Reduced-Order Modelling of Linear Time-Varying Systems

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Abstract

We present a theory for reduced-order modelling of linear time-varying systems, together with efficient numerical methods for application to large systems. The technique, called TVP (Time-Varying Padé), is applicable to deterministic as well as noise analysis of many types of communication subsystems, such as mixers and switched-capacitor filters, for which existing model reduction techniques cannot be used. TVP is therefore suitable for hierarchical verification of entire communication systems. We present practical applications in which TVP generates macromodels which are more than two orders of magnitude smaller, but still replicate the input-output behaviour of the original systems accurately. The size reduction results in a speedup of more than 500.

1 Introduction

An important task in communication system design is hierarchical verification of functionality at different levels, starting from individual circuits up to block representations of full systems. A key step in this process is the creation of small macromodels that abstract, to a given accuracy, the behaviour of much bigger subsystems. For systems with "nonlinear" blocks like mixers and switched-capacitor filters, this is typically achieved by using results from detailed nonlinear simulations to construct macromodels manually. This process has disadvantages. Nonlinear simulation does not provide parameters of interest (such as poles and zeros) directly; to obtain them by inspection, frequency-response plots with many points are often computed. This can be very time-consuming for large subsystems, since nonlinear blocks require a steady-state solution at each point. Also, the macromodelling step, critical for reliable verification, is heuristic, time-consuming and highly reliant on detailed internal knowledge of the system under consideration.

In this paper, we present an algorithmic technique for abstracting small macromodels from detailed descriptions of many kinds of "nonlinear" subsystems encountered in communication systems. Named TVP (Time-Varying Padé), the method reduces a large linear time-varying (LTV) system to a small one. The LTV model is adequate for many apparently nonlinear systems, like mixers and switched-capacitor filters, where the signal path is designed to be linear, even though other inputs (e.g., local oscillators, clocks) may cause "nonlinear" parametric changes to the system. TVP can also produce cyclostationary noise macromodels of time-varying systems. Extensions of TVP for macromodelling signal path nonlinearities and autonomous systems have been devised and will be described separately.

Reduced-order modelling is well established for circuit applications (e.g., AWE [PR90], PVL [FF95a, FF97], PRIMA [OCP97]), but to the best of our knowledge, existing methods are applicable only to linear time-invariant (LTI) systems. Hence they are inadequate for communication blocks with properties like frequency translation, that cannot be represented by LTI models. LTV descriptions of a system, on the other hand, can capture frequency translation and mixing/switching behaviour.

LTV transfer functions are often computed in the context of RF simulation (e.g., plotting frequency-responses or calculating cyclostationary noise [TKW96, Len97, RLF98]), but a formulation suitable for model reduction has not been available. The basic difficulty in generalizing LTI model-reduction techniques to the LTV case has been the interference of system time variations with input time variations. A key step in this work is to separate the two time-scales, using recent concepts of multiple time variables and the MPDE [BWLBG96, Roy97a, Roy98a], resulting in forms for the LTV transfer function that are suitable for model reduction. Padé approximation of this transfer function results in a smaller system, any desired number of moments of which match those of the original system.

TVP has several useful features. The computation/memory requirements of the method scale almost linearly with circuit size, thanks to the use of factored-matrix computations and iterative linear algebra [RA92, MFR95, RLF98]. TVP provides the reduced model as a LTI system followed by a memoryless mixing operation; this makes it easy to incorporate the macromodel in existing circuit and system level simulators. TVP itself can be implemented easily in a number of existing simulation tools, including nonlinear time-domain simulators like SPICE, nonlinear frequency-domain domain simulators using harmonic balance, as well as linear time-varying simulators like SWITCAP and SIMPLIS. Time-domain computations, moreover, do not necessarily require obtaining or using a steady state of the system. Existing LTI model reduction codes can be used as black boxes in TVP's implementation. Like its LTI counterparts, TVP based on Krylov methods (see Section 3.2) is numerically well-conditioned and can directly produce dominant poles and residues.

Most importantly, by providing an algorithmic means of generating reduced-order models to any desired accuracy, TVP enables macromodels of communication subsystems to be coupled to detailed realizations much more tightly and quickly than previously possible. This can significantly reduce the number of iterations it takes to settle on a final design.

The remainder of the paper is organized as follows. In Section 2, the MPDE is used to obtain the LTV transfer function in forms useful for model reduction. In Section 3, Padé approximation and reduced-order modelling of the LTV transfer function is presented. Applications of TVP are presented in Section 4.

2 The LTV transfer function

We consider a nonlinear system driven by a large signal $b_l(t)$ and a small input signal $u(t)$, to produce an output $z_t(t)$ (for simplicity, we take both $u(t)$ and $z(t)$ to be scalars; the generalization to the vector case is straightforward). The nonlinear system is modelled using vector differential-algebraic equations (DAEs), a description adequate for circuits [CL75] and many other applications:

$$\frac{\partial q(y(t))}{\partial t} + f(y(t)) = b_l(t) + bu(t), \quad z_t(t) = d^T y(t) \tag{1}$$

In the circuit context, $y(t)$ is a vector of node voltages and branch currents; $q()$ and $f()$ are nonlinear functions describing the charge/flux and resistive terms, respectively, in the circuit. b and d are vectors that link the input and output to the rest of the system.

We now move to the MPDE (multirate partial differential equation [BWLBG96, Roy97a, Roy98a]) form of (1). Doing so enables the input and system time-scales to be separated and, as will become apparent, leads to a form of the LTV transfer function useful for reduced-order modelling. The move to the MPDE (2), below, is justified by the fact (proved in, e.g., [Roy98a]) that (1) is exactly equivalent to (2).

$$\begin{aligned}&\frac{\partial q(\hat{y})}{\partial t_1} + \frac{\partial q(\hat{y})}{\partial t_2} + f(\hat{y}(t_1,t_2)) = b_l(t_1) + bu(t_2)\\ &\hat{z}_t(t_1,t_2) = d^T \hat{y}(t_1,t_2), \quad z_t(t) = \hat{z}_t(t,t)\end{aligned} \tag{2}$$

The hatted variables in (2) are bivariate (i.e., two-time) forms of the corresponding variables in (1).

To obtain the output component linear in u, we perform a linearization around the solution of (2) when $u(t_2) \equiv 0$. Let this solution be $\hat{y}^*(t_1)$ (note that we can always select $\hat{y}^*$ to be independent of t_2). Linearization about $\hat{y}^*$ yields the linear MPDE:

$$\begin{aligned}&\frac{\partial\left(C(t_1)\hat{x}(t_1,t_2)\right)}{\partial t_1} + \frac{\partial\left(C(t_1)\hat{x}(t_1,t_2))\right)}{\partial t_2} + G(t_1)\hat{x}(t_1,t_2) = bu(t_2)\\ &\hat{z}(t_1,t_2) = d^T \hat{x}(t_1,t_2), \quad z(t) = \hat{z}(t,t)\end{aligned} \tag{3}$$

In (3), the quantities $\hat{x}$, $\hat{z}$ and z are the small-signal versions of $\hat{y}$, $\hat{z}_t$ and z_t, respectively; $C(t_1) = \left.\frac{\partial q(\hat{y})}{\partial \hat{y}}\right|_{\hat{y}^*(t_1)}$ and $G(t_1) = \left.\frac{\partial f(\hat{y})}{\partial \hat{y}}\right|_{\hat{y}^*(t_1)}$ are time-varying matrices.

Note that the bi-variate output $\hat{z}(t_1,t_2)$ is linear in the input $u(t_2)$, but that the relationship is time-varying because of the presence of t_1. To obtain the time-varying transfer function from u to $\hat{z}$, we Laplace transform (3) with respect to t_2:

$$\begin{aligned} &\frac{\partial\left(C(t_1)\hat{X}(t_1,s)\right)}{\partial t_1} + sC(t_1)\hat{X}(t_1,s) + G(t_1)\hat{X}(t_1,s) = bU(s) \\ &\hat{Z}(t_1,s) = d^T\hat{X}(t_1,s) \end{aligned} \tag{4}$$

In (4), s denotes the Laplace variable along the t_2 time axis; the capital symbols denote transformed variables.

We can rewrite (4) as

$$\begin{aligned} &\left(\frac{D}{dt_1}[] + sC(t_1) + G(t_1)\right)\hat{X}(t_1,s) = bU(s) \\ &\hat{Z}(t_1,s) = d^T\hat{X}(t_1,s), \quad \text{where } \frac{D}{dt_1}[v] = \frac{\partial\left(C(t_1)v\right)}{\partial t_1} \end{aligned} \tag{5}$$

and obtain an *operator form* of the time-varying transfer function $H(t_1,s)$:

$$H(t_1,s) = d^T\left(\frac{D}{dt_1}[] + sC(t_1) + G(t_1)\right)^{-1}[b], \; \hat{Z}(t_1,s) = H(t_1,s)\,U(s) \tag{6}$$

Finally, the frequency-domain relation between the output $z(t)$ and its bi-variate form $\hat{z}$ is:

$$Z(s) = \int_{-\infty}^{\infty}\hat{Z}_s(s-s_2,s_2)\,ds_2 \tag{7}$$

where $Z(s)$ is the Laplace transform of $z(t)$ and $\hat{Z}_s(s_1,s_2)$ the two-dimensional Laplace transform of $\hat{z}(t_1,t_2)$, or equivalently, the Laplace transform of $\hat{Z}(t_1,s_2)$ with respect to t_1.

The operator form (6) is already useful for reduced-order modelling. We can proceed further, however, by expanding the t_1 dependence in a basis. This leads to matrix forms of the transfer function, to which existing model reduction codes can be applied – a very desirable feature for implementation. Frequency-domain basis functions, considered in Section 2.1, are natural for applications with relatively sinusoidal variations, while time-domain ones are better suited to systems with switching behaviour and for those that are not periodic.

2.1 Frequency-domain matrix form

Assume $C(t_1)$ and $G(t_1)$ to be periodic with angular frequency ω_0. Define $W(t_1,s)$ to be the operator-inverse in (6):

$$\begin{aligned} &W(t_1,s) = \left(\frac{D}{dt_1}[] + sC(t_1) + G(t_1)\right)^{-1}[b] \\ &\Rightarrow \frac{D}{dt_1}[W(t_1,s)] + [sC(t_1) + G(t_1)]\,W(t_1,s) = b \end{aligned} \tag{8}$$

Assume $W(t_1,s)$ also to be in periodic steady-state in t_1, and expand $C(t_1)$, $G(t_1)$, and $W(t_1,s)$ in Fourier series with coefficients C_i, G_i, and $W_i(s)$, respectively:

$$C(t_1) = \sum_{i=-\infty}^{\infty} C_i e^{ji\omega_0 t_1},\; G(t_1) = \sum_{i=-\infty}^{\infty} G_i e^{ji\omega_0 t_1},\; W(t_1,s) = \sum_{i=-\infty}^{\infty} W_i(s) e^{ji\omega_0 t_1} \tag{9}$$

Now define the following long vectors of Fourier coefficients: follows:

$$\begin{aligned} &\vec{W}_{FD}(s) = \left[\cdots, W_{-2}^T(s), W_{-1}^T(s), W_0^T(s), W_1^T(s), W_2^T(s), \cdots\right]^T \\ &\vec{B}_{FD} = \left[\cdots, 0, 0, b^T, 0, 0, \cdots\right]^T \end{aligned} \tag{10}$$

By putting the expressions (9) in (8) and equating coefficients of $e^{j\omega_0 t_1}$, it can be verified that the following block-matrix equation holds:

$$[sC_{FD} + \underbrace{(G_{FD} + \Omega C_{FD})}_{\text{denote this by } J_{FD}}]\vec{W}_{FD}(s) = \vec{B}_{FD} \tag{11}$$

where

$$\begin{aligned} &C_{FD} = \text{toeplitz}(\cdots, C_2, C_1, C_0, C_{-1}, C_{-2}, \cdots), \\ &G_{FD} = \text{toeplitz}(\cdots, G_2, G_1, G_0, G_{-1}, G_{-2}, \cdots), \text{ and} \\ &J_{FD} = G_{FD} + \Omega C_{FD}, \quad \Omega = j\omega_0\text{diag}(\cdots, -2I, -I, 0, I, 2I, \cdots) \end{aligned} \tag{12}$$

Now denote

$$E(t_1) = \left[\cdots, Ie^{-j2\omega_0 t_1}, Ie^{-j\omega_0 t_1}, I, Ie^{j\omega_0 t_1}, Ie^{j2\omega_0 t_1}, \cdots\right]^T \tag{13}$$

From (11), (8) and (6), we obtain the following matrix expression for $H(t_1,s)$:

$$H(t_1,s) = d^T E^T(t_1)\; [sC_{FD} + J_{FD}]^{-1}\; \vec{B}_{FD} \tag{14}$$

From (13), note that $E(t_1)$ can be written in the Fourier expansion:

$$E(t_1) = \sum_{i=-\infty}^{\infty} E_i e^{ji\omega_0 t_1},\; E_i = [\cdots, 0, 0, \underbrace{\;I\;}_{i^{\text{th}} \text{ position}}, 0, \cdots, 0, \cdots]^T \tag{15}$$

Hence we can rewrite (14) in a Fourier series:

$$H(t_1,s) = \sum_{i=-\infty}^{\infty} H_i(s) e^{ji\omega_0 t_1},\; H_i(s) = d^T E_i^T\; [sC_{FD} + J_{FD}]^{-1}\; \vec{B}_{FD} \tag{16}$$

(16) implies that any linear periodic time-varying system can be decomposed into LTI systems followed by memoryless multiplications with $e^{ji\omega_0 t}$. Hence the above constitutes an alternative derivation of Floquet theory ([Gri90]). In fact, it will be proved in (26) that (16) is equivalent to a much more compact representation, consisting of only *one* LTI block followed by a memoryless multiplication[1]. The quantities $H_i(s)$ will be called *baseband-referred transfer functions*.

Define:

$$\vec{H}_{FD}(s) = [\cdots, H_{-2}(s), H_{-1}(s), H_0(s), H_1(s), H_2(s), \cdots]^T \tag{17}$$

Then,

$$\vec{H}_{FD}(s) = D^T\,[sC_{FD} + J_{FD}]^{-1}\,\vec{B}_{FD},\; D = \text{diag}(\cdots, d, d, d, \cdots) \tag{18}$$

(18) is a block matrix equation for a single-input multi-output transfer function. If the size of the LTV system (3) is n, and N harmonics of the LTV system are considered in practice, then $\vec{B}_{FD}$ is a vector of size $Nn \times 1$, C_{FD} and J_{FD} are square matrices of size $Nn \times Nn$, D is a rectangular matrix of size $Nn \times N$, and $\vec{H}_{FD}(s)$ is a vector of size N.

3 Padé approximation of the LTV transfer function

The forms for the LTV transfer function (6) and (18) can be expensive to evaluate, since the dimension of the full system can be large in practice. In this section, methods are presented for approximating $H(t_1,s)$ using quantities of much smaller dimension.

The underlying principle is that of Padé approximation, i.e., for any of the forms of the LTV transfer function, to obtain a smaller form of size q whose first several moments match those of the original large system. This can be achieved in two broad ways, with correspondences in existing LTI model-reduction methods. TVP-E (TVP-Explicit), roughly analogous to AWE [PR90, CN94] for LTI systems, involves calculating moments of the large system explicitly and building the reduced order model from these moments. The method is outlined in Section 3.1. Another procedure, TVP-K (TVP-Krylov) replaces the large system directly with a smaller one, achieving moment-matching implicitly. This uses Krylov-subspace methods and is analogous to LTI model-reduction techniques based on the Lanczos- and Arnoldi-based processes (e.g., PVL [FF95a], operator-Lanczos methods [CC97], PRIMA [OCP97] and others). As in the LTI methods, TVP based on Krylov subspaces has several advantages over explicit moment matching, and is presented in Section 3.2. Operator- or matrix-based techniques can be applied to both explicit and Krylov-based TVP; Section 3.1 describes an operator-based procedure and Section 3.2 a matrix-based one.

[1]This is a new specialized form of the Floquet theorem.

3.1 TVP-E: TVP using explicit moment matching

Either of the forms (6) and (18) can be used for explicit moment matching. Here, we illustrate an operator procedure using (6). Rewrite $H(t_1,s)$ from (6) as:

$$H(t_1,s) = d^T \left(I\left[\right] + sL\left[\right]\right)^{-1}\left[r(t_1)\right], \; (I\left[\right] \text{ is the identity operator}),$$
$$L[v] = \left(\frac{D}{dt_1}\left[\right] + G(t_1)\right)^{-1}\left[C(t_1)\,v\right], r(t_1) = \left(\frac{D}{dt_1}\left[\right] + G(t_1)\right)^{-1}[b] \qquad (19)$$

$H(t_1,s)$ in (19) can be expanded as:

$$H(t_1,s) = d^T\left(r(t_1) - sL\left[r(t_1)\right] + s^2 L\left[L\left[r(t_1)\right]\right] + \cdots\right) = \sum_{i=0}^{\infty} m_i(t_1)\,s^i,$$
$$\text{where } m_i(t_1) = (-1)^i\, d^T\, \underbrace{L\left[L\left[\cdots L\left[r(t_1)\right]\cdots\right]\right]}_{i \text{ applications of } L[]} \qquad (20)$$

$m_i(t_1)$ in (20) are the *time-varying moments* of $H(t_1,s)$. Note that these moments can be calculated explicitly from their definition in (20), by repeated applications of $L\left[\right]$. From its definition in (19), applying $L\left[\right]$ corresponds to solving a linear time-varying differential equation. If the time-varying system is in periodic steady-state, as is often the case in applications, $L\left[\right]$ can be applied numerically by solving the equations that arise in the inner loop of harmonic balance or shooting methods. Recently-developed iterative methods (e.g., [RA92, MFR95, TKW95, FML96]) enable large systems of these equations to be solved in linear time, hence the time-varying moments can be calculated easily.

Once the moments $m_i(t_1)$ have been computed, t_1 can be fixed at a given value, and any existing LTI model reduction technique using explicit moments (e.g., AWE) can be run q steps to produce a q^{th}-order reduced model. This step can be repeated for all t_1 values of interest, to produce an overall reduced-order model for $H(t_1,s)$ in the form:

$$\tilde{H}^q(t_1,s) = \frac{\sum_{i=0}^{q-1} a_i(t_1)\,s^i}{\sum_{j=0}^{q} b_j(t_1)\,s^j} = \sum_{i=0}^{q} \frac{c_i(t_1)}{s + p_i(t_1)} \qquad (21)$$

The procedure outlined above, though simple, has two disadvantages. The first is that model reduction methods using explicit moments suffer from numerical ill-conditioning, making them of limited value for q more than 10 or so [FF95a]. The second is that the form (21) has time-varying poles. It can be shown using Floquet theory that the transfer function $H(t_1,s)$ has a potentially infinite number of poles *that are independent of* t_1 (these poles are simply the Floquet eigenvalues shifted by multiples of the system frequency), together with residues that do, in fact, vary with t_1. It is desirable to obtain a reduced-order model with similar properties. In fact, this requirement is equivalent to obtaining a reduced system in the time-domain form of (3), which is very desirable for system-level modelling applications. In Section 3.2, we present Krylov-subspace procedures for TVP that eliminate both problems.

3.2 TVP-K: TVP using Krylov subspace methods

In this section, we describe the application of the block-Lanczos algorithm [ABFH96, FGN91, FF95b, Fre98] to any multi-output matrix form of the LTV transfer function. Alternatively, operator-Lanczos or any Arnoldi-based method can also be applied. Using Krylov-subspace methods provides not only a numerically stable means of obtaining a reduced-order model, but in addition, the resulting reduced transfer function has a form similar to that of $H(t_1,s)$ in (6), with similar properties like a possibly infinite number of t_1-invariant poles.

(18) is in the form:

$$\vec{H}(s) = D^T\left[sC + J\right]^{-1}\vec{B} \qquad (22)$$

(22) can be used directly for reduced-order modelling by block-Lanczos methods. We recall briefly the relevant features of these techniques, which are described in detail in [ABFH96, FGN91, FF95b, Fre98]. Running the block-Lanczos algorithm q steps using the quantities D, C, J and $\vec{B}$ will produce the following matrices and vectors:

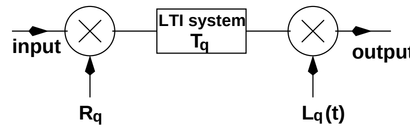

Figure 1: Specialized Floquet form of a LPTV system

L_q (of size $q \times N$), R_q (size $q \times 1$) and T_q (size $q \times q$). Define the q^{th}-order approximant $\vec{H}^q(s)$ by:

$$\vec{H}^q(s) = L_q^T\left[I_{q\times q} - sT_q\right]^{-1} R_q \qquad (23)$$

Then $\vec{H}^q(s) \approx \vec{H}(s)$, in the sense that a certain number of matrix-moments of the two quantities are identical – see [Fre98] for a precise description of the approximation. Further, it is typically the case in practice that even small values of q lead to excellent approximations.

Using the quantities in (23), an equivalent system of size q can be obtained easily. Define:

$$\vec{L}_q(t_1) = \sum_{i=-\infty}^{\infty} L_{q,i}\, e^{ji\omega_0 t_1} \qquad (24)$$

where $L_{q,i}$ is the i^{th} column of L_q. The approximate LTV transfer function $\tilde{H}^q(t_1,s)$ is given by

$$\tilde{H}^q(t_1,s) = \vec{L}_q(t_1)^T\left[I_{q\times q} - sT_q\right]^{-1} R_q \qquad (25)$$

(25) is the time-varying transfer function of the following q^{th}-order reduced system of time-domain equations:

$$-T_q\frac{\partial\tilde{x}}{\partial t} + \tilde{x} = R_q\,u(t), \qquad z(t) = \vec{L}_q(t)\tilde{x}(t) \qquad (26)$$

where $\tilde{x}(t)$ is a vector of size q, much smaller than that of the original system (3).

The TVP-K procedure above has a number of useful properties:

- Note that (26) represents a linear *time-invariant* system, with the time-variation appearing only in the output equation. The reduced system is illustrated in Figure 1. This feature makes the reduced model very easy to incorporate in existing simulation tools.
- In practice, only the baseband-referred transfer functions corresponding to harmonics of interest can be represented in (17), thereby reducing the number of columns of D.
- The form (25) can be shown to imply that $\tilde{H}^q(t_1,s)$ has a possibly infinite number of time-invariant poles, similar to $H(t_1,s)$. Further, the eigenvalues of T_q are the Floquet exponents of the reduced-order model, which approximate those of the original LTV system. The poles and residues of the reduced-order models of $H_i(s)$ can be easily calculated from the eigenvalues of T_q. The Floquet exponents are also useful in oscillator phase noise applications.
- Krylov-subspace algorithms such as Lanczos and Arnoldi require only matrix-vector products with C and linear system solutions with J. Though both matrices can be large, dense or difficult to factor, exploiting structure and using iterative linear algebra techniques can make these computations scale almost linearly with problem size [RA92, MFR95, TKW95, FML96, RLF98]. When these fast techniques are employed, the computation required by the TVP algorithm grows approximately linearly in circuit size and number of harmonics or time-points, making it usable for large problems.
- The numerical ill-conditioning problem with explicit moment matching in Section 3.1 is eliminated using Krylov methods, hence TVP can be run upto large values of q if necessary.
- A system with p_i inputs and p_o outputs can be handled easily, by stacking the extra outputs into $\vec{H}$ (resulting in D of size $nN \times p_oN$), and incorporating the inputs into $\vec{B}$ (to form a rectangular matrix of size $nN \times p_i$).

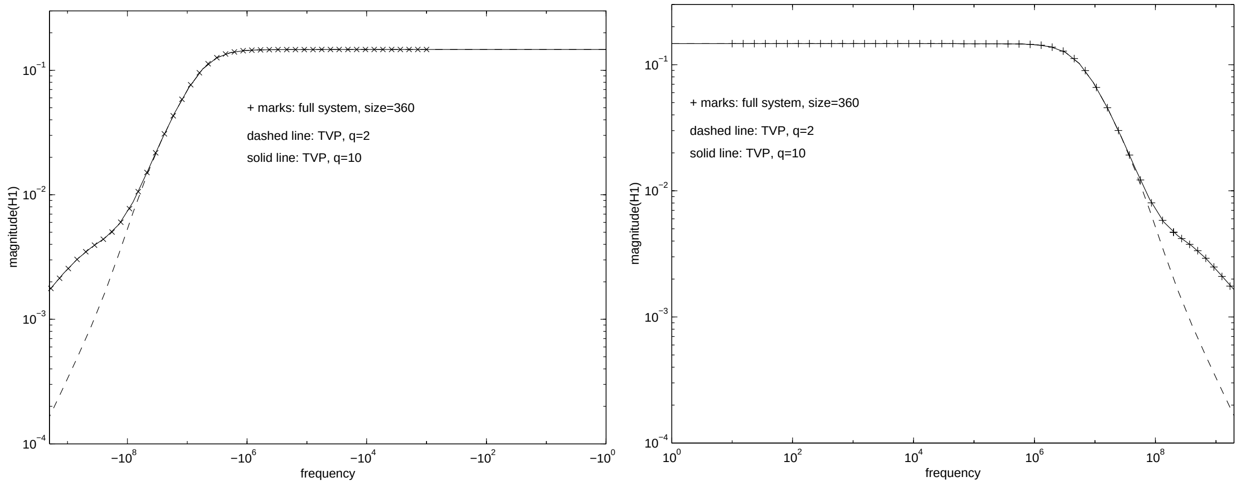

Figure 2: I-channel buffer/mixer $H_1(s)$: TVP vs full system, +ve and -ve frequencies

TVP, $q=2$	TVP, $q=10$
-2.95e-8	-2.95e-8
-2.30e-9 +j1.00e-11	-1.69e-8
	-2.81e-9 +j4.56e-9
	-2.81e-9 -j4.56e-9
	-2.54e-10 +j6.85e-13
	-2.85e-11 +j5.84e-11
	-2.53e-11 -j5.54e-11
	-2.12e-9 +j4.03e-13
	-3.16e-9 -j1.15e-14
	-3.06e-9 -j4.63e-11

Table 1: Poles of $H_1(s)$ for the I-channel buffer/mixer

4 Application to RF circuits

In this section, we apply TVP to a portion of the W2013 RFIC from Lucent Microelectronics, consisting of an I-channel buffer and mixer. The circuit consisted of about $n = 360$ nodes, and was excited by a local oscillator at 178Mhz driving the mixer, while the RF input was fed into the I-channel buffer. The time-varying system was obtained around a steady state of the circuit at the oscillator frequency; a total of $N = 21$ harmonics were considered for the time-variation.

Figure 3.2 shows frequency plots of $H_1(s)$, the upconversion transfer function. The points marked '+' were obtained by direct computation of (16), while the lines were computed using the TVP-reduced models with $q = 2$ and $q = 10$, respectively. Even with $q = 2$, a size reduction of two orders of magnitude, the reduced model provides a good match up to the LO frequency. When the order of approximation is increased to 10, the reduced model is identical upto well beyond the LO frequency. Evaluating the reduced models was more than *three orders of magnitude faster* than evaluating the transfer function of the original system.

The poles of the reduced models for $H_1(s)$ are shown in Table 1.

5 Conclusion

We have presented TVP, a theory and methods for reduced-order modelling of linear time-varying systems. The technique has applications in the macromodelling and hierarchical verification of communication systems, including noise. Applications to RF subsystems have been presented and size reductions of more than two orders of magnitude, resulting in similar speedups, have been obtained.

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