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REFLECTION AND TRANSMISSION OF FLUID TRANSIENTS AT AN ELBOW

by

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Summary.—A fundamental problem in the analysis of fluid-filled pipeline systems containing various types of joints and fittings (such as tees, elbows, U-joints and omega-bends) arises when the effects of such structural parts on the propagation of pressure transients through the system are considered. Often, all that is desired is a simple characterization of these "discontinuities" in terms of reflection and transmission pressure coefficients that can be incorporated into one-dimensional pressure transient algorithms or computer codes. In this context, the present paper considers the problem of characterizing elbow-like structural members (including elbows and U-joints).

The theoretical analysis is based on a Timoshenko-like treatment of a curved elastic tube element containing a compressible Newtonian fluid moving at low velocity. Four equations of motion (governing the fluid pressure and the axial force, transverse shear force and bending moment in the tube) and four associated constitutive equations are generated. For a given frequency ω, there are generally eight distinct wave numbers k and eight associated "modes" $\underline{u}_0$. Superposition of these modes and the matching of certain variables at interfaces allow the prediction of the strengths of waterhammer-like waves that are transmitted and reflected at a finite-radius elbow joining two straight sections of tube, when an input waterhammer-like wave is prescribed.

Theoretical results indicate that for high frequency waves, virtually all of the wave is transmitted through the elbow, regardless of elbow geometry. For low frequency waves, a partial negative pressure reflection occurs at the elbow and the strength of this reflection is of the order of 15% to 30% of that of the incoming wave; the amount of reflection depends strongly upon the ratio of the wall thickness to the mean tube radius, is somewhat dependent upon the total bend angle, but is relatively insensitive to the radius of curvature of the elbow.

Some simple experiments with two straight sections of water-filled aluminum electrical conduit welded to either end of a gentle 90°-elbow tend to confirm certain aspects of the theory. A short, impact-generated positive pressure pulse in one of the straight sections disperses somewhat at the elbow, producing a broad partial negative reflection; the rest is transmitted. By varying the static pressure head in the system, it has been ascertained that cavitation at the inlet side of the elbow can occur if the static head is not large enough to overcome the partial negative reflection.

1. Introduction

A consequence of the classical waterhammer theory for straight tubes[1] is that fluid transients propagate unattenuated with the Joukowski[2] wave speed

$$c_J = \left\{ \rho_f \left(\frac{1}{K} + 2\,\frac{a}{eE}\right)\right\}^{-1/2} , \tag{1}$$

where ρ_f and K are the mass density and bulk modulus of the fluid, and a, e and E are the inside radius, wall thickness and Young's modulus, respectively, of the tube. Many experiments, including those of Phillips and Walker,[3] have verified the applicability of the classical theory for non-cavitating, low-velocity flows in straight tubes. Discrepancies between the classical theory and experimental results do arise, however, when segments of curved tubing are inserted into piping networks.[3] An investigation of wave propagation in curved conduits was begun with the goal of predicting analytically the transmission and reflection characteristics of curved elements such as elbows and U-bends.

2. Analysis of waves in an unbounded conduit having initial curvature

The analysis begins with a consideration of the propagation of coupled, one-dimensional waves in a curved, fluid-filled conduit. For this purpose, a Timoshenko-like[4] theory[5] for the propagation of extensional-bending waves in a solid rod is modified to account for the presence of a compressible, non-cavitating fluid moving at a low velocity. While transverse momentum of entire tube elements, containing fluid, is accounted for, the radial momentum of the fluid and of the tube wall within the cross-section is neglected. It follows that the hoop stress σ_θ due to internal pressure p is given by

$$\sigma_\theta = p\,\frac{a}{e} . \tag{2}$$

There are four generalized constitutive equations governing the axial tube force N, the transverse shear force Q, the bending moment M, and the internal pressure p:

$$\frac{1}{EA_t} N - \frac{d}{R} - \frac{\partial u}{\partial s} - \frac{\nu a}{eE} p = 0; \qquad \frac{1}{\kappa^2 GA_t} Q - \frac{\partial d}{\partial s} + \phi + \frac{u}{R} = 0; \tag{3,4}$$

$$\frac{1}{EI_t} M - \frac{\partial \phi}{\partial s} = 0; \qquad \left(\frac{1}{K} + 2\,\frac{a}{e}\,\frac{1-\nu^2}{E}\right) \frac{\partial p}{\partial t} + 2\,\frac{R^2}{a^2}\left[1-(1-a^2/R^2)^{\frac{1}{2}}\right] \frac{\partial V}{\partial s} - 2\nu\,\frac{\partial \dot{u}}{\partial s} + (1-2\nu)\,\frac{\dot{d}}{R} = 0. \tag{5,6}$$

In these equations, A_t denotes the cross-sectional area of the tube wall, d denotes the transverse displacement of the tube centerline, R the radius of natural curvature of the tube centerline, u the axial displacement of the tube, E, G and ν the Young's modulus, shear modulus and Poisson's ratio, respectively, of the tube material, κ^2 the shear correction factor, ϕ the total rotation of a tube cross-section, I_t the moment of inertia of the tube cross-section, s the coordinate along the tube centerline, t the time, V the fluid centerline velocity; a superscript dot denotes differentiation with respect to time. There are also four equations of motion, which introduce additionally the mass densities ρ_f and ρ_t of the fluid and tube, respectively, and the cross-sectional area A_f of the fluid:

$$\frac{\partial N}{\partial s} + \frac{1}{R} Q - \rho_t A_t \frac{\partial \dot{u}}{\partial t} = 0; \tag{7}$$

$$\frac{\partial Q}{\partial s} - \frac{1}{R} N + \frac{A_f}{R} p - (\rho_f A_f + \rho_t A_t) \frac{\partial \dot{d}}{\partial t} = 0; \tag{8}$$

$$\frac{\partial M}{\partial s} + Q - \rho_t I_t \frac{\partial \dot{\phi}}{\partial t} = 0; \qquad \frac{\partial p}{\partial s} + \rho_f \frac{\partial V}{\partial t} = 0. \tag{9,10}$$

The constitutive equations (3)-(5) may be differentiated once with respect to time, rendering the eight governing equations (3)-(10) in first-order differential form,

$$\underset{\sim}{L}(\underline{u}) \equiv \underset{\sim}{A}^t \, \underline{u}_{,t} + \underset{\sim}{A}^s \, \underline{u}_{,s} + \underset{\sim}{B}\, \underline{u} = \underline{0}, \tag{11}$$

where $\underline{u}$ is the solution vector with components $(N, Q, M, p, \dot{u}, \dot{d}, \dot{\phi}, V)$ and $\underset{\sim}{A}^t$, $\underset{\sim}{A}^s$ and $\underset{\sim}{B}$ are 8×8 constant coefficient matrices.

Harmonic solutions to eqs. (11), in the form $\underline{u} = \underline{u}_0 e^{i(\omega t - ks)}$, where ω is the circular frequency and k is the wave number, may be assumed; it follows that $\underline{u}_0$ must satisfy

$$(i\omega \underset{\sim}{A}^t - ik \underset{\sim}{A}^s + \underset{\sim}{B}) \underline{u}_0 = \underline{0} \tag{12}$$

where

$$(i\omega \underset{\sim}{A}^t - ik \underset{\sim}{A}^s + \underset{\sim}{B}) = \tag{13}$$

$$\begin{bmatrix}
i\omega\frac{1}{EA_t} & 0 & 0 & -i\omega \frac{\nu a}{eE} & ik & -\frac{1}{R} & 0 & 0 \\
0 & i\omega\frac{1}{\kappa^2 GA_t} & 0 & 0 & \frac{1}{R} & ik & 1 & 0 \\
0 & 0 & i\omega\frac{1}{EI_t} & 0 & 0 & 0 & ik & 0 \\
0 & 0 & 0 & i\omega\left(\frac{1}{K} + 2\,\frac{a}{e}\,\frac{1-\nu^2}{E}\right) & ik\; 2\nu & \frac{1}{R}(1-2\nu) & 0 & -ik\; 2\frac{R^2}{a^2}[1-(1-a^2/R^2)^{\frac{1}{2}}] \\
-ik & \frac{1}{R} & 0 & 0 & -i\omega\, \rho_t A_t & 0 & 0 & 0 \\
-\frac{1}{R} & -ik & 0 & \frac{1}{R} A_f & 0 & -i\omega(\rho_f A_f + \rho_t A_t) & 0 & 0 \\
0 & 1 & -ik & 0 & 0 & 0 & -i\omega\, \rho_t A_t & 0 \\
0 & 0 & 0 & -ik & 0 & 0 & 0 & i\omega\, \rho_f
\end{bmatrix}$$

In order for a right null vector (eigenvector) $\underline{u}_0$ to exist, the determinant of $(i\omega \underset{\sim}{A}^t - ik\underset{\sim}{A}^s + \underset{\sim}{B})$ must vanish, i.e.

$$F(k) \equiv \det(i\omega \underset{\sim}{A}^{L} - ik \underset{\sim}{A}^{S} + \underset{\sim}{B}) = 0. \tag{14}$$

On physical grounds, it can be argued that, if k is a solution to $F(k) = 0$, then so is $-k$; and consequently $F(k)$, being a polynomial in k, must be of the form

$$F(k) = c_1 k^8 + c_2 k^6 + c_3 k^4 + c_4 k^2 + c_5 ,$$

where the c_i are functions of the frequency ω. Youngdahl[6] has evaluated the $c_i(\omega)$ by direct expansion of the determinant.[7] The general form of k is complex, i.e. $k = Re(k) + i\, Im(k)$ so that $\underline{u}$ is of the form $\underline{u} = \underline{u}_0\, e^{i[\omega t - Re(k)\, s]}\, e^{Im(k)\, s}$. There are eight, generally distinct, roots to eq. (14), and radiation conditions[7] at $s = \pm\infty$ may be invoked to select the appropriate four of these eight roots for problems involving semi-infinite conduits. Generally the roots satisfying the radiation conditions lie in the fourth quadrant of the k-plane, as depicted in the phase curves given schematically in Figs. 1 and 2 for straight and curved conduits, respectively.

In the case of the straight tube, the right null vectors $\underline{u}_0$ have distinct characteristics that depend upon the particular branch of the ω-k curves under consideration, and the branches in Fig. 1 have been named accordingly. The "fluid" wave will be of particular interest in this paper; it corresponds closely with the usual waterhammer wave described by elementary theory.[1] In a curved tube, all modes involve all the dependent variables, and it is difficult to give meaningful names to all the branches in ω-k space. One exception to this last statement, however, concerns the portions of the two branches that would correspond to the one "fluid" branch in the straight tube case; these portions have strong pressure components in their associated eigenvectors and are consequently labelled 'primary "fluid" wave segments' in Fig. 2.

Fig. 1. Phase space (ω-k curves) for harmonic disturbances in a straight fluid-filled conduit. Curves are not necessarily drawn to scale.

Fig. 2. Phase space (ω-k curves) for harmonic disturbances in a curved fluid-filled conduit. Curves are not necessarily drawn to scale.

The cutoff frequencies ω_c (in Fig. 1) and ω_{c1}, ω_{c2} (in Fig. 2) can be calculated by setting k = 0 in eq. (14). The result for a straight tube is that there is one cutoff value,

$$\omega_c = \sqrt{\frac{\kappa^2 G A_t}{\rho_t I_t}} , \tag{15}$$

above which there are four real roots and below which there are three real roots and one imaginary root associated with bending and shear only. The result for a curved conduit is there are two cutoff frequencies, given by

$$\omega_{c1} = \sqrt{\frac{EA_t + (1-2\nu)\dfrac{A_f - \nu \frac{a}{e} A_t}{\frac{1}{K} + 2\frac{a}{e}\frac{1-\nu^2}{E}}}{R^2 (\rho_t A_t + \rho_f A_f)}} \quad \text{and} \quad \omega_{c2} = \sqrt{\left(\frac{1}{R^2}\frac{I_t}{A_t} + 1\right)\frac{\kappa^2 G A_t}{\rho_t I_t}} . \tag{16a,b}$$

In addition, in a curved conduit, there are two other saddle-point frequencies ω' and ω'' where the wave numbers change from complex to real, or from complex to imaginary, respectively. Expressions for these frequencies have not been found.

3. Analysis of waves propagating through an elbow of arbitrary bend angle θ

In this section, the problem of wave propagation through an elbow joined with continuous tangent to two straight tubes on either side is studied. It is assumed that the incoming wave is a "fluid" wave possessing a strong pressure component and no bending or shear. The elbow is assumed to have the same cross-sectional dimensions and material properties that the straight tubes have.

Let $\underline{u}_1$ be the solution for all (eight) waves in the first straight tube; let $\underline{u}_2$ be the solution for all (eight) waves in the curved tube; and let $\underline{u}_3$ denote the solution for the four waves, satisfying the radiation conditions at infinity, in the second straight tube. Then the assumed continuity conditions require that

$$\underline{u}_1 = \underline{u}_2 \text{ at } s = 0 \quad \text{and} \quad \underline{u}_2 = \underline{u}_3 \text{ at } s = R\theta \tag{17a,b}$$

for all t, where θ is the total bend angle. Details can be found in Reference 7. Of all the dependent variables that could be studied, only two will be discussed in this paper, namely, the strength of the pressure term in the "fluid" wave reflected in the first straight tube, and that of the pressure term in the "fluid" wave transmitted into the second straight tube.

4. Numerical results for elbows

Computer results for transmission and reflection coefficients have been generated for comparison with experiments in progress. In the experiments, water-filled aluminum electrical conduit of 3/4-inch trade dimension is employed, and the following parameters are used: a = 10.6 mm, A_t = 190 mm², e = 2.54 mm, ν = 0.33, E = 72.4 GPa, K = 2.09 GPa, $\rho_t = 2.69\times10^3$ kg/m³, $\rho_f = 1.00\times10^3$ kg/m³, $\kappa^2 = \pi^2/12$. Calculations for various bend angles, including 90°, have been made for two bend radii, namely R = 120 mm (R/a = 11.3) and

Fig. 3. Transmission and reflection coefficients for a 90°-elbow having the parameters given in the text, as a function of the frequency ω. Results are for a gentle elbow (R = 120 mm).

Fig. 4. Transmission and reflection coefficients for a 90°-elbow having the parameters given

Fig. 5. Transmission coefficient for a gentle elbow (R = 120 mm) as a function of the total bend angle θ, for the parameters given in the text.

Fig. 6. Transmission coefficient for a sharp elbow (R = 12 mm) as a function of the total bend angle θ, for the parameters given in the text.

R = 12.0 mm (R/a = 1.13). The results for a 90°-elbow are shown in Figs. 3 and 4 for the gentle and sharp bends, respectively. It will be seen that, for low frequencies, the strength of the transmitted wave is virtually independent of the frequency and is in phase with the incoming wave; its magnitude is about 85% of that of the input wave. In the same frequency range, the reflected wave is also approximately constant, but is 180° out-of-phase with the input wave, meaning that it is negative if the input wave is regarded as positive; its magnitude is about 15% of that of the input wave. For higher frequencies, the results indicate that more of the pulse should propagate through the elbow. It is remarkable that the transmission and reflection coefficients do not appear to be strongly influenced by the magnitude of the radius of curvature of the elbow.

The effect of altering the elbow angle θ has been investigated and the results are shown in Figs. 5 and 6 for R/a = 11.3 and 1.13, respectively. For a gentle elbow (Fig. 5), it will be seen that, for low frequencies, the transmission coefficient drops quickly with θ for small values of θ, but generally levels out for values of θ exceeding 90°. For higher frequencies, virtually all the wave is transmitted regardless of the value of θ. The trends for a sharp elbow (Fig. 6) are similar, except that at higher frequencies not all the wave is transmitted. The results in Figs. 5 and 6 suggest that the transmission and reflection characteristics for a U-bend are about the same as those for a 90°-elbow.

It has been shown theoretically that when a waterhammer-type wave or pulse enters a typical elbow joining two straight conduits, a non-dispersed pulse having an amplitude about 85% of that of the incoming pulse should be detected in the downstream tube, as long as the dominant frequencies associated with the pulse are sufficiently low. A negative reflected pulse having a strength of approximately 15% of the incoming pulse is also generated at the elbow. Generally speaking, the transmission coefficient increases with increasing frequency and with decreasing bend angle. On the basis of other results (not shown), it can be stated that transmission coefficients decrease significantly with decreasing wall-thickness-to-tube-radius ratio.

Although the model will not be applicable if cavitation occurs, the model can still be used to predict the initiation of cavitation. Consider, for example, the experimental arrangement described in previous work,[3] where a short pressure pulse is propagated toward an elbow. According to the theory presented here, if the static pressure head in the fluid does not equal or exceed the magnitude of the negative reflected pulse, then cavitation can occur. Preliminary experimental results tend to verify these findings. In particular, negative, long-wavelength pressure reflections have been observed at the inlet to a gentle 90°-elbow, but only with a sufficiently high static pressure head present.

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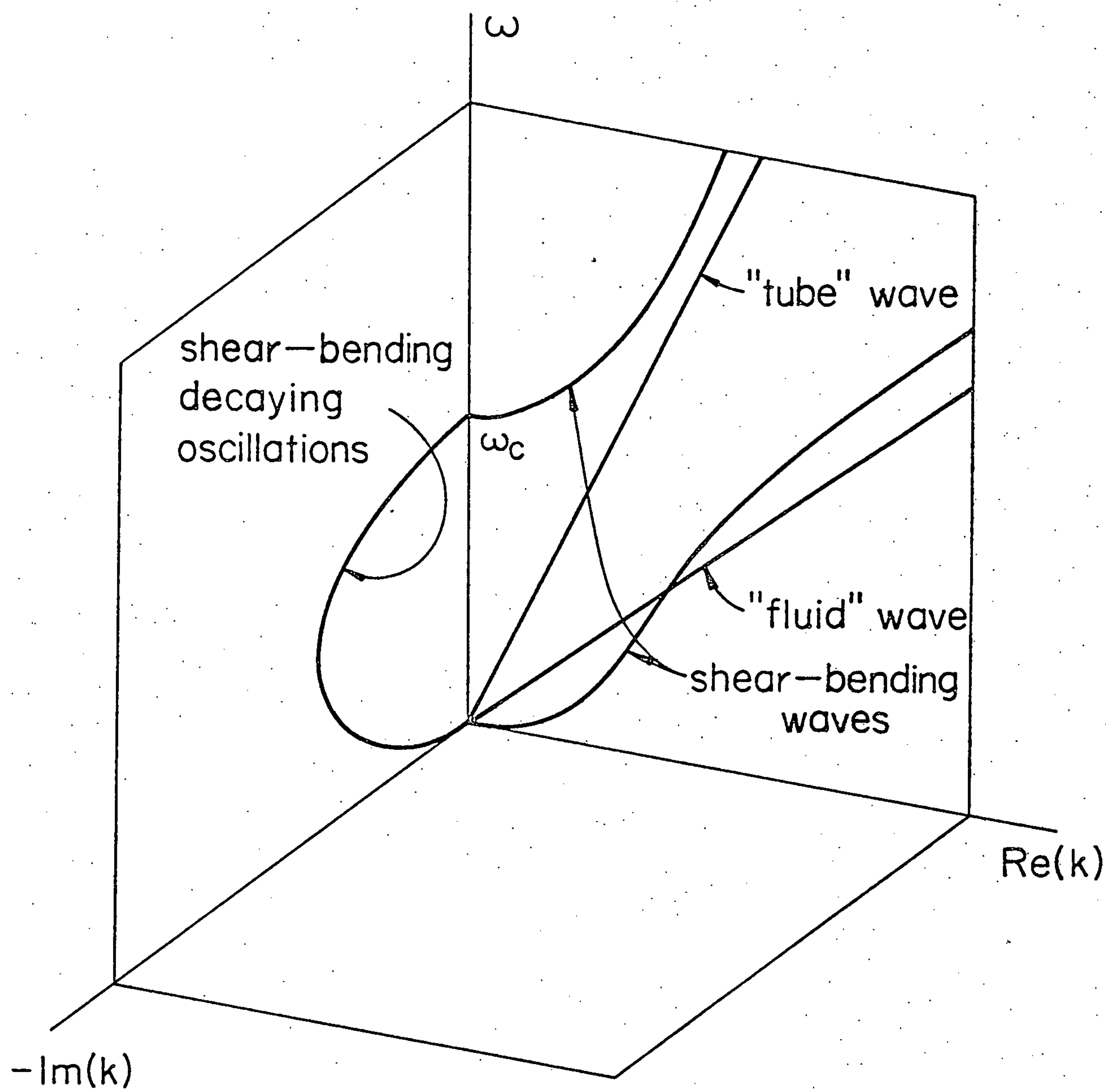
ω
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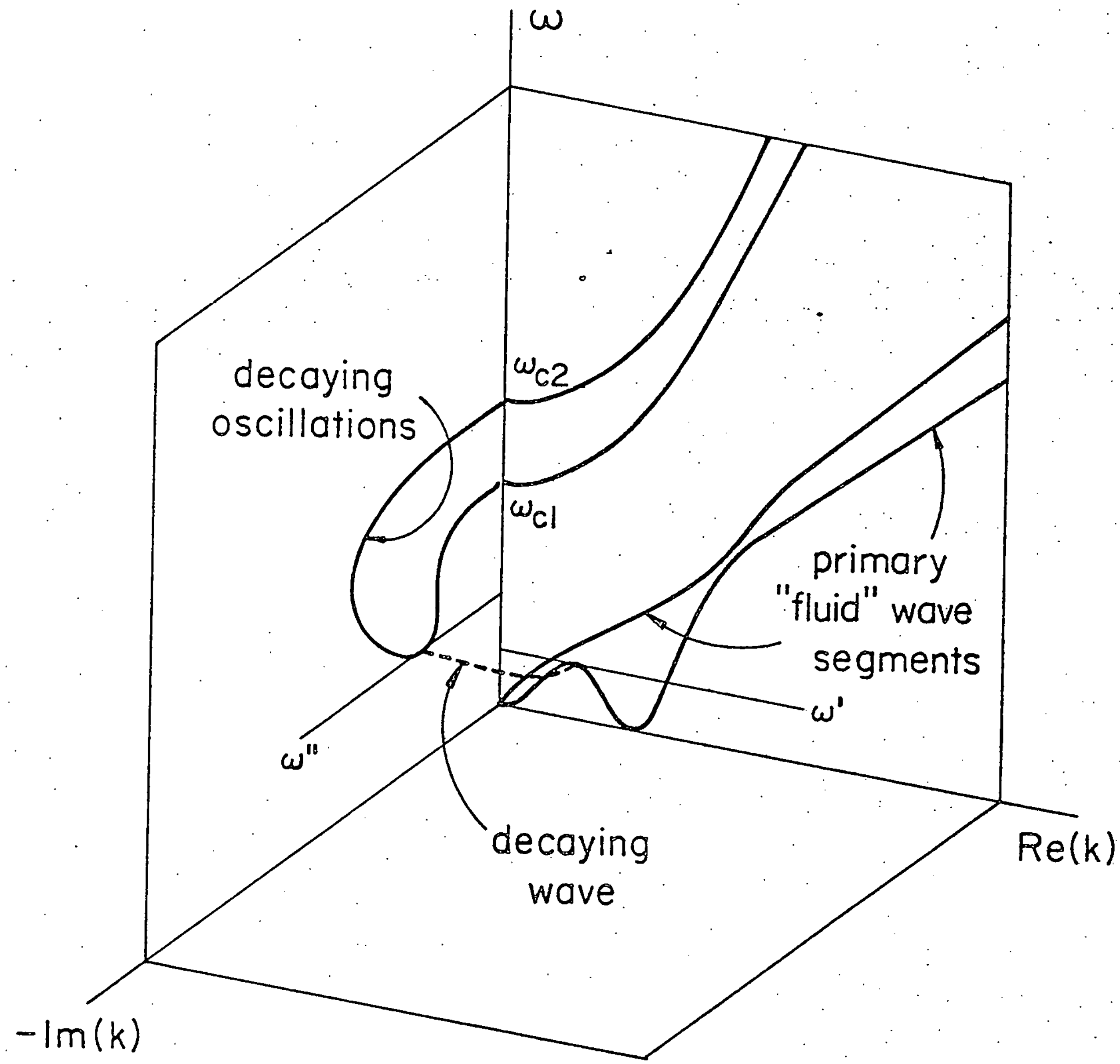
shear—bending
decaying
oscillations
ωc
"fluid" wave
shear—bending
waves
Re(k)
-Im(k)


ω
ω_{c2}
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oscillations
ω_{c1}
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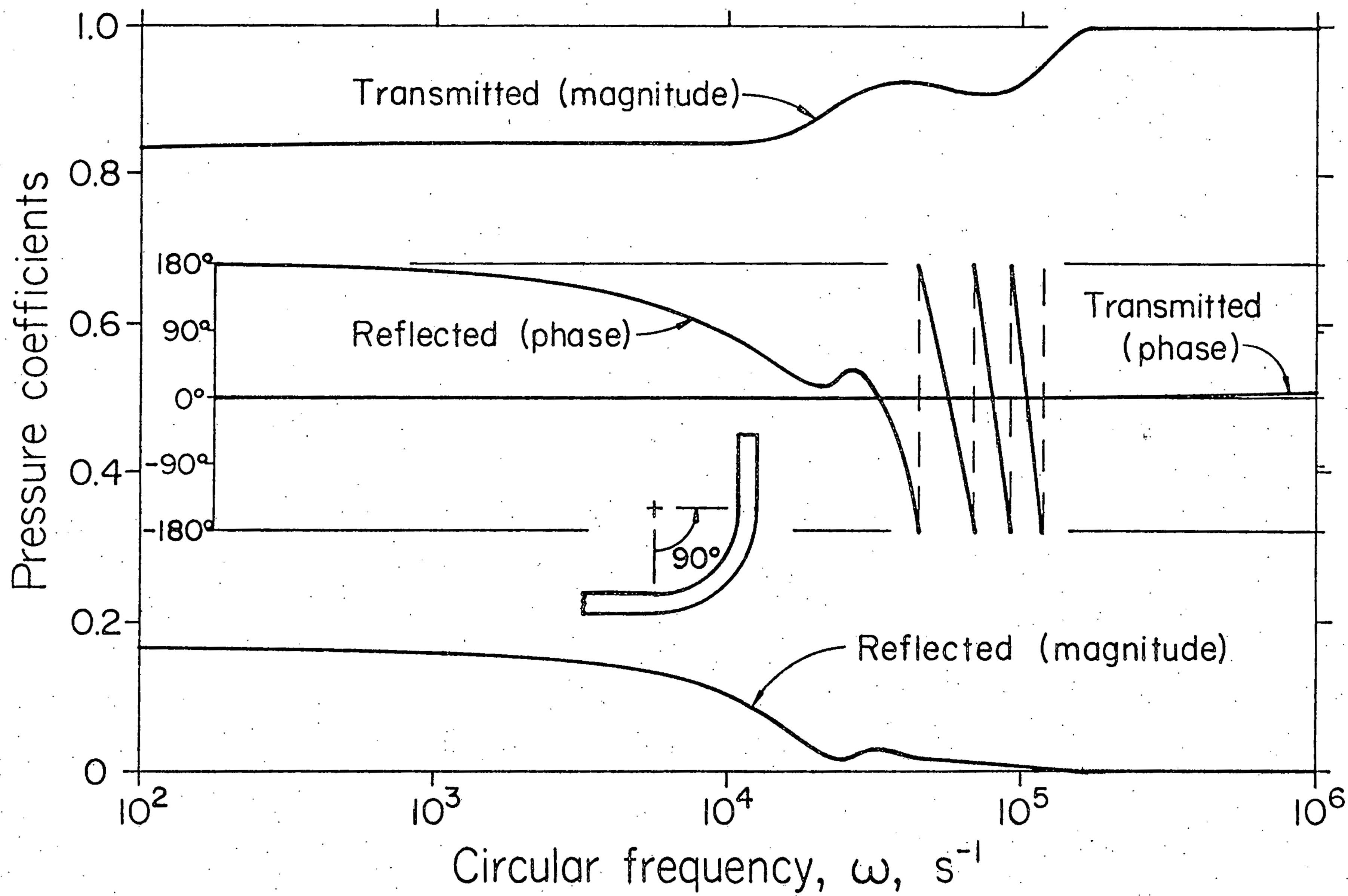
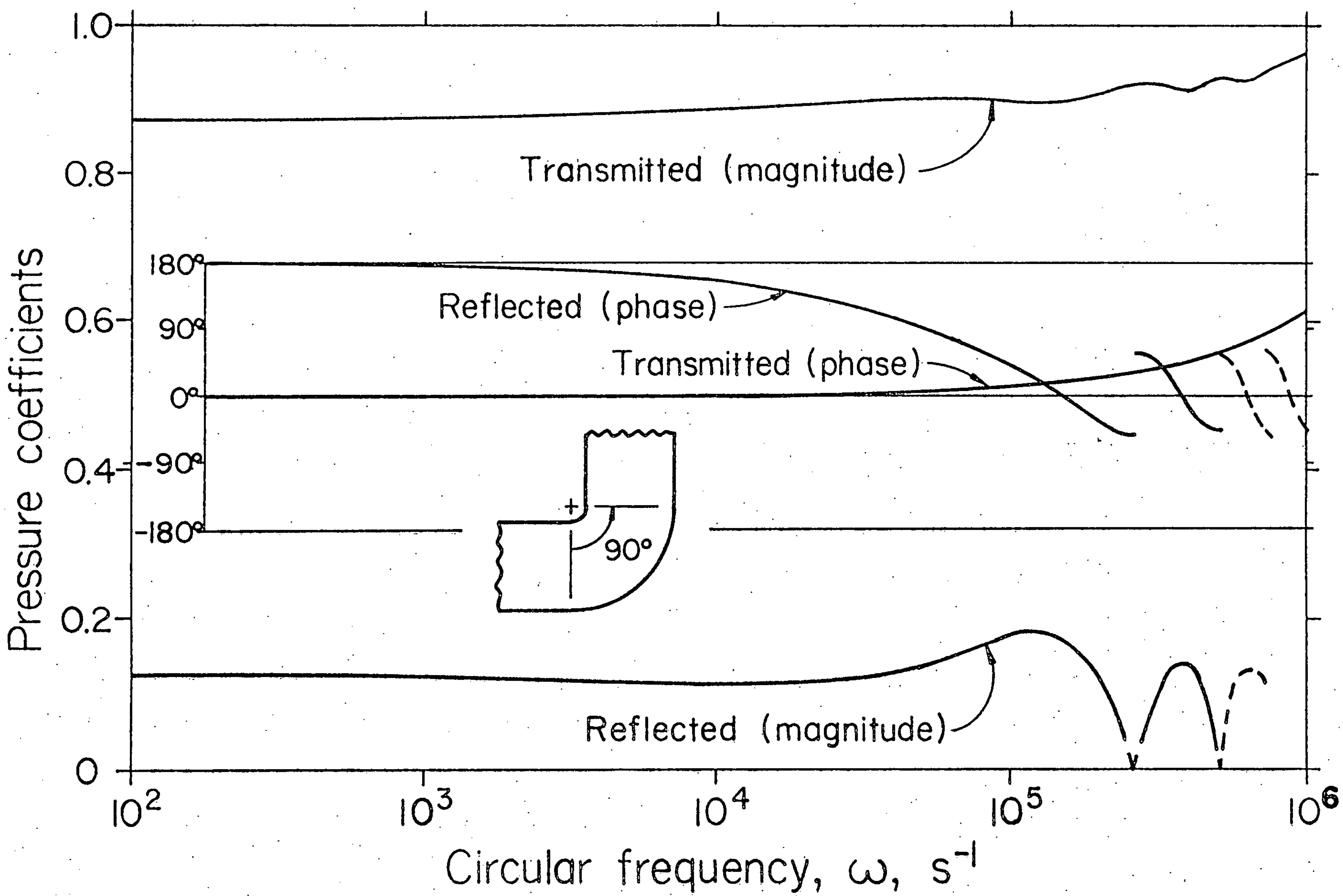
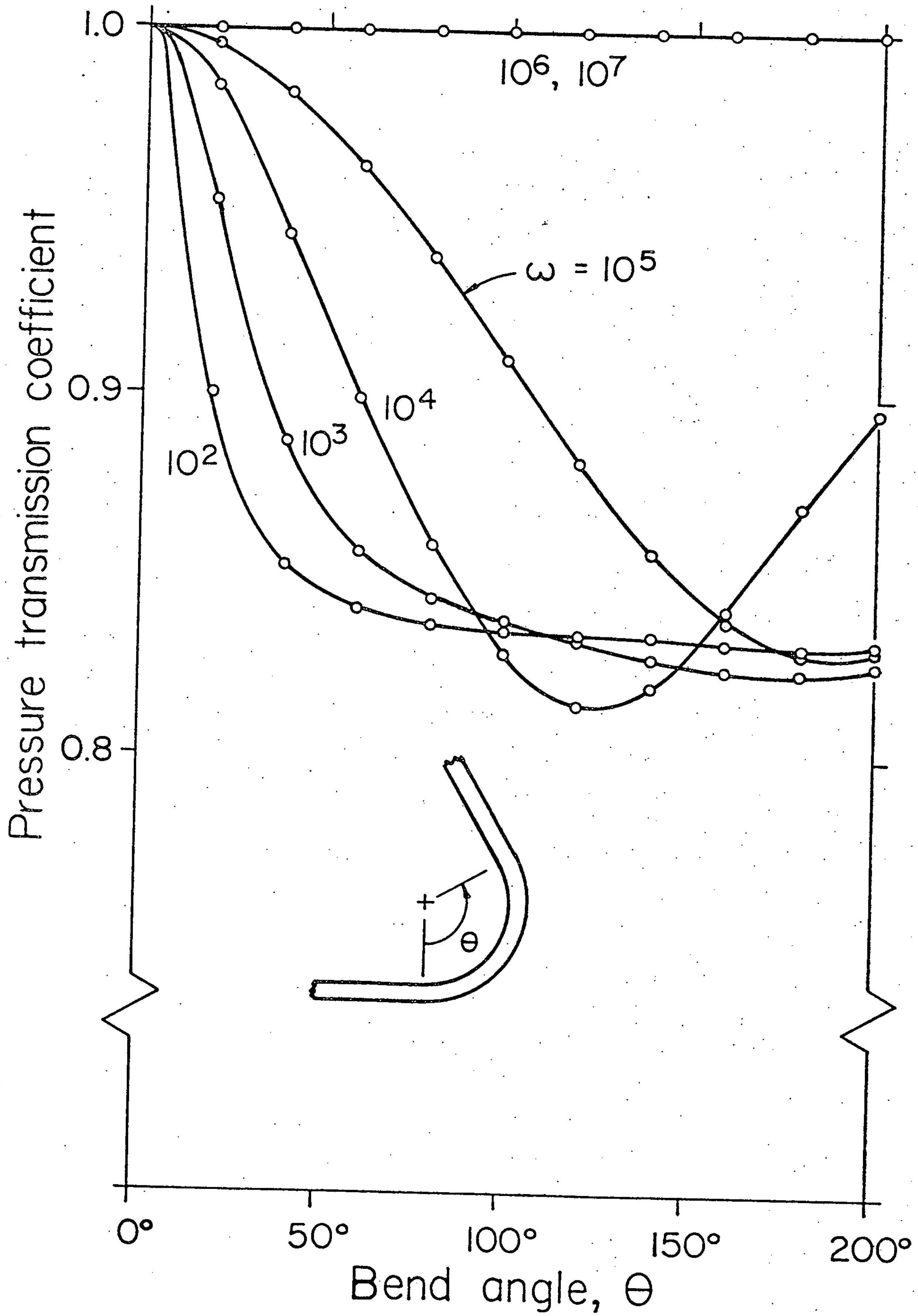
segments
ω'
ω"
decaying
wave
Re(k)
-Im(k)



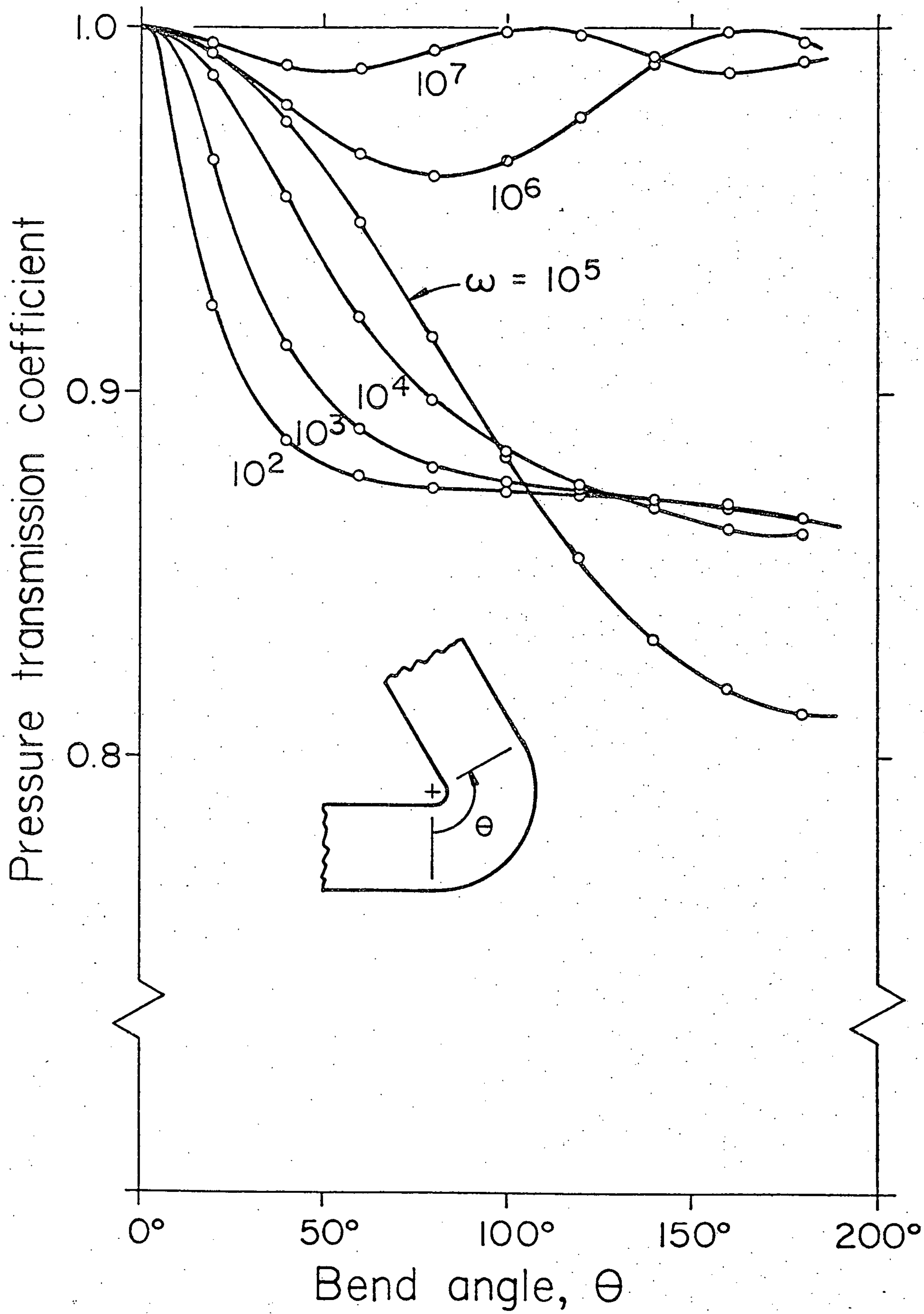
1.0
0.8
0.6
0.4
0.2
0
Pressure coefficients
Transmitted (magnitude)
180°
90°
0°
-90°
-180°
Reflected (phase)
Transmitted
(phase)
90°
Reflected (magnitude)
10^2
10^3
10^4
10^5
10^6
Circular frequency, ω, s^{-1}



1.0
0.8
0.6
0.4
0.2
0
Pressure coefficients
Transmitted (magnitude)
180°
90°
0°
-90°
-180°
Reflected (phase)
Transmitted (phase)
+
90°
Reflected (magnitude)
10^2
10^3
10^4
10^5
10^6
Circular frequency, ω, s^{-1}



1.0
0.9
0.8
Pressure transmission coefficient
10^6, 10^7
$\omega = 10^5$
10^4
10^3
10^2
θ
0°
50°
100°
150°
200°
Bend angle, θ



Pressure transmission coefficient
1.0
0.9
0.8
10^7
10^6
$\omega = 10^5$
10^4
10^3
10^2
θ
0°
50°
100°
150°
200°
Bend angle, θ