Reducing Location Update and Paging Costs in a PCS Network

Pablo García Escalle, Vicente Casares Giner, and Jorge Mataix Oltra

***Abstract*—*Mobility Tracking* operations in a *personal communication service (PCS)* system are signaling consuming. Several strategies have been proposed in the literature to reduce both, the location update (LU) and the paging (PG) costs. In this paper, we propose a location-tracking algorithm called three-location area (TrLA), combined with selective paging. In the TrLA, the mobile terminal (MT) allocates the identification of three neighboring location areas (LAs) in its local memory. We call this set of three LAs, a big-location area (BLA). Each time the MT exits the BLA, it triggers an LU message to the system *Databases* via a *base station*, in order to maintain the *Databases* up to date. The MT also updates its *caché* memory. A two-step selective PG is also considered and compared with the single-step (or nonselective PG). An analytical model based on a semi-Markov process has been used to evaluate our proposal. This scheme is compared with the classical Global System for Mobile Communications (GSM) standard and the Two-Location Algorithm proposal. As a result, this new scheme outperforms the cited strategies, and in our opinion, it can easily be implemented in existing standard cellular and personal communication systems.**

***Index Terms*—Global strategy, location update, mobility tracking, paging, personal communications services systems.**

I. Introduction

In a personal communication services (PCS) system, an incoming call to a mobile terminal (MT) must be delivered on time and at a minimum cost. The *mobility tracking* is performed by a set of procedures whose main goal is to locate an MT. In Global System for Mobile Communications (GSM) terminology, these procedures are called location update (LU) and Call Delivery (CD). First, the location of an MT must be reported to the system *Databases* and maintained up to date (LU procedure). Second, the CD procedure facilitates the search of an MT whenever there is an incoming call addressed to it. The *cell* where an MT is roaming must be identified in a short period, while keeping the LU and CD costs under certain constraints.

Roughly speaking, two LU strategies are envisaged in the literature. In the static or global strategy, usually implemented in current cellular systems, the whole coverage area is divided into several *location areas* (LAs).[1] The borders of the LAs are fixed, and an LA is composed of one or several *cells*. Each time an MT crosses the border between two LAs, it triggers an LU message to the system *databases* via a *base station*. Considering a location *database* (i.e., visitor location register or VLR) is assigned to exact one LA, every LU message triggered undergoes a registration at the VLR and home location register (HLR). However, it should be noted that a VLR might cover several LAs in the existing PCS systems.

On the other hand, by dynamic or local strategies [7]–[9], the LU procedure depends on the MT. No LAs exist. In [7], three dynamic strategies were proposed. An MT updates its location according to a policy based on 1) the time elapsed; 2) the number of *cells* visited; or 3) the distance to the *cell* in which the last LU message was triggered. In this paper, we focus our study on a global strategy.

The CD procedure is divided in two major functions: the *interrogation* and the paging (PG) procedures. The *interrogation* procedure is entirely supported by the *fixed network*. The MT is searched in the system *databases*. The output of a successful *interrogation* is the area where the MT is registered (the MT had its last interaction with the system *databases*). For instance, in the case of an LU global strategy, the output of this query would be an LA. In the case of an LU local strategy the output would be a *cell*. After a successful *Interrogation*, PG procedure is started. Then, the MT is searched according to some selective or nonselective PG algorithms. The output of the PG procedure is the *cell* where the MT is currently roaming.

This paper deals with *mobility tracking* procedures carried out in the common air interface (CAI), i.e., LU messages triggered by the MTs, and PG procedure. No attention is paid to the *Interrogation* procedure. A static LA layout is considered. The location-tracking algorithm proposed provides a hysteresis effect that avoids the "ping-pong" effect caused by MTs roaming in the surrounding of LA borders. In fact, the multilayer LU scenario proposed in [4] is the starting point for our proposal. Selective or multistep PG procedures are considered [6], [10]–[12] and a single-step PG or a two-step PG procedure is followed.

The work is organized as follows. In Section II, the scenario is introduced; LU and PG procedures are described. In Section III,

Manuscript received November, 1999; revised May, 2001; accepted May 30, 2001. The editor coordinating the review of this paper and approving it for publication is J. Lin. This work was supported by the Comisión Interministerial de Ciencia y Tecnología Spain (CICYT) under Project TIC98-0495-C02-02. This work was presented in part at the 9th MPRG/VT Symposium on Wireless Personal Communications, Blacksburg, VA, June, 1999.

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Publisher Item Identifier S 1536-1276(02)00195-2.

[1]The terms *registration area* and *location registration* are used in the *IS-41* standard, instead of *location area* and *location update*

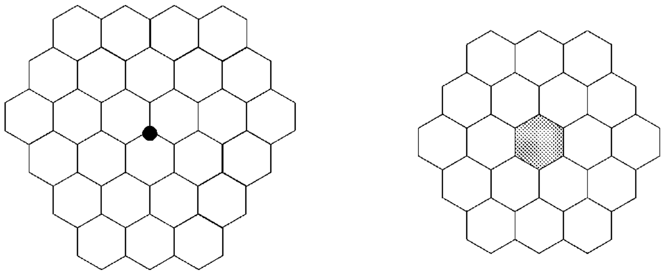

Fig. 1. Mosaic graphs (a) M_2 and (b) T_2.

TABLE I
NUMBER OF *CELLS* IN RING q, L_q, NUMBER OF *CELLS* IN THE MOSAIC X_q, N_q, WHERE $X = M$ OR T, AND PERIMETER OF THE MOSAIC X_q (IN TERMS OF NUMBER OF EDGES), $0 \leq q \leq n$

Mosaic graph		Ring number			
		Ring $q = 0$	Ring $q = 1$	...	Ring $q = n$
Mosaic T_n	L_q	1	6	...	$6n$
	N_q	1	7	...	$3n^2 + 3n + 1$
	Perimeter	6	18	...	$12n + 6$
Mosaic M_n	L_q	3	9	...	$6n + 3$
	N_q	3	12	...	$3n^2 + 6n + 3$
	Perimeter	12	24	...	$12n + 12$

the mathematical analysis is carried out. Some illustrative examples and discussions are reported in Section IV. Finally, in Section V conclusions are described.

II. Scenario and Procedure Descriptions

The configuration chosen has been the hexagonal *cell* layout. All *cells* have the same size. The *cell* dwell time of an MT has been characterized with a generalized gamma distribution, [15]. After leaving a *cell*, the MT enters one of its 6 neighboring hexagonal *cells* with a 1/6 probability. This is in fact a two-dimensional (2-D) random walk mobility model already used in several studies, [12].

As in current cellular networks, the whole coverage area is partitioned into a number of static LAs. We have considered that all the LAs have the same size. LAs are configured with mosaic graphs [5], which are constructed by arranging *cells* in concentric cycles around a starting point or a *cell*. With a vertex as starting point, Fig. 1(a), we obtain the so-called mosaic graphs M_n, where n denotes the number of rings minus one; i.e., the mosaic M_0 is composed of three *cells* (a ring of three *cells*), the mosaic M_1 is composed of 12 *cells* (a first ring of three *cells*, and a second ring of 9 *cells*), and so on. When the starting point is a *cell*, Fig. 1(b), we obtain mosaic graphs, T_n. For T_n, $n = 0$ corresponds to a single hexagon, and $n = 1$ corresponds to a cluster of seven *cells*. For both mosaic graphs, Table I illustrates: 1) the number of *cells* in ring q; 2) the number of *cells* in the mosaic X_q (where $X = T$ or M); and 3) the perimeter of the mosaic X_q (in terms of number of edges), $0 \leq q \leq n$.

A. Location Update Procedure

In current cellular networks, each MT has a local memory with a single record used to store the *Location Area Identifier* (LAI) where it is currently roaming. Each time an MT visits a new LA, it performs an LU procedure and updates the record with the new LAI. One of the main drawbacks of this option is the "ping-pong" effect caused by the MTs moving back and forth in the surroundings of LA borders; they generate an excessive signaling load because too many LU messages are sent.

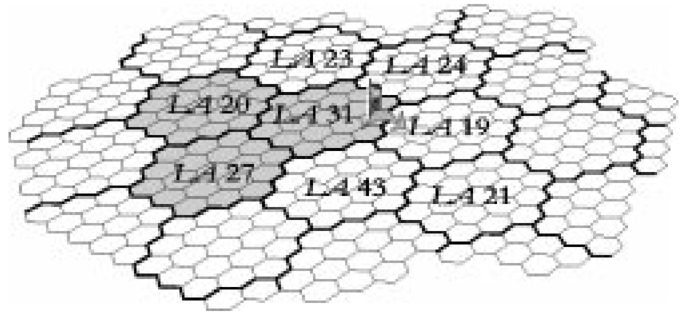

Fig. 2. The MT is registered at the BLA formed out of the LAs 20, 27, and 31. The LAs are mosaics T_2.

To mitigate this effect, the philosophy adopted in [4] is used: the MT keeps the identification of three neighboring LAs in its local memory. This set of three LAs is called *Big Location Area*, BLA. In fact, a BLA is a mosaic M_0 where the basic elements are LAs instead of *cells*. Each time an MT visits an LA, it checks the LAI: if it is coincident with one of the three LAIs stored in its memory, no LU message is triggered by the MT. Otherwise, the MT sends an LU message containing the identifier of the new visited LA toward the system *databases*. The PCS system sends the new set of LAs to the MT, for them to be stored. Whenever there is a change of BLA, the MT always overwrites one or two records. The new BLA always overlaps the old BLA, and the fact that at least one LA is common to both (the old and the new) BLAs is clear. Observe that the LAs that conform a BLA change dynamically according to the movement of the MT.

For convenience, we consider a VLR is assigned to exact one LA. Therefore, an MT is registered in three VLRs at a time, i.e., the three LAs of the BLA. For each MT, three registers have to be maintained up to date in the HLR. Whenever an LU message is sent by an MT, a registration procedure is started at the VLR receiving it. This VLR also starts a registration procedure at the HLR. The HLR knows the three VLRs where the MT was registered, and the new VLR where the registration procedure started. Considering the HLR knows the VLR layout (i.e., the LA layout), it can evaluate the three most suitable VLRs where the MT should be registered, and then send the corresponding LAIs of its new BLA to the MT. Notice also that, the VLR where the registration procedure started is stored in the first place at the HLR, because in case of a selective PG, the MT will be paged first in this LA (VLR).

In Fig. 2, the MT is registered at the BLA formed out of the LAs 20, 27, and 31 (these are the LAIs). A VLR is assigned to exact one LA. When the MT enters the LA 19, it searches the LAI number 19 in its three local registers. This search is not successful, so it triggers an LU message. The VLR corresponding to LA 19 receives the message, and registers the MT at its area. The VLR also forward the message to the HLR. The HLR knows that: 1) the MT was registered in the VLRs corresponding to LAs 20, 27, and 31; and 2) the MT is now registered at LA 19. Since the HLR knows the VLR layout, it registers the

MT at the VLRs corresponding to LAs 19, 43, and 31 (the BLA formed out of the LAs 19, 24, and 31 is also valid), and properly informs the MT. Notice that LA 31 is common to the old and the new BLA. A deregistration message to both VLRs (corresponding to LA 20 and LA 27) is forwarded from the HLR.

B. Paging Procedure

After each successful *Interrogation* procedure, the system initiates the PG process. When a single-step process is used, the total number of *cells* per BLA, N_c, ($N_c = N_{\mathrm{LA}\,0} + N_{\mathrm{LA}\,1} + N_{\mathrm{LA}\,2}$) is paged simultaneously, and a minimum delay in the PG process is achieved. $N_{\mathrm{LA}\,k}$ is the total number of *cells* that configures LA k, where $k = 0$, 1, or 2 is a state number used in the mathematical model (it is not an LAI), see Section III. Considering LA k is a mosaic X_n (where $X = T$ or M), $N_{\mathrm{LA}\,k} = N_n$, see Table I. The HLR determines the three LAs where the MT is registered, and orders to search the MT in these three LAs simultaneously.

The second alternative is the multistep or selective PG. In this work, a two-step PG procedure has been considered, [6]. The MT will first be paged in the LA corresponding to the VLR where the last interaction with the HLR occurred. As mentioned at the end of Section II-A, this LA corresponds to the VLR first stored at the HLR (corresponding also to state $k = 0$, 1, or 2). If the first PG is not successful, the MT is paged in the two remaining LAs simultaneously.

III. Mathematical Model

In this section we provide the mathematical analysis needed to evaluate the total cost per call arrival. The evolution of Fig. 4 is the mathematical model proposed in Fig. 5. Each state k represents an LA of the BLA where the MT is registered. A BLA looks like a mosaic M_0 formed out of three LAs (the three states). Therefore, state k corresponds to LA k, $k = 0$, 1, or 2. k is the state number and not the LAI. In the mathematical model, only state numbers are referred to (that is, LA numbers), LAIs are not used.

While the MT keeps moving from one state to another in the same BLA (the grey BLA in Figs. 4 and 5), no LU message is triggered. If the MT exits these grey states, it triggers an LU message and enters a new BLA. Obviously, the new BLA has yet again three states; LA 0, LA 1, and LA 2. The LA most recently visited can be LA 0, LA 1, or LA 2.

The *cell* dwell time of an MT has been characterized with a generalized gamma distribution, [15], with probability density function (pdf) given by

$$\begin{aligned} f_{\mathrm{cell}}(t; a, b, c) =& \frac{c}{b^{ac}\Gamma(a)} t^{ac-1} e^{-(t/b)^c} \\ =& f_{\mathrm{cell}}(t); \quad t, a, b, c > 0 \end{aligned} \tag{1}$$

where

- $\Gamma(a)$ is the gamma function, defined as $\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx$ being a any real and positive number;
- a, b, and c, are fixed according to [15]. In the cited article, the pdf of the *cell* dwell time is studied, and the suitable values of these variables are $a = 2.31$, $b = 1.22\ R$, and $c = 1.72$ where R is the equivalent *cell* radius (considering an average speed of 50 km/h and zero drift).

We denote its Laplace transform (LT) as $f^*_{\mathrm{cell}}(s)$. Its mean value is given by

$$-f^{*'}_{\mathrm{cell}}(0) = b\frac{\Gamma\left(\frac{a+1}{c}\right)}{\Gamma(a)} = \frac{1}{\lambda_m}. \tag{2}$$

The variables a and c are fixed. Obviously, there is a relation between b, R, and λ_m, only one of them needs to be fixed. We will use λ_m as a parameter (the mobility parameter in a *cell*), and then evaluate b and R.

The pdf of the sojourn time in LA k is denoted by $f_{k,n,X}(t) = f_k(t)$; the LA k is a mosaic X_n of *cells*, $X = T$ or M. The LT of $f_{k,n,X}(t)$ is denoted by $f^*_{k,n,X}(s) = f^*_k(s)$, and the pdf of the residual sojourn time is denoted by $f^*_{kr,n,X}(s) = f^*_{kr}(s)$. In order to evaluate the mean sojourn time in an LA, we use the state transition diagram shown in Fig. 3 with the transition probabilities of Table II. This transition diagram is obtained assuming that the MT is roaming in a *cell* associated to the qth ring, $0 \le q \le n$, and taking into account the random walk mobility model discussed at the beginning of Section II, [12]. As illustrated in Fig. 3, we assume that the LAs are mosaics X_n formed out of *cells*, where $X = T$ or M. Based on this assumption, Appendix A-1, illustrates that the mean sojourn time in state k (or LA k), $-f^{*'}_{k,n,X}(0) = 1/\lambda_{\mathrm{LA}\,k,\,n,\,X} = 1/\lambda_{\mathrm{LA}\,k}$, is given by

$$\begin{aligned} \frac{1}{\lambda_{\mathrm{LA}\,k}} =& \begin{cases} \frac{3n^2+3n+1}{2n+1}\frac{1}{\lambda_m}, & \text{for } X = T, \\ \frac{3}{2}(n+1)\frac{1}{\lambda_m}, & \text{for } X = M, \end{cases} \quad \text{for} \\ n =& 0, 1, 2, 3, \ldots. \end{aligned} \tag{3}$$

Expression (3) will be used to evaluate the LU cost to be derived in Section III-A. In Section III-B, the PG cost is also derived by assuming both, single-step and two-step PG procedures.

A. Location Update Cost

Let m be the number of LU messages triggered by the MT between two call arrivals, conditioned by the fact that the MT received its last call in LA k, $k = 0$, 1, 2, see Fig. 5. States $ex0$, $ex1$, and $ex2$ are absorbing states. At time $t = 0^+$, i.e., just after an incoming call arrives to the MT, the system is in state k with probability $P_k(0^+)$ to be determined, $k = 0$, 1, 2. The MT will reside in the initial state k following the residual time distribution of $f^*_k(s)$; i.e., $f^*_{kr}(s) = \lambda_{\mathrm{LA}\,k}\left(1 - f^*_k(s)\right)/s$ [1] (we recall the short notations $f^*_k(s) = f^*_{k,n,X}(s)$; $\lambda_{\mathrm{LA}\,k} = \lambda_{\mathrm{LA}\,k,\,n,\,X}$ used in previous paragraphs). This is so, because the time distribution between two call arrivals is exponential (Poisson arrival with rate λ_c). Afterwards, we want to find the probability $\mathrm{pr}_{k,i}(\lambda_c)$ of the MT being visiting state i at the instant of the next call arrival, given that the MT has not exit the BLA. That is, $\mathrm{pr}_{k,i}(\lambda_c)$ ($k, i = 0$, 1, or 2) is the conditional probability that the system starting from state k will be visiting state i *when* the exponential time expires (the system is not absorbed into states exj, $j = 0$, 1, or 2). We also denote by $\mathrm{pr}_{k,exj}(\lambda_c)$, the absorption probability, into state exj *before* the observation time expires. Parallel definition applies to $p_{k,i}(\lambda_c)$ and $p_{k,exi}(\lambda_c)$ *when* the first sojourn time in state k is defined by $f^*_k(s)$, instead of

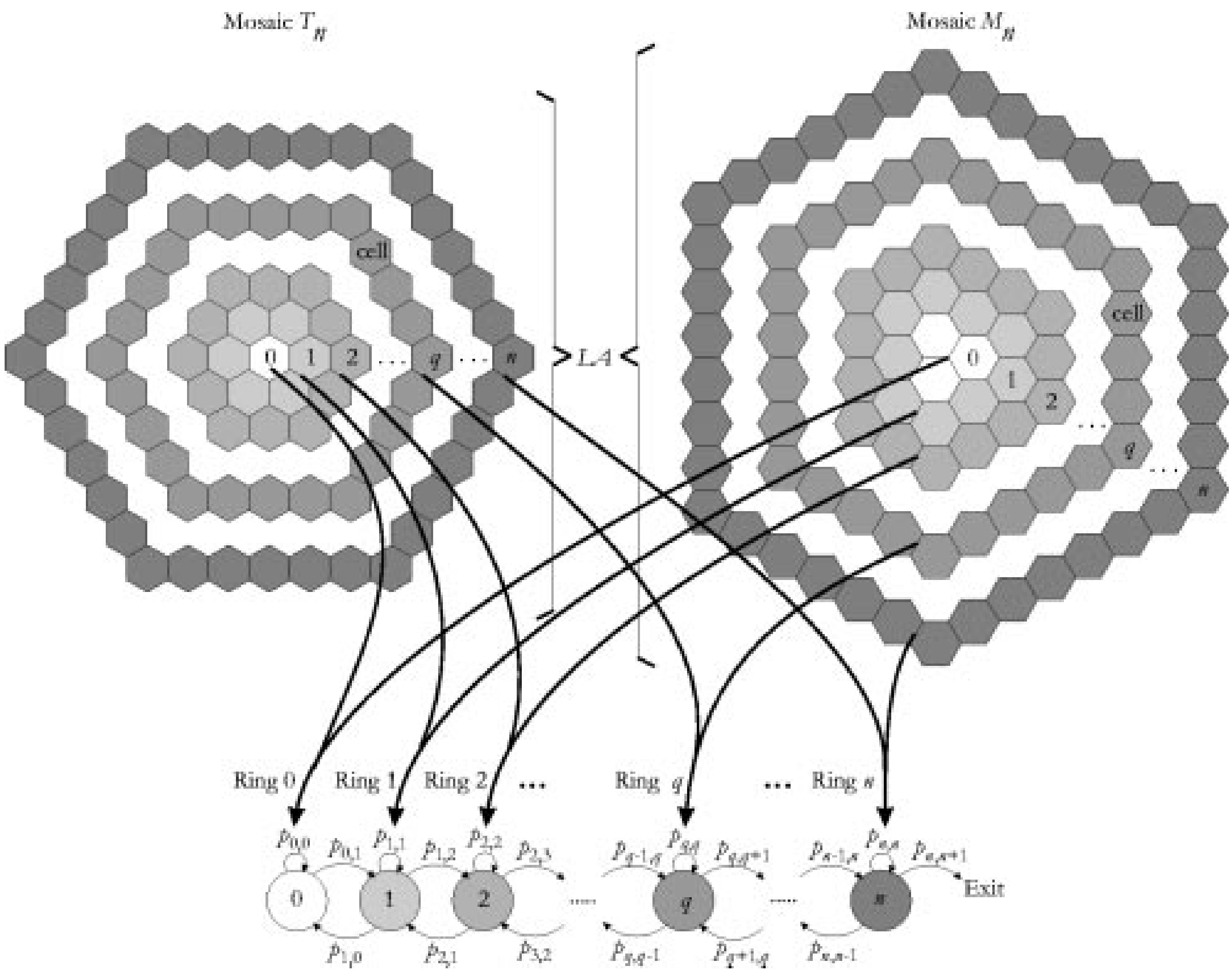

Fig. 3. State transition diagram of 2-D random walk mobility model used with hexagonal configurations.

TABLE II
TRANSITION PROBABILITIES FROM RING q TO RINGS $q-1$ AND $q+1$ ($p_{q,q-1}$ AND $p_{q,q+1}$) IN DIFFERENT HEXAGONAL *CELL* LAYOUT CONFIGURATIONS. SEE FIG. 3

Mosaic graph	$p_{0,1}$	$p_{1,0}$	...	$p_{q,q-1}$	$p_{q,q+1}$	...	$p_{n,n-1}$	$p_{n,n+1}$
Mosaic T_n	1	$\frac{1}{6}$	...	$\frac{2q-1}{6q}$	$\frac{2q+1}{6q}$	...	$\frac{2n-1}{6n}$	$\frac{2n+1}{6n}$
Mosaic M_n	$\frac{2}{3}$	$\frac{2}{9}$	...	$\frac{2q}{6q+3}$	$\frac{2(q+1)}{6q+3}$	...	$\frac{2n}{6n+3}$	$\frac{2(n+1)}{6n+3}$

the residual sojourn time $f^*_{kr}(s)$. Notice that we are using the memoryless property of the exponential distribution.

Then, no LU message (no absorptions) is triggered with probability

$$\Pr(\text{no LU}/k) = 1 - \sum_{i=0}^{2} \text{pr}_{k,exi}(\lambda_c) = \sum_{i=0}^{2} \text{pr}_{k,i}(\lambda_c); \quad k = 0, 1, \text{ or } 2. \tag{4}$$

Each time the system visits the absorbing state exi ($i = 0, 1, 2$), an instantaneous transition to state j is produced, $j \neq i$ ($i, j = 0$, 1, or 2), and an LU message is triggered by the MT. To be more precise, when an absorption into state exi is produced, an instantaneous transition to state $j = i+1 \pmod 2$ with probability $\alpha_i/(\alpha_i+\omega_i)$, or to state $j = i+2 \pmod 2$ with probability $\omega_i/(\alpha_i+\omega_i)$ is produced, see, also, Fig. 4. Probabilities α_i and ω_i are given in terms of the common perimeter between two neighboring LAs. In matrix notation to be used later, these probabilities can be written as

$$\mathbf{T} = \begin{bmatrix} 0 & \frac{\alpha_0}{\alpha_0+\omega_0} & \frac{\omega_0}{\alpha_0+\omega_0} \\ \frac{\omega_1}{\alpha_1+\omega_1} & 0 & \frac{\alpha_1}{\alpha_1+\omega_1} \\ \frac{\alpha_2}{\alpha_2+\omega_2} & \frac{\omega_2}{\alpha_2+\omega_2} & 0 \end{bmatrix}. \tag{5}$$

After each absorption and instantaneous transition to state j, the sojourn time in state j is clearly defined by $f^*_j(s)$, instead of the residual sojourn time $f^*_{jr}(s)$.

We denote by matrixes $\mathbf{Sr}$ and $\mathbf{S}$

$$\mathbf{Sr} = \begin{bmatrix} \text{pr}_{0,ex0}(\lambda_c) & \text{pr}_{0,ex1}(\lambda_c) & \text{pr}_{0,ex2}(\lambda_c) \\ \text{pr}_{1,ex0}(\lambda_c) & \text{pr}_{1,ex1}(\lambda_c) & \text{pr}_{1,ex2}(\lambda_c) \\ \text{pr}_{2,ex0}(\lambda_c) & \text{pr}_{2,ex1}(\lambda_c) & \text{pr}_{2,ex2}(\lambda_c) \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} p_{0,ex0}(\lambda_c) & p_{0,ex1}(\lambda_c) & p_{0,ex2}(\lambda_c) \\ p_{1,ex0}(\lambda_c) & p_{1,ex1}(\lambda_c) & p_{1,ex2}(\lambda_c) \\ p_{2,ex0}(\lambda_c) & p_{2,ex1}(\lambda_c) & p_{2,ex2}(\lambda_c) \end{bmatrix}. \tag{6}$$

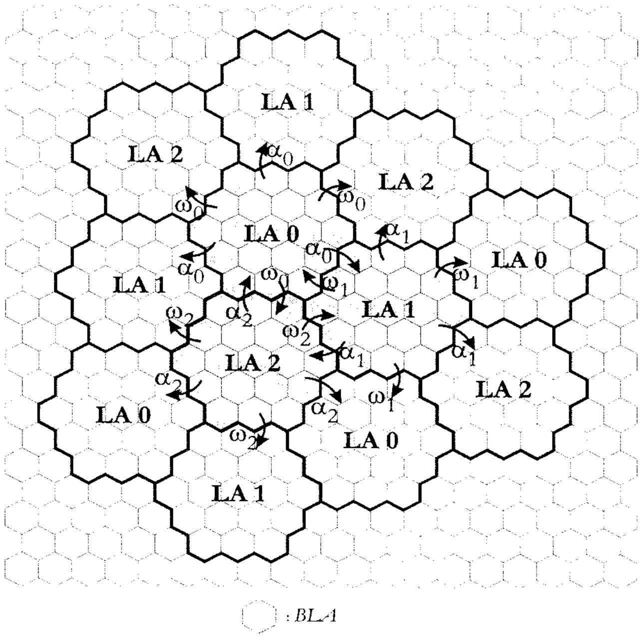

Fig. 4. Three LAs each one characterized with a mosaic graph T_2, with their exit probabilities toward neighboring LAs.

In matrix $\mathbf{Sr}$ $(\mathbf{S})$, $\mathrm{pr}_{k,exi}(\lambda_c)$ $(p_{k,exi}(\lambda_c))$ is the absorption probability into state exi *before* the observation time expires when the first sojourn time in state k is defined by $fr_k^*(s)$ $(f_k^*(s))$. Notice that because of the memoryless property of the exponential distribution, $p_{k,exi}(\lambda_c)$ is calculated in a similar way as $\mathrm{pr}_{k,exi}(\lambda_c)$.

When absorption happens, an LU message is triggered toward the system *databases*. It is easy to show that the probability of m LU messages (absorptions) being triggered during the exponential observation time, in vector/matrix notation, is given by

$$\begin{bmatrix} \Pr(m\mathrm{LUs}/0) \\ \Pr(m\mathrm{LUs}/1) \\ \Pr(m\mathrm{LUs}/2) \end{bmatrix} = \mathbf{SrT}[\mathbf{ST}]^{m-1}[\mathbf{I}-\mathbf{S}]\mathbf{e}; \quad m>0 \tag{7}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{e}$ is the column vector $\mathbf{e}^t = [1,1,1]$. The elements $st_{i,j}$ of $\mathbf{ST}$ matrix are the conditional probabilities that just after an LU has been triggered by the MT, it will be visiting state j, given the fact that the MT started visiting state i at $t=0^+$.

We denote by m_k $(k=0,1,2)$ the number of LU messages triggered between two call arrivals, given that just after the last call arrival, the system is in state k. Taking into account (7), the mean value of m_k can be calculated according to

$$\overline{m} = \begin{bmatrix} \overline{m_0} \\ \overline{m_1} \\ \overline{m_2} \end{bmatrix} = \sum_{m=1}^{\infty} m \begin{bmatrix} \Pr(m\mathrm{LUs}/0) \\ \Pr(m\mathrm{LUs}/1) \\ \Pr(m\mathrm{LUs}/2) \end{bmatrix} = \mathbf{SrT}[\mathbf{I}-\mathbf{ST}]^{-1}\mathbf{e}. \tag{8}$$

Therefore, assuming the MT starts its movement from LA k, the expected LU cost per call arrival is given by

$$C_{\mathrm{up-mt}}(k) = P_U\overline{m_k}; \quad k=0,1,2 \tag{9}$$

where P_U is the LU cost per message triggered by the MT.

Taking into account the Poisson Arrivals See Time Averages property, [2], an incoming call will find the MT roaming in $LA\,k$, $k=0$, 1, or 2, with probability $P_k(0^+)$, given by

$$P_k(0^+) = \frac{\frac{1}{\lambda_{LA\,k}}\pi_k}{\frac{1}{\lambda_{\mathrm{LA}\,0}}\pi_0 + \frac{1}{\lambda_{\mathrm{LA}\,1}}\pi_1 + \frac{1}{\lambda_{\mathrm{LA}\,2}}\pi_2} \tag{10}$$

where π_k and $1/\lambda_{\mathrm{LA}\,k}$ are the portion of visits and the mean sojourn time in state k respectively, [3]. The probability vector $\pi = [\pi_0\ \pi_1\ \pi_2]$ is obtained by solving $\pi = \pi\mathbf{T}$. Therefore, unconditioning (9), the LU cost per call arrival is given by

$$C_{\mathrm{up-mt}} = \sum_{k=0}^{2} P_k(0^+)C_{\mathrm{up-mt}}(k). \tag{11}$$

B. Terminal Paging Cost

Between two consecutive incoming calls the MT may trigger LU messages toward the system *databases*. The terminal PG cost is obtained by considering two excluding events: the MT triggers at least an LU message since the last incoming call was received, or the MT remains in the BLA since the last incoming call was received.

With probability given by (4) no LU message is triggered between two call arrivals. The probabilities of the last LU message having been triggered in state i $(i=0,1,2)$ are given by

$$\begin{bmatrix} \Pr(\text{last LU} = i/0) \\ \Pr(\text{last LU} = i/1) \\ \Pr(\text{last LU} = i/2) \end{bmatrix} = \mathbf{SrT}[\mathbf{I}-\mathbf{ST}]^{-1}\mathbf{H}_i[\mathbf{I}-\mathbf{S}]\mathbf{e}; \quad i = 0,1,2 \tag{12}$$

where $\mathbf{H}_i$ $(i=0,1,2)$ are square matrixes

$$\mathbf{H}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{H}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \mathbf{H}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

Therefore, given that the observation starts in state k ($k =$ 0, 1, or 2), the probability of no LU messages being triggered between two call arrivals and the MT being visiting state l ($l = 0,1,2$) at the end of the observation interval, is given by

$$\Pr(\text{no LU}, l/k) = \mathrm{pr}_{k,l}(\lambda_c); \quad k,l = 0,1,2. \tag{14}$$

Analogously, given that the observation starts in state k and that at least one LU message has been triggered, the probability of the last LU being triggered from state i and the MT being visiting state l $(k,i,l = 0,1,2)$, is given by

$$\begin{bmatrix} \Pr(\text{last LU} = i, l/0) \\ \Pr(\text{last LU} = i, l/1) \\ \Pr(\text{last LU} = i, l/2) \end{bmatrix} = \mathbf{SrT}[\mathbf{I}-\mathbf{ST}]^{-1}\mathbf{H}_i\mathbf{e}p_{i,l}(\lambda_c); \quad i,l = 0,1,2. \tag{15}$$

When two-step PG is applied, the terminal PG cost assuming the MT starts its movement from LA k can be formulated as

$$\begin{aligned} C_{\text{pg-mt}}(k) =& P_V \Bigg[N_{\text{LA}\,k} \Pr(\text{no LU}, k/k) \\ &+ N_c \sum_{j=0, j\neq k}^{2} \Pr(\text{no LU}, j/k) \\ &+ \sum_{i=0}^{2} N_{\text{LA}\,i} \Pr(\text{last LU} = i, i/k) \\ &+ N_c \sum_{i=0}^{2} \sum_{j=0, j\neq i}^{2} \Pr(\text{last LU} = i, j/k) \Bigg] \end{aligned} \tag{16}$$

where P_V is the terminal PG cost per *cell*, $N_{\text{LA}\,i}$ is the total number of *cells* that configures LA i, and N_c is the total number of *cells* per BLA.

In the right-hand side of (16), i.e., the terms within brackets, the first and third terms take into account the *hit* probability and the second and fourth terms take into account the *miss* probability, [14]. There is another reading of (16); the first and second terms are the mean number of *cells* paged in order to find the MT when no LU has been triggered since the last incoming call. The third and fourth terms (second line) are the mean number of *cells* paged in order to find the MT when an LU has been triggered, at least, since the last incoming call.

And finally, the PG cost per call arrival is given by

$$C_{\text{pg-mt}} = \sum_{k=0}^{2} P_k(0^+) C_{\text{pg-mt}}(k). \tag{17}$$

Obviously, when single-step PG (nonselective) algorithm is applied, the terminal PG cost can be written as

$$C_{\text{pg-mt}} = C_{\text{pg-mt}}(k) = P_V N_c. \tag{18}$$

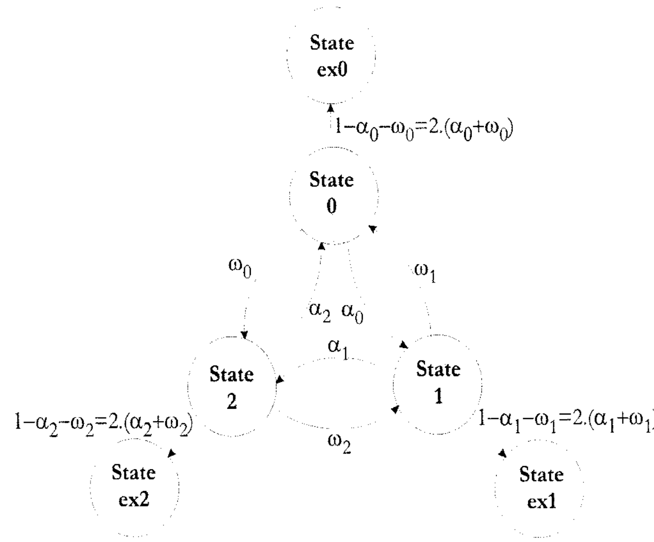

Fig. 5. State transition diagram of three LAs.

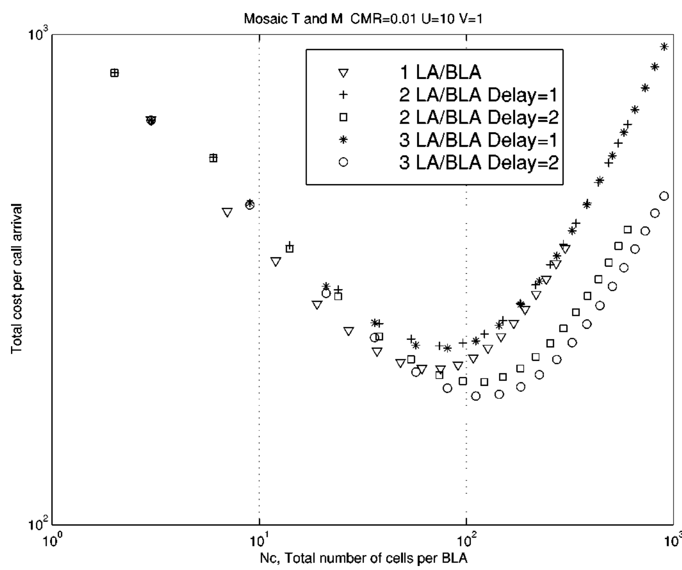

Fig. 6. Total cost (location + paging) per call arrival. $\text{CMR} = 0.01$, $P_U = 10$, and $P_V = 1$.

IV. Some Examples and Discussion

The analytical model presented in previous section allows obtaining the total LU and PG cost per call arrival C_T under various parameters and with three LAs/BLA. It is defined as, $C_T = C_{\text{up-mt}} + C_{\text{pg-mt}}$. We have also found out the LU and PG costs in scenarios with two LAs/BLA and one LA/BLA (GSM scheme). Further details about these particular models can be read in our internal report [16].

The parameters used are the total number of *cells* per BLA, N_c, the LU cost, P_U, the PG cost, P_V, the call arrival rate λ_c, and the mean *cell* dwell time $1/\lambda_m$ (and, therefore, the call-to-mobility ratio defined as $\text{CMR} = \lambda_c/\lambda_m$). The dwell time for a *cell* has been assumed to be characterized by a generalized gamma distribution. To that end, we have followed the approach proposed in [15]. Finally we have assumed a random walk mobility model, therefore, $\alpha_i = \omega_i = 1/6$, $\pi_i = 1/3$, $i = 0$, 1, or 2, see Fig. 5, and (10).

In Figs. 6–9 the total cost per call arrival, C_T, is reported for some $\text{CMR} = 0.01$, 0.1, 1, and 10. As in [12], $P_U = 10$ and $P_V = 1$ have been chosen. All LAs have the same size. Mosaic graph M and T have been considered (LA configuration). For low CMR, let us say $\text{CMR} < 1$, an optimum value for the LA size, N_c^* is observed. As N_c exceeds its optimum value N_c^*, the terminal PG cost dominates and C_T is an increasing function of N_c. And *vice versa*, as N_c decreases from its optimum value, the LU cost dominates. When CMR becomes higher than one, the optimum LA size is reduced to one, because of the low mobility of the MT. For a fixed value of CMR a significant saving is also achieved when the paging delay increases from single step to two steps.

For low CMR, CMR= 0.01 and single-step PG, delay one, the optimum configuration is one LA/BLA, mosaic graph T_4, $N_c = 61$. The total cost is approximately $C_T = C_{\text{up-mt}} + C_{\text{pg-mt}} \cong 147 + 61 = 208$, Fig. 6. It is worth to note that two LAs/BLA and three LAs/BLA configurations are worst than the one LA/BLA configuration. It can be explained by comparing a) one LA/BLA, mosaic graph M_4, $N_c = 75$, with b)

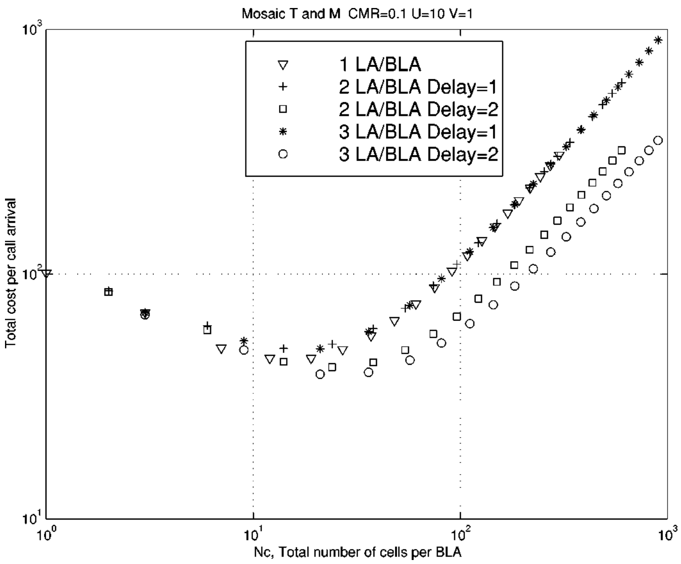

Fig. 7. Total cost per call arrival. $\mathrm{CMR} = 0.1$, $P_U = 10$, and $P_V = 1$.

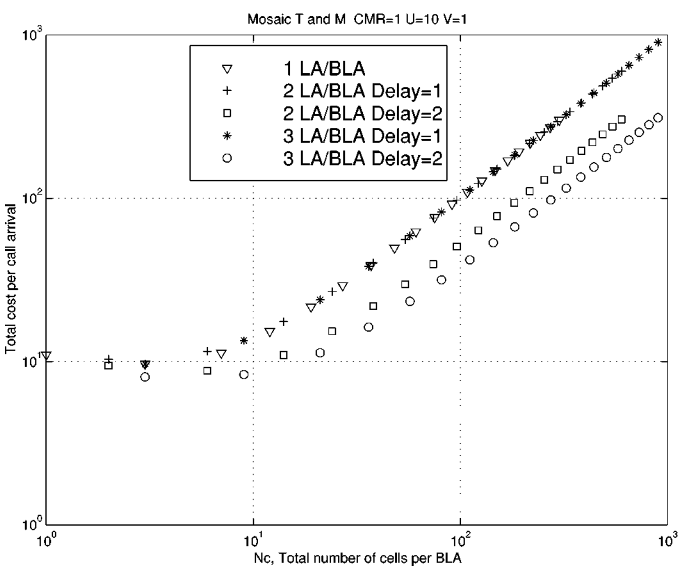

Fig. 8. Total cost per call arrival. $\mathrm{CMR} = 1$, $P_U = 10$, and $P_V = 1$.

two LAs/BLA mosaic graph T_3, $N_c = 2 \times 37 = 74$. Both configurations roughly present the same total number of *cells*, and so, roughly the same PG cost. However the LU cost in case b) is higher than LU cost in case a), due to the fact that the perimeters are, 60 edges for one LA/BLA and 70 edges for two LAs/BLA, see Table I. We recall that the perimeter plays an important role in the LU rate [5].

When two-step PG, delay two, is considered, the conclusions are quite different. According to the Two-Location Algorithm strategy presented in [14], two LAs/BLA, mosaic graph T_4, $N_c = 2 \times 61 = 122$, shows the optimum total cost, equal to $C_T = C_{\mathrm{up-mt}} + C_{\mathrm{pg-mt}} \cong 123 + 73 = 196$. A better result is obtained for three LAs/BLA scenario, mosaic graph T_3, $N_c = 3 \times 37 = 111$, where the optimum total cost, equals to $C_T = C_{\mathrm{up-mt}} + C_{\mathrm{pg-mt}} \cong 126 + 57 = 183$. When comparing with the optimum value of one LA/BLA, single-step, we achieve a saving total cost of $(208 - 196)/208 \cong 0.0577$, (5.77%), and $(208 - 183)/208 \cong 0.12$, (12.02%), respectively.

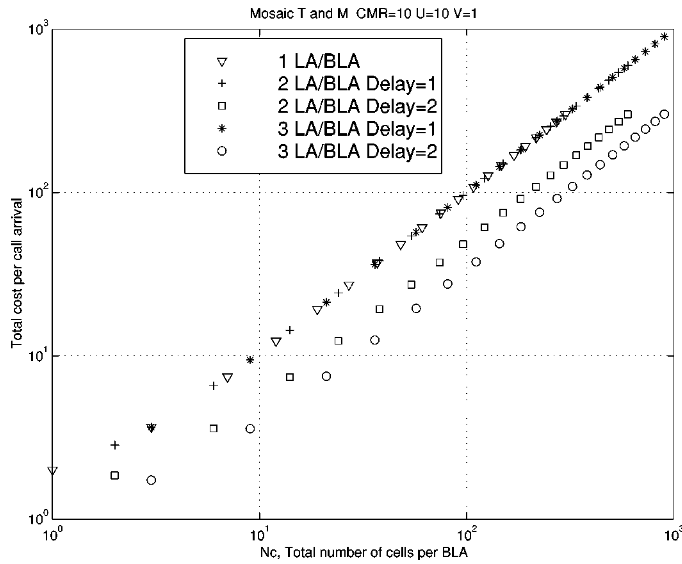

Fig. 9. Total cost per call arrival. $\mathrm{CMR} = 10$, $P_U = 10$, and $P_V = 1$.

V. Conclusion

This paper deals with *mobility tracking* strategies. In the literature they are classified as static or global and dynamic or local. We have studied a new LU strategy combined with selective PG, and we have compared it to the classical GSM scheme (one LA/BLA) and the two LAs/BLA strategy. The paging areas of the selective PG algorithm are LAs. Our new strategy can be seen as a hybrid between the global and local strategies.

When single-step PG algorithm is used, the hysteresis effect obtained in two LAs/BLA and three LAs/BLA policies is not enough to reduce the total signaling cost in the *common air interface*. In those cases, the classical GSM scheme (one LA/BLA) is more suitable than the two LAs/BLA and three LAs/BLA strategies. On the other hand, when a two-step PG algorithm is used, the total signaling cost in the *common air interface* is reduced, compared with the classical GSM scheme. In this case, the proposed strategy of three LAs/BLA obtains the best performance. Good results are also obtained in the two LAs/BLA.

The contribution of this paper can be seen as an option to improve the performance of *location management* strategies of actual real cellular systems, such as *IS-54*, *IS-95*, and GSM, to name a few, [13, page 140]. In fact, we believe that our proposal can be easily implemented in those real cellular systems.

Appendix

A. Probability Density Function of the Sojourn Time in a Location Area

Let us consider the Markov chain of Fig. 5. We consider that the MT starts its movement from the outer ring of the mosaic X_n (the LA). The pdf of the residual sojourn time in state k (or LA k), $k = 0$, 1, or 2, in the transformed domain, is given by [1]

$$f^*_{kr,n,X}(s) = f^*_{kr}(s) = \lambda_{\mathrm{LA}\,k} \frac{1 - f^*_k(s)}{s}. \qquad \text{(A-1)}$$

In the following lines, we show $f_k^*(s)$ for two scenarios; the mosaic T *cell* layout, and the mosaic M *cell* layout. Let us now consider the one–dimensional Markov chain of Fig. 3. Each state (of the chain) corresponds to a ring (of the mosaic —LA—). The pdfs of the time an MT spends in ring q in the transformed domain, denoted by $f_{\text{ring}\, q,X}^*(s)$, is

$$
\begin{aligned}
f_{\text{ring}\,0,X}^*(s) &= \begin{cases} f_{\text{cell}}^*(s), & \text{for } X = T_n, \forall\, n \\ \frac{2 f_{\text{cell}}^*(s)}{3 - f_{\text{cell}}^*(s)}, & \text{for } X = M_n, \forall\, n \end{cases} \\
f_{\text{ring}\,q,X}^*(s) &= \frac{2}{3} f_{\text{cell}}^*(s) \sum_{i=0}^{\infty} \left[\frac{1}{3} f_{\text{cell}}^*(s)\right]^i \\
&= \frac{2 f_{\text{cell}}^*(s)}{3 - f_{\text{cell}}^*(s)}, \text{ for } X = T_n \text{ or } M_n, \forall\, n \\
\text{and } q &= 1, 2, \ldots, n.
\end{aligned} \tag{A-2}
$$

As to its LT, the pdf of the sojourn time of an MT in LA k, $f_{k,n,X}^*(s)$, can be evaluated now assuming that its movement begins from the outer ring since an MT starts roaming in an LA from the outer ring

$$
\begin{aligned}
f_{k,0,X}^*(s) &= \begin{cases} f_{\text{cell}}^*(s), & \text{for } X = T_0 \\ \frac{2 f_{\text{cell}}^*(s)}{3 - f_{\text{cell}}^*(s)}, & \text{for } X = M_0 \end{cases} \\
f_{k,n,X}^*(s) &= \alpha_{n,X} f_{\text{ring}\,n,X}^*(s) \\
&\quad + \omega_{n,X} \left[f_{\text{ring}\,n,X}^*(s) f_{k,n-1,X}^*(s) f_{k,n,X}^*(s) \right] \\
&= \frac{\alpha_{n,X} f_{\text{ring}\,n,X}^*(s)}{1 - \omega_{n,X} f_{\text{ring}\,n,X}^*(s) f_{k,n-1,X}^*(s)}, \text{ for} \\
X &= T_n \text{ or } M_n, \text{ and } n = 1, 2, \ldots
\end{aligned} \tag{A-3}
$$

where

$$
\begin{aligned}
\alpha_{n,X} &= 1 - \omega_{n,X} = \frac{p_{n,n+1}}{p_{n,n-1} + p_{n,n+1}} \\
&= \begin{cases} \frac{2n+1}{4n}, & \text{for } X = T, \\ \frac{n+1}{2n+1}, & \text{for } X = M, \end{cases} \text{ for} \\
n &= 1, 2, \ldots \text{ (see Table II).}
\end{aligned} \tag{A-4}
$$

The mean sojourn time of an MT in ring q of mosaic X_n and in state k $(1/\lambda_{\text{LA}k})$ can be easily obtained taking the first derivatives of (A-2) and (A-3) evaluated at $s = 0$. Therefore, expression (3) is obtained where $-f_{\text{cell}}^{*'}(0) = 1/\lambda_m$.

B. Calculus of Probabilities

Let $g(t)$ be the pdf of the duration of an activity with *LT* $g^*(s)$. This activity is observed during an exponential time of mean value $1/\lambda_c$. Then, the probability that the activity ends *before* the observation interval ends is given by

$$
\begin{aligned}
&P(\text{activity ends } \textit{before} \text{ the observation interval ends}) \\
&\quad = \int_{r=0}^{\infty} \int_{t=r}^{\infty} g(r) \lambda_c e^{-\lambda_c t} dr dt = g^*(\lambda_c).
\end{aligned} \tag{A-5}
$$

Probabilities $\text{pr}_{k,exi}(\lambda_c)$, $\text{pr}_{k,i}(\lambda_c)$, $p_{k,exi}(\lambda_c)$, and $p_{k,i}(\lambda_c)$ can be evaluated by using the following arguments. We define a box of two states as $\text{box}(k,l) \equiv \text{box}(l,k)$, $(k,l = 0,1,2)$. Therefore, we will use the following boxes of states, $\text{box}(0,1)$, $\text{box}(1,2)$, and $\text{box}(2,0)$. Let us denote by $rr_{i,j}^*(k,l,s)$ the LT of the sojourn time in $\text{box}(k,l)$ conditioned to the fact that at time $t = 0^+$ the MT is in state i and the exit is produced via state j toward state n, $(n \neq k,l)$, $(i,j = k,l)$; and the initial sojourn time in state i is defined by the residual sojourn time $f_{ir}^*(s)$. In the same way, we denote by $r_{i,j}^*(k,l,s)$ the same *LT* but with initial sojourn time characterized by $f_i^*(s)$.

Then, for $rr_{i,j}^*(k,l,s)$ and $r_{i,j}^*(k,l,s)$ $(i,j,k,l = 0,1)$, the following equations can be written (in the sequel, only equations related to $\text{box}(0,1)$ will be shown. Similar expressions for $\text{box}(1,2)$ and $\text{box}(2,0)$ can be deduced.)

$$
\begin{aligned}
rr_{0,0}^*(0,1,s) &= \omega_0 f_{0r}^*(s) + \alpha_0 f_{0r}^*(s) r_{1,0}^*(0,1,s); \\
rr_{1,0}^*(0,1,s) &= \omega_1 f_{1r}^*(s) r_{0,0}^*(0,1,s) \\
rr_{1,1}^*(0,1,s) &= \alpha_1 f_{1r}^*(s) + \omega_1 f_{1r}^*(s) r_{0,1}^*(0,1,s); \\
rr_{0,1}^*(0,1,s) &= \alpha_0 f_{0r}^*(s) r_{1,1}^*(0,1,s).
\end{aligned} \tag{A-6}
$$

First, expressions for $r_{i,j}^*(k,l,s)$ are derived from the above equations, simply by replacing $f_{ir}^*(s)$ by $f_i^*(s)$. After some simple algebra we obtain

$$
\begin{aligned}
r_{0,0}^*(0,1,s) &= \frac{\omega_0 f_0^*(s)}{1 - \alpha_0 \omega_1 f_0^*(s) f_1^*(s)}; \\
r_{1,0}^*(0,1,s) &= \frac{\omega_0 \omega_1 f_0^*(s) f_1^*(s)}{1 - \alpha_0 \omega_1 f_0^*(s) f_1^*(s)} \\
r_{1,1}^*(0,1,s) &= \frac{\alpha_1 f_1^*(s)}{1 - \alpha_0 \omega_1 f_0^*(s) f_1^*(s)}; \\
r_{0,1}^*(0,1,s) &= \frac{\alpha_0 \alpha_1 f_0^*(s) f_1^*(s)}{1 - \alpha_0 \omega_1 f_0^*(s) f_1^*(s)}.
\end{aligned} \tag{A-7}
$$

Second, expressions for $rr_{i,j}^*(k,l,s)$ are derived when inserting expressions (A-7) into (A-6).

The remaining task is the calculus of $\text{pr}_{k,i}(\lambda_c)$, $p_{k,i}(\lambda_c)$, $\text{pr}_{k,exi}(\lambda_c)$, and $p_{k,exi}(\lambda_c)$. To that end, we first define the following *LTs*

$$
\begin{aligned}
\Delta_0^*(s) &= \alpha_0 \left[r_{1,1}^*(1,2,s) + r_{1,2}^*(1,2,s) \right]; \\
\Omega_0^*(s) &= \omega_0 \left[r_{2,1}^*(1,2,s) + r_{2,2}^*(1,2,s) \right] \\
\Delta_1^*(s) &= \alpha_1 \left[r_{2,2}^*(2,0,s) + r_{2,0}^*(2,0,s) \right]; \\
\Omega_1^*(s) &= \omega_1 \left[r_{0,2}^*(2,0,s) + r_{0,0}^*(2,0,s) \right] \\
\Delta_2^*(s) &= \alpha_2 \left[r_{0,0}^*(0,1,s) + r_{0,1}^*(0,1,s) \right]; \\
\Omega_2^*(s) &= \omega_2 \left[r_{1,0}^*(0,1,s) + r_{1,1}^*(0,1,s) \right].
\end{aligned} \tag{A-8}
$$

Therefore, and starting with $\text{pr}_{i,exi}(\lambda_c)$ and $\text{pr}_{i,i}(\lambda_c)$, we can write

$$
\begin{aligned}
\text{pr}_{i,exi}(\lambda_c) &= [1 - \alpha_i - \omega_i] f_{ir}^*(\lambda_c) \\
&\quad + f_{ir}^*(\lambda_c) \left[\Delta_i^*(\lambda_c) + \Omega_i^*(\lambda_c) \right] p_{i,exi}(\lambda_c); \\
\text{for } i &= 0, 1, \text{and } 2
\end{aligned} \tag{A-9}
$$

and

$$
\begin{aligned}
\text{pr}_{i,i}(\lambda_c) &= 1 - f_{ir}^*(\lambda_c) \\
&\quad + f_{ir}^*(\lambda_c) \left[\Delta_i^*(\lambda_c) + \Omega_i^*(\lambda_c) \right] p_{i,i}(\lambda_c); \\
&\quad \text{for } i = 0, 1, \text{and } 2.
\end{aligned} \tag{A-10}
$$

$$p_{1,ex0}(\lambda_c) = \frac{(1-\alpha_0-\omega_0) f_1^*(\lambda_c) f_0^*(\lambda_c)(\omega_1+\alpha_1\alpha_2 f_2^*(\lambda_c))}{(1-\omega_2 f_2^*(\lambda_c)\alpha_1 f_1^*(\lambda_c))\left[1-f_0^*(\lambda_c)(\Delta_0^*(\lambda_c)+\Omega_0^*(\lambda_c))\right]}$$
$$p_{2,ex0}(\lambda_c) = \frac{(1-\alpha_0-\omega_0) f_2^*(\lambda_c) f_0^*(\lambda_c)(\alpha_2+\omega_2\omega_1 f_1^*(\lambda_c))}{(1-\omega_2 f_2^*(\lambda_c)\alpha_1 f_1^*(\lambda_c))\left[1-f_0^*(\lambda_c)(\Delta_0^*(\lambda_c)+\Omega_0^*(\lambda_c))\right]}. \quad \text{(A-14)}$$

$$p_{1,0}(\lambda_c) = \frac{(1-f_0^*(\lambda_c)) f_1^*(\lambda_c)(\omega_1+\alpha_1\alpha_2 f_2^*(\lambda_c))}{(1-\omega_2 f_2^*(\lambda_c)\alpha_1 f_1^*(\lambda_c))\left[1-f_0^*(\lambda_c)\left(\Delta_0^*(\lambda_c)+\Omega_0^*(\lambda_c)\right)\right]}$$
$$p_{2,0}(\lambda_c) = \frac{(1-f_0^*(\lambda_c)) f_2^*(\lambda_c)\left(\alpha_2+\omega_2\omega_1 f_1^*(\lambda_c)\right)}{(1-\omega_2 f_2^*(\lambda_c)\alpha_1 f_1^*(\lambda_c))\left[1-f_0^*(\lambda_c)\left(\Delta_0^*(\lambda_c)+\Omega_0^*(\lambda_c)\right)\right]}. \quad \text{(A-16)}$$

Clearly, from (A-9) we can write

$$p_{i,exi}(\lambda_c) = (1-\alpha_i-\omega_i)\frac{f_i^*(\lambda_c)}{1-\left[\Delta_i^*(\lambda_c)+\Omega_i^*(\lambda_c)\right] f_i^*(\lambda_c)}; \quad \text{for } i=0,1, \text{and } 2. \quad \text{(A-11)}$$

Inserting expression (A-11) in (A-9) we get $\mathrm{pr}_{i,exi}(\lambda_c)$. And from (A-10) we have

$$p_{i,i}(\lambda_c) = \frac{1-f_i^*(\lambda_c)}{1-\left[\Delta_i^*(\lambda_c)+\Omega_i^*(\lambda_c)\right] f_i^*(\lambda_c)}; \quad \text{for } i=0,1, \text{and } 2. \quad \text{(A-12)}$$

Inserting expression (A-12) in (A-10), we get $\mathrm{pr}_{i,i}(\lambda_c)$.

The probabilities $\mathrm{pr}_{i,exj}(\lambda_c)$ and $p_{i,exj}(\lambda_c)$, $i,j = 0,1,2$ $(i \neq j)$, can be obtained from the following equations (only equations related to exit toward state $exi = ex0$ are shown. Similar expressions for absorbing states $ex1$ and $ex2$ can be deduced):

$$\mathrm{pr}_{1,ex0}(\lambda_c) = f_{1r}^*(\lambda_c)[\alpha_1 p_{2,ex0}(\lambda_c)+\omega_1 p_{0,ex0}(\lambda_c)]$$
$$\mathrm{pr}_{2,ex0}(\lambda_c) = f_{2r}^*(\lambda_c)[\alpha_2 p_{0,ex0}(\lambda_c)+\omega_2 p_{1,ex0}(\lambda_c)]. \quad \text{(A-13)}$$

First, we deduce expressions for $p_{i,exj}(\lambda_c)$, $i \neq j$. Replacing $\mathrm{pr}_{i,exj}(\lambda_c)$, by $p_{i,exj}(\lambda_c)$ and $f_{ir}^*(\lambda_c)$ by $f_i^*(\lambda_c)$ in (A-13), and solving the set of two equations we get (A-14), shown at the top of the page.

Inserting expressions (A-11) and (A-14) in (A-13) we obtain the probabilities $\mathrm{pr}_{i,exj}(\lambda_c)$ for $i \neq j$.

In the same way as in (A-13) and (A-14), probabilities $\mathrm{pr}_{i,j}(\lambda_c)$ and $p_{i,j}(\lambda_c)$, for $i,j=0,1,2$ $(i \neq j)$ can be obtained (Only equations related to final state $i=0$ are shown. Similar equations can be obtained for final states 1 and 2.)

$$\mathrm{pr}_{1,0}(\lambda_c) = f_{1r}^*(\lambda_c)[\alpha_1 p_{2,0}(\lambda_c)+\omega_1 p_{0,0}(\lambda_c)]$$
$$\mathrm{pr}_{2,0}(\lambda_c) = f_{2r}^*(\lambda_c)[\alpha_2 p_{0,0}(\lambda_c)+\omega_2 p_{1,0}(\lambda_c)]. \quad \text{(A-15)}$$

First, we deduce expressions for $p_{i,j}(\lambda_c)$, $i \neq j$. Replacing $\mathrm{pr}_{i,j}(\lambda_c)$, by $p_{i,j}(\lambda_c)$ and $f_{ir}^*(\lambda_c)$ by $f_i^*(\lambda_c)$ in (A-15), and solving the set of two equations we obtain (A-16), shown at the top of the page.

Finally, inserting expressions (A-12) and (A-16) in (A-15) we get the probabilities $\mathrm{pr}_{i,j}(\lambda_c)$, for $i \neq j$.

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