IPhT-t16/080

Refining the boundaries of the classical de Sitter landscape

David Andriot[1,2] and Johan Blåbäck[3]

[1] *Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, Am Mühlenberg 1, 14467 Potsdam-Golm, Germany*

[2] *Institut für Mathematik, Humboldt-Universität zu Berlin, IRIS-Adlershof, Zum Großen Windkanal 6, 12489 Berlin, Germany*

[3] *Institut de Physique Théorique, Université Paris Saclay, CEA, CNRS, F-91191 Gif sur Yvette, France*

`david.andriot@aei.mpg.de, johan.blaback@cea.fr`

Abstract

We derive highly constraining no-go theorems for classical de Sitter backgrounds of string theory, with parallel sources; this should impact the embedding of cosmological models. We study ten-dimensional vacua of type II supergravities with parallel and backreacted orientifold O_p-planes and D_p-branes, on four-dimensional de Sitter space-time times a compact manifold. Vacua for $p = 3$, 7 or 8 are completely excluded, and we obtain tight constraints for $p = 4$, 5, 6. This is achieved through the derivation of an enlightening expression for the four-dimensional Ricci scalar. Further interesting expressions and no-go theorems are obtained. The paper is self-contained so technical aspects, including conventions, might be of more general interest.

Contents

1 Introduction

Recent high precision cosmological observations [1–4] have set important constraints on models describing the early universe. The coming measures (see e.g. [5]) could even put a quantum gravity theory such as string theory under pressure [6]. It is thus a timely moment to address crucial pending questions of string cosmology, among which finding a metastable de Sitter vacuum. Our present accelerating universe is well described as a four-dimensional de Sitter space-time, and without a proposal for an evolution mechanism, this shape should remain when going back to the early times. For instance, the end-point or vacuum for inflation scenarios are commonly considered to be de Sitter (see also [7]). In addition, having all details of this vacuum is of prime importance to be able to embed the scenario in a string compactification. In supergravity, many inflationary models have been proposed recently and been compared to the new experimental data, but few of them are realised completely within string theory (see e.g. [8]). This prevents from connecting them to U.V. and quantum gravity aspects. To achieve this, one should know the de Sitter vacuum, the internal compact geometry, etc. in detail. In addition, this would allow to verify that all aspects of the compactification, e.g. moduli stabilisation, are under control and do not spoil the inflation mechanism. With these motivations in mind, in the present paper we focus on the question of finding de Sitter vacua.

Several ideas have been proposed for how to construct de Sitter within string theory. A problem with these proposals is often the use of features that lack a full understanding in ten dimensions. A famous idea to achieve de Sitter is given in [9] where anti-branes are used to uplift the value of the cosmological constant. Attempts to construct the underlying ten-dimensional solution supporting this scenario have encountered several challenges, starting with [10]. While this has been an active subject for years, the final outcome of [9] remains unclear, see e.g. [11]. Other remarks can be found e.g. in [12, 13].

While further proposals have been made to obtain a positive cosmological constant at the four-dimensional level (in particular by the use of non-geometric fluxes), we prefer here to remain in the simpler and somewhat safer (in terms of control on the compactification) setting of ten-dimensional classical de Sitter vacua. We consider standard ten-dimensional type II supergravities without α' corrections, supplemented by the Ramond-Ramond (RR) sources D_p-branes and orientifold O_p-planes; no Neveu-Schwarz source such as NS_5-branes or Kaluza-Klein (KK) monopoles are included. Relevant to us are vacua where the space-time is the warped product of a four-dimensional de Sitter space-time and a six-dimensional compact internal manifold: with a controlled value of the dilaton, this would be a valid classical background of string theory. This ten-dimensional setting could be the only one where a classical de Sitter string background exists: indeed, such vacua have been ruled-out recently in supersymmetric heterotic string [14–17].

In our context, no-go theorems have also been established. To start with, standard ones for classical de Sitter vacua with compact internal geometries [18–21] are circumvented by requiring orientifolds. This is however far from being enough, and many refined no-go theorems have been worked-out [22–33], most of them studying the corresponding four-dimensional scalar potential inspired by [34], sometimes considering as well constraints on the slow-roll parameter for inflation or on the vacuum metastability. Note that the four-dimensional approach always has the drawback of considering smeared sources, and thus neglecting (or averaging) their backreaction (see e.g. [35, 36] on this topic); the effectiveness of these models is also often debatable. In the present paper, we avoid such questions by working purely in ten dimensions and keeping the dependence on the warp factor and dilaton explicitly during our computation. From this whole literature, an outcome is that very few classical de Sitter vacua have been found, and none of them is metastable [24–26, 28, 37, 38]. Further work was dedicated directly to the stability problem [39–42], but no systematic explanation has been found for the tachyons appearing.

In this paper, we work in ten dimensions and focus on the existence of classical de Sitter vacua of type II supergravities with D_p and O_p sources, without ever considering the four-dimensional stability. We aim to provide general statements that would clarify the situation and refine the boundaries of the classical de Sitter landscape. To that end, we consider sources of one fixed dimension at a time, $3 \leq p \leq 8$, which are also parallel, i.e. not intersecting, or equivalently, having the same transverse subspace; see Section 2 for the detailed specifications on the sources. Such a ten-dimensional setting would lead, after a standard dimensional reduction, to a four-dimensional $\mathcal{N} = 4$ gauged supergravity, for which, to the best of our knowledge, no de Sitter vacuum is known. In addition, all known (unstable) classical de Sitter vacua mentioned above have intersecting sources. A natural guess is then that a no-go theorem exists for parallel sources: the outcome of this work is very close to such a result. We first prove the following

$$\boxed{\text{There is no de Sitter vacuum for } p = 3,\ 7,\ \text{or } 8.} \tag{1.1}$$

The $p = 3$ result was already derived in [35], whose methods act as an inspiration for the generalization to other p. Note that for $p = 3$, sources are always parallel, making this result very general. In the other cases, we first reproduce in Section 3 some results previously obtained in four dimensions. More importantly, we then derive the following

$$\boxed{\text{There is no de Sitter vacuum for } p = 4,\ 5,\ \text{or } 6,\ \text{if some curvature terms are} \geq 0.} \quad (1.2)$$

These curvature terms are related to curvatures of internal subspaces. Their value is further constrained as discussed in Section 4.2 (see in particular the range of values (4.34)), leaving eventually very little room for de Sitter vacua, with parallel sources. These terms also vanish in many examples of Minkowski vacua. Finally, as a side result, we prove two more no-go theorems (4.6) and (4.9) in the smeared limit, building on the interesting expression (4.5).

These results are derived thanks to appropriate combinations of ten-dimensional equations of motion and flux Bianchi identities, that isolate the unwarped four-dimensional curvature $\tilde{\mathcal{R}}_4$. For a de Sitter vacuum, we require the latter to be positive. On this aspect, the main result of the paper is the expression (4.21) schematically given by

$$\boxed{\tilde{\mathcal{R}}_4 = -\ (\text{BPS-like})^2\ -\ (\text{flux})^2\ -\ \text{curvature terms} + \text{total derivative}} \quad (1.3)$$

It is inspired by the $p = 3$ case of [35] and generalizes [43]. This expression makes the sign contributions to $\tilde{\mathcal{R}}_4$ apparent, and the above no-go theorems for $p = 3, 4, 5, 6$ are then easy to obtain; in particular, the curvature terms (and flux terms) vanish for $p = 3$, leading to (4.31). For $p = 7, 8$, we followed [8] to derive the appropriate expressions (3.6) and (3.7). What is denoted "BPS-like" in (1.3) are interesting combinations: setting them to zero would fix the sourced RR flux F_k (with $k = 8 - p$), and relate the flux F_{k-2} to the H-flux, or at least components thereof. It generalizes the conditions obtained in [43] for $p = 3$, in particular the imaginary self-dual condition. This will be the topic of a companion paper [44], where we focus on Minkowski vacua.

The paper is organised as follows. Conventions on ten-dimensional type II supergravities are given in the self-contained Appendix A, and those are applied to our compactification setting as detailed in Section 2. Then, we derive the no-go theorems for $p = 7, 8$ and further results for other p values in Section 3. Different equation manipulations are then presented in Section 4 to conclude and discuss the no-go theorems for $p = 3, 4, 5, 6$. We end with an outlook in Section 5. Useful formulas and details of computations are given in Appendix B.

2 Compactification setting

We consider ten-dimensional type IIA and IIB supergravities and use the conventions given in Appendix A. We allow for Ramond-Ramond (RR) sources, namely D_p-branes and orientifold O_p-planes, but for no further ingredient, in particular no NS_5-brane or KK-monopole. In this section, we specify to a compactification setting and detail our notations. The ten-dimensional space-time is a warped product of a four-dimensional maximally symmetric space-time (anti-de Sitter, Minkowski, de Sitter) along directions $\mathrm{d}x^\mu$ and a six-dimensional (internal) compact manifold $\mathcal{M}$ along directions $\mathrm{d}y^m$. The metric is written accordingly

$$\mathrm{d}s^2 = e^{2A(y)} \tilde{g}_{\mu\nu}(x) \mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(y) \mathrm{d}y^m \mathrm{d}y^n \ . \quad (2.1)$$

The warp factor is e^A. A tilde denotes quantities without the warp factor, i.e. where it has been explicitly extracted; we also dub such quantities as "smeared". Looking for a vacuum, we will require to preserve Lorentz invariance in four dimensions. A first consequence is that the dilaton is restricted to depend only on internal coordinates. Further, the fluxes F_0, F_1, F_2, F_3, H have to be purely internal (in components and coordinate dependence), and F_4^{10} and F_5^{10} can have four-dimensional components in a constrained manner. With the unwarped, and warped, four-dimensional volume form denoted $\widetilde{\mathrm{vol}}_4 = \sqrt{|\tilde{g}_4|}\mathrm{d}^4x$, and vol_4, respectively, one can have

$$F_4^{10} = F_4^4 + F_4 \text{ with } F_4^4 = \widetilde{\mathrm{vol}}_4\, f_4\ ,\quad F_5^{10} = F_5^4 + F_5 \text{ with } F_5^4 = \widetilde{\mathrm{vol}}_4 \wedge f_5\ , \tag{2.2}$$

with an internal scalar f_4, and internal forms F_4, F_5, f_5. We introduce as a notation an internal 6-form F_6 such that $f_4 = e^{4A} *_6 F_6$; because $F_5^4 = - *_{10} F_5$, one obtains $f_5 = -e^{4A} *_6 F_5$, so

$$F_4^4 = \mathrm{vol}_4 \wedge *_6 F_6\ ,\ \ F_5^4 = -\mathrm{vol}_4 \wedge *_6 F_5\ . \tag{2.3}$$

Since $| *_6 F_6|^2 = |F_6|^2$, we deduce $|F_4^{10}|^2 = |F_4|^2 - |F_6|^2$, and $|F_5|^2 = -|F_5^4|^2 = | *_6 F_5|^2$. Finally, let us list the properties of the sources D_p and O_p:

1. Because of four-dimensional Lorentz invariance, the sources have to be space-time filling, meaning that their world-volume spans the whole four-dimensional space-time, and possibly wraps some internal subspace; this restricts $p \geq 3$, and we consider $p \leq 8$.

2. We assume for each source that there exists one basis (possibly flat) where the metric is block diagonal in the directions parallel and transverse to it, allowing to identify the world-volume directions in the ten-dimensional space-time (see Appendix A for more details). With (2.1), the metric is given by three diagonal blocks in that basis, and we denote the three volume forms by $\mathrm{vol}_4, \mathrm{vol}_{||}, \mathrm{vol}_{\perp}$. The two internal ones, $\mathrm{vol}_{||}$ and $\mathrm{vol}_{\perp}$, are respectively $(p-3)$- and $(9-p)$-forms. With our ordering conventions, one has

$$\mathrm{vol}_4 \wedge \mathrm{vol}_{||} \wedge \mathrm{vol}_{\perp} = \mathrm{vol}_{10} = \mathrm{d}^{10}x\sqrt{|g_{10}|}\ , \tag{2.4}$$
$$\mathrm{vol}_{||} \wedge \mathrm{vol}_{\perp} = \mathrm{vol}_6 = \mathrm{d}^6y\sqrt{|g_6|}\ ,\ *_6\mathrm{vol}_{\perp} = (-1)^{9-p}\mathrm{vol}_{||}\ ,\ *_6\mathrm{vol}_{||} = \mathrm{vol}_{\perp}\ . \tag{2.5}$$

3. We consider for each source that $-\imath^*[b] + \mathcal{F} = 0$ (see Appendix A for more details). We also consider them to be BPS, giving $\mu_p = T_p$.

4. We restrict ourselves to sources of only one fixed size p.

5. We finally consider all sources to be parallel, meaning having the same transverse directions. Note that for $p = 3$, this is not an assumption. As a consequence, the basis where the metric is block diagonal basis is the same for all sources, and the metric can be written as[1]

$$\mathrm{d}s_{10}^2 = e^{2A}(\mathrm{d}\tilde{s}_4^2 + \mathrm{d}\tilde{s}_{6||}^2) + e^{-2A}\mathrm{d}\tilde{s}_{6\perp}^2\ . \tag{2.6}$$

   The warp factor, by definition of D_p and O_p, only depends on transverse directions, i.e. $A(y_{\perp})$. Actually, this standard metric does not assume anything apart from the block structure and (2.1), since we do not specify the coordinate dependence of $\mathrm{d}\tilde{s}_{6||}^2$, $\mathrm{d}\tilde{s}_{6\perp}^2$.

6. In the very end, we will require the transverse subspace to be a compact manifold without boundary.

[1]The physically relevant six-dimensional internal manifold, i.e. the one whose geometry could be discussed, is the underlying smeared one, i.e. the one corresponding to the metric with $A = 0$; this manifold can be simply viewed as deformed by the warp factor.

Due to these properties, the Bianchi identities (BI) for F_4^{10} and F_5^{10} impose f_4 and f_5 to be closed; then, the RR BI can be restricted to the internal forms F_k only, towards

$$
\begin{aligned}
& \mathrm{d}F_k - H \wedge F_{k-2} = -\varepsilon_p\, 2\kappa_{10}^2\, T_p\ \mathrm{d}^{(9-p)\perp}y \sum_{p-\text{sources}} c_p \delta(y_\perp) = \varepsilon_p\, \frac{T_{10}}{p+1} \mathrm{vol}_\perp \ , \qquad (2.7) \\
& \text{for } 0 \leq k = 8-p \leq 5 \ , \ \varepsilon_p = (-1)^{p+1}(-1)^{\left[\frac{9-p}{2}\right]} \ ,
\end{aligned}
$$

where $F_{-1} = F_{-2} = 0$, and one uses (A.12) for T_{10}. Given the right-hand side of (2.7), we will need to project forms on the transverse directions. To that end, we introduce the following notations: for a form G, we denote its projection on the transverse directions with $G|_\perp$ or $(G)|_\perp$, i.e. the form obtained by keeping only its components entirely along those directions. In addition, if G is a $(9-p)$-form, $(G)_\perp$ denotes the coefficient of this form on the transverse world-volume, i.e. $G|_\perp = (G)_\perp \mathrm{vol}_\perp$; one has equivalently $(G)_\perp = *_\perp G|_\perp$. We deduce that the BI (2.7) gives after projection

$$
(\mathrm{d}F_k)_\perp - (H \wedge F_{k-2})_\perp = \varepsilon_p\, \frac{T_{10}}{p+1} \ . \qquad (2.8)
$$

Note that $(H \wedge F_{k-2})|_\perp = H|_\perp \wedge F_{k-2}|_\perp$. Using that $A \wedge *B = B \wedge *A$ for forms A and B of same degree, we can show that $*_\perp H|_\perp \wedge *_\perp F_{k-2}|_\perp = F_{k-2}|_\perp \wedge *_\perp^2 H|_\perp = H|_\perp \wedge F_{k-2}|_\perp$. From this we conclude, for any sign ε

$$
\left| *_\perp H|_\perp + \varepsilon e^{\phi} F_{k-2}|_\perp \right|^2 = |H|_\perp|^2 + e^{2\phi} |F_{k-2}|_\perp|^2 + 2\varepsilon e^{\phi} (H \wedge F_{k-2})_\perp \ , \qquad (2.9)
$$

where the definition of the square is given below (A.5). This formula and reasoning will be useful. For completeness, we give the fluxes equations of motion (e.o.m.) expressed in terms of internal quantities, considering no source contribution to the b-field e.o.m.

$$
\begin{aligned}
& e^{-4A} \mathrm{d}(e^{4A} *_6 F_q) + H \wedge *_6 F_{q+2} = 0 \quad (1 \leq q \leq 4) \ , \qquad (2.10) \\
& e^{-4A} \mathrm{d}(e^{4A-2\phi} *_6 H) - \sum_{0 \leq q \leq 4} F_q \wedge *_6 F_{q+2} = 0 \ . \qquad (2.11)
\end{aligned}
$$

We turn to the dilaton e.o.m. and Einstein equation. We denote $\mathcal{R}_{10} = g^{MN} \mathcal{R}_{MN}$, and

$$
\mathcal{R}_4 = g^{MN} \mathcal{R}_{MN=\mu\nu} \ , \ \mathcal{R}_6 = g^{MN} \mathcal{R}_{MN=mn} = \mathcal{R}_{10} - \mathcal{R}_4 \ , \ (\nabla\partial\phi)_4 = g^{MN=\mu\nu} \nabla_M \partial_N \phi \ . \quad (2.12)
$$

The dilaton e.o.m., the ten-dimensional Einstein trace, and the four-dimensional one,[2] are

$$
2\mathcal{R}_{10} + e^{\phi} \frac{T_{10}}{p+1} - |H|^2 + 8(\Delta\phi - |\partial\phi|^2) = 0 \ , \qquad (2.13)
$$

$$
4\mathcal{R}_{10} + \frac{e^{\phi}}{2} T_{10} - |H|^2 - \frac{e^{2\phi}}{2} \sum_{q=0}^{6} (5-q)|F_q|^2 - 20|\partial\phi|^2 + 18\Delta\phi = 0 \ , \qquad (2.14)
$$

$$
\mathcal{R}_4 - 2\mathcal{R}_{10} - \frac{2e^{\phi}}{p+1} T_{10} + |H|^2 + e^{2\phi} \sum_{q=0}^{6} |F_q|^2 + 2(\nabla\partial\phi)_4 + 8|\partial\phi|^2 - 8\Delta\phi = 0 \ , \qquad (2.15)
$$

[2] Let us detail the indices counting for F_5^{10}: the four-dimensional trace selects the F_5^4 piece giving

$$
\frac{g^{\mu\nu}}{2 \cdot 4!} F^4_{5\ \mu PQRS} F^{4\ \ PQRS}_{5\ \nu} = \frac{g^{\mu\nu}}{2 \cdot 3!} F^4_{5\ \mu\pi\rho\tau s} F^{4\ \ \pi\rho\tau s}_{5\ \nu} = \frac{2}{4!} F^4_{5\ \mu\pi\rho\tau s} F_5^{4\ \mu\pi\rho\tau s} = \frac{2}{5 \cdot 4!} F^4_{5\ MPQRS} F_5^{4\ MPQRS} = 2|F_5^4|^2 \ .
$$

where one should only consider even/odd RR fluxes in IIA/IIB, and we used the above properties (we only used Point 1 through 4 for the sources), giving in particular $g^{MN}T_{MN=\mu\nu} = 4T_{10}/(p+1)$. These scalar equations will be combined to express $\mathcal{R}_4$ in terms of a limited number of ingredients.

3 No de Sitter vacuum for O_7, O_8, and more no-go theorems

Given the context presented in Section 2, we prove here that there cannot be any de Sitter vacuum for $p = 7, 8$ sources, and get constraints for the other p, that can be viewed as no-go theorems. We derive these results in ten dimensions; for $p = 7, 8$, this is done *without smearing*. This reproduces known results obtained in [26, 29] from a four-dimensional approach, that uses conditions for a vacuum but also for its stability.

We proceed as in [8]: we first use the dilaton e.o.m. to eliminate T_{10} in respectively the ten- and four-dimensional traces; we get (with even/odd RR fluxes in IIA/IIB)

$$(p-3)\left(-2\mathcal{R}_{10} + |H|^2 + 8|\partial\phi|^2 - 8\Delta\phi\right) + 2|H|^2 - e^{2\phi}\sum_{q=0}^{6}(5-q)|F_q|^2 - 2e^{2\phi}\Delta e^{-2\phi} = 0 \quad (3.1)$$

$$3\mathcal{R}_4 = -2\mathcal{R}_6 + |H|^2 - e^{2\phi}\sum_{q=0}^{6}|F_q|^2 - 2(\nabla\partial\phi)_4 + 8|\partial\phi|^2 - 8\Delta\phi \ . \quad (3.2)$$

with $-2|\partial\phi|^2 + \Delta\phi = -\frac{1}{2}e^{2\phi}\Delta e^{-2\phi}$. We now multiply (3.2) by $(p-3)$, insert (3.1) and get

$$(p-3)\mathcal{R}_4 = -2|H|^2 + e^{2\phi}\sum_{q=0}^{6}(8-q-p)|F_q|^2 + 2e^{2\phi}\Delta e^{-2\phi} - 2(p-3)(\nabla\partial\phi)_4 \ . \quad (3.3)$$

Now, the warp factor and dilaton terms need to be computed using the metric (2.6): this is done in Appendix B.1. As mentioned there, we deduce from (3.3) the standard dilaton value

$$e^{\phi} = g_s e^{A(p-3)} \ , \quad (3.4)$$

where g_s is a constant. This allows to obtain

$$(p-3)\mathcal{R}_4 - 2e^{2\phi}\Delta e^{-2\phi} + 2(p-3)(\nabla\partial\phi)_4 = (p-3)e^{-2A}\tilde{\mathcal{R}}_4 \ , \quad (3.5)$$

where $\tilde{\mathcal{R}}_4$ is the four-dimensional Ricci scalar built from $\tilde{g}_{\mu\nu}$. We conclude in IIA and IIB

$$\boxed{\begin{aligned} \frac{(p-3)}{e^{2A}}\tilde{\mathcal{R}}_4 &= -\,2|H|^2 + e^{2\phi}\left((8-p)|F_0|^2 + (6-p)|F_2|^2 + (4-p)|F_4|^2 + (2-p)|F_6|^2\right), \quad (3.6)\\ \frac{(p-3)}{e^{2A}}\tilde{\mathcal{R}}_4 &= -\,2|H|^2 + e^{2\phi}\left((7-p)|F_1|^2 + (5-p)|F_3|^2 + (3-p)|F_5|^2\right). \quad (3.7)\end{aligned}}$$

These equations have an interesting interpretation for $p \neq 3$: if the D_p and O_p source the flux F_k, the coefficient in front of F_k precisely vanishes [8]; $\tilde{\mathcal{R}}_4$ is then only given by the non-sourced fluxes.

We now study the possibility of getting a de Sitter vacuum, i.e. $\tilde{\mathcal{R}}_4 > 0$. From (3.6) and (3.7), the result is clear for $p = 7, 8$:

$$\boxed{\text{There is no de Sitter vacuum for } p = 7 \text{ or } p = 8.} \quad (3.8)$$

Let us make a comment: we only used combinations of e.o.m., which until (3.3) only required Point 1 through 4, from Section 2, on the sources. Point 5, i.e. the assumption of parallel sources, was only used in the computation of warp factor and dilaton contributions. So this result on $p = 7, 8$ can be extended to intersecting sources, at least in the smeared limit.

We now turn to the sources with $3 \leq p \leq 6$ in the smeared limit, in which the dilaton and warp factor are taken constant. We denote collectively (ϕ) the dilaton terms to be neglected.

- $p = 6$: equating (3.3) with (3.2), we get (as in [45])

$$\frac{9}{2}\mathcal{R}_4 = 3\left(e^{2\phi}\left(|F_0|^2 - |F_4|^2 - 2|F_6|^2\right) - |H|^2\right) + (\phi) \tag{3.9}$$
$$= -2\mathcal{R}_6 - e^{2\phi}\left(|F_2|^2 + 2|F_4|^2 + 3|F_6|^2\right) + (\phi) \ .$$

  For de Sitter, one needs $F_0 \neq 0$ and $\mathcal{R}_6 < 0$ of sufficient magnitude to overtake the remaining possible non-zero terms, as pointed-out already in [23].

- $p = 5$: equating three halves of (3.3) with (3.2), we get

$$4\mathcal{R}_4 = 4\left(e^{2\phi}(|F_1|^2 - |F_5|^2) - |H|^2\right) + (\phi) = -2\mathcal{R}_6 - e^{2\phi}(|F_3|^2 + 2|F_5|^2) + (\phi) \ . \tag{3.10}$$

  For de Sitter, one needs $F_1 \neq 0$ and $\mathcal{R}_6 < 0$, of sufficient magnitude.

- $p = 4$: equating three times (3.3) with (3.2), we get (as in [8])

$$\frac{7}{2}\mathcal{R}_4 = 7\left(e^{2\phi}(2|F_0|^2 + |F_2|^2 - |F_6|^2) - |H|^2\right) + (\phi) \tag{3.11}$$
$$= -2\mathcal{R}_6 + e^{2\phi}(|F_0|^2 - |F_4|^2 - 2|F_6|^2) + (\phi) \ .$$

  For de Sitter, one needs $F_0 \neq 0$, or $F_2 \neq 0$ and $\mathcal{R}_6 < 0$, all of sufficient magnitude.

- $p = 3$: (3.3) and (3.2) give (using (3.4) for the dilaton)

$$3\mathcal{R}_4 = -2\mathcal{R}_6 + e^{2\phi}(|F_1|^2 - |F_5|^2) \ , \quad 2e^{2\phi}|F_1|^2 = |H|^2 - e^{2\phi}|F_3|^2 \ . \tag{3.12}$$

  For de Sitter, one needs $\mathcal{R}_6 < 0$, or $F_1 \neq 0$ and $H \neq 0$, all of sufficient magnitude.

These are limited results, valid in the smeared limit. In the next section we will make use of the BI which will allow us to put further restrictions on the possibility of de Sitter vacua.

4 No de Sitter vacuum for O_3, no-go theorems for O_4, O_5, O_6

4.1 First manipulations

In Section 3, we combined the e.o.m. to eliminate T_{10}. Here we will eliminate $\mathcal{R}_{10}$ (or $\mathcal{R}_6$), and make a further step by using the BI for T_{10}. Finally, we will use another equation, the trace of the Einstein equation along the internal parallel directions, to rewrite the result more conveniently: this will bring us to the no-go theorems.

We start by combining the dilaton e.o.m. and the four-dimensional trace to get

$$\mathcal{R}_4 = e^{\phi}\frac{T_{10}}{p+1} - e^{2\phi}\sum_{q=0}^{6}|F_q|^2 - 2(\nabla\partial\phi)_4 \ , \tag{4.1}$$

with even/odd RR fluxes in IIA/IIB. Note that in smeared limit where the dilaton and warp factor are constant, one concludes that de Sitter needs $T_{10} > 0$ [20]; this requirement not only means having O_p, but also that they contribute more than D_p. We now combine the dilaton e.o.m. with the ten-dimensional trace and get

$$(p-3)e^{\phi}\frac{T_{10}}{p+1} + 2|H|^2 - e^{2\phi}\sum_{q=0}^{6}(5-q)|F_q|^2 - 8|\partial\phi|^2 + 4\Delta\phi = 0 \ . \tag{4.2}$$

Equation (4.1) is multiplied by $-(p+1)$, and added to (4.2), giving

$$\mathcal{R}_4 + 2(\nabla\partial\phi)_4 = -\frac{1}{p+1}\left(-8|\partial\phi|^2 + 4\Delta\phi - 4e^{\phi}\frac{T_{10}}{p+1} + 2|H|^2 + e^{2\phi}\sum_{q=0}^{6}(p+q-4)|F_q|^2\right) \ . \tag{4.3}$$

From now on, we use notations of (2.7), where the sourced flux is F_k with $0 \leq k = 8-p \leq 5$, and $F_{-1} = F_{-2} = F_7 = F_8 = F_9 = F_{10} = F_{11} = 0$. Then, (4.3) gets rewritten as

$$\begin{aligned} \mathcal{R}_4 + 2(\nabla\partial\phi)_4 = -\frac{2}{p+1}\bigg(& -4|\partial\phi|^2 + 2\Delta\phi - 2e^{\phi}\frac{T_{10}}{p+1} + |H|^2 \\ & + e^{2\phi}(|F_{k-2}|^2 + 2|F_k|^2 + 3|F_{k+2}|^2 + 4|F_{k+4}|^2 + 5|F_{k+6}|^2)\bigg) \ . \end{aligned} \tag{4.4}$$

We now use the BI projected on transverse directions (2.8) to replace T_{10}. With (2.9), we get

$$\boxed{\begin{aligned} \mathcal{R}_4 + 2(\nabla\partial\phi)_4 = -\frac{2}{p+1}\bigg(& -4|\partial\phi|^2 + 2\Delta\phi - 2\varepsilon_p e^{\phi}(\mathrm{d}F_k)_{\perp} + \Big|*_{\perp}H|_{\perp} + \varepsilon_p e^{\phi}F_{k-2}|_{\perp}\Big|^2 \\ & + |H|^2 - |H|_{\perp}|^2 + e^{2\phi}(|F_{k-2}|^2 - |F_{k-2}|_{\perp}|^2) \\ & + e^{2\phi}(2|F_k|^2 + 3|F_{k+2}|^2 + 4|F_{k+4}|^2 + 5|F_{k+6}|^2)\bigg) \ . \end{aligned}} \tag{4.5}$$

Let us make a few comments. In the smeared limit, the only term in the right-hand side with indefinite sign is $(\mathrm{d}F_k)_{\perp}$. There are thus two interesting subcases to mention. First, we get

$$\boxed{\text{There is no (smeared) de Sitter vacuum if in the smeared limit } (\mathrm{d}F_k)_{\perp} \to 0 \ .} \tag{4.6}$$

For instance, in the Minkowski vacua of [43, 46], F_k is only given by a ∂A which vanishes in the smeared limit. Deformations of the vacuum preserving this property will then not give de Sitter. Second, Minkowski vacua with calibrated sources [47–49] verify (in our conventions)

$$F_k = (-1)^p \varepsilon_p e^{-4A} *_6 \mathrm{d}\left(e^{4A-\phi}\mathrm{vol}_{||}\right) \ . \tag{4.7}$$

This calibration condition, or source energy minimization condition, is e.g. automatically satisfied in Minkowski supersymmetric vacua [48]. From (4.7), one can show

$$\int_{\mathcal{M}} 2e^{\phi}\varepsilon_p f\,(\mathrm{d}F_k)_{\perp}\mathrm{vol}_6 = \int_{\mathcal{M}} 2e^{2\phi} f\,|F_k|^2\,\mathrm{vol}_6 \ , \tag{4.8}$$

with $f = e^{4A-2\phi}$. Upon integration, the $(\mathrm{d}F_k)_{\perp}$ term in (4.5) is then compensated by the $|F_k|^2$ one, which leads us to conclude on de Sitter in the smeared limit

There is no (smeared) de Sitter vacuum if sources are Minkowski-calibrated, i.e. if holds. (4.9)

For instance, deforming a supersymmetric Minkowski vacuum while preserving (4.7), by e.g. adding more fluxes or changing part of the geometry, will not give de Sitter.

To go further, we need to characterise $({\rm d}F_k)_\perp$. To that end, we use flat indices: the internal metric is expressed with vielbeins as $g_{mn} = e^a{}_m e^b{}_n \eta_{ab}$ and we denote $\partial_a = e^m{}_a \partial_m,\ e^a = e^a{}_m {\rm d}y^m$. The "geometric flux" $f^a{}_{bc}$ is defined as

$$ {\rm d}e^a = -\frac{1}{2} f^a{}_{bc} e^b \wedge e^c \ \Leftrightarrow\ f^a{}_{bc} = 2 e^a{}_m \partial_{[b} e^m{}_{c]} = -2 e^m{}_{[c}\partial_{b]} e^a{}_m \ . \tag{4.10} $$

The metric has been assumed block diagonal in one basis, given in (2.6), one can always further diagonalise it to the flat basis, such that having parallel and transverse flat indices makes sense. We thus decompose F_k on its parallel or transverse (flat) components

$$ F_k = \frac{1}{k!} F^{(0)}_{k\ a_{1\perp}\dots a_{k\perp}} e^{a_{1\perp}} \wedge \dots \wedge e^{a_{k\perp}} + \frac{1}{(k-1)!} F^{(1)}_{k\ a_{1||}\dots a_{k\perp}} e^{a_{1||}} \wedge e^{a_{2\perp}} \wedge \dots \wedge e^{a_{k\perp}} + \dots \ , \tag{4.11} $$

where terms with at least two parallel directions have been left out. By definition, $F_k^{(0)} = F_k|_\perp$; we also take for convenience $F_0 = F_0|_\perp$ and $F_0^{(1)} = 0$. One deduces

$$ ({\rm d}F_k)|_\perp = ({\rm d}F_k^{(0)})|_\perp + ({\rm d}F_k^{(1)})|_\perp \ , \quad ({\rm d}F_k^{(1)})|_\perp = (\iota_{\partial_{a_{||}}} F_k^{(1)}) \wedge ({\rm d}e^{a_{||}})|_\perp \ , \tag{4.12} $$

with ι_V the contraction by a vector V, e.g. $\iota_{\partial_{a_{||}}} e^{b_{||}} = \delta^{b_{||}}_{a_{||}}$, and $({\rm d}e^{a_{||}})|_\perp = -\frac{1}{2} f^{a_{||}}{}_{b_\perp c_\perp} e^{b_\perp} \wedge e^{c_\perp}$. Proceeding similarly to (2.9), we further have

$$ \begin{aligned} \sum_{a_{||}} \left| *_\perp ({\rm d}e^{a_{||}})|_\perp - \varepsilon_p e^\phi (\iota_{\partial_{a_{||}}} F_k^{(1)}) \right|^2 = & \sum_{a_{||}} e^{2\phi} |(\iota_{\partial_{a_{||}}} F_k^{(1)})|^2 + \sum_{a_{||}} |({\rm d}e^{a_{||}})|_\perp|^2 \\ & - 2\varepsilon_p e^\phi ((\iota_{\partial_{a_{||}}} F_k^{(1)}) \wedge ({\rm d}e^{a_{||}})|_\perp)_\perp \end{aligned} \tag{4.13} $$

$$ \text{with} \ \sum_{a_{||}} e^{2\phi} |(\iota_{\partial_{a_{||}}} F_k^{(1)})|^2 = e^{2\phi} |F_k^{(1)}|^2 \ , \quad \sum_{a_{||}} |({\rm d}e^{a_{||}})|_\perp|^2 = \frac{1}{2} \eta^{be} \eta^{cf} \eta_{ad} f^{a_{||}}{}_{b_\perp c_\perp} f^{d_{||}}{}_{e_\perp f_\perp} \ . $$

We thus reconstruct interesting squares from $({\rm d}F_k)_\perp$ at the cost of introducing the geometric contributions $|({\rm d}e^{a_{||}})|_\perp|^2$. Those actually appear in curvature terms, present in the trace of the Einstein equation (A.15) or (A.16) along internal parallel directions. So we turn to this trace, and denote $\mathcal{R}_{6||}$ the trace of $\mathcal{R}_{MN}$ along internal parallel directions. We obtain

$$ \begin{aligned} \mathcal{R}_{6||} + 2(\nabla\partial\phi)_{6||} = & \frac{p-3}{4} \left(\mathcal{R}_4 + 2(\nabla\partial\phi)_4 + 2e^{2\phi}|F_6|^2 \right) \\ & + \frac{1}{2} \left(|H|^2 - |H|_\perp|^2 + e^{2\phi}(|F_2|^2 - |F_2|_\perp|^2 + |F_4|^2 - |F_4|_\perp|^2 \right) \end{aligned} \tag{4.14} $$

$$ \begin{aligned} \mathcal{R}_{6||} + 2(\nabla\partial\phi)_{6||} = & \frac{p-3}{4} \left(\mathcal{R}_4 + 2(\nabla\partial\phi)_4 + e^{2\phi}|F_5|^2 \right) \\ & + \frac{1}{2} \left(|H|^2 - |H|_\perp|^2 + e^{2\phi}(|F_1|^2 - |F_1|_\perp|^2 + |F_3|^2 - |F_3|_\perp|^2 \right) \\ & + \frac{1}{4} e^{2\phi} \left(|F_5|^2 - |F_5|_\perp|^2 - |*_6 F_5|^2 + |(*_6 F_5)|_\perp|^2 \right) \ , \end{aligned} \tag{4.15} $$

where we used the four-dimensional trace of the Einstein equation for terms in $p-3$. The above is valid for $0 \leq k = 8-p \leq 5$; for $p=3$ where all internal directions are transverse, we take as a definition of the left-hand side that it vanishes. A generic rewriting of the above is

$$\begin{aligned} 2\mathcal{R}_{6||} + 4(\nabla\partial\phi)_{6||} - \frac{p-3}{2}\left(\mathcal{R}_4 + 2(\nabla\partial\phi)_4\right) &= |H|^2 - |H|_\perp|^2 + e^{2\phi}\left(|F_{k-2}|^2 - |F_{k-2}|_\perp|^2\right) \\ &+ e^{2\phi}\Big(|F_k|^2 - |F_k|_\perp|^2 + |F_{k+2}|^2 + (9-p)|F_{k+4}|^2 + 5|F_{k+6}|^2 + \frac{1}{2}(|(*_6 F_5)|_\perp|^2 - |F_5|_\perp|^2)\Big) \end{aligned} \tag{4.16}$$

where the F_5 terms should only be considered in IIB, we took the same conventions as for (4.4), and used that $F_5|_\perp = 0$ for $p=5,7$, $F_4|_\perp = 0$ for $p=6,8$, $F_3|_\perp = 0$ for $p=7$, $F_2|_\perp = 0$ for $p=8$. We now combine (4.16) with (4.5) to get

$$\begin{aligned} 2\mathcal{R}_4 + 4(\nabla\partial\phi)_4 = -\Big(& -4|\partial\phi|^2 + 2\Delta\phi + \Big|*_\perp H|_\perp + \varepsilon_p e^{\phi} F_{k-2}|_\perp\Big|^2 \\ & + 2\mathcal{R}_{6||} + 4(\nabla\partial\phi)_{6||} - 2\varepsilon_p e^{\phi}(\mathrm{d}F_k)_\perp + e^{2\phi}\left(|F_k|^2 + |F_k|_\perp|^2\right) \\ & + e^{2\phi}\Big(2|F_{k+2}|^2 + (p-5)|F_{k+4}|^2 + \frac{1}{2}(|F_5|_\perp|^2 - |(*_6 F_5)|_\perp|^2)\Big)\Big) \ . \end{aligned} \tag{4.17}$$

One can verify that the last line of (4.17) is always positive; we are now interested in the second line. First, we determine in Appendices B.1 and B.2 the expression for $\mathcal{R}_{6||}$, which combined to the other warp factor and dilaton contributions gives

$$\begin{aligned} & 2\mathcal{R}_4 + 4(\nabla\partial\phi)_4 - 4|\partial\phi|^2 + 2\Delta\phi + 4(\nabla\partial\phi)_{6||} + 2\mathcal{R}_{6||} \\ & = 2e^{-2A}\tilde{\mathcal{R}}_4 + 2\mathcal{R}_{||} + 2\mathcal{R}_{||}^{\perp} + \sum_{a_{||}} |(\mathrm{d}e^{a_{||}})|_\perp|^2 + 2e^{6A}\tilde{\Delta}_\perp e^{-4A} - 2e^{10A}|\widetilde{\mathrm{d}e^{-4A}}|^2 \ . \end{aligned} \tag{4.18}$$

This is derived in (B.15) and (B.30), and the curvature terms $\mathcal{R}_{||}$ and $\mathcal{R}_{||}^{\perp}$ are defined in (B.23) and (B.24). We used, for simplicity, Point 6 on sources in Section 2 requiring the transverse subspace to be a compact manifold, without boundary. Combining (4.18) with $(\mathrm{d}F_k^{(1)})_\perp$ will simplify, given (4.13) and as initially motivated there. The remaining $(\mathrm{d}F_k^{(0)})_\perp$ will combine interestingly with the warp factor terms, and $|F_k|_\perp|^2 = |F_k^{(0)}|^2$. To that end we introduce $(G)_{\tilde{\perp}}$, an analogous coefficient to $(G)_\perp$ (defined above (2.8)) on the smeared transverse subspace (i.e. for $A=0$)

$$G|_\perp = (G)_{\tilde{\perp}} \widetilde{\mathrm{vol}}_\perp \ , \ (G)_{\tilde{\perp}} = \tilde{*}_\perp G|_\perp \ , \ (G)_{\tilde{\perp}} = (G)_\perp e^{A(p-9)} \ . \tag{4.19}$$

We then have $\tilde{\Delta}_\perp e^{-4A} = \tilde{*}_\perp \mathrm{d}(\tilde{*}_\perp \mathrm{d}e^{-4A}) = \left(\mathrm{d}(\tilde{*}_\perp \mathrm{d}e^{-4A})\right)_{\tilde{\perp}}$, and $e^{\phi}(\mathrm{d}F_k^{(0)})_\perp = e^{6A} g_s (\mathrm{d}F_k^{(0)})_{\tilde{\perp}}$. We also make use of the rewriting (B.17) that gives

$$\begin{aligned} e^{2\phi}|F_k^{(0)}|^2 &= e^{2\phi}|g_s^{-1}\tilde{*}_\perp \mathrm{d}e^{-4A} - \varepsilon_p F_k^{(0)}|^2 + e^{10A}|\widetilde{\mathrm{d}e^{-4A}}|^2 \\ &+ e^{-2A}\left(\mathrm{d}\left(e^{8A}\tilde{*}_\perp \mathrm{d}e^{-4A} - e^{8A}\varepsilon_p g_s F_k^{(0)}\right)\right)_{\tilde{\perp}} - e^{6A}\left(\mathrm{d}\left(\tilde{*}_\perp \mathrm{d}e^{-4A} - \varepsilon_p g_s F_k^{(0)}\right)\right)_{\tilde{\perp}} \ . \end{aligned} \tag{4.20}$$

Combining (4.17) with (4.12), (4.13), (4.18) and (4.20) finally leads to

$$\boxed{\begin{aligned} 2e^{-2A}\tilde{\mathcal{R}}_4 = & - \left| *_\perp H|_\perp + \varepsilon_p e^{\phi} F_{k-2}|_\perp \right|^2 - 2e^{2\phi} \left| g_s^{-1} \tilde{*}_\perp \mathrm{d}e^{-4A} - \varepsilon_p F_k^{(0)} \right|^2 \\ & - \sum_{a_{||}} \left| *_\perp (\mathrm{d}e^{a_{||}})|_\perp - \varepsilon_p e^{\phi} (\iota_{\partial_{a_{||}}} F_k^{(1)}) \right|^2 - 2\mathcal{R}_{||} - 2\mathcal{R}_{||}^{\perp} \\ & - 2e^{-2A} \left(\mathrm{d} \left(e^{8A} \tilde{*}_\perp \mathrm{d}e^{-4A} - e^{8A} \varepsilon_p g_s F_k^{(0)} \right) \right)_{\tilde{\perp}} \\ - e^{2\phi} \Big(|F_k|^2 - |F_k^{(0)}|^2 - |F_k^{(1)}|^2 & + 2|F_{k+2}|^2 + (p-5)|F_{k+4}|^2 + \frac{1}{2}(|F_5|_\perp|^2 - |(*_6 F_5)|_\perp|^2) \Big) \end{aligned}} \tag{4.21}$$

For clarity, we detail the last line of (4.21), i.e. the $-(\text{flux})^2$ contribution

$$p=3: \ -e^{2\phi}(\text{fluxes}) = 0 \tag{4.22}$$

$$p=4: \ -e^{2\phi}(\text{fluxes}) = -2e^{2\phi}|F_6|^2 \tag{4.23}$$

$$p=5: \ -e^{2\phi}(\text{fluxes}) = -e^{2\phi}\Big(|F_3|^2 - |F_3^{(0)}|^2 - |F_3^{(1)}|^2 + 2|F_5|^2 - \frac{1}{2}|(*_6 F_5)|_\perp|^2\Big) \tag{4.24}$$

$$p=6: \ -e^{2\phi}(\text{fluxes}) = -e^{2\phi}\Big(|F_2|^2 - |F_2^{(0)}|^2 - |F_2^{(1)}|^2 + 2|F_4|^2 + |F_6|^2\Big) \tag{4.25}$$

$$p=7: \ -e^{2\phi}(\text{fluxes}) = -2e^{2\phi}\Big(|F_3|^2 + |F_5|^2 - \frac{1}{4}|(*_6 F_5)|_\perp|^2\Big) \tag{4.26}$$

$$p=8: \ -e^{2\phi}(\text{fluxes}) = -e^{2\phi}\Big(2|F_2|^2 + 3|F_4|^2\Big) \ . \tag{4.27}$$

One has $|F_k|^2 - |F_k^{(0)}|^2 - |F_k^{(1)}|^2 \geq 0$ and $|F_5|^2 = | *_6 F_5|^2 \geq |(*_6 F_5)|_\perp|^2$, so this line always gives a negative (semi-)definite contribution to $\tilde{\mathcal{R}}_4$. Thanks again to Point 6 on the sources in Section 2, i.e. the transverse subspace is a compact manifold without boundary, the total derivative in (4.21) is integrated to give zero, resulting in

$$\boxed{\begin{aligned} \tilde{\mathcal{R}}_4 \int_{\tilde{\perp}} \widetilde{\mathrm{vol}}_\perp = - \int_{\tilde{\perp}} \widetilde{\mathrm{vol}}_\perp \frac{e^{2A}}{2} \Bigg(& \left| *_\perp H|_\perp + \varepsilon_p e^{\phi} F_{k-2}|_\perp \right|^2 + 2e^{2\phi} \left| g_s^{-1} \tilde{*}_\perp \mathrm{d}e^{-4A} - \varepsilon_p F_k^{(0)} \right|^2 \\ & + \sum_{a_{||}} \left| *_\perp (\mathrm{d}e^{a_{||}})|_\perp - \varepsilon_p e^{\phi} (\iota_{\partial_{a_{||}}} F_k^{(1)}) \right|^2 + 2\mathcal{R}_{||} + 2\mathcal{R}_{||}^{\perp} \\ + e^{2\phi} \Big(|F_k|^2 - |F_k^{(0)}|^2 - |F_k^{(1)}|^2 + 2|F_{k+2}|^2 & + (p-5)|F_{k+4}|^2 + \frac{1}{2}(|F_5|_\perp|^2 - |(*_6 F_5)|_\perp|^2) \Big) \Bigg) \end{aligned}} \tag{4.28}$$

4.2 No-go theorems

From (4.28), we conclude straightforwardly on the no-go theorem

$$\boxed{\begin{array}{l} \text{There is no de Sitter vacuum for } p = 4, 5, \text{ or } 6, \text{ if the curvature terms vanish} \\ \text{or are positive, i.e. for } \mathcal{R}_{||} + \mathcal{R}_{||}^{\perp} \geq 0. \end{array}} \tag{4.29}$$

We recall that $\mathcal{R}_{||}$ and $\mathcal{R}_{||}^{\perp}$ are defined in (B.23) and (B.24). This no-go theorem (4.29) is actually valid for all $3 \leq p \leq 8$, as is (4.28). But we proved the complete absence of de Sitter vacuum for $p=7,8$ in (3.8), while for $p=3$, since all directions are transverse, one has by definition $\mathcal{R}_{||}=\mathcal{R}_{||}^{\perp}=0$. This leads us to

$$\boxed{\text{There is no de Sitter vacuum for } p=3.} \tag{4.30}$$

This result was already obtained in [35]. In type IIB, the $\tilde{\mathcal{R}}_4$ expression (4.21) has been obtained combining (4.5) with (4.15), with various rewritings. As indicated below (4.15), that equation is however completely vanishing for $p=3$, so one can verify that (4.21) and (4.5) are then identical, and boil down to

$$\begin{aligned} p=3: \quad 2e^{-2A}\tilde{\mathcal{R}}_4 = & -\left|*_6 H + \varepsilon_3 e^{\phi} F_3\right|^2 - 2\left|\tilde{*}_6 \mathrm{d}e^{-4A} - \varepsilon_3 g_s F_5\right|^2 \\ & - 2e^{-2A}\left(\mathrm{d}\left(e^{8A}\tilde{*}_6 \mathrm{d}e^{-4A} - e^{8A}\varepsilon_3 g_s F_5\right)\right)_{\tilde{\perp}} \ . \end{aligned} \tag{4.31}$$

Integrating the above makes (4.30) even more apparent.

We now comment on $\mathcal{R}_{||}$ and $\mathcal{R}_{||}^{\perp}$ for $p=4,5,6$. These two quantities rather tend to be negative, so the no-go theorem would apply for them vanishing. In common examples of vacua, they do vanish: $\mathcal{R}_{||}$ encodes the curvature of the wrapped subspace, often taken to be a torus, which would make $\mathcal{R}_{||}$ vanish. An extreme case is $p=4$ where there is only one internal parallel direction, implying $\mathcal{R}_{||}=0$. $\mathcal{R}_{||}^{\perp}$ is encoding the fibration of the transverse space over the (parallel) base subspace: this is also unusual, since common vacua have sources wrapping fibers, which would set $\mathcal{R}_{||}^{\perp}=0$. It is also fair that most of these examples are obtained by T-duality: we comment on this below. We now list constraints on these curvatures terms.

- The most important constraint comes from (4.16) that imposes

$$2\mathcal{R}_{6||} + 4(\nabla\partial\phi)_{6||} - \frac{p-3}{2}\left(\mathcal{R}_4 + 2(\nabla\partial\phi)_4\right) \geq 0 \ . \tag{4.32}$$

We compute this quantity from results of Appendix B and find it equal to $2\mathcal{R}_{6||}|_{(\partial A=0)} - \frac{p-3}{2}e^{-2A}\tilde{\mathcal{R}}_4$. We deduce the following requirement for a de Sitter vacuum when $p>3$

$$2\mathcal{R}_{6||}|_{(\partial A=0)} = 2\mathcal{R}_{||} + 2\mathcal{R}_{||}^{\perp} + \frac{1}{2}\eta^{ch}\eta^{dj}\eta_{ab} f^{a_{||}}{}_{c_\perp j_\perp} f^{b_{||}}{}_{h_\perp d_\perp} > 0 \ , \tag{4.33}$$

where we recall that $\sum_{a_{||}} |(\mathrm{d}e^{a_{||}})|_\perp|^2 = \frac{1}{2}\eta^{ch}\eta^{dj}\eta_{ab} f^{a_{||}}{}_{c_\perp j_\perp} f^{b_{||}}{}_{h_\perp d_\perp}$. Combined with the no-go theorem (4.29), we conclude the following

$$\boxed{\begin{array}{l}\text{There is no de Sitter vacuum for } p=4,5, \text{ or } 6, \text{ if the inequalities} \\ -\frac{1}{2}\sum_{a_{||}} |(\mathrm{d}e^{a_{||}})|_\perp|^2 < \mathcal{R}_{||} + \mathcal{R}_{||}^{\perp} < 0 \ \text{ are } \textit{not} \text{ satisfied.}\end{array}} \tag{4.34}$$

This narrow window which would allow de Sitter can easily be checked on concrete examples.[3] The requirement $f^{a_{||}}{}_{b_\perp c_\perp} \neq 0$ is also interesting.

[3] For $p=8$ where there is only one transverse direction, the left-hand side of (4.34) vanishes, making us recover the no-go theorem (3.8) in that case.

- $\tilde{f}^{a_\perp}{}_{b_{||}c_{||}}$, and thus the second line of $\mathcal{R}_{||}^{\perp}$, vanishes in several instances: for $p=4$, that has only one parallel direction, for $\widetilde{\mathrm{dvol}}_\perp = 0$, or on group manifolds where the $\tilde{f}^a{}_{bc}$ are constant; for the latter, the orientifold projection sets $\tilde{f}^{a_\perp}{}_{b_{||}c_{||}} = 0$.[4] This vanishing can also be viewed as the "T-dual" condition to $H_{a_{||}b_{||}c_{||}} = 0$, required to avoid the Freed-Witten anomaly (see e.g. [50] and references therein), and should then be imposed.

- Given the interpretation of the curvature terms, they should vanish if the metric (or rather here vielbeins) does not depend on internal parallel directions. This notion of parallel directions is discussed at the end of Appendix B.1. Let us prove this idea more precisely: for $\partial_{a_{||}} e^b{}_m = 0$, one has $f^{a_{||}}{}_{b_{||}c_{||}} = 0$ so $\mathcal{R}_{||} = 0$, see (B.23); with $\partial_{a_{||}} e^b{}_m = 0$ and the "gauge-fixing" (B.18), one also gets $f^{a_\perp}{}_{b_{||}c_{||}} = 0$ and $f^{a_\perp}{}_{b_\perp c_{||}} = 0$ hence $\mathcal{R}_{||}^{\perp} = 0$, see (B.24). We therefore conclude

There is no de Sitter vacuum for $p = 4, 5$, or 6, if the vielbeins are independent of internal parallel directions.	(4.35)

Remarks on T-dual vacua

We prove the following below: if a vacuum metric and b-field are independent of internal parallel directions, the same holds for the metric and b-field of T-dual vacua. If this can be brought to the initial and T-dual vielbeins, while respecting the "gauge-fixing" (B.18), we can then apply (4.35) and deduce: if a vacuum vielbeins and b-field are independent of internal parallel directions, then neither this vacuum nor its T-dual are de Sitter. This implies that there is no de Sitter vacuum T-dual to a vacuum with O_3. All this agrees with the four-dimensional point of view on T-duality: the scalar potential of four-dimensional gauged supergravity is invariant under T-duality (its terms and scalar fields are covariant, making the whole invariant), so its vacuum value, related to $\tilde{\mathcal{R}}_4$, is not changed by T-duality. In other words, one does not make a de Sitter vacuum appear by T-dualizing.

We now prove the above point. We start with a vacuum metric and b-field depending only on a set of transverse directions denoted collectively $y_\perp^0$. Because of the isometry requirement of T-duality, one is not allowed to T-dualize along $y_\perp^0$. Let us first T-dualize along a different transverse direction (assuming one exists) denoted $y_\perp^1$: sources get extended along this direction, that becomes in the T-dual vacuum a parallel direction. So the new sources are still transverse to $y_\perp^0$. As the fields coordinate dependence is not changed by T-duality, they still depend on $y_\perp^0$, so are independent of (new) parallel directions. If we rather initially T-dualize along a parallel direction, sources lose this dimension which becomes a transverse one; this has no impact on the fields dependence either. We conclude that the T-dual metric and b-field do not depend on parallel directions.

5 Outlook

In this paper, we study classical de Sitter vacua of ten-dimensional type II supergravities, where the sources D_p and O_p have only one size p and are parallel. As summarized in the Introduction, we show that there is no such de Sitter vacuum for $p = 3, 7, 8$; for $p = 4, 5, 6$, we

[4] Generally in this paper, the orientifold projection is not helping since most objects are a priori functions and not constant, and are thus only constrained to be even or odd.

cannot completely exclude these vacua, but still set high constraints on them, which amounts to having restricted values for some curvature terms of internal subspaces. These results provide clearer and tighter boundaries for the de Sitter string landscape. In addition, they can be applied concretely on various cosmological scenarios to test if those can be uplifted to string theory through a compactification. For instance, de Sitter vacua required to embed the monodromy inflation mechanism of [51] are fully excluded, completing the no-go theorem [8]: indeed, this model needs an O_4 and ${f^{a_{||}}}_{b_\perp c_\perp} = 0$, which violates the requirement (4.34). Finally, all technical tools are presented here in a self-contained manner, and can be used to pursue the search for de Sitter vacua in more involved settings.

While restrictions on the curvature terms for $p = 4, 5, 6$ are discussed in details in Section 4.2, one may wonder if additional information could be brought to further constrain them, and exclude completely de Sitter vacua. An idea would be to use calibration of D_p and O_p. The conditions for calibrated sources correspond to a minimization of their energy and are thus physically relevant. Using a condition derived for sources along Minkowski (4.8), we already obtain a no-go theorem (4.9). The corresponding condition for anti-de Sitter was derived in [52] and differs by a boundary term. In both cases, although not mandatory, supersymmetry serves as an interesting guideline, making the study of the de Sitter case more difficult. It would still be interesting to derive analogous conditions for de Sitter. Related geometric conditions could constrain the curvature terms further. Another idea would be to study the stability of a vacuum with such terms present. The work [41] could be useful to that end: the four-dimensional scalar fields introduced there are relevant to reproduce our results in the smeared limit, and determine the stability. Proving that the curvature terms generically lead to tachyons would be an important result.

The complete exclusion of classical de Sitter vacua with parallel sources would have two important consequences. On the one hand, having parallel sources is the only setting where a complete type II supergravity description of the vacuum is possible. Indeed, having either intersecting sources, or trying to add *NS*-sources, forces one to a partial or total smearing of the sources, at least in the current state of the art. Neglecting the backreaction of the sources in such a manner cannot always be properly justified. Progress on this is then required for any string cosmology. On the other hand, we have only focused on the shape of our universe without considering its content: matter should arise from the open string sector. In this context, the standard model would arise from intersecting branes rather than parallel branes. In addition, intersecting branes would break more supersymmetries. There is thus an optimistic view on an exclusion of classical de Sitter vacua with parallel sources: if string theory requires (specific?) intersecting branes settings to admit metastable de Sitter backgrounds, it could turn-out to be predictive when describing our universe. An application of a classical de Sitter vacuum supporting an intersecting brane model would be the description of the reheating phase after inflation. To that end, further development of intersecting brane models beyond simple torus geometries, as e.g. in [53], is crucial for a connection to string cosmology.

Acknowledgements

We would like to thank T. Van Riet, without whom this paper would not exist. We would also like to thank U. H. Danielsson, G. Dibitetto, F. F. Gautason, L. Martucci and F. Wolf for helpful discussions. The work of D. A. is part of the Einstein Research Project "Gravitation and High Energy Physics", funded by the Einstein Foundation Berlin. The work of J. B. is supported by John Templeton Foundation Grant 48222, and the CEA Eurotalents program.

A Type II supergravities

We consider (massive) type II supergravities in string frame, supplemented with the Ramond-Ramond (RR) sources D_p-branes and orientifold O_p-planes. The bosonic part of the ten-dimensional action can be decomposed as follows

$$S = S_{\text{bulk}} + S_{\text{sources}} \text{ where } S_{\text{bulk}} = S_0 + S_{CS}\,,\ S_{\text{sources}} = S_{DBI} + S_{WZ}\ . \tag{A.1}$$

The bulk fields are first the metric g_{MN} ($_{M,N}$ denote ten-dimensional curved indices), the dilaton ϕ and the Kalb-Ramond two-form b. In addition, the IIA p-form potentials are C_1, C_3 and the IIB ones are C_0, C_2 and C_4. The fluxes are $H = \mathrm{d}b$, and the Romans mass F_0, $F_2 = \mathrm{d}C_1 + bF_0$, $F_4^{10} = \mathrm{d}C_3 - H \wedge C_1 + \frac{1}{2} b \wedge bF_0$ in IIA, $F_1 = \mathrm{d}C_0$, $F_3 = \mathrm{d}C_2 - H \wedge C_0$ and F_5^{10} in IIB. The corresponding action in IIA is

$$S_0 = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{|g_{10}|} \left(e^{-2\phi}(\mathcal{R}_{10} + 4|\partial\phi|^2 - \frac{1}{2}|H|^2) - \frac{1}{2}(|F_0|^2 + |F_2|^2 + |F_4^{10}|^2) \right)\ , \tag{A.2}$$

with $2\kappa_{10}^2 = (2\pi)^7(\alpha')^4$, $\alpha' = l_s^2$, and $|g_{10}|$ the absolute value of the determinant of the metric. For a p-form A_p, we denote $|A_p|^2 = A_{p\,M_1\dots M_p}\, g^{M_1N_1}\dots g^{M_pN_p} A_p^{N_1\dots N_p}/p!$. In IIB, one has

$$S_0 = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{|g_{10}|} \left(e^{-2\phi}(\mathcal{R}_{10} + 4|\partial\phi|^2 - \frac{1}{2}|H|^2) - \frac{1}{2}(|F_1|^2 + |F_3|^2 + \frac{1}{2}|F_5^{10}|^2) \right)\ . \tag{A.3}$$

This is a pseudo-action for the flux F_5^{10}, that has to satisfy the following constraint on-shell

$$F_5^{10} = -*_{10} F_5^{10}\ . \tag{A.4}$$

The Hodge star in dimension D is defined as follows, with the Levi-Civita symbol $\epsilon_{0\dots D-1} = 1$,

$$*_D\,(\mathrm{d}x^{m_1} \wedge \dots \wedge \mathrm{d}x^{m_p}) = \frac{\sqrt{|g_D|}}{(D-p)!}\, g^{m_1n_1}\dots g^{m_pn_p} \epsilon_{n_1\dots n_p r_{p+1}\dots r_D} \mathrm{d}x^{r_{p+1}} \wedge \dots \wedge \mathrm{d}x^{r_D}\ . \tag{A.5}$$

One has $A_p \wedge *_D A_p = \mathrm{d}^D x \sqrt{|g_D|}\ |A_p|^2$, and we recall that $*_D^2 A_p = s(-1)^{p(D-p)} A_p$ for a signature s. From the constraint (A.4), one gets on-shell

$$F_5^{10} \wedge *_{10} F_5^{10} = -*_{10} F_5^{10} \wedge F_5^{10} = -F_5^{10} \wedge *_{10} F_5^{10} \Rightarrow |F_5^{10}|^2 = 0\ . \tag{A.6}$$

This would imply that F_5^{10} vanishes for a positive definite metric, which is not the case here. We will not need to specify the Chern-Simons term S_{CS}, so we turn to the Dirac-Born-Infeld action

$$S_{DBI} = -c_p\, T_p \int_{\Sigma_{p+1}} \mathrm{d}^{p+1}\xi\ e^{-\phi} \sqrt{|\imath^*[g_{10} - b] + \mathcal{F}|}\ , \tag{A.7}$$

where Σ_{p+1} is the source world-volume and $\imath^*[\cdot]$ the pull-back to it. The tension T_p is given by $T_p^2 = \frac{\pi}{\kappa_{10}^2}(4\pi^2\alpha')^{3-p}$. For a D_p, $c_p = 1$; for an O_p, $c_p = -2^{p-5}$ and $\mathcal{F} = 0$. Finally, the Wess-Zumino term is given by

$$S_{WZ} = c_p\, \mu_p \int_{\Sigma_{p+1}} \sum_q \imath^*[C_q] \wedge e^{-\imath^*[b] + \mathcal{F}}\ , \tag{A.8}$$

where the charge $\mu_p = T_p$ for BPS sources as we consider here. One also has $\mathrm{d}\mathcal{F} = 0$.

We now impose two restrictions on the sources that allow to promote their action to a ten-dimensional one. We first consider $-\imath^*[b]+\mathcal{F}=0$; doing so at the level of the action instead of the equations of motion (e.o.m.) can only generate a difference in the b-field e.o.m.. In addition, for each source, we consider that there exists one basis (possibly flat) where the metric is block diagonal in the directions parallel and transverse to it, allowing to identify the world-volume directions in the ten-dimensional space-time (see Section 2). So we take a $\delta(y_\perp)$ localizing the source in the transverse directions, and use the projection $P[\cdot]$ to its directions

$$S_{DBI} \overset{\text{(here)}}{=\!=\!=} -c_p\, T_p \int \mathrm{d}^{10}x\, e^{-\phi}\sqrt{|P[g_{10}]|}\,\delta(y_\perp)\ , \quad S_{WZ} \overset{\text{(here)}}{=\!=\!=} c_p\, \mu_p \int C_{p+1}{\wedge}\mathrm{d}^{\perp}y\, \delta(y_\perp)\ , \tag{A.9}$$

where the form ordering is a convention choice.

We now derive the Einstein equation and dilaton e.o.m.. S_{CS} and S_{WZ} are topological terms that do not depend on g_{MN} or ϕ, so they do not contribute. We define the energy momentum tensor as

$$\frac{1}{\sqrt{|g_{10}|}} \sum_{\text{sources}} \frac{\delta S_{DBI}}{\delta g^{MN}} \equiv -\frac{e^{-\phi}}{4\kappa_{10}^2} T_{MN}\ . \tag{A.10}$$

It is given here, together with its trace, by

$$T_{MN} = -\frac{2\kappa_{10}^2}{\sqrt{|g_{10}|}} \sum_{\text{sources}} c_p\, T_p\ P[g_{MN}]\sqrt{|P[g_{10}]|}\ \delta(y_\perp)\ , \tag{A.11}$$

$$T_{10} = g^{MN}T_{MN} = -\frac{2\kappa_{10}^2}{\sqrt{|g_{10}|}} \sum_{\text{sources}} c_p\, T_p\ (p+1)\sqrt{|P[g_{10}]|}\ \delta(y_\perp) \equiv \sum_{\text{sources}} (p+1)\ t_p\ . \tag{A.12}$$

One can then verify

$$\frac{1}{\sqrt{|g_{10}|}} \sum_{\text{sources}} \frac{\delta S_{DBI}}{\delta \phi} = -\frac{e^{-\phi}}{2\kappa_{10}^2} \sum_{\text{sources}} t_p\ . \tag{A.13}$$

We deduce the dilaton equation of motion and the Einstein equation[5] in type IIA and IIB

$$2\mathcal{R}_{10} - |H|^2 + 8(\Delta\phi - |\partial\phi|^2) = -e^{\phi} \sum_{\text{sources}} t_p\ , \tag{A.14}$$

$$\begin{aligned}\mathcal{R}_{MN} - \frac{g_{MN}}{2}\mathcal{R}_{10} = &\ \frac{1}{4}H_{MPQ}H_N{}^{PQ} + \frac{e^{2\phi}}{2}\left(F_{2\ MP}F_{2\ N}{}^{P} + \frac{1}{3!}F^{10}_{4\ MPQR}F^{10}_{4\ N}{}^{PQR}\right)\\ &+ \frac{e^{\phi}}{2}T_{MN} - \frac{g_{MN}}{4}\left(|H|^2 + e^{2\phi}(|F_0|^2+|F_2|^2+|F_4^{10}|^2)\right)\\ &- 2\nabla_M\partial_N\phi + 2g_{MN}(\Delta\phi - |\partial\phi|^2)\ ,\end{aligned} \tag{A.15}$$

$$\begin{aligned}\mathcal{R}_{MN} - \frac{g_{MN}}{2}\mathcal{R}_{10} = &\ \frac{1}{4}H_{MPQ}H_N{}^{PQ} + \frac{e^{2\phi}}{2}\left(F_{1\ M}F_{1\ N} + \frac{1}{2!}F_{3\ MPQ}F_{3\ N}{}^{PQ} + \frac{1}{2\cdot 4!}F^{10}_{5\ MPQRS}F^{10}_{5\ N}{}^{PQRS}\right)\\ &+ \frac{e^{\phi}}{2}T_{MN} - \frac{g_{MN}}{4}\left(|H|^2 + e^{2\phi}(|F_1|^2+|F_3|^2)\right)\\ &- 2\nabla_M\partial_N\phi + 2g_{MN}(\Delta\phi - |\partial\phi|^2)\ ,\end{aligned} \tag{A.16}$$

where we imposed the constraint (A.6).

[5]On the dilaton terms in the Einstein equation, we refer to Footnote 30 of [54]. We also recall the Laplacian on a function φ: $\Delta\varphi = g^{MN}\nabla_M\partial_N\varphi = \frac{1}{\sqrt{|g|}}\partial_M(\sqrt{|g|}g^{MN}\partial_N\varphi)$; Δ stands here for the ten-dimensional one.

We now turn to the fluxes. As pointed-out in the seminal paper [55], the Wess-Zumino action (A.9) is problematic for the higher D_p-branes, and the magnetic coupling. To derive the fluxes Bianchi identities (BI) and e.o.m. in presence of sources, one should then use the democratic formalism, that has in addition the advantage of avoiding the Chern-Simons terms. One replaces the previous RR action for the following pseudo-action

$$\frac{1}{2\kappa_{10}^2}\int\left(-\frac{1}{4}\right)\sum_q F_{q+1}\wedge *_{10}F_{q+1}+\sum_{\text{sources},\,q} c_{q-1}\mu_{q-1}\int\frac{1}{2}C_q\wedge\delta^{\perp}_{10-q}\ , \tag{A.17}$$

where $q=1,3,5,7,9$ for IIA and $q=0,2,4,6,8$ for IIB, with $F_p=\mathrm{d}C_{p-1}-H\wedge C_{p-3}+F_0e^b|_p$ consistently with above, and $\delta^{\perp}_{10-q}\equiv \mathrm{d}^{(10-q)\,\perp}y\ \delta(y_{\perp})$. One should then impose on-shell the following constraint

$$F_p=(-1)^{\left[\frac{p+1}{2}\right]} *_{10} F_{10-p}\ , \tag{A.18}$$

where the integer part of $\frac{p}{2}$ can be rewritten as $(-1)^{\left[\frac{p}{2}\right]}=(-1)^{\frac{p(p-1)}{2}}$, giving $(-1)^{\left[\frac{p+1}{2}\right]}=(-1)^{\frac{(p+1)p}{2}}=(-1)^p(-1)^{\left[\frac{p}{2}\right]}$. The e.o.m. for C_q is now

$$\mathrm{d}(*_{10}F_{q+1})+H\wedge *_{10}F_{q+3}=2\kappa_{10}^2\ (-1)^{q+1}\sum_{(q-1)-\text{sources}} c_{q-1}\mu_{q-1}\,\delta^{\perp}_{10-q}\ . \tag{A.19}$$

Imposing the constraint gives the equivalent equation

$$\mathrm{d}(F_{9-q})-H\wedge F_{7-q}=2\kappa_{10}^2\ (-1)^{q+1}\sum_{(q-1)-\text{sources}} c_{q-1}\mu_{q-1}\,\lambda(\delta^{\perp}_{10-q})\ , \tag{A.20}$$

where for a p-form A_p, $\lambda(A_p)=(-1)^{\left[\frac{p}{2}\right]}A_p$. To get respectively the standard e.o.m. and BI, we restrict to the standard fluxes, giving in IIA and IIB

$$\begin{aligned}
&\mathrm{d}(*_{10}F_2)+H\wedge *_{10}F_4^{10}=2\kappa_{10}^2\sum_{0-\text{sources}} c_0\mu_0\,\delta_9^{\perp}\ ,\quad \mathrm{d}(*_{10}F_4^{10})+H\wedge F_4^{10}=2\kappa_{10}^2\sum_{2-\text{sources}} c_2\mu_2\,\delta_7^{\perp}\ ,\\
&\mathrm{d}(F_0)=2\kappa_{10}^2\sum_{8-\text{sources}} c_8\mu_8\,\delta_1^{\perp}\ ,\quad \mathrm{d}(F_2)-H\wedge F_0=-2\kappa_{10}^2\sum_{6-\text{sources}} c_6\mu_6\,\delta_3^{\perp}\ ,\\
&\mathrm{d}(F_4^{10})-H\wedge F_2=2\kappa_{10}^2\sum_{4-\text{sources}} c_4\mu_4\,\delta_5^{\perp}\ ;
\end{aligned}\tag{A.21}$$

$$\begin{aligned}
&\mathrm{d}(*_{10}F_1)+H\wedge *_{10}F_3=0\ ,\quad \mathrm{d}(*_{10}F_3)+H\wedge *_{10}F_5^{10}=-2\kappa_{10}^2\sum_{1-\text{sources}} c_1\mu_1\,\delta_8^{\perp}\ ,\\
&\mathrm{d}(*_{10}F_5^{10})+H\wedge F_3=-2\kappa_{10}^2\sum_{3-\text{sources}} c_3\mu_3\,\delta_6^{\perp}\ \Leftrightarrow\ \mathrm{d}(F_5^{10})-H\wedge F_3=2\kappa_{10}^2\sum_{3-\text{sources}} c_3\mu_3\,\delta_6^{\perp}\ ,\\
&\mathrm{d}(F_1)=2\kappa_{10}^2\sum_{7-\text{sources}} c_7\mu_7\,\delta_2^{\perp}\ ,\quad \mathrm{d}(F_3)-H\wedge F_1=-2\kappa_{10}^2\sum_{5-\text{sources}} c_5\mu_5\,\delta_4^{\perp}\ .
\end{aligned}\tag{A.22}$$

Finally, with the above pseudo-action, the b-field e.o.m. is given by

$$\mathrm{d}(e^{-2\phi} *_{10} H)-\sum_{1\leq q\leq 4} F_{q-1}\wedge *_{10}F_{q+1}-\frac{1}{2}F_4^{10}\wedge F_4^{10}=\text{sources}\ , \tag{A.23}$$

where the democratic formalism constraint (A.18) has been applied. The right-hand side "sources" denotes collectively the contribution from S_{sources} as well as from the source term in the right-hand side of (A.19) that has been used. The latter seems to cancel the contribution from S_{WZ}, leaving only the contribution from S_{DBI}, as pointed-out in [49]. We will not use this e.o.m.. In absence of NS_5-branes as here, the BI is

$$\mathrm{d}H = 0 \ . \tag{A.24}$$

Relation to other conventions

We follow the democratic formalism conventions [55] (the same as [56]), except for the Hodge star definition, where we get a sign $(-1)^{(D-p)p} = (-1)^{(D-1)p}$. We thus make this sign explicit, as in the constraint (A.18). Also, in [55] is not considered a b-field in the sources action, for which we then follow consistently [57].

Another set of conventions in the literature are those of e.g. [48, 49, 58, 59]. These conventions differ from the democratic formalism ones by a change of sign of H in IIB, and the change $C_q \to (-1)^{\frac{q-1}{2}} C_q$ in IIA. The latter is equivalent to *no change of* C_q, but a change of sign of H together with $F_{q+1} \to (-1)^{\frac{q-1}{2}} F_{q+1}$ for $q \geq 0$, leading to rewriting the constraint by replacing $(-1)^{\left[\frac{p}{2}\right]}$ by $(-1)^{\left[\frac{p-1}{2}\right]}$. That replacement is neutral in IIB, so the constraint can be rewritten in both theories. The map from our conventions to those of [48, 49, 58, 59], for both IIA and IIB, is then to change the sign of H, or actually of the b-field, rewriting the constraint (A.18) by replacing $(-1)^{\left[\frac{p}{2}\right]} \to (-1)^{\left[\frac{p-1}{2}\right]}$, and changing the Hodge star by an appropriate sign.[6] Upon this map, one can verify that the e.o.m. (A.19) and (B.6) of [49] match, using $\delta^{\perp}_{10-(p+1)} = \lambda(j_{(\Sigma_p,0)})$. But since the constraint differs by a sign, the BI differ by a sign (in IIA), making them not equivalent. That sign has however no physical relevance: it could be avoided by changing the sign of the WZ term in the brane action, which amounts to change the definition of brane versus anti-brane, or equivalently change which of the two type II supersymmetries are preserved (and its projector), or change the orientation of the world-volume. Note that the calibration (poly)form, related to the volume form of the sources, would then also pick a sign in IIA.

B Computational details

B.1 Warp factor and dilaton contributions

In this appendix, we compute various terms involving the warp factor and the dilaton, using the ten-dimensional metric (2.6). This metric is the standard one for (non-intersecting) D_p-branes and orientifold O_p-planes, defined in supergravity as ten-dimensional solutions characterized by the warp factor in a specific manner. This warp factor only depends on transverse directions: $A(y_\perp)$. The D_p and O_p are localized generically by $\delta(y_\perp)$, as in T_{10}. In our conventions, these δ functions are typically obtained analytically from $\tilde{\Delta}_\perp e^{-4A}$, produced by $\mathrm{d}F_k$ in the BI (see e.g. [46] for explicit examples); we denote by $\tilde{\Delta}_\perp$ the Laplacian on the transverse subspace involving the (smeared) metric $\mathrm{d}\tilde{s}^2_{6\perp}$. This e^{-4A} will guide us in the following. Finally, in D_p and O_p solutions, the dilaton is typically related to the warp factor, in a way derived below.

[6] Another difference is the value of the Levi-Civita symbol, which is opposite. This has no impact here since this symbol is considered only formally, defining e.g. $\mathrm{d}^{10}x \equiv 1/10!\ \epsilon_{M_1\dots M_{10}} \mathrm{d}x^{M_1} \wedge \ldots \wedge \mathrm{d}x^{M_{10}}$. It may matter if one computes explicit duals of forms, as e.g. the RR fluxes in [46, 56].

We first compute the ten-dimensional Ricci tensor in curved indices along parallel directions, for the Levi-Civita connection; see e.g. [8, 60] for relevant formulas. We extract the warp factor dependence and obtain

$$\begin{aligned}\mathcal{R}_{M_{||}N_{||}} =& \tilde{\mathcal{R}}_{||\, M_{||}N_{||}} + \frac{1}{4}\partial_{N_{||}}\tilde{g}_{P_\perp Q_\perp}\partial_{M_{||}}\tilde{g}^{P_\perp Q_\perp} + \frac{1}{2}e^{4A}\tilde{g}^{P_\perp Q_\perp}\tilde{g}^{R_{||}S_{||}}\partial_{P_\perp}\tilde{g}_{N_{||}R_{||}}\partial_{Q_\perp}\tilde{g}_{M_{||}S_{||}} \\ & - \partial_{N_{||}}\partial_{M_{||}}\ln\sqrt{|\tilde{g}_{6\perp}|} + \tilde{\Gamma}^{P_{||}}_{M_{||}N_{||}}\partial_{P_{||}}\ln\sqrt{|\tilde{g}_{6\perp}|} - \frac{1}{2}\frac{e^{4A}}{\sqrt{|\tilde{g}_6|}}\partial_{P_\perp}\left(\sqrt{|\tilde{g}_6|}\tilde{g}^{P_\perp Q_\perp}\partial_{Q_\perp}\tilde{g}_{M_{||}N_{||}}\right) \\ & - \frac{p-5}{2}e^{2A}\tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{2A}\partial_{Q_\perp}\tilde{g}_{M_{||}N_{||}} \\ & + e^{2A(p-2)}\tilde{g}_{M_{||}N_{||}}\left(e^{2A}\frac{\tilde{\Delta}_6 e^{-2A(p-3)}}{2(p-3)} + \tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{2A}\partial_{Q_\perp}e^{-2A(p-3)}\right)\ ,\end{aligned} \tag{B.1}$$

where the last line, valid for $p \neq 3$, should be replaced for $p = 3$ by $\frac{1}{2}e^{4A}\tilde{g}_{M_{||}N_{||}}\tilde{\Delta}_6 \ln e^{-2A}$. This computation required the following connection coefficient, to be used again later

$$\Gamma^{P_\perp}_{M_{||}N_{||}} = -\frac{1}{2}e^{2A}\tilde{g}^{P_\perp Q_\perp}\partial_{Q_\perp}(e^{2A}\tilde{g}_{M_{||}N_{||}})\ . \tag{B.2}$$

$\tilde{\mathcal{R}}_{||\, M_{||}N_{||}}$ denotes the purely parallel Ricci tensor (i.e. parallel smeared metric and parallel derivatives). This, with the first and second line of (B.1), can be denoted collectively the $(\partial A = 0)$ terms. Since $\mathcal{R}_4 = g^{MN}\mathcal{R}_{MN=\mu\nu} = g^{MN}\mathcal{R}_{M_{||}N_{||}=\mu\nu}$, we deduce from (B.1)

$$\mathcal{R}_4 = e^{-2A}\tilde{\mathcal{R}}_4 + 4e^{2A(p-3)}\left(e^{2A}\frac{\tilde{\Delta}_6 e^{-2A(p-3)}}{2(p-3)} + \tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{2A}\partial_{Q_\perp}e^{-2A(p-3)}\right) \quad \text{for } p \neq 3 \tag{B.3}$$

$$= e^{-2A}\tilde{\mathcal{R}}_4 + 2e^{2A}\tilde{\Delta}_6 \ln e^{-2A} \quad \text{for } p = 3\ , \tag{B.4}$$

where $\tilde{\mathcal{R}}_4$ is the four-dimensional Ricci scalar built from $\tilde{g}_{\mu\nu}$. We finally compute the quantities

$$2(\nabla\partial\phi)_4 \equiv 2g^{MN=\mu\nu}\left(\partial_M\partial_N\phi - \Gamma^P_{MN}\partial_P\phi\right) = -2e^{2\phi}\tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{2A}\partial_{Q_\perp}e^{-2\phi}\ , \tag{B.5}$$

$$4|\partial\phi|^2 - 2\Delta\phi = e^{2\phi}\Delta e^{-2\phi} = e^{2\phi+2A}\tilde{\Delta}_6 e^{-2\phi} + (p-3)e^{2\phi}\tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{2A}\partial_{Q_\perp}e^{-2\phi}\ , \tag{B.6}$$

where the second equations made use that the dilaton depends only on transverse coordinates; one cannot reconstruct a $\tilde{\Delta}_6$ otherwise, while this will be required. We also used (B.2).

We now use these results in equation (3.3): they involve the quantity

$$(p-3)\mathcal{R}_4 - 2e^{2\phi}\Delta e^{-2\phi} + 2(p-3)(\nabla\partial\phi)_4 = (p-3)e^{-2A}\tilde{\mathcal{R}}_4 - 2e^{2\phi-2A(p-4)}\tilde{\Delta}_6\, e^{2A(p-3)-2\phi}\ . \tag{B.7}$$

The last term vanishes for the standard value of a D_p-brane solution

$$e^{\phi} = g_s e^{A(p-3)}\ , \tag{B.8}$$

where g_s is a constant. For $p = 7, 8$, this value is even imposed by equation (3.3): indeed, (3.3) becomes $\tilde{\Delta}_6\, e^{2A(p-3)-2\phi} = X$, where $X \geq 0$ for a Minkowski or de Sitter vacuum. The Laplacian is integrated to give zero on the internal manifold. The integral of X then vanishes, and so does X. We deduce $\tilde{\Delta}_6\, e^{2A(p-3)-2\phi} = 0$. Harmonic functions on a compact manifold without boundary are constant, so we derive the value (B.8). We conclude that the quantity (B.7) is equal to $(p-3)e^{-2A}\tilde{\mathcal{R}}_4$, and use this result in Section 3.

We now turn to (4.17) that involves the quantity

$$2\mathcal{R}_4 + 4(\nabla\partial\phi)_4 - 4|\partial\phi|^2 + 2\Delta\phi + 4(\nabla\partial\phi)_{6||} + 2\mathcal{R}_{6||} \ . \tag{B.9}$$

Let us consider temporarily $p \neq 3$. To compute the last two terms, we use for now curved indices, meaning $\mathcal{R}_{6||} = g^{MN}\mathcal{R}_{M_{||}N_{||}=m_{||}n_{||}}$; we come back to this computation in flat indices in Appendix B.2. Using (B.2), we first get

$$4(\nabla\partial\phi)_{6||} = -e^{2\phi}\tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{-2\phi}\left(2e^{2A}\partial_{Q_\perp}\ln\sqrt{|\tilde{g}_{6||}|} + (p-3)\partial_{Q_\perp}e^{2A}\right) \ . \tag{B.10}$$

Further, using (B.1), we get

$$\begin{aligned} 2\mathcal{R}_{6||} &= 2\mathcal{R}_{6||}|_{(\partial A=0)} + e^{2A(p-2)}\tilde{\Delta}_6 e^{-2A(p-3)} \\ &\quad + 2e^{2A(p-3)}\tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{-2A(p-3)}\left(\frac{p-5}{p-3}e^{2A}\partial_{Q_\perp}\ln\sqrt{|\tilde{g}_{6||}|} + (p-3)\partial_{Q_\perp}e^{2A}\right) \ . \end{aligned} \tag{B.11}$$

With the above results, in particular the dilaton expression (3.4), and extracting $\sqrt{|\tilde{g}_{6||}|}$ from $\tilde{\Delta}_6$ to cancel such contributions, we conclude

$$\begin{aligned} &2\mathcal{R}_4 + 4(\nabla\partial\phi)_4 - 4|\partial\phi|^2 + 2\Delta\phi + 4(\nabla\partial\phi)_{6||} + 2\mathcal{R}_{6||} \\ &= 2e^{-2A}\tilde{\mathcal{R}}_4 + 2\mathcal{R}_{6||}|_{(\partial A=0)} + \frac{4}{p-3}e^{2\phi+2A}\tilde{\Delta}_\perp e^{-2\phi} + 4e^{2\phi}\tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{-2\phi}\partial_{Q_\perp}e^{2A} \ . \end{aligned} \tag{B.12}$$

We further rewrite

$$2e^{2\phi+2A}\tilde{\Delta}_\perp e^{-2\phi} = (p-3)e^{6A}\tilde{\Delta}_\perp e^{-4A} + 2(5-p)e^{2\phi}\tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{-2\phi}\partial_{Q_\perp}e^{2A} \ , \tag{B.13}$$

$$e^{2\phi}\tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{-2\phi}\partial_{Q_\perp}e^{2A} = -\frac{(p-3)}{4}e^{10A}|\widetilde{\mathrm{d}e^{-4A}}|^2 \ , \tag{B.14}$$

where by definition $|\widetilde{\mathrm{d}e^{-4A}}|^2 = \tilde{g}^{P_\perp Q_\perp}\partial_{P_\perp}e^{-4A}\partial_{Q_\perp}e^{-4A}$. We finally get

$$\begin{aligned} &2\mathcal{R}_4 + 4(\nabla\partial\phi)_4 - 4|\partial\phi|^2 + 2\Delta\phi + 4(\nabla\partial\phi)_{6||} + 2\mathcal{R}_{6||} \\ &= 2e^{-2A}\tilde{\mathcal{R}}_4 + 2\mathcal{R}_{6||}|_{(\partial A=0)} + 2e^{6A}\tilde{\Delta}_\perp e^{-4A} - 2e^{10A}|\widetilde{\mathrm{d}e^{-4A}}|^2 \ . \end{aligned} \tag{B.15}$$

This last equation is actually also valid for $p = 3$: this can be shown using that the dilaton is constant, the internal parallel quantities are set to zero, and one uses (B.4) for $\mathcal{R}_4$ with

$$2e^{2A}\tilde{\Delta}_6\ln e^{-2A} = e^{6A}\tilde{\Delta}_\perp e^{-4A} - e^{10A}|\widetilde{\mathrm{d}e^{-4A}}|^2 \ . \tag{B.16}$$

To make use of (B.15) for (4.17), one is left to compute $\mathcal{R}_{6||}|_{(\partial A=0)}$. While this can be read from (B.1), it is more convenient to obtain the result in flat indices: this is done in Appendix B.2. We give further motivation in the final remark of this appendix.

We now detail the following rewriting useful for (4.17). One has $e^{2\phi}|F_k^{(0)}|^2 = e^{10A}|\widetilde{g_s F_k^{(0)}}|^2$

thanks to the dilaton (B.8), and analogously to (2.9) with (4.19),

$$
\begin{aligned}
|\widetilde{g_s F_k^{(0)}}|^2 &= |\varepsilon_p g_s F_k^{(0)} - \tilde{*}_\perp \mathrm{d}e^{-4A} + \tilde{*}_\perp \mathrm{d}e^{-4A}|^{\tilde{2}} \qquad (B.17)\\
&= |\varepsilon_p g_s F_k^{(0)} - \tilde{*}_\perp \mathrm{d}e^{-4A}|^{\tilde{2}} + |\tilde{*}_\perp \mathrm{d}e^{-4A}|^{\tilde{2}} + 2\left(\mathrm{d}e^{-4A} \wedge \left(\varepsilon_p g_s F_k^{(0)} - \tilde{*}_\perp \mathrm{d}e^{-4A}\right)\right)_{\tilde{\perp}}\\
&= e^{2\phi-10A}|\varepsilon_p F_k^{(0)} - g_s^{-1}\tilde{*}_\perp \mathrm{d}e^{-4A}|^2 + |\widetilde{\mathrm{d}e^{-4A}}|^2 - e^{-12A}\left(\mathrm{d}e^{8A} \wedge \left(\varepsilon_p g_s F_k^{(0)} - \tilde{*}_\perp \mathrm{d}e^{-4A}\right)\right)_{\tilde{\perp}}\\
&= e^{2\phi-10A}|g_s^{-1}\tilde{*}_\perp \mathrm{d}e^{-4A} - \varepsilon_p F_k^{(0)}|^2 + |\widetilde{\mathrm{d}e^{-4A}}|^2 + e^{-12A}\left(\mathrm{d}\left(e^{8A}\tilde{*}_\perp \mathrm{d}e^{-4A} - e^{8A}\varepsilon_p g_s F_k^{(0)}\right)\right)_{\tilde{\perp}}\\
&- e^{-4A}\left(\mathrm{d}\left(\tilde{*}_\perp \mathrm{d}e^{-4A} - \varepsilon_p g_s F_k^{(0)}\right)\right)_{\tilde{\perp}} \ .
\end{aligned}
$$

Remark on the notions of parallel and transverse directions

As mentioned below (4.10), it makes sense to consider parallel and transverse flat indices: those can always be defined globally given the block diagonal metric (2.6). On the contrary, parallel or transverse coordinates do not necessarily make sense globally. There is a true coordinate separation between four-dimensional coordinates and internal ones, but the same does not hold a priori for internal parallel or transverse. This gets reflected in the vielbeins: while in four dimensions one has only $e^{\alpha}{}_{\mu}$, internally one can have a priori $e^{a_{||}}{}_{m_{||}}, e^{a_\perp}{}_{m_\perp}, e^{a_{||}}{}_{m_\perp}, e^{a_\perp}{}_{m_{||}}$ (see notations of Appendix B.2), indicating possible fibrations. Given the metric (2.6), saying that the warp factor only depends on transverse coordinates then amounts to $\partial_{a_{||}}A = 0$ (and $\partial_\alpha A = 0$). This condition is equivalent to $\partial_{m_{||}}A = 0$ if one considers in addition

$$e^{a_\perp}{}_{m_{||}} = 0 \Leftrightarrow e^{m_\perp}{}_{a_{||}} = 0 \ . \qquad (B.18)$$

This restriction on the vielbein may be viewed as a gauge choice, i.e. fixing by Lorentz transformation whether the vielbeins are upper or lower block diagonal. To derive (B.1) and (B.2), we used $\partial_{M_{||}}A = 0$. While this is equivalent to the flat condition in four dimensions, one would then require (B.18) to hold for the internal directions, in particular when computing $\mathcal{R}_{6||}$. Another way to see this requirement is when computing the internal parallel trace on flat indices, and comparing the results: for $\mathcal{R}_{MN=m_{||}n_{||}} = e^{A}{}_{M}e^{B}{}_{N}\mathcal{R}_{AB=a_{||}b_{||}}$ to hold, one also needs the condition (B.18). In any case, $\mathcal{R}_{6||}$ is computed in Appendix B.2 in flat indices, allowing to get the corresponding expression for $\mathcal{R}_{6||}|_{(\partial A=0)}$. We do obtain a matching of the results using (B.18).

B.2 Curvature terms

To compute the trace of the Einstein equation along internal parallel directions (see above (4.14)), we need the Ricci tensor in flat indices (definitions around (4.10)): for the Levi-Civita connection, the spin connection is related to $f^a{}_{bc}$, so that one has generically (see e.g. [61])

$$
\begin{aligned}
2\ \mathcal{R}_{cd} &= \partial_a f^a{}_{cd} + 2\eta^{ab}\partial_a f^g{}_{b(c}\eta_{d)g} - 2\partial_c f^b{}_{bd} \qquad (B.19)\\
&+ f^a{}_{ab}\left(f^b{}_{cd} + 2\eta^{bg}f^h{}_{g(c}\eta_{d)h}\right) - f^b{}_{ac}f^a{}_{bd} - \eta^{bg}\eta_{ah}f^h{}_{gc}f^a{}_{bd} + \frac{1}{2}\eta^{ah}\eta^{bj}\eta_{ci}\eta_{dg}f^i{}_{aj}f^g{}_{hb} \ .
\end{aligned}
$$

From now on, we denote respectively $_{A}, _{\alpha}, _{a}$ the flat ten-, four- and six-dimensional indices, and refer to the metric (2.6). Given the discussion below (4.10), we take $\partial_\alpha A = 0$, $\partial_{a_{||}}A = 0$. It implies

$$f^{a_{||}}{}_{BC} = \delta^b_B\delta^c_C f^{a_{||}}{}_{bc} \ , \ f^A{}_{Bc_{||}} = \delta^A_a\delta^b_B f^a{}_{bc_{||}} \ , \ f^A{}_{Bc_\perp} = \delta^A_a\delta^b_B f^a{}_{bc_\perp} + \delta^A_\alpha\delta^\beta_B\delta^\alpha_\beta e^{-A}\partial_{c_\perp}e^A \ . \qquad (B.20)$$

This allows to compute $\mathcal{R}_{6||} = \eta^{AB}\mathcal{R}_{AB=a_{||}b_{||}}$, giving

$$
\begin{aligned}
2\mathcal{R}_{6||} &= 2\eta^{cd}\partial_c f^{a_{||}}{}_{da_{||}} + 8\eta^{cd} f^{a_{||}}{}_{c_\perp a_{||}} e^{-A}\partial_{d_\perp} e^A - 2\eta^{ab}\partial_{a_{||}} f^c{}_{cb_{||}} + 2\eta^{cd} f^{a_{||}}{}_{ca_{||}} f^e{}_{ed} \\
&\quad - \eta^{ab} f^d{}_{ca_{||}} f^c{}_{db_{||}} - \eta^{ab}\eta^{dg}\eta_{ch} f^h{}_{ga_{||}} f^c{}_{db_{||}} + \frac{1}{2}\eta^{ch}\eta^{dj}\eta_{ab} f^{a_{||}}{}_{cj} f^{b_{||}}{}_{hd} \qquad \text{(B.21)} \\
&= 2\eta^{cd}\partial_{c_\perp} f^{a_{||}}{}_{d_\perp a_{||}} + 8\eta^{cd} f^{a_{||}}{}_{c_\perp a_{||}} e^{-A}\partial_{d_\perp} e^A - 2\eta^{ab}\partial_{a_{||}} f^c{}_{cb_{||}} + 2\eta^{cd} f^{a_{||}}{}_{ca_{||}} f^e{}_{ed} \\
&\quad + 2\mathcal{R}_{||} + 2\mathcal{R}_{||}^\perp + \frac{1}{2}\eta^{ch}\eta^{dj}\eta_{ab} f^{a_{||}}{}_{c_\perp j_\perp} f^{b_{||}}{}_{h_\perp d_\perp} \ , \qquad \text{(B.22)}
\end{aligned}
$$

$$
\begin{aligned}
\text{where } \ 2\mathcal{R}_{||} &= 2\eta^{cd}\partial_{c_{||}} f^{a_{||}}{}_{d_{||}a_{||}} - \eta^{ab} f^{d_{||}}{}_{c_{||}a_{||}} f^{c_{||}}{}_{d_{||}b_{||}} - \frac{1}{2}\eta^{ch}\eta^{dj}\eta_{ab} f^{a_{||}}{}_{c_{||}j_{||}} f^{b_{||}}{}_{h_{||}d_{||}} \ , \qquad \text{(B.23)} \\
2\mathcal{R}_{||}^\perp &= -\eta^{ab} f^{d_\perp}{}_{c_\perp a_{||}} f^{c_\perp}{}_{d_\perp b_{||}} - \eta^{ab}\eta^{dg}\eta_{ch} f^{h_\perp}{}_{g_\perp a_{||}} f^{c_\perp}{}_{d_\perp b_{||}} \qquad \text{(B.24)} \\
&\quad - 2\eta^{ab} f^{d_\perp}{}_{c_{||}a_{||}} f^{c_{||}}{}_{d_\perp b_{||}} - \eta^{ab}\eta^{dg}\eta_{ch} f^{h_\perp}{}_{g_{||}a_{||}} f^{c_\perp}{}_{d_{||}b_{||}} \ .
\end{aligned}
$$

We now extract the warp factor with $e^{a_{||}}{}_m = e^A \tilde{e}^{a_{||}}{}_m$, $e^{a_\perp}{}_m = e^{-A}\tilde{e}^{a_\perp}{}_m$. We first obtain

$$
\begin{aligned}
& f^{a_{||}}{}_{b_{||}c_{||}} = e^{-A}\tilde{f}^{a_{||}}{}_{b_{||}c_{||}} \ , \ f^{a_\perp}{}_{b_{||}c_{||}} = e^{-3A}\tilde{f}^{a_\perp}{}_{b_{||}c_{||}} \ , \ f^{a_\perp}{}_{b_\perp c_{||}} = e^{-A}\tilde{f}^{a_\perp}{}_{b_\perp c_{||}} \ , \qquad \text{(B.25)} \\
& f^{a_{||}}{}_{b_\perp c_{||}} = e^A \tilde{f}^{a_{||}}{}_{b_\perp c_{||}} - \delta^{a_{||}}_{c_{||}}\partial_{\tilde{b}_\perp} e^A \ , \ f^{a_\perp}{}_{b_\perp c_\perp} = e^A \tilde{f}^{a_\perp}{}_{b_\perp c_\perp} + 2e^{2A}\delta^{\tilde{a}_\perp}_{[\tilde{b}_\perp}\partial_{\tilde{c}_\perp]} e^{-A} \ .
\end{aligned}
$$

With in addition $f^{a_{||}}{}_{c_\perp j_\perp} = e^{3A}\tilde{f}^{a_{||}}{}_{c_\perp j_\perp}$, one shows that the last line in (B.22) does not produce any ∂A, so contributes to what we denote $\mathcal{R}_{6||}|_{(\partial A=0)}$. We turn to the other line. For a compact manifold (without boundary), one generically has $f^a{}_{ab} = 0$. Here the relevant manifold is the smeared one (see Footnote 1), meaning the correct condition is $\tilde{f}^a{}_{ab} = 0$. From the above, we deduce

$$
f^a{}_{ab_{||}} = 0 \ , \ f^a{}_{ab_\perp} = (2p-11)\partial_{\tilde{b}_\perp} e^A \ . \qquad \text{(B.26)}
$$

We then compute

$$
\begin{aligned}
& 2\eta^{cd}\partial_{c_\perp} f^{a_{||}}{}_{d_\perp a_{||}} + 8\eta^{cd} f^{a_{||}}{}_{c_\perp a_{||}} e^{-A}\partial_{d_\perp} e^A - 2\eta^{ab}\partial_{a_{||}} f^c{}_{cb_{||}} + 2\eta^{cd} f^{a_{||}}{}_{ca_{||}} f^e{}_{ed} \qquad \text{(B.27)} \\
&= 2\eta^{cd} e^{2A}\partial_{\tilde{c}_\perp}\tilde{f}^{a_{||}}{}_{d_\perp a_{||}} + e^{2A(p-2)}\eta^{cd}\partial_{\tilde{c}_\perp}\partial_{\tilde{d}_\perp} e^{-2A(p-3)} \\
&\quad + 2\eta^{cd} e^{2A(p-3)}\partial_{\tilde{c}_\perp} e^{-2A(p-3)}\left(-e^{2A}\tilde{f}^{a_{||}}{}_{d_\perp a_{||}} + (p-3)\partial_{\tilde{d}_\perp} e^{2A}\right) \ .
\end{aligned}
$$

Generically, $\nabla_a V_b = \partial_a V_b - \omega_a{}^c{}_b V_c$ and $\omega_{(a}{}^c{}_{b)} = \eta^{cd} f^e{}_{d(a}\eta_{b)e}$, so we deduce, with $\tilde{f}^a{}_{d_\perp a} = 0$,

$$
\begin{aligned}
& \tilde{\Delta}_\perp e^{-2A(p-3)} = \eta^{cd}\partial_{\tilde{c}_\perp}\partial_{\tilde{d}_\perp} e^{-2A(p-3)} + \eta^{cd}\tilde{f}^{a_{||}}{}_{d_\perp a_{||}}\partial_{\tilde{c}_\perp} e^{-2A(p-3)} \ , \qquad \text{(B.28)} \\
& 4(\nabla\partial\phi)_{6||} = 4\eta^{AB=a_{||}b_{||}}\nabla_A\partial_B\phi = 2\eta^{cd} e^{2\phi+2A}\tilde{f}^{a_{||}}{}_{d_\perp a_{||}}\partial_{\tilde{c}_\perp} e^{-2\phi} - (p-3)\eta^{cd} e^{2\phi}\partial_{\tilde{c}_\perp} e^{-2\phi}\partial_{\tilde{c}_\perp} e^{2A} \ .
\end{aligned}
$$

Point 6 regarding sources in Section 2 requires a compact transverse subspace without boundaries, implying $\tilde{f}^{a_\perp}{}_{d_\perp a_\perp} = -\tilde{f}^{a_{||}}{}_{d_\perp a_{||}} = 0$. Setting this to zero, we eventually obtain

$$
\begin{aligned}
& 2\mathcal{R}_{6||} = 2\mathcal{R}_{6||}|_{(\partial A=0)} + e^{2A(p-2)}\tilde{\Delta}_\perp e^{-2A(p-3)} + 2(p-3)\eta^{cd} e^{2A(p-3)}\partial_{\tilde{c}_\perp} e^{-2A(p-3)}\partial_{\tilde{d}_\perp} e^{2A} \qquad \text{(B.29)} \\
& \text{where } \ 2\mathcal{R}_{6||}|_{(\partial A=0)} = 2\mathcal{R}_{||} + 2\mathcal{R}_{||}^\perp + \frac{1}{2}\eta^{ch}\eta^{dj}\eta_{ab} f^{a_{||}}{}_{c_\perp j_\perp} f^{b_{||}}{}_{h_\perp d_\perp} \ , \qquad \text{(B.30)} \\
& 4(\nabla\partial\phi)_{6||} = -(p-3)\eta^{cd} e^{2\phi}\partial_{\tilde{c}_\perp} e^{-2\phi}\partial_{\tilde{d}_\perp} e^{2A} \ . \qquad \text{(B.31)}
\end{aligned}
$$

Using (B.18), namely $e^{m_\perp}{}_{a_{||}} = 0$, one has $\tilde{f}^{a_{||}}{}_{d_\perp a_{||}} = -\partial_{\tilde{d}_\perp} \ln\sqrt{|\tilde{g}_{6||}|}$, set to zero here. We then get the matching of (B.29) with (B.11) and of (B.31) with (B.10).

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