Progress of Theoretical Physics, Vol. 40, No. 2, August 1968

Reflection Symmetry of Conspiring Fermion Trajectories

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(Received March 30, 1968)

It is shown by making use of the analytic property of the Khuri amplitude near $W=0$, W being the c.m. energy of πN scattering, that two fermion trajectories $\alpha^{\pm}(W)$ as well as their reduced residues $\bar{\beta}^{\pm}(W)$, with parity $\pm$ and with the same signature, are related with each other by reflection of their argument W, i.e. $\alpha^{+}(W)=\alpha^{-}(-W)$, and $\bar{\beta}^{+}(W)=\bar{\beta}^{-}(-W)$. This symmetry cannot be obtained from the analytic property of the partial-wave amplitude, contrary to the conventional assertion.

§ 1. Introduction and summary

One of the recent topics in the theory of relativistic Regge poles is the so-called "conspiracy" among trajectories with different quantum numbers. At present, the conspiracy relations are obtained elegantly by the group theoretical methods.[1] These methods, however, are not applicable to fermion trajectories, but only to boson trajectories relating to reactions in which the initial and final states contain equal-mass particles (e.g. $N\bar{N}\to\pi\pi$). Furthermore, the group theoretical methods cannot give any conspiracy relations at finite mass, but only at zero mass.

Gribov[2] has first suggested that two fermion trajectories with opposite parity should intersect each other at zero mass. His argument, however, contains the unpleasant assumption on the behavior of helicity partial-wave amplitudes at $u=0$ that one of helicity partial-wave amplitudes should remain finite at $u=0$, where u is the total c.m. energy squared of πN scattering. At $u=0$, the point $\cos\theta_u=-1$ corresponds to the infinity on the Mandelstam diagram (stu plot). Hence, the behavior of helicity partial-wave amplitudes at $u=0$ is controlled by the Regge pole in the direct, *not crossed*, channel. It is, therefore, not very clear that the amplitude is finite at $u=0$.

Sakmar[3] has proposed another approach: He asserts that the conspiracy is a straightforward consequence of the "MacDowell reflection Symmerty" for the

Froissart-Gribov partial-wave amplitudes $f^{\pm}(W, J)$:[4),*)]

$$f^{+}(W, J) = f^{-}(-W, J), \tag{1·1}$$

where J is the total angular momentum in πN scattering, W is the c.m. energy, and the index $\pm$ refers to the parity $(-1)^{J\pm(1/2)}$. To any pole trajectory of f^{+}, say $\alpha^{+}(W)$, there corresponds some trajectory of f^{-}, say $\alpha^{-}(W)$, which is related to $\alpha^{+}(W)$ by

$$\alpha^{+}(W) = \alpha^{-}(-W) \quad \text{(for the same signature)}. \tag{1·2}$$

The two trajectories α^{+} and α^{-} then intersect each other at $W=0$.

We should, however, reconsider the meaning of Eq. (1·1). The amplitudes $f^{\pm}(\sqrt{u}, J)$ are related to the helicity amplitudes $G(u, J)$ and $H(u, J)$ through

$$f^{\pm}(\sqrt{u}, J) = \tfrac{1}{2}[G(u, J) \pm H(u, J)/\sqrt{u}], \tag{1·3}$$

where $\sqrt{u} = W$, and $G(u, J)$ and $H(u, J)$ are the Froissart-Gribov interpolations of

$$G_J(u) = \tfrac{1}{2}\int_{-1}^{+1} G(u, t)\,[P_{J-(1/2)}(z) + P_{J+(1/2)}(z)]\,dz\,, \tag{1·4}$$

$$H_J(u) = \tfrac{1}{2}\int_{-1}^{+1} H(u, t)\,[P_{J-(1/2)}(z) - P_{J+(1/2)}(z)]\,dz\,. \tag{1·5}$$

Since the full helicity amplitudes $G(u, t)$ and $H(u, t)$ defined by (2·3) have the ordinary Mandelstam analyticity, $G(u, J)$ and $H(u, J)$ have a cut passing through $u=0$ in the u plane, or equivalently, a vertical cut along the whole imaginary axis of the W plane. If such a cut were absent, the functions $G(u, J)$ and $H(u, J)$ could be *analytic* even functions of W $(=\sqrt{u})$, and the symmetry (1·1) follows immediately. At first sight, Eq. (1·4) or (1·5) seems to define *one analytic* even function of W, but in fact, due to the vertical cut, it defines *two* analytic functions, one in the right and one in the left half of the W plane.[**)] Consequently, each of the functions $f^{\pm}(W, J)$ defined by (1·3) is also a "hybrid" of two analytic functions.

If we denote by $f^{\pm R}$ and $f^{\pm L}$ the values of $f^{\pm}$ in the right- and the left-half planes, respectively, the relation (1·1) means

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*) Note that our partial-wave amplitudes $f^{\pm}$ are different from those of MacDowell's by the factor W^{-1}. See Eqs. (2·2).

**) A simple example to illustrate this situation is the function defined by

$$f(W) = \frac{1}{i\pi}\int_{-i\infty}^{+i\infty} \frac{W'^2}{1-W'^2}\frac{dW'}{W'^2-W^2}\,.$$

For $\mathrm{Re}\,W>0$, we have $f(W) = 1/(1+W) \equiv f^{R}(W)$, while $f(W) = 1/(1-W) \equiv f^{L}(W)$ for $\mathrm{Re}\,W<0$. Clearly this function is not one analytic even function, but is a "hybrid" of two analytic functions,

$$f^{+R}(W, J) = f^{-L}(-W, J) \quad \text{for} \quad \text{Re}\, W > 0\,, \tag{1·6}$$

and

$$f^{+L}(W, J) = f^{-R}(-W, J) \quad \text{for} \quad \text{Re}\, W < 0\,. \tag{1·7}$$

Equations (1·6) and (1·7) imply, instead of (1·2),

$$\alpha^{+R}(W) = \alpha^{-L}(-W) \quad \text{for} \quad \text{Re}\, W > 0\,, \tag{1·8}$$

$$\alpha^{+L}(W) = \alpha^{-R}(-W) \quad \text{for} \quad \text{Re}\, W < 0\,. \tag{1·9}$$

As the function f^{+L} is not the analytic continuation of f^{+R}, there is no reason to believe that the α^{+L} coincides with the α^{+R} to yield (1·2), and the same is true for α^{-}.

In § 3 of this paper, we show that the relation (1·2) actually follows from the analytic properties of full amplitudes. In order to enjoy the analytic properties of *full* amplitudes maximally, we employ, in § 2, the Khuri amplitudes instead of the partial-wave amplitudes because as will be shown in § 3, the vertical cuts of the Khuri amplitudes do not cover the whole imaginary axis of the W plane and these amplitudes are analytic in the vicinity of $W=0$, so that we can continue the amplitudes analytically from the right-half W plane into the left-half W plane.

To sum up, we obtain the relation (1·2) by making use of the analytic property of the Khuri amplitudes near $u=0$. The same reflection symmetry is also obtained for their reduced Regge residues. This relation shows not only that each α^{+} trajectory must intersect some α^{-} trajectory at $u=0$, but also that these two trajectories are obtained from single analytic function of W. Recent experimental observations of parity doublets of baryon resonances strongly suggest that the relation (1·2) holds.[5)]

§ 2. The Khuri amplitudes

We first summarise the relevant notations and formulas. Let p_i and q_i be the four-momenta of the incoming ($i=1$) and outgoing ($i=2$) nucleon and pion. We take $u=(p_1+q_1)^2$ to be the c.m. energy squared and denote $t=(p_1-p_2)^2$, the invariant momentum transfer, and $s=(p_1-q_2)^2=2m^2-2\mu^2-u-t$, the crossed channel c.m. energy squared. The differential cross section in the c.m. system can be written[4)]

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2 u} \sum_{\text{spins}} \left| \left\langle f \middle| f_1 + \frac{\boldsymbol{\sigma}\cdot\boldsymbol{q}_1 \boldsymbol{\sigma}\cdot\boldsymbol{q}_2}{q_1 q_2} f_2 \middle| i \right\rangle \right|^2, \tag{2·1}$$

where the matrix element is taken between the two component spinors. (We consider only a definite isospin state.) The functions f_1 and f_2 are related to the two invariant amplitudes $A(u, t)$ and $B(u, t)$ which are assumed to have the ordinary Mandelstam analyticity:

$$f_1(Wt) = (E+m)[A+(W-m)B], \tag{2·2·1}$$

$$f_2(Wt) = (E-m)[-A+(W+m)B], \tag{2·2·2}$$

where $W=\sqrt{u}$ and $E=(W^2+m^2-\mu^2)/2W$ (the c.m. energy of nucleon). The two helicity amplitudes are given by

$$f_1+f_2=2mA+(u-m^2-\mu^2)B\equiv G(u,t), \tag{2·3·1}$$

$$f_1-f_2=W^{-1}[(u+m^2-\mu^2)A+m(u-m^2+\mu^2)B]\equiv W^{-1}H(u,t). \tag{2·3·2}$$

The functions G and H defined here possess the Mandelstam analyticity, and, in particular, holomorphic in the vicinity of $u=0$ for Re t sufficiently large.

The helicity amplitudes $G(u,t)$ and $W^{-1}H(u,t)$ are related to the partial-wave amplitudes $f_J{}^{\pm}(W)$ by

$$G(u,t)=\sum_{J=1/2}^{\infty}[f_J{}^+(W)+f_J{}^-(W)][P'_{J+(1/2)}(z)-P'_{J-(1/2)}(z)], \tag{2·4·1}$$

$$W^{-1}H(u,t)=\sum_{J=1/2}^{\infty}[f_J{}^+(W)-f_J{}^-(W)][P'_{J+(1/2)}(z)+P'_{J-(1/2)}(z)], \tag{2·4·2}$$

where $z=1+t/2q^2$, $q^2=E^2-m^2$, and the superscript $\pm$ refers to the parity $(-1)^{J\pm 1/2}$. Equations (2·4) can be inverted by

$$f_J{}^+(W)+f_J{}^-(W)=\frac{1}{2}\int_{-1}^{+1}G(u,t)[P_{J-(1/2)}(z)+P_{J+(1/2)}(z)]dz\,, \tag{2·5·1}$$

$$f_J{}^+(W)-f_J{}^-(W)=\frac{1}{2}\int_{-1}^{+1}W^{-1}H(u,t)[P_{J-(1/2)}(z)-P_{J+(1/2)}(z)]dz\,. \tag{2·5·2}$$

Let us define the "Khuri amplitudes"[6] associated with the t cut by

$$g(u,\nu)=\frac{t_0{}^{\nu-(1/2)}}{\pi}\int_{t_0}^{\infty}dt\,t^{-\nu-(1/2)}G_t(u,t), \tag{2·6·1}$$

$$h(u,\nu)=\frac{t_0{}^{\nu-(1/2)}}{\pi}\int_{t_0}^{\infty}dt\,t^{-\nu-(1/2)}H_t(u,t), \tag{2·6·2}$$

where $t_0=(2\mu)^2$ is the t-channel threshold and G_t and H_t are the discontinuities across the t cut of G and H, respectively. The integrals in Eqs. (2·6) converge, according to the conventional hypothesis of the polynomial bound, for Re ν greater than some real number N. For Re $\nu<N$ the functions g and h are defined by analytic continuation. Corresponding to the usual assumptions, g and h are assumed to contain only moving poles in the plane for Re $\nu>-L$ where L is some constant.

If $\xi_i(u)$ $(i=1, 2, \cdots)$ are the poles of $g(u,\nu)$ in Re $\nu>-L$, we may write

$$g(u,\nu)=\sum_i\frac{\gamma_i(W)}{\nu-\xi_i(W)}+\bar{g}(u,\nu), \tag{2·7}$$

where $\bar{g}(u, \nu)$ is a function regular in $\mathrm{Re}\,\nu > -L$. The inversion of (2·6·1) is given by

$$G_t(u, t) = \frac{1}{2i}\int_{N-i\infty}^{N+i\infty}\left(\frac{t}{t_0}\right)^{\nu-(1/2)} g(u, \nu)\, d\nu\,. \tag{2·8}$$

Equations (2·8) and (2·7) yield

$$G_t(u, t) = \sum_i \gamma_i(W)\left(\frac{t}{t_0}\right)^{\xi_i(W)-(1/2)} + \frac{1}{2i}\int_{-L-i\infty}^{-L+i\infty}\left(\frac{t}{t_0}\right)^{\nu-(1/2)} \bar{g}(u, \nu)\, d\nu\,. \tag{2·9}$$

On the other hand Eq. (2·5·1) implies that $G_t(u, t)$ contributes to the Froissart-Gribov amplitudes as

$$\frac{1}{2}\{[f^{+(e)}(W, J) + f^{-(e)}(W, J)] + [f^{+(o)}(W, J) + f^{-(o)}(W, J)]\}$$

$$= \frac{1}{2\pi q^2}\int_{t_0}^{\infty} G_t(u, t)\left[Q_{J+(1/2)}\left(1+\frac{t}{2q^2}\right) + Q_{J-(1/2)}\left(1+\frac{t}{2q^2}\right)\right] dt\,. \tag{2·10}$$

The superscript in the bracket refers to the signature. Inserting Eq. (2·9) and using a truncated asymptotic expansion of the Q_l functions we get for the r.h.s. of (2·10)

$$\sum_i \sum_{k=0}^{N_i(W)} \frac{(-1)^k \Gamma(J+k+\frac{1}{2})^2}{k!\,\Gamma(2J+2+k)} \cdot \frac{(2J+1)\gamma_i(W)}{J-\xi_i(W)+k}\left(\frac{4q^2}{t_0}\right)^{J+k-(1/2)}$$

$$+ \text{terms regular in } \mathrm{Re}\,J > -L\,. \tag{2·11}$$

The integers $N_i(W)$ are determined so as to satisfy $-L < \mathrm{Re}[\xi_i(W) - N_i(W)] < -L+1$. We compare the l.h.s. of (2·10) with (2·11) to conclude: (i) The Regge poles located in the region $\mathrm{Re}\,J > -L$ should be sought among the Khuri poles $\xi_i(W)$. The Khuri poles which coincide with one of the Regge poles are called "principal Khuri poles". (ii) For each Regge pole, say $\alpha(W)$, there should exist a family of "satellite Khuri poles" to cancel out the redundant poles $\alpha(W)-k$ $(k=1, 2, 3, \cdots)$.

Given a set of Regge poles $\alpha_n^{\pm}(W)$ $(n=1, 2, 3, \cdots)$ we relabel the Khuri poles according to the criterion introduced above, and rewrite Eq. (2·7) as

$$g(u, \nu) = \sum_n \sum_{\pm} \sum_{r=0}^{K_n^{\pm}(W)} \frac{\gamma_{n,r}^{\pm}(W)}{\nu - \alpha_n^{\pm}(W) + r} + \bar{g}(W, \nu)\,. \tag{2·12·1}$$

Here and henceforth we suppress the signature in order to simplify the notation. By an identical procedure we get

$$h(u, \nu) = \sum_n \sum_{\pm} \sum_{r=0}^{K_n^{\pm}(W)} \frac{\pm W\gamma_{n,r}'^{\pm}(W)}{\nu - \alpha_n^{\pm}(W) + r} + \bar{h}(u, \nu)\,. \tag{2·12·2}$$

In Eqs. (2·12), $K_n^{\pm}(W)$ denotes the greatest integer smaller than $\mathrm{Re}(\alpha_n^{\pm}(W)$

$+L)$. The residues γ and γ' can be expressed in terms of the Regge residues $\beta_n^{\pm}(W)$. In particular, for $r=0$ we have

$$\gamma^{\pm}_{n,0}(W)=\gamma'^{\pm}_{n,0}(W)=\frac{\Gamma(2\alpha_n^{\pm}+1)}{\Gamma(\alpha_n^{\pm}+(1/2))}\left(\frac{t_0}{4q^2}\right)^{\alpha_n^{\pm}-(1/2)}\beta_n^{\pm}(W). \quad (2\cdot13)$$

§ 3. Proof of conspiracy

We begin with examining the analyticity of the Khuri amplitudes $g(u,\nu)$ and $h(u,\nu)$ in the u plane. Apart from a finite number of subtractions the absorptive part $G_t(u,t)$ has the following integral representation due to the assumption of the Mandelstam analyticity on G:

$$G_t(u,t)=\frac{1}{\pi}\int_{u_0}^{\infty}\frac{\rho_{tu}(t,u')}{u'-u}du+\frac{1}{\pi}\int_{s_0}^{\infty}\frac{\rho_{ts}(t,s')}{s'+t+u-2m^2-2\mu^2}ds', \quad (3\cdot1)$$

with $u_0=s_0=(m+\mu)^2$. Then, in view of Eq. (2·6·1) one sees that, for Re $\nu>N$, $g(u,\nu)$ is regular in the u plane with the right-hand and the left-hand cuts along the real axis. The right-hand cut starts at u_0, while the end point of the left-hand cut corresponds to the maximum value of u on the Landau curves of the diagrams shown in Fig 1. This maximum u_1 is attained by the curve of the diagram (a) and is calculated to be $-1.5\,\mu^2$. It is a crucial point for the following discussion that u_1 is *negative* and that $g(u,\nu)$ turns out to be regular at $u=0$. In the W plane the functions g and h have four cuts along the real and imaginary axes as is shown in Fig. 2.

With this analytic property of g and h in mind, we *assume* that the trajectory functions $\alpha_n^{\pm}(W)$ are regular in some region in the W plane containing the

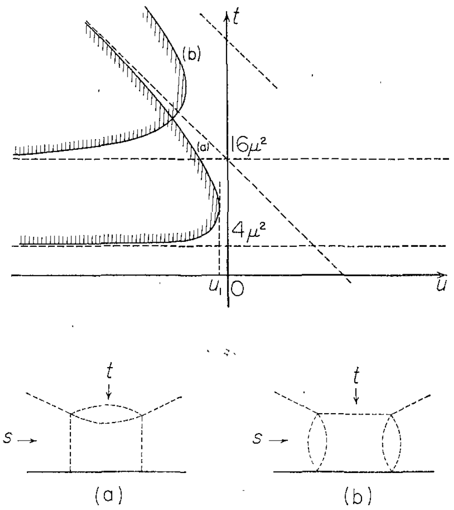

Fig. 1. The diagrams and their Landau curves The u_1 corresponds to the left-hand branch point of Khuri amplitudes ($u_1\approx-1.5\mu^2$).

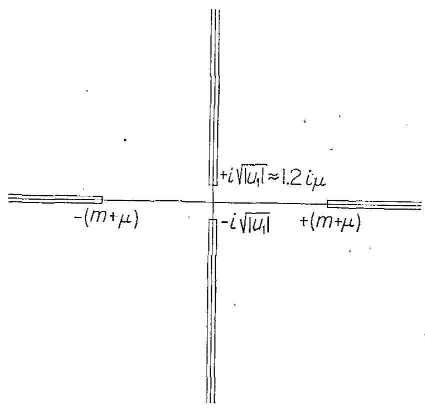

Fig. 2. The analyticity of Khuri amplitudes.

point $W=0$. In the most optimistic case, as suggested by the N/D method, this region may be the whole W plane with unitary cuts. Now, we define the region $\mathscr{D}$ around $W=0$ such that no principal Khuri poles cross the line $\mathrm{Re}\,\nu=-L$ when W takes all values of $\mathscr{D}$. (Here it is assumed that no principal poles lie on the line $\mathrm{Re}\,\nu=-L$ as $W=0$.)

Next we revise the r.h.s. of Eqs. (2·12) and let the summation include all the satellite terms that appear in $\mathrm{Re}\,\nu>-L$ for some value in $\mathscr{D}$, that is, we replace $K_n^{\pm}(W)$ by constant integers satisfying

$$0<K_n^{\pm}\leq \mathrm{Re}[\alpha_n^{\pm}(W)+L]. \tag{3·2}$$

After this modification the separation of the pole terms in Eqs. (2·12) is now analytic with respect to W.

The first step of our proof is to show that the residue functions $\gamma_{n,r}^{\pm}$ and $\gamma_{n,r}'^{\pm}$ cannot have cuts in $\mathscr{D}$. Suppose that the $\gamma_{n,r}^{\pm}$'s and/or $\bar{g}$ have the vertical cuts $W=\pm i\rho\,(\rho\geq\sqrt{|u_1|})$ as $g(u,\nu)$ does. Then Eq (2·12·1) gives, for $\mathrm{Re}\,\nu>N$,

$$\underset{u\leq u_1}{\mathrm{Disc}}\, g(u,\nu)=\sum_n\sum_{\pm}\sum_{r=0}^{K_n^{\pm}}\frac{\mathrm{Disc}\,\gamma_{n,r}^{\pm}(W)}{\nu-\alpha_n^{\pm}(W)+r}+\underset{W}{\mathrm{Disc}}\,\bar{g}(W,\nu). \tag{3·3}$$

The l.h.s. of this equation can be computed by using Eqs. (3·1) and (2·6·1):

$$\underset{u\leq u_1}{\mathrm{Disc}}\, g(u,\nu)=\frac{t_0^{\nu-(1/2)}}{\pi}\int_{t_0}^{2m^2+2\mu^2-u-S_0} t^{-\nu-(1/2)}\rho_{ts}(t,2m^2+2\mu^2-t-u)\,dt\,. \tag{3·4}$$

Since the integration range is finite, $\mathrm{Disc}\,g(u,\nu)$ is an entire function of ν. Each term in Eq. (3·3) can now be analytically continued into the region $-L<\mathrm{Re}\,\nu\leq N$ where $\mathrm{Disc}\;\bar{g}(W,\nu)$ is regular, and we see that $\mathrm{Disc}\,\gamma_{n,r}^{\pm}(W)$ should vanish as far as $\mathrm{Re}\,\alpha_n^{\pm}(W)>-L$. In a similar way we can show that the $\gamma_{n,r}^{\pm}$'s cannot have branch points anywhere else in $\mathscr{D}$. The $\gamma_{n,r}'^{\pm}$'s also should have the same analyticity.*)

As is seen from the previous discussion the functions $g(u,\nu)$ and $h(u,\nu)$ are regular at $u=0$ and therefore regular even functions of W as far as $\mathrm{Re}\,\nu>N$. Then we have, due to Eqs. (2·12),

$$\sum_{n,\pm,r}\frac{\gamma_{n,r}^{\pm}(W)}{\nu-\alpha_n^{\pm}(W)+r}+\bar{g}(W,\nu)=\sum_{n,\pm,r}\frac{\gamma_{n,r}^{\pm}(-W)}{\nu-\alpha_n^{\pm}(-W)+r}+\bar{g}(-W,\nu), \tag{3·5·1}$$

*) We remark here that the above result could not be obtained if one had worked directly with the (reduced) Froissart-Gribov amplitudes: In unequal-mass cases the amplitudes have a circular cut which unables us to continue the functions analytically to the point $u=0$. In other words the usual integral

$$\frac{1}{2\pi\,(q^2)^{l+1}}\int_{t_0}^{\infty}dt\,A_t(u,t)\,Q_l\left(1+\frac{t}{2q^2}\right)$$

defines different analytic functions inside and outside the circular cut respectively. This point has not been considered by Freedman and Wang in Appendix A of their paper[7] where they tried to show the absence of cuts around $u=0$.

$$\sum_{n,\pm,r}\frac{\pm W\gamma'^{\pm}_{n,r}(W)}{\nu-\alpha_n{}^{\pm}(W)+r}+\bar{h}(W,\nu)=\sum_{n,\pm,r}\frac{\pm(-W)\gamma'^{\pm}_{n,r}(-W)}{\nu-\alpha_n{}^{\pm}(-W)+r}+\bar{h}(-W,\nu). \quad (3\cdot5\cdot2)$$

By continuing both sides of these equations analytically into $N\geqslant \mathrm{Re}\,\nu>-L$, one sees that there should occur a pairing between the families of trajectories such that

$$\alpha_n{}^{p}(W)=\alpha_{n'}^{p'}(-W), \quad (3\cdot6\cdot1)$$

$$\gamma^{p}_{n,r}(W)=\gamma^{p'}_{n',r}(-W) \quad (3\cdot6\cdot2)$$

and

$$p\gamma'^{p}_{n,r}(W)=-p'\gamma'^{p'}_{n',r}(-W), \quad (3\cdot6\cdot3)$$

where p denotes the parity $\pm$. The last two equations imply, with a special choice $r=0$ and with the help of Eq. $(2\cdot13)$,

$$p=-p' \quad \text{and} \quad \gamma^{p}_{n,0}(W)=\gamma^{p'}_{n',0}(-W). \quad (3\cdot7)$$

That is, after suitable relabelling of the trajectories, we get finally for Regge poles

$$\alpha_n{}^{+}(W)=\alpha_n{}^{-}(-W) \quad \text{and} \quad \bar{\beta}_n{}^{+}(W)=\bar{\beta}_n{}^{-}(-W), \quad (3\cdot8)$$

where $\bar{\beta}(W)=(q^2)^{-\alpha+(1/2)}\beta(W)$ is the reduced Regge residue.

We have so far suppressed the signature. At this stage, the relations in $(3\cdot8)$ may be the relations between the same signature as well as between the different signature. However, if we consider the Khuri amplitude associated with the s cut, we can show that the relations $(3\cdot8)$ are true only for the *same* signature. Define the Khuri amplitude associated with the s cut by

$$\hat{g}(u,\nu)=\frac{s_0{}^{\nu-(1/2)}}{\pi}\int_{s_0}^{\infty}ds\, s^{-\nu-(1/2)}G_s(u,s), \quad \mathrm{Re}\,\nu>N, \quad (3\cdot9)$$

where G_s is the discontinuity across the s cut of G. By an argument similar to the case of $g(u,\nu)$ one can see that $\hat{g}(u,\nu)$ is also regular at $u=0$. The equation corresponding to $(2\cdot10)$ is

$$\frac{1}{2}\{[f^{+(e)}(W,J)+f^{-(e)}(W,J)]-[f^{+(o)}(W,J)+f^{-(o)}(W,J)]\}$$

$$=\frac{1}{2\pi q^2}\int_{s_0}^{\infty}G_s(u,s)\left[Q_{J-(1/2)}\left(\frac{s+u-2m^2-2\mu^2}{2q^2}-1\right)\right.$$

$$\left.+Q_{J+(1/2)}\left(\frac{s+u-2m^2-2\mu^2}{2q^2}-1\right)\right]ds. \quad (3\cdot10)$$

By quite the same procedure as that of the t-cut case, we have for the pole terms of $\hat{g}(u,\nu)$

$$\sum_{n,p,r} \frac{\hat{\gamma}_r^{(e)}}{\nu-\alpha^{(e)}+r} - \sum_{n,p,r} \frac{\hat{\gamma}_r^{(o)}}{\nu-\alpha^{(o)}+r}, \tag{3·11}$$

while the pole terms of $g(u, \nu)$ are of the form

$$\sum_{n,p,r} \frac{\gamma_r^{(e)}}{\nu-\alpha^{(e)}+r} + \sum_{n,p,r} \frac{\gamma_r^{(o)}}{\nu-\alpha^{(o)}+r}, \tag{3·12}$$

where the other summation indices n and p are omitted. For $r=0$, one obtains the relations

$$\hat{\gamma}_0^{(e)}=\gamma_0^{(e)}, \quad \hat{\gamma}_0^{(o)}=\gamma_0^{(o)}. \tag{3·13}$$

Now, let us assume that the relations (3·8) are the relations between the different signature, i.e.

$$\alpha^{(e)}(W)=\alpha^{(o)}(-W), \quad \gamma_0^{(e)}(W)=\gamma_0^{(o)}(-W). \tag{3·14}$$

On the other hand, since $\hat{g}(u, \nu)$ is a regular even function of W, we have, from (3·11) and (3·13),

$$\gamma_0^{(e)}(W)=-\gamma_0^{(o)}(-W). \tag{3·15}$$

This contradicts (3·14), and hence we reach the conclusion that the conspiracy relations (3·8) hold for the same signature.

References

1) D. Z. Freedman and J. M. Wang, Phys. Rev. **160** (1967), 1560.
M. Toller, Nuovo Cim. **37** (1965), 631.
2) V. N. Gribov, JETP **43** (1962), 1529 [Soviet Phys.-JETP **16** (1963), 1080].
3) I. A. Sakmar, Phys. Rev. **135B** (1964), 249.
4) S. W. MacDowell, Phys. Rev. **116** (1960), 774.
5) V. Barger and D. Cline, Phys. Letters **26B** (1967), 85; Phys. Rev. Letters **20** (1968), 298.
6) N. N. Khuri, Phys. Rev. Letters **10** (1963), 420; Phys. Rev. **132** (1963), 914.
7) D. Z. Freedman and J. M. Wang, Phys. Rev. **153** (1967), 1596.