

Reflection of quasi-P and quasi-SV waves at the free and rigid boundaries of a fibre-reinforced medium

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Abstract. The propagation of plane waves in fibre-reinforced media is discussed. The expressions of phase velocities of quasi-P (qP) and quasi-SV (qSV) waves propagating in plane symmetry are obtained in terms of propagation vectors. We have established a relation from which the displacement vector can be obtained in terms of the propagation vector. Expressions for the reflection coefficients of qP and qSV waves are obtained. Numerical results of reflection coefficients are obtained and presented graphically. The partition of energy between qP and qSV waves reflected on free and rigid boundaries due to incident qP and qSV waves are also obtained and presented graphically.

Keywords. Reflection of waves; quasi-P waves; quasi-SV waves; quasi-SH waves; fibre-reinforced media; reflection coefficients

1. Introduction

Fibre-reinforced composite materials have become very attractive in many engineering applications recently due to their superiority over other structural materials in applications requiring high strength and stiffness in lightweight components. Consequently the characterisation of their mechanical behaviour is of particular importance for structural design using these materials.

Effects of earthquakes on artificial structures are of prime importance to engineers and architects. During an earthquake and similar disturbances a structure is excited into a more or less violent vibration, with resulting oscillatory stresses, which depend both upon the ground vibration and physical properties of the structure (Richter 1958). Most concrete construction includes steel reinforcing, at least nominally. Thus wave propagation in a reinforced medium plays a very important role in civil engineering and geophysics.

The propagation of body waves in anisotropic media is fundamentally different from their propagation in isotropic media, although the differences may be comparatively subtle and difficult to observe (Crampin 1975). In general, for any type of anisotropy, there are always three types of body waves propagating with three different velocities. Choosing the three components of displacement adequately, they are called quasi-P(qP), quasi-SV(qSV) and quasi-SH

(qSH) waves. The velocities of these three waves change according to the type of symmetry present in the medium. Owing to these properties, anisotropy is detected by observations of change in P-wave velocity along two perpendicular directions and by observations of S-wave splitting. For both these effects it is not necessary that the whole medium be anisotropic; only some part of it need be so (Udias 1999). Generally, particle motion is neither purely longitudinal nor purely transverse. For this reason, the three types of body waves in an anisotropic medium are referred as qP, qSV and qSH instead of P, SV and SH.

The problem of reflection and refraction of elastic have been discussed by several authors. Without going into the details of the research work in this field, we mention the papers by Knott (1899), Gutenberg (1944), Thapliyal (1974), Keith & Crampin (1977), Dey & Addy (1979), Tolstoy (1982), Norris (1983), Pal & Chattopadhyay (1984), Achenbach (1976), Henneke (1972), Chattopadhyay *et al* (1995) and Singh *et al* (2002), as giving a broad picture of the work done so far.

Crampin & Taylor (1971) studied surface wave propagation in examples of unlayered and multi-layered anisotropic media which is examined numerically with a program using an extension of the Thompson-Haskell matrix formulation. They studied some examples of surface wave propagation in anisotropic media to interpret a possible geophysical structure. Crampin (1975) showed that the surface waves have distinct particle motion when propagating in a structure having a layer of anisotropic material with certain symmetry relations. Chattopadhyay & Saha (1996) have studied the problem of reflection of qSV-wave at free and rigid boundary in a medium of monoclinic type.

The above mentioned authors have not studied the reflection behaviour at a free and rigid boundaries of a fibre-reinforced medium. The reflection of qP and qSV waves in a fibre-reinforced medium is discussed. In this paper we have computed the reflection coefficients of qP and qSV waves at the free and rigid boundary of a fibre-reinforced medium. It is well known that in an anisotropic medium the direction of particle motion is neither perpendicular nor parallel to the direction of propagation. Considering this fact, a relation has been established to calculate the displacement vector in terms of propagation vector. The expressions for phase velocity of qP and qSV waves are obtained in terms of the propagation vector. The partition of energy between qP and qSV waves reflected for qP and qSV waves incident on a free and rigid boundaries have been derived and presented graphically.

2. Formulation of the problem

The constitutive equations for fibre-reinforced linearly elastic medium whose preferred direction is that of \mathbf{a} are (Spencer 1972)

$$\begin{aligned} \tau_{ij} = & \lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha (a_k a_m e_{km} \delta_{ij} + e_{kk} a_i a_j) \\ & + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta (a_k a_m e_{km} a_i a_j), \end{aligned}$$

where τ_{ij} are components of stress, e_{ij} are components of infinitesimal strain, a_j are components of \mathbf{a} , all referred to cartesian coordinates. The vector \mathbf{a} may be a function of position. The coefficients λ , μ_L , μ_T , α and β are elastic constants with the dimension of stress.

If \mathbf{a} is so chosen that its components are (1,0,0). The stress components (1) become

$$\begin{aligned} \tau_{11} = & (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)e_{11} + (\lambda + \alpha)e_{22} + (\lambda + \alpha)e_{33}, \\ \tau_{22} = & (\lambda + \alpha)e_{11} + (\lambda + 2\mu_T)e_{22} + \lambda e_{33}, \end{aligned} \quad (1)$$

$$\begin{aligned}\tau_{33} &= (\lambda + \alpha)e_{11} + \lambda e_{22} + (\lambda + 2\mu_T)e_{33}, \\ \tau_{12} &= 2\mu_L e_{12}, \tau_{13} = 2\mu_L e_{13}, \tau_{23} = 2\mu_T e_{23},\end{aligned}\quad (2)$$

where $2e_{ij} = u_{i,j} + u_{j,i}$ and $u_i (i = 1, 2, 3)$ are the displacement components.

We take the plane of symmetry of the fibre-reinforced medium as the x_1x_2 -plane and x_2 -axis vertically upwards. For the plane wave propagation in x_1x_2 -plane, we have $\partial/\partial x_3 = 0$.

The non-vanishing equations of motion without body forces are

$$\begin{aligned}\frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} &= \rho \frac{\partial^2 u_1}{\partial t^2}, \\ \frac{\partial \tau_{21}}{\partial x_1} + \frac{\partial \tau_{22}}{\partial x_2} &= \rho \frac{\partial^2 u_2}{\partial t^2}, \\ \frac{\partial \tau_{31}}{\partial x_1} + \frac{\partial \tau_{32}}{\partial x_2} &= \rho \frac{\partial^2 u_3}{\partial t^2},\end{aligned}\quad (3)$$

The stress equations of motion (3) with the help of (2) become

$$(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta) \frac{\partial^2 u_1}{\partial x_1^2} + \mu_L \frac{\partial^2 u_1}{\partial x_2^2} + (\lambda + \alpha + \mu_L) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} = \rho \frac{\partial^2 u_1}{\partial t^2}, \quad (4)$$

$$\mu_L \frac{\partial^2 u_2}{\partial x_1^2} + (\lambda + 2\mu_T) \frac{\partial^2 u_2}{\partial x_2^2} + (\lambda + \mu_L + \alpha) \frac{\partial^2 u_1}{\partial x_1 \partial x_2} = \rho \frac{\partial^2 u_2}{\partial t^2}, \quad (5)$$

$$\mu_L \frac{\partial^2 u_3}{\partial x_1^2} + \mu_T \frac{\partial^2 u_3}{\partial x_2^2} = \rho \frac{\partial^2 u_3}{\partial t^2}. \quad (6)$$

From (4) to (6), it is obvious that qSH wave which is represented by u_3 motion in (6) is decoupled from (u_1, u_2) motion representing qP and qSV waves. The phase velocity of qSH wave is

$$\rho c_n^2 = \mu_L \{p_1^{(n)}\}^2 + \mu_T \{p_2^{(n)}\}^2, \quad (7)$$

where $\mathbf{p} (p_1^{(n)}, p_2^{(n)}, 0)$ denote the unit propagation vector, c_n is the phase velocity and k_n is the wave number of plane waves propagating in the x_1x_2 -plane. We consider plane wave solutions of (4) and (5) as

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = A \begin{pmatrix} d_1^{(n)} \\ d_2^{(n)} \end{pmatrix} \exp [ik_n(\mathbf{x} \cdot \mathbf{p} - c_n t)], \quad (8)$$

where $\mathbf{d} (d_1^{(n)}, d_2^{(n)}, 0)$ is the unit displacement vector.

Using the expressions of (8) for u_1 and u_2 in the equations of motion (4) and (5), we obtain

$$\frac{d_1^{(n)}}{d_2^{(n)}} = \frac{S}{\rho c_n^2 - R} = \frac{\rho c_n^2 - T}{S}, \quad (9)$$

where

$$\begin{aligned}R &= (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta) \{p_1^{(n)}\}^2 + \mu_L \{p_2^{(n)}\}^2, \\ S &= (\lambda + \alpha + \mu_L) p_1^{(n)} p_2^{(n)}, \\ T &= \mu_L \{p_1^{(n)}\}^2 + (\lambda + 2\mu_T) \{p_2^{(n)}\}^2.\end{aligned}\quad (10)$$

Equation (9) may be used to find the \mathbf{d} in terms of \mathbf{p} .

From the above equation, we have

$$\rho^2 c_n^4 - (R + T)\rho c_n^2 + (RT - S^2) = 0.$$

The solutions of the above equation are

$$2\rho c_n^2 = (R + T) \pm [(R - T)^2 + 4S^2]^{1/2}.$$

Velocities of qP wave and qSV waves are

$$2\rho c_L^2 = (R + T) + ((R - T)^2 + 4S^2)^{1/2}, \quad (11)$$

$$2\rho c_T^2 = (R + T) - ((R - T)^2 + 4S^2)^{1/2}, \quad (12)$$

From (4) and (5), we obtain

$$\begin{aligned} & [(\lambda + 2\alpha + 3\mu_L - 2\mu_T + \beta)d_1^{(n)}d_2^{(n)}\{p_1^{(n)}\}^2] + (\mu_L - \lambda - 2\mu_T)d_1^{(n)}d_2^{(n)}\{p_2^{(n)}\}^2 \\ & + (\lambda + \alpha + \mu_L)[\{d_2^{(n)}\}^2 - \{d_1^{(n)}\}^2]p_1^{(n)}p_2^{(n)} = 0. \end{aligned} \quad (13)$$

Pure longitudinal and shear waves can propagate only in certain specific directions. Longitudinal and transverse specific directions are found by taking $\mathbf{d} = \mathbf{p}$ and \mathbf{d} perpendicular to \mathbf{p} . In the anisotropic case no such relations can be considered between the displacement vector and the propagation vector.

We consider a homogeneous fibre-reinforced half-space occupying the region $x_2 \leq 0$ and the plane of symmetry is taken as the x_1x_2 -plane. Plane qP wave is incident at the traction-free boundary $x_2 = 0$ and will generate reflected qP and qSV waves. Let $n = 0, 1, 2$ be assumed for incident qP, reflected qP and reflected qSV waves respectively. We consider the plane strain problem and hence

$$u_1 = u_1(x_1, x_2, t), u_2 = u_2(x_1, x_2, t), u_3 = 0.$$

The displacement field may be represented by

$$u_1 = \sum_{j=0}^2 A_j d_1^{(j)} e^{i\eta_j}, \quad u_2 = \sum_{j=0}^2 A_j d_2^{(j)} e^{i\eta_j}, \quad (14)$$

where

$$\eta_n = k_n(x_1 p_1^{(n)} + x_2 p_2^{(n)} - c_n t). \quad (15)$$

For incident qP wave, which makes an angle θ_0 , we have

$$p_1^{(0)} = \sin \theta_0, \quad p_2^{(0)} = \cos \theta_0, \quad c_0 = c_L.$$

In the plane $x_2 = 0$, the displacement and stress components due to incident qP-wave may be written as

$$\begin{aligned} u_1^{(0)} &= A_0 d_1^{(0)} e^{i\eta_0}, \quad u_2^{(0)} = A_0 d_2^{(0)} e^{i\eta_0}, \\ \tau_{22}^{(0)} &= i A_0 k_0 [(\lambda + \alpha) d_1^{(0)} \sin \theta_0 + (\lambda + 2\mu_T) d_2^{(0)} \cos \theta_0] e^{i\eta_0}, \\ \tau_{21}^{(0)} &= i A_0 k_0 \mu_L [d_1^{(0)} \cos \theta_0 + d_2^{(0)} \sin \theta_0] e^{i\eta_0}, \end{aligned} \quad (16)$$

where

$$\eta_0 = k_0(x_1 p_1^{(0)} - c_L t). \quad (17)$$

For a reflected qP wave which makes an angle θ_1 we have

$$p_1^{(1)} = \sin \theta_1, p_2^{(1)} = -\cos \theta_1, c_1 = c'_L.$$

In the plane $x_2 = 0$, the displacement and stress components due to reflected qP-wave may be written as

$$\begin{aligned} u_1^{(1)} &= A_1 d_1^{(1)} e^{i\eta_1}, u_2^{(1)} = A_1 d_2^{(1)} e^{i\eta_1}, \\ \tau_{22}^{(1)} &= i A_1 k_1 [(\lambda + \alpha) d_1^{(1)} \sin \theta_1 - (\lambda + 2\mu_T) d_2^{(1)} \cos \theta_1] e^{i\eta_1}, \\ \tau_{21}^{(1)} &= i A_1 k_1 \mu_L [-d_1^{(1)} \cos \theta_1 + d_2^{(1)} \sin \theta_1] e^{i\eta_1}, \end{aligned} \quad (18)$$

where

$$\eta_1 = k_1(x_1 p_1^{(1)} - c'_L t).$$

If the reflected qSV wave makes an angle θ_2 , we have

$$p_1^{(2)} = \sin \theta_2, p_2^{(2)} = -\cos \theta_2, c_2 = c_T.$$

In the plane $x_2 = 0$, the displacement and stress components due to reflected qSV-wave may be written as

$$\begin{aligned} u_1^{(2)} &= A_2 d_1^{(2)} e^{i\eta_2}, u_2^{(2)} = A_2 d_2^{(2)} e^{i\eta_2}, \\ \tau_{22}^{(2)} &= i A_2 k_2 [(\lambda + \alpha) d_1^{(2)} \sin \theta_2 - (\lambda + 2\mu_T) d_2^{(2)} \cos \theta_2] e^{i\eta_2}, \\ \tau_{21}^{(2)} &= i k_2 A_2 \mu_L [-d_1^{(2)} \cos \theta_2 + d_2^{(2)} \sin \theta_2] e^{i\eta_2}, \end{aligned} \quad (19)$$

where

$$\eta_2 = k_2(x_1 p_1^{(2)} - c_T t).$$

3. Boundary conditions and solution of the problem for incident qP-waves

Case 1: Reflection reflection of qP-wave at a free boundary.

When $x_2 = 0$ is a free surface, the sum of the three tractions must vanish at $x_2 = 0$ and we can write the boundary conditions as:

$$\tau_{22}^{(0)} + \tau_{22}^{(1)} + \tau_{22}^{(2)} = 0,$$

and

$$\tau_{21}^{(0)} + \tau_{21}^{(1)} + \tau_{21}^{(2)} = 0. \quad (20)$$

Substituting in (20), the values of $\tau_{22}^{(n)}, \tau_{21}^{(n)}$ (for $n = 0, 1, 2$) from (16), (18) and (19), we obtain:

$$\begin{aligned} & i k_0 A_0 [(\lambda + \alpha) d_1^{(0)} \sin \theta_0 + (\lambda + 2\mu_T) d_2^{(0)} \cos \theta_0] \exp(i\eta_0) \\ & + i k_1 A_1 [(\lambda + \alpha) d_1^{(1)} \sin \theta_1 - (\lambda + 2\mu_T) d_2^{(1)} \cos \theta_1] \exp(i\eta_1) \\ & + i k_2 A_2 [(\lambda + \alpha) d_1^{(2)} \sin \theta_2 - (\lambda + 2\mu_T) d_2^{(2)} \cos \theta_2] \exp(i\eta_2) = 0, \end{aligned} \quad (21)$$

and

$$\begin{aligned}
 & ik_0 A_0 [d_1^{(0)} \cos \theta_0 + d_2^{(0)} \sin \theta_0] \exp(i\eta_0) \\
 & + ik_1 A_1 [-d_1^{(1)} \cos \theta_1 + d_2^{(1)} \sin \theta_1] \exp(i\eta_1) \\
 & + ik_2 A_2 [-d_1^{(2)} \cos \theta_2 + d_2^{(2)} \sin \theta_2] \exp(i\eta_2) = 0.
 \end{aligned} \tag{22}$$

Equations (21) and (22) must be valid for all values of x_1 and t , hence

$$\eta_0 = \eta_1 = \eta_2, \tag{23}$$

which means

$$k_0(x_1 \sin \theta_0 - c_L t) = k_1(x_1 \sin \theta_1 - c'_L t) = k_2(x_1 \sin \theta_2 - c_T t).$$

This gives

$$k_0 \sin \theta_0 = k_1 \sin \theta_1 = k_2 \sin \theta_2 = \phi$$

and

$$k_0 c_L = k_1 c'_L = k_2 c_T = \omega, \tag{24}$$

where ϕ is the apparent wave number, and ω is the circular frequency.

From the above relations we have

$$\frac{k_1}{k_0} = \frac{c_L}{c'_L} = \frac{\sin \theta_0}{\sin \theta_1},$$

and

$$\frac{k_2}{k_0} = \frac{c_L}{c_T} = \frac{\sin \theta_0}{\sin \theta_2}. \tag{25}$$

Equations (21) and (22) after using (23) may be written as

$$\begin{aligned}
 P_0 A_0 + P_1 A_1 + P_2 A_2 &= 0, \\
 P_3 A_0 + P_4 A_1 + P_5 A_2 &= 0,
 \end{aligned} \tag{26}$$

where

$$\begin{aligned}
 P_0 &= k_0 [(\lambda + \alpha) d_1^{(0)} \sin \theta_0 + (\lambda + 2\mu_T) d_2^{(0)} \cos \theta_0], \\
 P_1 &= k_1 [(\lambda + \alpha) d_1^{(1)} \sin \theta_1 - (\lambda + 2\mu_T) d_2^{(1)} \cos \theta_1], \\
 P_2 &= k_2 [(\lambda + \alpha) d_1^{(2)} \sin \theta_2 - (\lambda + 2\mu_T) d_2^{(2)} \cos \theta_2], \\
 P_3 &= k_0 [d_1^{(0)} \cos \theta_0 + d_2^{(0)} \sin \theta_0], \\
 P_4 &= k_1 [-d_1^{(1)} \cos \theta_1 + d_2^{(1)} \sin \theta_1], \\
 P_5 &= k_2 [-d_1^{(2)} \cos \theta_2 + d_2^{(2)} \sin \theta_2].
 \end{aligned} \tag{27}$$

Solving the above two equations, we have,

$$\frac{A_1}{A_0} = \frac{a_2 - b_2}{a_1 b_2 - a_2 b_1}, \quad \frac{A_2}{A_0} = -\frac{a_1 - b_1}{a_1 b_2 - a_2 b_1}, \tag{28}$$

where

$$a_1 = \frac{P_1}{P_0}, a_2 = \frac{P_2}{P_0}, b_1 = \frac{P_4}{P_3}, b_2 = \frac{P_5}{P_3}.$$

$d_1^{(i)}/d_2^{(i)}$ ($i = 0, 1, 2$) can be calculated from (9) and are as under:

$$d_1^{(0)}/d_2^{(0)} = (\rho c_L^2 - T)/S \quad (29)$$

where R , S and T can be calculated after putting $p_1 = p_1^{(0)} = \sin \theta_0$ and $p_2 = p_2^{(0)} = \cos \theta_0$ in (10).

$$d_1^{(1)}/d_2^{(1)} = (\rho c_L^2 - T_1)/S_1, \quad (30)$$

where R_1 , S_1 and T_1 can be calculated after putting $p_1 = p_1^{(1)} = \sin \theta_1$ and $p_2 = p_2^{(1)} = -\cos \theta_1$ in (10).

$$d_1^{(2)}/d_2^{(2)} = (\rho c_T^2 - T_2)/S_2 \quad (31)$$

where R_2 , S_2 and T_2 can be calculated after putting $p_1 = p_1^{(2)} = \sin \theta_2$ and $p_2 = p_2^{(2)} = -\cos \theta_2$ in (10).

From (11) and (12), the velocities of incident qP, reflected qP and reflected qSV may be defined by

$$\begin{aligned} 2\rho c_L^2 &= (R + T) + ((R - T)^2 + 4S^2)^{1/2}, \\ 2\rho c_L^2 &= (R_1 + T_1) + ((R_1 - T_1)^2 + 4S_1^2)^{1/2}, \\ 2\rho c_T^2 &= (R_2 + T_2) - ((R_2 - T_2)^2 + 4S_2^2)^{1/2}, \end{aligned} \quad (32)$$

where R , R_1 , R_2 , S , S_1 , S_2 , T , T_1 and T_2 are defined in (29) to (31).

Using the following values of reinforced-free medium

$$\mu_L = \mu_T = \mu, \alpha = \beta = 0,$$

equations (28) reduce to

$$\frac{A_1}{A_0} = \frac{\sin 2\theta_0 \sin 2\theta_2 - \bar{K}^2 \cos^2 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \bar{K}^2 \cos^2 2\theta_2} \quad (33)$$

$$\frac{A_2}{A_0} = \frac{2\bar{K} \sin 2\theta_0 \cos 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \bar{K}^2 \cos^2 2\theta_2} \quad (34)$$

where,

$$\bar{K} = [(\lambda + 2\mu)/\mu]^{1/2}$$

which are the reflection coefficients of P and SV waves respectively for free boundary in isotropic case (Achenbach 1976, p. 175).

The partition of energy between reflected qP and qSV waves for incident qP wave is given by

$$\left(\frac{A_1}{A_0}\right)^2 \frac{c_L m_1 \cos \theta_1}{c_L' m_0 \cos \theta_0} + \left(\frac{A_2}{A_0}\right)^2 \frac{c_L m_2 \cos \theta_2}{c_T m_0 \cos \theta_0} = 1, \quad (35)$$

where

$$\begin{aligned}
 m_0 &= (ap_1^{(0)}d_1^{(0)} + bp_2^{(0)}d_2^{(0)}) \sin \theta_0 d_1^{(0)} + \mu_L(d_1^{(0)}p_2^{(0)} + d_2^{(0)}p_1^{(0)}) \\
 &\quad \times (d_1^{(0)} \cos \theta_0 + d_2^{(0)} \sin \theta_0) + (bd_1^{(0)}p_1^{(0)} + cd_2^{(0)}p_2^{(0)})d_2^{(0)} \cos \theta_0, \\
 m_1 &= (ap_1^{(1)}d_1^{(1)} + bp_2^{(1)}d_2^{(1)}) \sin \theta_1 d_1^{(1)} + \mu_L(d_1^{(1)}p_2^{(1)} + d_2^{(1)}p_1^{(1)}) \\
 &\quad \times (-d_1^{(1)} \cos \theta_1 + d_2^{(1)} \sin \theta_1) - (bp_1^{(1)}d_1^{(1)} + cp_2^{(1)}d_2^{(1)})d_2^{(1)} \cos \theta_1, \\
 m_2 &= (ap_1^{(2)}d_1^{(2)} + bp_2^{(2)}d_2^{(2)}) \sin \theta_2 d_1^{(2)} + \mu_L(d_1^{(2)}p_2^{(2)} + d_2^{(2)}p_1^{(2)}) \\
 &\quad \times (-d_1^{(2)} \cos \theta_2 + d_2^{(2)} \sin \theta_2) - (bp_1^{(2)}d_1^{(2)} + cp_2^{(2)}d_2^{(2)})d_2^{(2)} \cos \theta_2. \\
 a &= \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta, \quad b = \lambda + \alpha, \quad c = \lambda + 2\mu_T.
 \end{aligned}$$

For an isotropic case, (35) becomes

$$\left(\frac{A_1}{A_0}\right)^2 + \left(\frac{A_2}{A_0}\right)^2 \frac{c_T \cos \theta_2}{c_L \cos \theta_0} = 1,$$

which is same as that of Achenbach (1976, p. 182).

The velocity of surface wave can be obtained from (28) by equating the denominator to zero. It has been observed that the surface wave velocity at $\theta = 8.8^\circ$ in the case of a fibre-reinforced medium is 1.58 times more than the Rayleigh wave in the classical case (values of λ , α , β , μ_T and μ_L are defined in (6).

Case 2: Reflection of qP -wave at a rigid boundary.

Since the boundary $x_2 = 0$ is bounded by a rigid layer, the boundary conditions may be taken as

$$u_1^{(0)} + u_1^{(1)} + u_1^{(2)} = 0,$$

and

$$u_2^{(0)} + u_2^{(1)} + u_2^{(2)} = 0. \tag{36}$$

Substituting the values of $u_1^{(n)}$, $u_2^{(n)}$ for $n = 0, 1, 2$ from (16), (18) and (19) in (36), we get

$$A_0 d_1^{(0)} \exp(i\eta_0) + A_1 d_1^{(1)} \exp(i\eta_1) + A_2 d_1^{(2)} \exp(i\eta_2) = 0, \tag{37}$$

$$A_0 d_2^{(0)} \exp(i\eta_0) + A_1 d_2^{(1)} \exp(i\eta_1) + A_2 d_2^{(2)} \exp(i\eta_2) = 0. \tag{38}$$

Solving the above two equations, we have

$$\begin{aligned}
 \frac{A_1}{A_0} &= \frac{d_1^{(2)}d_2^{(0)} - d_2^{(2)}d_1^{(0)}}{d_1^{(1)}d_2^{(2)} - d_2^{(1)}d_1^{(2)}}, \\
 \frac{A_2}{A_0} &= \frac{d_1^{(0)}d_2^{(1)} - d_1^{(1)}d_2^{(0)}}{d_1^{(1)}d_2^{(2)} - d_2^{(1)}d_1^{(2)}}.
 \end{aligned} \tag{39}$$

The above equations are the reflection coefficients of qP and qSV waves for rigid boundaries.

4. Reflection of qSV waves at a free boundary

Incident qSV wave will generate reflected qP and qSV waves. Let $n = 0, 1, 2$ be assumed for incident qSV, reflected qP and reflected qSV waves respectively. For incident qSV wave, which makes an angle θ_0 , we have

$$p_1^{(0)} = \sin \theta_0, \quad p_2^{(0)} = \cos \theta_0, \quad c_0 = c'_T. \quad (40)$$

In the plane $x_2 = 0$, the displacement and stress components of incident wave and reflected waves are same as in (16), (18) and (19). Equation (17) to be replaced by

$$\eta_0 = k_0(x_1 p_1^{(0)} - c'_T t). \quad (41)$$

5. Boundary conditions and solution of the problem for qSV waves

5.1 Reflection of qSV wave at a free boundary

Substituting in (20), the values of $\tau_{22}^{(n)}, \tau_{21}^{(n)}$ (for $n = 0, 1, 2$) from (41), (18) and (19), we obtain the same expressions as (21) and (22) except that c_L is replaced by c'_T . The ratio $d_1^{(0)}/d_2^{(0)}$ is mentioned in (43), other ratios are the same as (30) and (31).

Equations (21) and (22) must be valid for all values of x_1 and t , hence

$$\begin{aligned} k_0 \sin \theta_0 &= k_1 \sin \theta_1 = k_2 \sin \theta_2 = \phi, \\ \text{and} \quad k_0 c'_T &= k_1 c'_L = k_2 c_T = \omega, \end{aligned} \quad (42)$$

where ϕ and ω are defined in (24). Solving (21) and (22) we have the same sets of equations as (26) with some changes as mentioned below:

$d_1^{(i)}/d_2^{(i)}$ ($i = 0, 1, 2$), may be calculated from (9) and are as under.

$$d_1^{(0)}/d_2^{(0)} = (\rho c_T^2 - T)/S, \quad (43)$$

where R, S and T can be calculated after taking $p_1 = p_1^0 = \sin \theta_0$ and $p_2 = p_2^0 = \cos \theta_0$ in (10). $d_1^{(i)}/d_2^{(i)}$ ($i = 1, 2$) are defined in (30) and (31).

From (12) and (11), the velocity of incident qSV may be defined by

$$2\rho c_T^2 = (R + S) - ((R - S)^2 + 4T^2)^{1/2}. \quad (44)$$

Reflected qP and qSV waves velocities are already defined in (32).

Using the following values of reinforced- free medium (values are mentioned in section 3), we obtain the reflection coefficients for isotropic case as

$$\begin{aligned} \frac{A_1}{A_0} &= -\frac{\bar{K} \sin 4\theta_0}{\sin 2\theta_0 \sin 2\theta_1 + \bar{K}^2 \cos^2 2\theta_0}, \\ \frac{A_2}{A_0} &= \frac{\sin 2\theta_0 \sin 2\theta_1 - \bar{K}^2 \cos^2 2\theta_0}{\sin 2\theta_0 \sin 2\theta_1 + \bar{K}^2 \cos^2 2\theta_0}, \end{aligned} \quad (45)$$

where $\bar{K} = [(\lambda + 2\mu)/\mu]^{1/2}$.

The partition of energy between reflected qP and qSV waves due to incident qSV wave is given by

$$\left(\frac{A_1}{A_0}\right)^2 \frac{c'_T m_1 \cos \theta_1}{c'_L m_0 \cos \theta_0} + \left(\frac{A_2}{A_0}\right)^2 \frac{c'_T m_2 \cos \theta_2}{c_T m_0 \cos \theta_0} = 1 \tag{46}$$

where all definitions are as in (35).

5.2 *Reflection of qSV wave at a rigid boundary*

Since the boundary $x_2 = 0$ is a rigid layer, the boundary conditions are the same as for (36).

After substituting the values of $u_1^{(n)}, u_2^{(n)}$ for $n = 0, 1, 2$ from (41), (18) and (19) in (36), and solving, we have the same sets of equations as in (39) for reflection coefficients of qP and qSV waves.

Here $d_1^{(i)}/d_2^{(i)}$ ($i = 0, 1, 2$) are as defined in (43), (30) and (31).

6. Numerical calculations and discussions

The material constants for fibre-reinforced medium have been considered as per Markham (1970).

$$\begin{aligned} \mu_T &= 2.46 \times 10^9 \text{ N/m}^2, \quad \mu_L = 5.66 \times 10^9 \text{ N/m}^2, \\ \lambda &= 5.65 \times 10^9 \text{ N/m}^2, \quad \beta = 220.90 \times 10^9 \text{ N/m}^2, \\ \alpha &= -1.28 \times 10^9 \text{ N/m}^2, \quad \rho = 7800 \text{ kg/m}^3. \end{aligned}$$

6.1 *Reflection of qP waves*

In figure 1, curve II corresponds to reflection coefficient of qP-wave in fibre-reinforced medium. All the values of curve II are negative except from 0° to 10° and from 83° to 90° . In

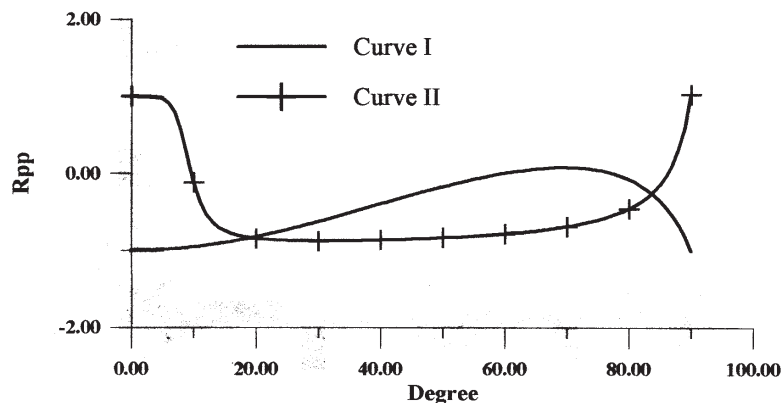


Figure 1. Amplitude ratios of qP waves due to incident qP waves.

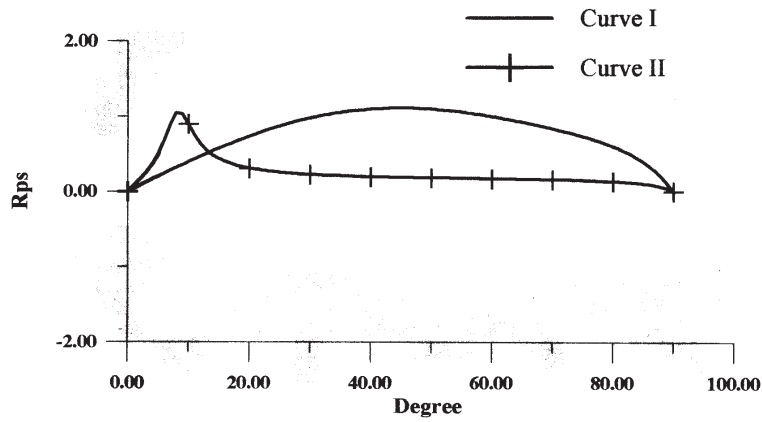


Figure 2. Amplitude ratios of qSV waves due to incident qP waves.

the isotropic case (curve I), the values of (A_1/A_0) are all negative except from $\theta_0 = 57^\circ$ to 78° . Significant differences of values exist from 0° to 5° and from 80° to 90° in fibre-reinforced case compared to isotropic case. The values from 21° to 80° are greater in the isotropic case compared to those in the fibre-reinforced case.

In figure 2, the reflection coefficients of qSV waves for a free boundary of fibre-reinforced medium at different angles of incidence have plotted along with the curve for isotropic medium. The values of (A_2/A_0) are all positive and equal for curves I and II at $\theta_0 = 0^\circ, 15^\circ$ and 90° . The difference in values at $\theta_0 = 50^\circ$ in isotropic case is significantly more compared to fibre-reinforced medium.

Figure 3 shows the comparison of partition of energy between reflected P and qP waves for incident qP waves. In this case $A_1/A_0 = 0$, for angle of incidence at 60° and 78° , and $A_1/A_0 = 1$ for angle of incidence at 0° and 90° in case of isotropic media. For fibre-reinforced medium (curve II), $A_1/A_0 = 1$ at 0° only.

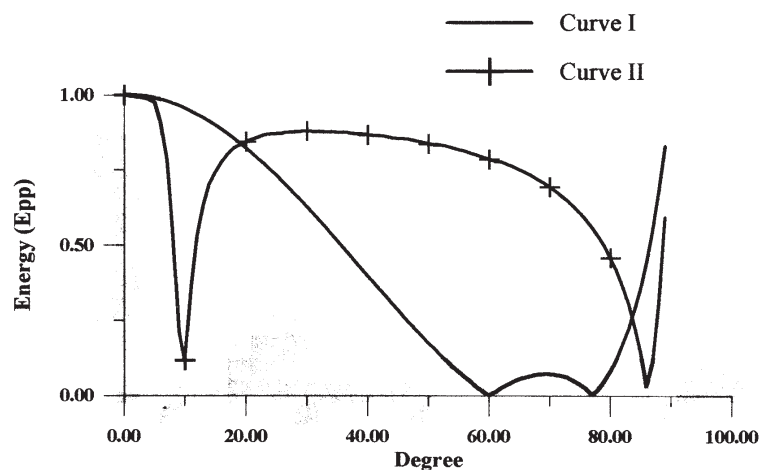


Figure 3. Partition of energy of qP waves due to incident qP waves.

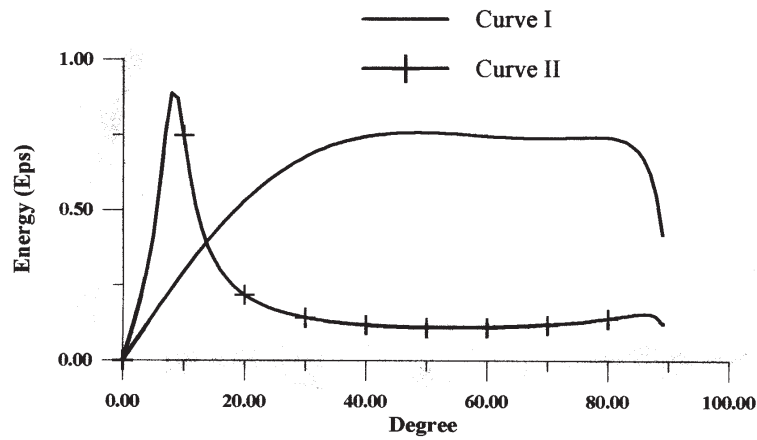


Figure 4. Partition of energy of qSV waves due to incident qP waves.

Figure 4 shows the comparison of partition of energy between reflected SV and qSV waves for incident qP waves. In this case $A_2/A_0 = 0$ at 0° for both isotropic and fibre-reinforced (curve II) media. The critical point exists at 13° .

In figure 5, the values of reflection coefficients of qP-wave for isotropic (curve I) and fibre-reinforced media (curve II) have been plotted for rigid boundaries. The values of A_1/A_0 for fibre-reinforced media sharply increase from 0° to 18° and then remain constant from 19° to 90° . The value of A_1/A_0 at 90° is significantly more in case of a fibre-reinforced medium compared to the isotropic case, and at 0° the value in the isotropic case is more compared to that in the fibre-reinforced case. The critical point exists at 18° .

In figure 6, the values of reflection coefficient of qSV (A_2/A_0) have been plotted for rigid boundary in curves II and I. It has been observed that all the values of the reflection

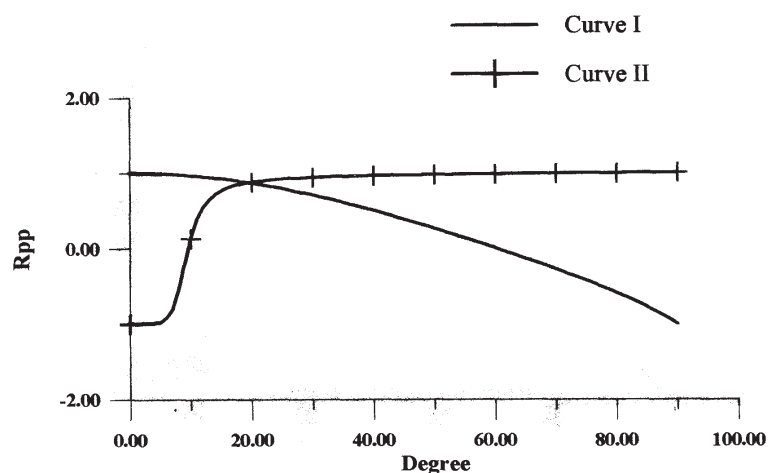


Figure 5. Amplitude ratios of qP waves due to incident qP waves in rigid boundary.

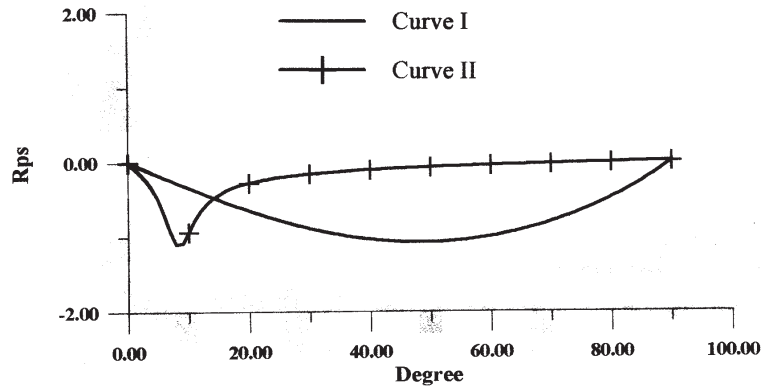


Figure 6. Amplitude ratios of qSV waves due to incident qP waves in rigid boundary.

coefficients of qSV are negative except those at 0° and 90° . The values are greater in case of fibre-reinforced media as compared to isotropic media from 15° to 89° . The maximum difference exists at 10° . The critical point exists at 15° .

Figures 7 and 8 show the partition of energy between reflected qP and qSV waves for incidence of a qP wave in a rigid boundary.

6.2 Reflection of qSV waves

In figure 9, curve I corresponds to isotropic medium and agrees with the result of Achenbach (1976). Curve II corresponds to fibre-reinforced medium. The value of A_1/A_0 in a fibre-reinforced medium is greater compared to that in isotropic medium but the difference is greater at 33° .

In figure 10, the reflection coefficient of qSV-waves (curve II) for reinforced medium for different values of θ_0 ranging from 0° to 33° have been plotted which are permissible of θ_0

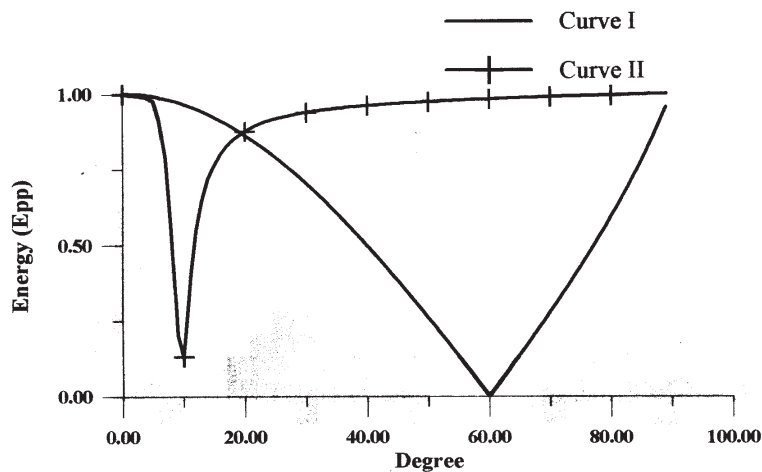


Figure 7. Partition of energy of qP waves due to incident qP waves in rigid boundary.

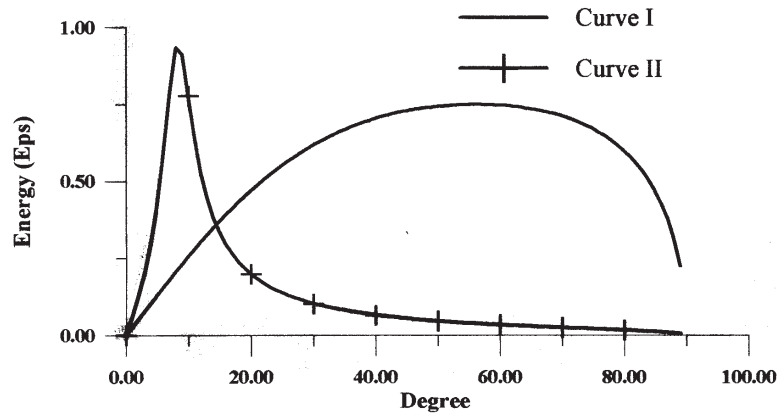


Figure 8. Partition of energy of qSV waves due to incident qP waves in rigid boundary.

for A_2/A_0 in fibre reinforced medium and compared with those in isotropic media (curve I). In this case, due to the effect of fibre-reinforced medium the values of A_2/A_0 are greater compared to the isotropic case.

This rigid boundary plays a very important role in case of reflection phenomena. All the values of A_1/A_0 (figure 13) in case of fibre-reinforced material in a rigid boundary are less compared to the isotropic case.

Figure 14 shows the reflection coefficient A_2/A_0 for qSV wave. The values of A_2/A_0 for fibre-reinforced media with rigid boundaries coincide with the values of A_2/A_0 for isotropic media at 32° . This is the critical point. The values for fibre-reinforced media (curve II) are less compared to those for isotropic media from 5° to 31° .

Figures 11, 12, 15 and 16 show the partition of energy for incident qSV waves due to free and rigid boundaries.

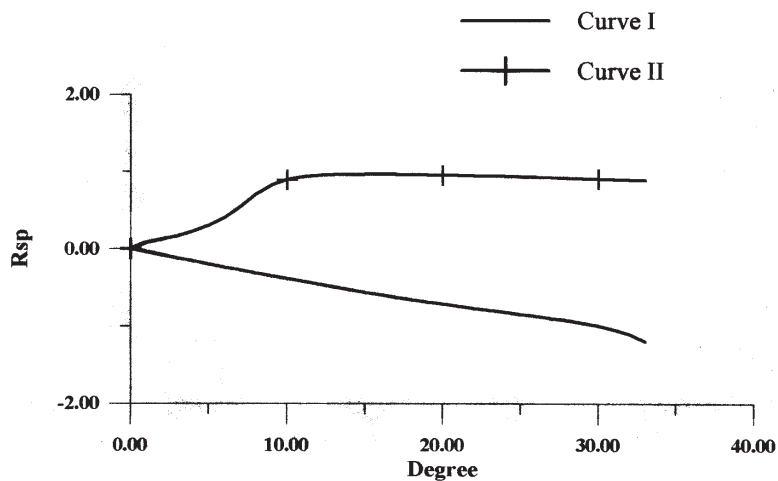


Figure 9. Amplitude ratios of qP waves due to incident qSV waves.

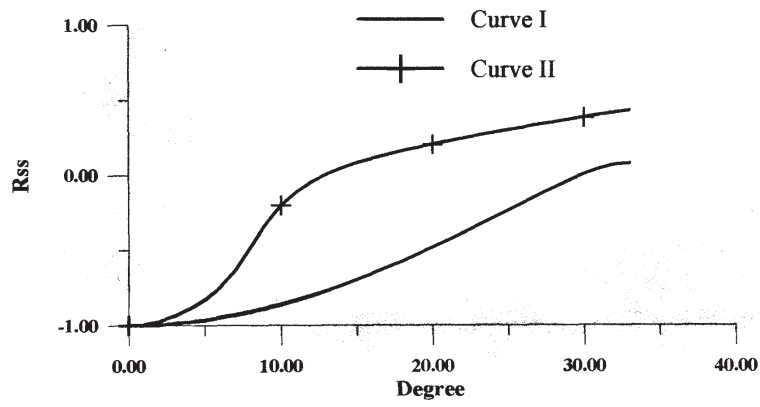


Figure 10. Amplitude ratios of qSV waves due to incident qSV waves.

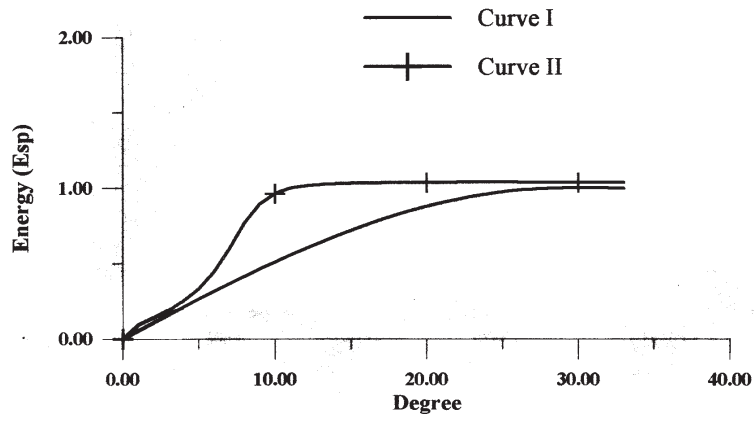


Figure 11. Partition of energy of qP waves due to incident qSV waves.

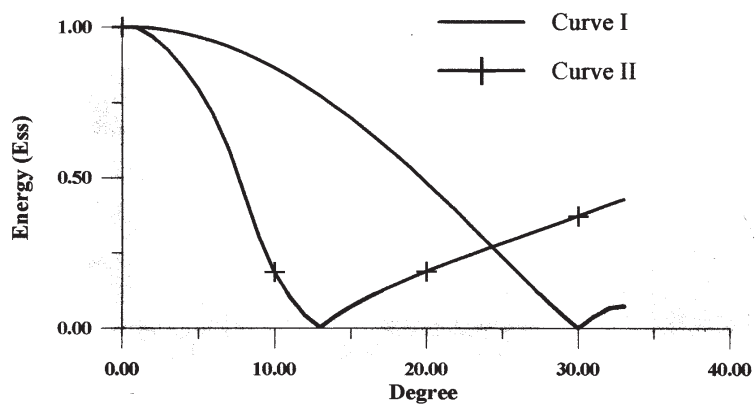


Figure 12. Partition of energy of qSV waves due to incident qSV waves.

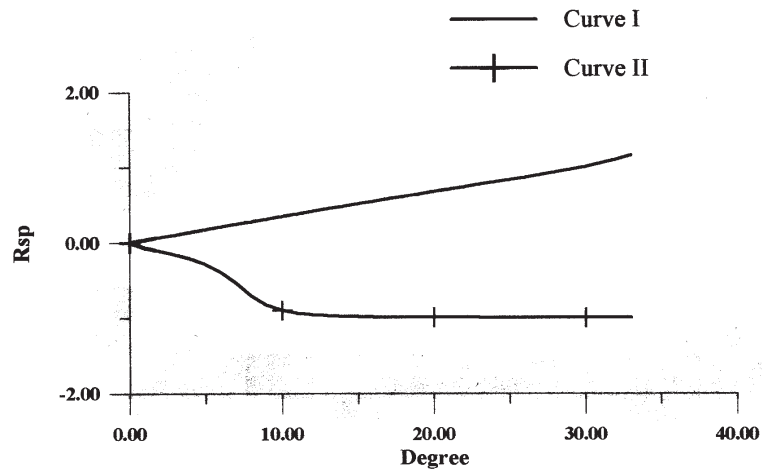


Figure 13. Amplitude ratios of qP waves due to incident qSV waves in rigid boundary.

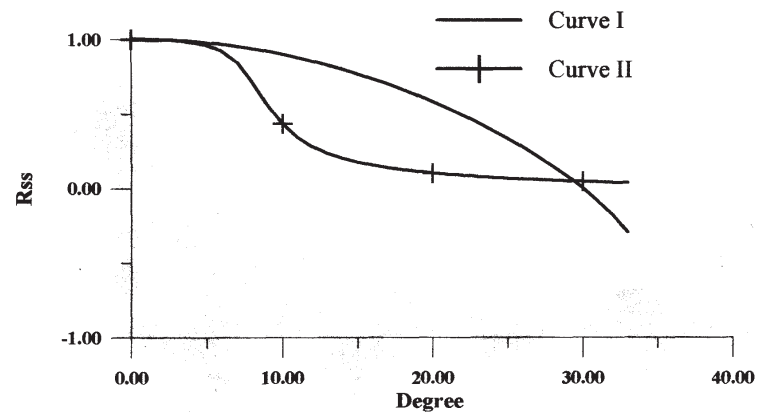


Figure 14. Amplitude ratios of qSV waves due to incident qSV waves in rigid boundary.

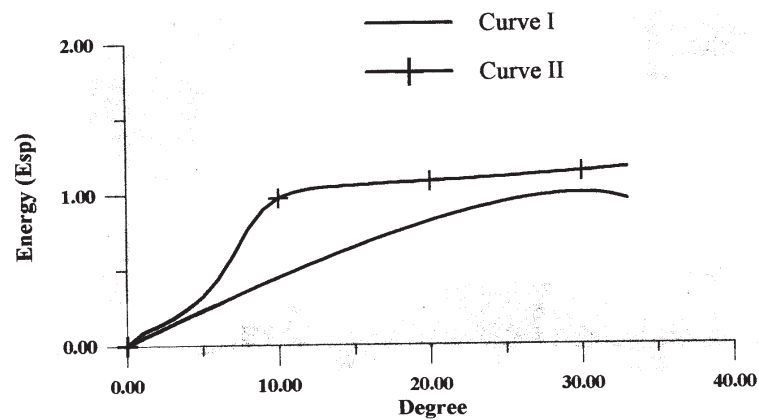


Figure 15. Partition of energy of qP waves due to incident qSV waves in rigid boundary.

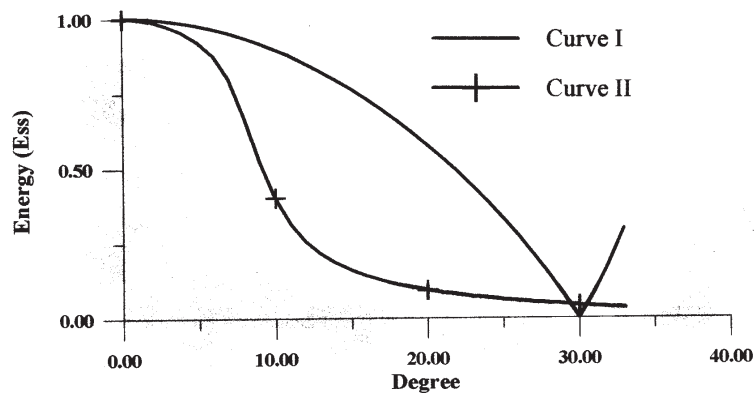


Figure 16. Partition of energy of qSV waves due to incident qSV waves in rigid boundary.

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