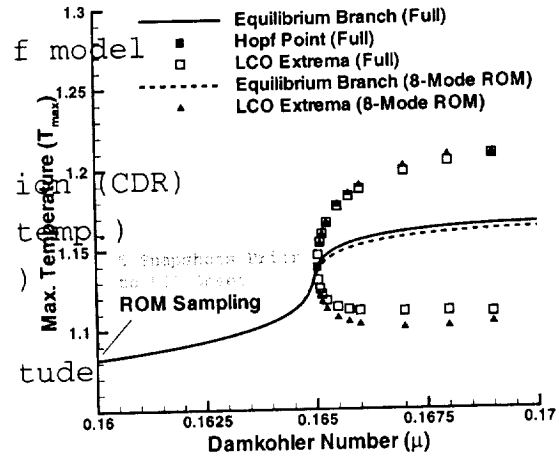


Fig. 15 Comparison of full-system and ROM solutions for an 8-mode ROM trained at $\mu^0 = 0.16$. Equilibrium and LCO branches.

described elsewhere,⁶³ the CDR equations take the form $\frac{dw}{dt} = \mathbf{R}(w; \mu)$.

The CDR system experiences a supercritical Hopf bifurcation at $\mu^* = \mu = 0.16504$,⁶³ which is accurately predicted by a POD-based ROM. The stability properties of the CDR system are shown in 15, where it is seen that stability of the equilibrium branch is lost beyond the bifurcation point. Solutions are characterized by the maximum value of temperature computed over the domain, T_{max} . The ROM is developed by sampling the CDR system as it evolves towards steady-state ($0 \leq t \leq 2.5$) for a value of μ leading to system stability and starting with $\mu^0 = 0.16$ and $\mu^0 < \mu^*$. Following the procedure described above, 8 modes are computed and retained, representing a 15-fold reduction in problem size. Solutions of the full system are explicitly computed via time integration.

Equilibrium solutions of the full system and the ROM are observed to be in excellent agreement. As seen in Figure 15, agreement is nearly exact at $\mu^0 = 0.16$, where the POD is constructed, and is excellent for the remaining values of μ shown. Beyond the Hopf point, LCO amplitude is well predicted with the ROM.

The critical value of μ at which the CDR system loses stability is also very accurately predicted using reduced-order modeling. Stability loss is observed at $\mu = 0.16503$, nearly the same value predicted with the full-system equations ($\mu = 0.16504$).

It is clear that POD and related methods provide insight into the dominant dynamics of a given system. Future research may focus on the coupling of Volterra methods with POD methods for a hybrid approach.

Concluding Remarks

The development of ROMs based on the time-domain and frequency-domain versions of the Volterra theory of nonlinear systems has been described, including continuous- and discrete-time versions of the theory. The basic objective of the theory is the identification of linearized and nonlinear kernel functions that capture the dominant response features of a nonlinear system. The method provides a very natural and intuitive extension of well-understood linear phenomena into the nonlinear domain. The results presented highlight two primary advantages of the theory: improved computational efficiency and insight into the nonlinear dynamics of the system of interest.

The status of the Volterra-based ROM approach can be summarized as follows. The method has been used to show that discrete-time concepts, indeed digital signal processing concepts such as unit pulses and step inputs, are directly applicable to CFD codes. The method has also been shown to be a higher-level generalization of the standard linear methods in use today. In addition, the nature of the method is such that it requires minimal modification to the CFD code of interest. Most unsteady aerodynamic or aeroelastic CFD codes already have various excitation inputs (e.g., sinusoidal) and extension to a Volterra-based ROM approach simply involves adding a pulse (or step) input to the suite of available inputs - the CFD code itself remains unchanged.

As for POD-based ROM developments (and related methods), it is clear by the quantity and quality of the research effort devoted to this approach that a great deal of insight can be gained into the system being analyzed. For example, a POD-based ROM of a discretized convection-diffusion-reaction (CDR) system was described and shown capable of determining a variety of important characteristics of the nonlinear system, including nonlinear static behavior, bifurcation to limit-cycle behavior, and sensitivity to changes in system parameters. The CDR problem serves as an analog for the study of the aeroelastic properties of a wing, including static analysis, dynamic analysis, bifurcation analysis, and sensitivity analysis. Several of the POD references mentioned thus far provide detailed discussions regarding future extensions and applications of the method.

Future Directions

As for the challenges associated with the Volterra-based ROM approach, there is much work to be done. An important issue that needs to be addressed is the issue of modal superposition with respect to nonlinear effects. Although it is clear that a mode-by-mode excitation is a linearization of the aeroelastic process, it is important

to understand the limitations of this approach. In addition, work continues on the development of a technique that provides simultaneous excitation to all modes, eliminating the linearization issue. Linearized state-space models have been developed using the CFD-based pulse responses that can be incorporated directly into control system analysis, for example. These state-space matrices also sidestep the need to transform time-domain CFD loads into the frequency-domain only to transform the frequency-domain loads back into the time domain via rational function approximations. Using the Volterra approach, time-domain CFD-based information goes directly into creating time-domain state-space matrices, a more efficient process. But the ultimate challenge lies in the creation of nonlinear (bilinear) state-space matrices which are mathematically related to the higher-order Volterra kernels. Some work has been done in this area, but there is significantly more work that needs to be done. With respect to the frequency-domain version of the theory, additional research is needed to develop precise interpretations of the higher-order spectra as they apply to nonlinear aeroelastic systems. Additional research in the analytical derivation of these functions as well as experimental application of these techniques promises to provide great insights to all these questions.

Acknowledgements

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