

## REFINEMENTS OF THE SHAFER–FINK INEQUALITY OF ARBITRARY UNIFORM PRECISION

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*Abstract.* A method of producing refinements of the Shafer–Fink ([5]) inequality

$$\frac{3x}{1+2\sqrt{1+x^2}} \leq \arctan x \leq \frac{\pi x}{1+2\sqrt{1+x^2}}$$

is given. We prove, for instance:

$$\frac{\pi(3+8\sqrt{2})x}{7+6\sqrt{1+x^2}+16\sqrt{2}\sqrt{1+x^2}+\sqrt{1+x^2}} \leq \arctan x \leq \frac{45x}{7+6\sqrt{1+x^2}+16\sqrt{2}\sqrt{1+x^2}+\sqrt{1+x^2}}.$$

Other algebraic approximations for the arctangent functions are, rather informally, presented.

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### REFERENCES

- [1] G. E. ANDREWS, R. ASKEY, R. ROY, *Special functions*, Encyclopedia of Mathematics and Its Applications, Cambridge University Press, 1999.
- [2] D. BAILEY, P. BORWEIN, S. PLOUFFE, *On the rapid computation of various polylogarithmic constants*, Math. Comp. 66 (1997), no. 218, 903–913.
- [3] A. M. FINK, *Two inequalities*, Univ. Beograd. Publ. Elektrotehn. Fak., Ser. Mat. 6 (1995), 48–49. (<http://pefmath.etf.bg.ac.yu/>)
- [4] D. S. MITRINOVIĆ, *Elementary Inequalities*, Groningen, 1964.
- [5] R. E. SHAFER, *E 1867*, Amer. Math. Monthly 73 (1966), no. 3, 309.
- [6] FENG QI, SHI-QIN ZHANG, BAI-NI GUO, *Sharpening and generalizations of Shafer’s inequality for the arc tangent function*, J. Inequal. Appl. 2009, Art. ID 930294, 9 pp.