Reflection from Layered Surfaces due to Subsurface Scattering

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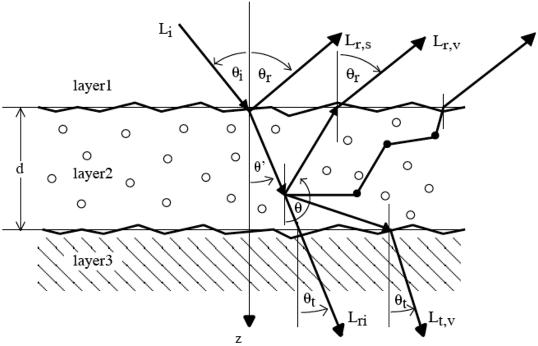
SIGGRAPH 1993

Outlines

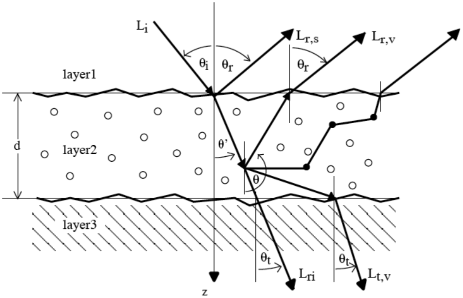
Reflected and Transmitted Radiances



$$L_r(\theta_r, \phi_r) = L_{r,s}(\theta_r, \phi_r) + L_{r,v}(\theta_r, \phi_r) \quad (1)$$

$$L_t(\theta_t, \phi_t) = L_{ri}(\theta_t, \phi_t) + L_{t,v}(\theta_t, \phi_t) \quad (2)$$

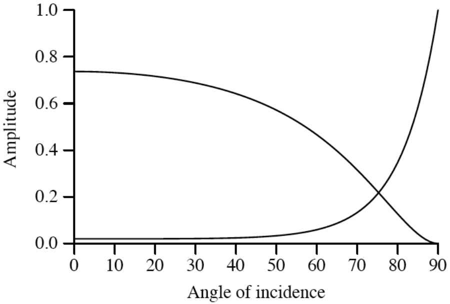
BRDF and BTDF



$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) \equiv \frac{L_r(\theta_r, \phi_r)}{L_i(\theta_i, \phi_i) \cos\theta_i dw_i} \quad (BRDF) \tag{3}$$

$$f_t(\theta_i, \phi_i; \theta_t, \phi_t) \equiv \frac{L_t(\theta_t, \phi_t)}{L_i(\theta_i, \phi_i) \cos\theta_i dw_i} \quad (BTDF) \tag{4}$$

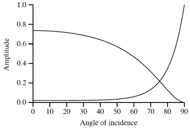
Fresnel transmission and reflection



- For planar surface

$$
\begin{aligned}
L_r(\theta_r, \phi_r) &= R^{12}(n_i, n_t; \theta_i, \phi_i \rightarrow \theta_r, \phi_r) L_i(\theta_i, \phi_i) \quad (5) \\
L_t(\theta_t, \phi_t) &= T^{12}(n_i, n_t; \theta_i, \phi_i \rightarrow \theta_t, \phi_t) L_i(\theta_i, \phi_i) \quad (6)
\end{aligned}
$$

Fresnel transmission and reflection



- For planar surface

$$L_r(\theta_r,\phi_r) = R^{12}(n_i,n_t;\theta_i,\phi_i \to \theta_r,\phi_r)L_i(\theta_i,\phi_i) \quad (5)$$

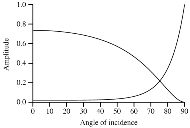
$$L_t(\theta_t,\phi_t) = T^{12}(n_i,n_t;\theta_i,\phi_i \to \theta_t,\phi_t)L_i(\theta_i,\phi_i) \quad (6)$$

where

$$R^{12}(n_i,n_t;\theta_i,\phi_i \to \theta_r,\phi_r) = R(n_i,n_t,\cos\theta_i,\cos\theta_t)$$

$$T^{12}(n_i,n_t;\theta_i,\phi_i \to \theta_t,\phi_t) = \frac{n_t^2}{n_i^2}T = \frac{n_t^2}{n_i^2}(1-R)$$

Fresnel transmission and reflection



- For planar surface

$$L_r(\theta_r,\phi_r) = R^{12}(n_i,n_t;\theta_i,\phi_i \to \theta_r,\phi_r)L_i(\theta_i,\phi_i) \quad (5)$$
$$L_t(\theta_t,\phi_t) = T^{12}(n_i,n_t;\theta_i,\phi_i \to \theta_t,\phi_t)L_i(\theta_i,\phi_i) \quad (6)$$

- In our model of reflection:

$$f_r = Rf_{r,s} + Tf_{r,v} = Rf_{r,s} + (1-R)f_{r,v} \quad (7)$$

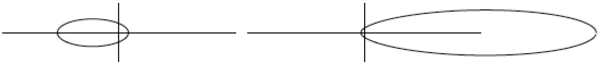
Description of Materials

- Index of Refraction
- Absorption and scattering cross section

$$\sigma_t = \sigma_a + \sigma_s$$

- Scattering phase function Henyey-Greenstein

$$p_{HG}(\cos j) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g \cos j)^{3/2}}$$



Light Transport Equations

- Transport theory models the distribution of light in a volume by

$$\frac{\partial L(\vec{x},\theta,\phi)}{\partial s} = -\sigma_t L(\vec{x},\theta,\phi) + \sigma_s \int p(\vec{x};\theta,\phi;\theta',\phi') L(\vec{x},\theta',\phi') d\theta' d\phi' \quad (8)$$

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$$\cos\theta \frac{\partial L(\theta,\phi)}{\partial z} = -\sigma_t L(\theta,\phi) + \sigma_s \int p(\theta,\phi;\theta',\phi')L(\theta',\phi')d\theta' d\phi' \tag{9}$$

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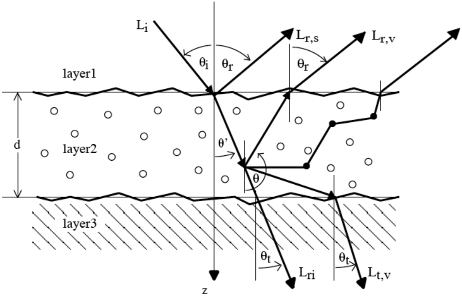
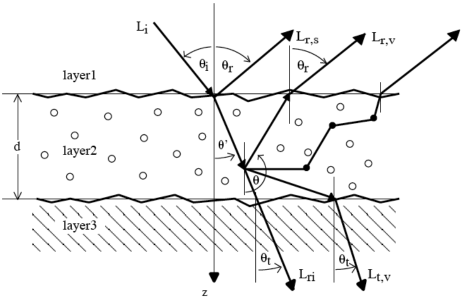
$$L(z;\theta,\phi) = \int_0^z e^{-\int_0^{z'} \sigma_t \frac{dz''}{\cos\theta}} \int \sigma_s(z')p(z';\theta,\phi;\theta',\phi')L(z';\theta';\phi')dw' \frac{dz'}{\cos\theta} \tag{10}$$

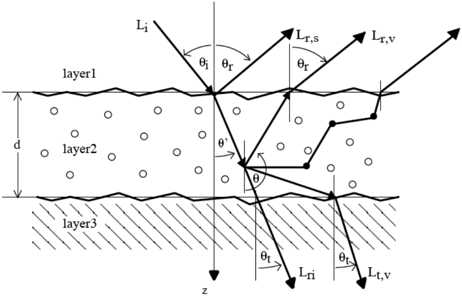


$$L(\theta, \phi) \quad = \quad L_{+}(\theta, \phi) + L_{-}(\pi - \theta, \phi) \tag{11}$$



$$L(\theta,\phi) = L_+(\theta,\phi) + L_-(\pi-\theta,\phi) \tag{11}$$

$$L_+(z=0;\theta',\phi') = \int f_{t,s}(\theta,\phi;\theta',\phi')L_i(\theta,\phi)dw_i \tag{12}$$



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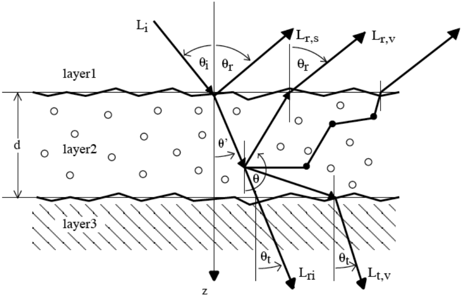
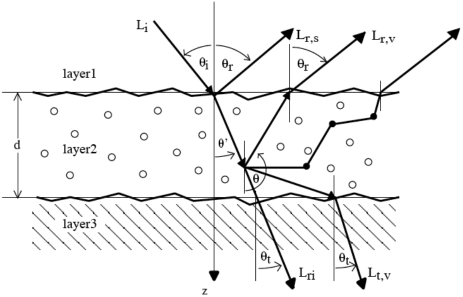
$$= T^{12}(n_i,n_t;\theta_i,\phi_i \rightarrow \theta',\phi')L_i(\theta_i,\phi_i) \quad (13)$$

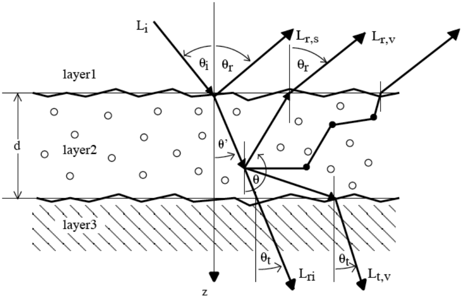


$$L_{r,v}(\theta_r, \phi_r) = \int f_{t,s}(\theta, \phi; \theta_r, \phi_r) L_-(z = 0; \theta, \phi) dw \qquad (14)$$



$$L_{r,v}(\theta_r,\phi_r) = \int f_{t,s}(\theta,\phi;\theta_r,\phi_r)L_-(z=0;\theta,\phi)dw \quad (14)$$

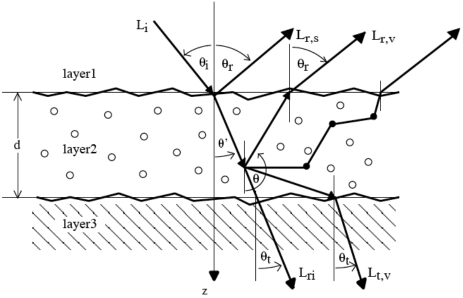
$$= T^{21}(n_2,n_1;\theta,\phi \rightarrow \theta_r,\phi_r)L_-(\theta,\phi) \quad (15)$$



$$L_{r,v}(\theta_r, \phi_r) = \int f_{t,s}(\theta, \phi; \theta_r, \phi_r) L_-(z = 0; \theta, \phi) dw \quad (14)$$

$$= T^{21}(n_2, n_1; \theta, \phi \rightarrow \theta_r, \phi_r) L_-(\theta, \phi) \quad (15)$$

$$L_{t,v}(\theta_t, \phi_t) = \int f_{t,s}(\theta, \phi; \theta_t, \phi_t) L_+(z = d; \theta, \phi) dw \quad (16)$$



$$L_{r,v}(\theta_r,\phi_r) = \int f_{t,s}(\theta,\phi;\theta_r,\phi_r)L_-(z=0;\theta,\phi)dw \quad (14)$$

$$= T^{21}(n_2,n_1;\theta,\phi \rightarrow \theta_r,\phi_r)L_-(\theta,\phi) \quad (15)$$

$$L_{t,v}(\theta_t,\phi_t) = \int f_{t,s}(\theta,\phi;\theta_t,\phi_t)L_+(z=d;\theta,\phi)dw \quad (16)$$

$$= T^{23}(n_2,n_3;\theta,\phi \rightarrow \theta_t,\phi_t)L_+(z=d;\theta,\phi)$$

Solving the Intergral Equation

$$L = \sum_{i=0}^{\infty} L^{(i)}$$

$$L^{(i+1)}(z;\theta,\phi) = \int_0^z e^{-\int_0^{z'} \sigma_t \frac{dz''}{\cos\theta}} \int \sigma_s(z')p(z';\theta,\phi;\theta',\phi')L^{(i)}(z';\theta';\phi')dw' \frac{dz'}{\cos\theta} \tag{17}$$

First-Order Approximation

$$L_+^{(0)} = L_+(z=0)e^{-\tau/\cos\theta} \tag{18}$$

where

$$\tau(z) = \int_0^z \sigma_t dz \tag{19}$$

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$$\begin{aligned} L_{t,v}^{(0)}(\theta_t, \phi_t) &= T^{23}(n_2, n_3; \theta, \phi \rightarrow \theta_t, \phi_t) L_+^{(0)}(\theta, \phi) \\ &= T^{12} T^{23} e^{-\tau_d} L_i(\theta_i, \phi_i) \end{aligned} \quad (20)$$

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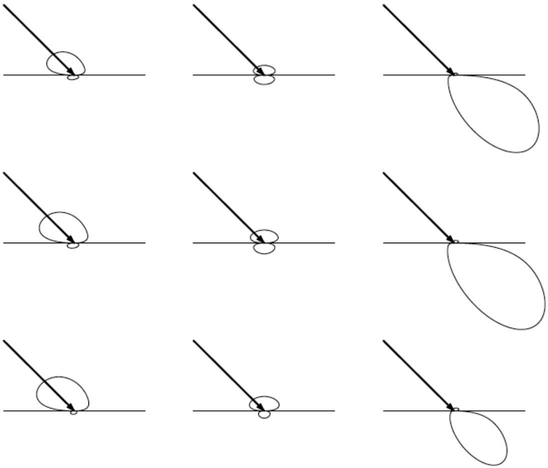
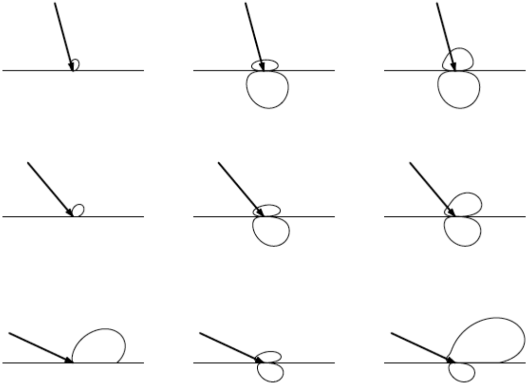
$$\begin{aligned} L_{t,v}^{(0)}(\theta_t,\phi_t) &= T^{23}(n_2,n_3;\theta,\phi \to \theta_t,\phi_t)L_+^{(0)}(\theta,\phi) \\ &= T^{12}T^{23}e^{-\tau_d}L_i(\theta_i,\phi_i) \end{aligned} \quad (20)$$

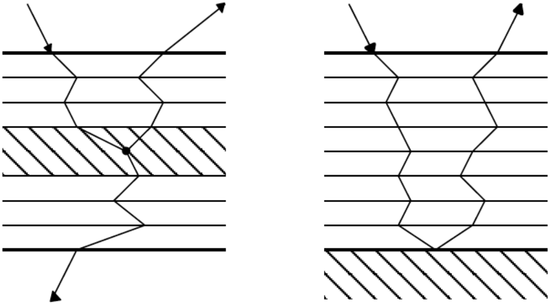
$$\begin{aligned} L_{r,v}^{(1)}(\theta_r,\phi_r) &= WT^{12}T^{21}p(\phi-\theta_r,\phi_r;\theta_i,\phi_i)\frac{\cos\theta_i}{\cos\theta_i+\cos\theta_r} \\ &\quad (1-e^{-\tau_d(1/\cos\theta_i+1/\cos\theta_r)})L_i(\theta_i,\phi_i) \end{aligned} \quad (21)$$



The reflection steadily increases as the layer becomes thicker.



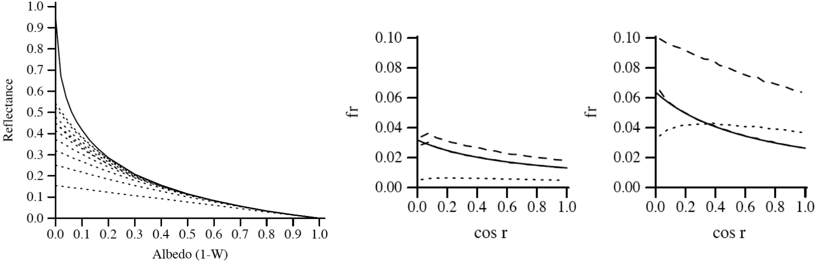
The distributions vary as a function of reflection direction.
Lambert's Law predicts a constant reflectance in all directions.



Multiple Scattering

An Monte Carlo Algorithm:

- Initialize:
- Events:
  - Step:
  - Scatter:
- Score:



$$L_{r,v}(\theta_r, \phi_r) = L^{(1)}(\theta_r, \phi_r) + L^m \quad (22)$$