REFLECTED STABLE SUBORDINATORS FOR FRACTIONAL CAUCHY PROBLEMS

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In this talk we will show how to explicitly compute the transition densities of a spectrally negative stable process with index greater than one, reflected at its infimum. First we derive the forward equation using the theory of sun-dual semigroups. The resulting forward equation is a boundary value problem on the positive half-line that involves a negative Riemann-Liouville fractional derivative in space, and a fractional reflecting boundary condition at the origin. Then we apply numerical methods to explicitly compute the transition density of this space-inhomogeneous Markov process, for any starting point, to any desired degree of accuracy. Finally, we discuss an application to fractional Cauchy problems, which involve a positive Caputo fractional derivative in time.

Theorem 1. *Let Y_t be a stable Lévy process with no positive jumps, with characteristic function $\mathbb{E}[e^{ikY_t}] = e^{t(ik)^\alpha}$ and index $1 < \alpha < 2$. Then the probability density $p(x,t)$ of the reflected stable process $Z_t = Y_t - \inf\{Y_s : 0 \le s \le t\}$ started at the origin solves the fractional boundary value problem:*

$$\begin{aligned} \partial_t p(x,t) &= D^\alpha_{-x} p(x,t) \quad \text{for } t>0 \text{ and } x>0; \\ 0 &= D^{\alpha-1}_{-x} p(x,t) \quad \text{for } t>0 \text{ and } x=0. \end{aligned} \tag{1}$$

Here we use the negative Riemann-Liouville fractional derivative

$$D^\alpha_{-x} f(x) := \frac{(-1)^n}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_x^\infty f(y)(y-x)^{n-\alpha-1}\, dy$$

where $n-1 < \alpha < n$.

References

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