CHAPTER 29

Refraction of Finite-Height and Breaking Waves
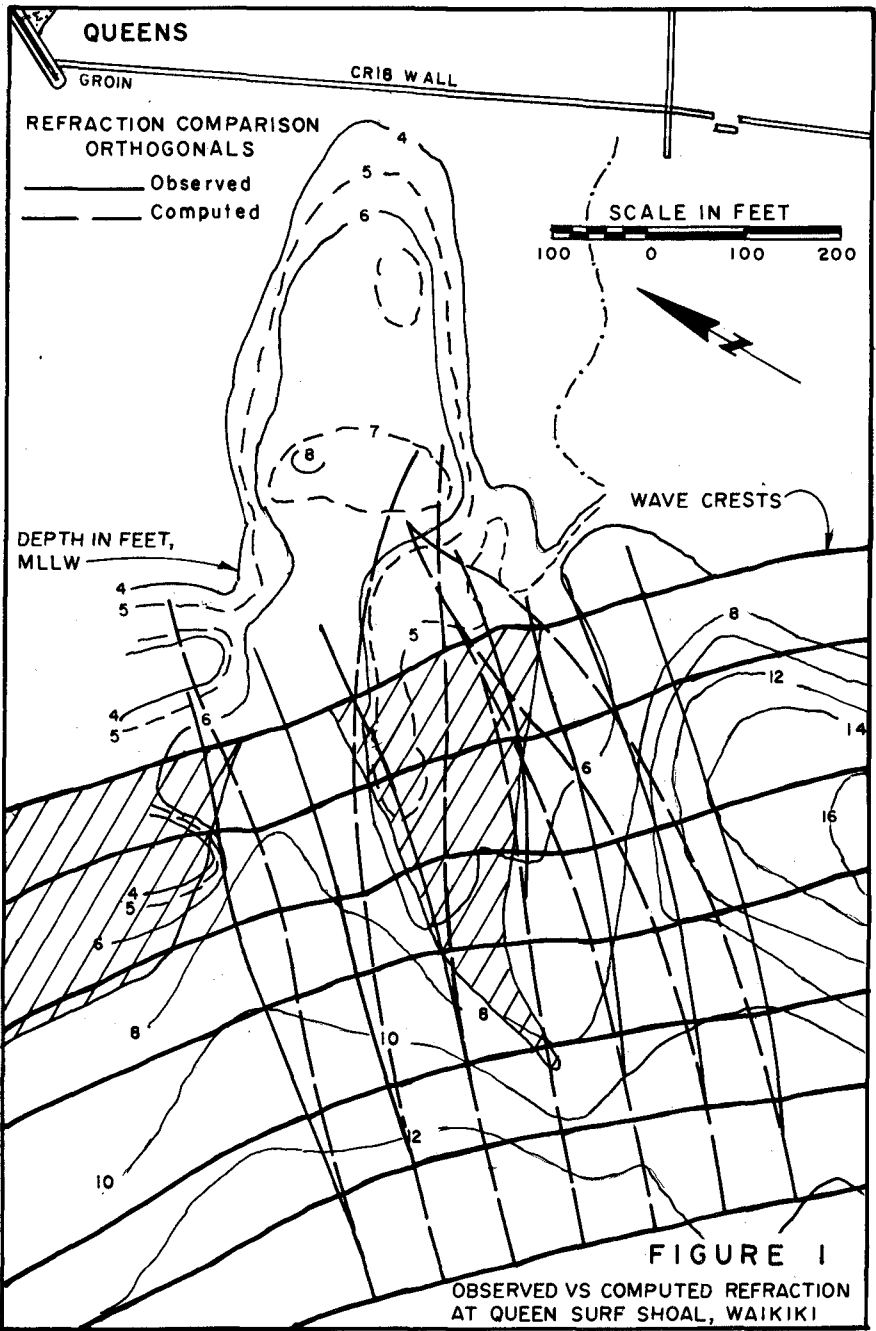
by James R. Walker, Ph.D., P.E.*

ABSTRACT

The primary objective of this study was to ascertain the influence of wave height and breaking on wave refraction over a three-dimensional shoal. The subject wave transformations were studied in an hydraulic model. Wave shoaling, decay in the breaker zone, and phase velocities were analyzed in a base test series over a bottom slope of 1:30. A second test series was conducted over a three-dimensional shoal. Wave patterns were photographed and wave heights and celerities were measured. The measurements were compared with wave refraction patterns and coefficients computed by analytical methods. Wave shoaling observed over the constant 1:30 slope was 25 percent greater than predicted by Airy theory at the breaking point for wave steepness $H_0/L_0 = .030$ and 50 percent greater than predicted for $H_0/L_0 = .002$. Shoaling measurements were compared with other empirical data sets, confirming the inadequacy of commonly used practice using linear wave theory near the breaker zone. The celerity measurements indicated that the non-breaking celerity was given by $C = (1 + .25 \ H/d)C_a$, where C_a is the Airy celerity. The discussion and results give a basic understanding of wave refraction near the breaker zone, supplementing analytical papers on refraction procedures using finite amplitude wave theories.

INTRODUCTION

The topic for this study evolved from field investigations of wave transformations over recreational surfing shoals in Hawaii. Walker, Palmer, and Kukea (1972) noted through observation of aerial photographs that, in some cases, breaking waves had a diverging pattern over the centerline of the shoal where linear refraction theory predicted a strong convergence. Figure 1 shows an observed wave at 5-second intervals propagating over Queens surf shoal in Waikiki. Orthogonals drawn from the wave crests are compared with those computed by linear refraction theory. The observed orthogonals tended to be considerably less affected by the bathymetry than were the theoretical orthogonals. This observation raised several questions concerning the applicability of extending refraction analysis into the breaker zone, especially over an irregular shoal. Whalin (1971) observed effects of finite height altering wave refraction patterns, but focused his attention on diffraction. Weigel and Arnold (1957) state that refraction theory is valid within two to three wave lengths seaward of the breaking position. Unfortunately, the coastal engineer often extends the limits of theory

---

*Moffatt & Nichol, Engineers, Long Beach, California.

QUEENS
GROIN   CRIB WALL

REFRACTION COMPARISON
  ORTHOGONALS
_____ Observed
___ ___ Computed

SCALE IN FEET
100   0   100   200

WAVE CRESTS

DEPTH IN FEET,
MLLW

FIGURE 1
OBSERVED VS COMPUTED REFRACTION
AT QUEEN SURF SHOAL, WAIKIKI

into the surf to acquire the required design wave or frequency of occurrence of a given wave height.

The purpose of this study is to determine the influence of finite height and breaking on wave refraction over a three-dimensional shoal for application to surf shoal design and prediction of the influence of the shoal on adjacent beaches. The effects of diffraction, wave-induced currents, reflection, and energy dissipation other than breaker decay are assumed negligible in this study. This study was conducted in an hydraulic model of a shoal of specific shape; however, the results also have general application in understanding the nature of error and order-of-magnitude of error when conventional refraction techniques are employed near the breaking region.

THE MODEL

The primary objective of the model was to study the transformation of waves propagating from deep water into the surf zone over a three-dimensional shoal. Several compromises were made in design of the model basin and test shoal. Since wave propagation was to be followed through the surf zone, the scale should be as large as possible to reduce scale effects due to viscous dissipation and air entrainment. The bottom slope should be constant for a distance of at least a wave length seaward of the test shoal according to criteria developed by Camfield and Street (1966). The bottom slope should be representative of conditions typically found in nature. The wave generator should be located in water depth to wave length ratio $d/L > .1$ such that wave shoaling could be measured from relatively deep water and to ensure that secondary wave crests described by Galvin (1968) would not be generated.

Space limitations restricted the basin length to 50 feet and the wave generator established the basin width at 20.6 feet. Figure 2 shows the model basin dimensions. The basin side walls were 2 feet and the water depth was 1.8 feet. The test shoal was modeled after a general concept of a surf site discussed by Walker and Palmer (1971).

The bathymetry of the test shoal was required to:

a. induce a theoretical wave orthogonal convergence by application of linear refraction theory;

b. induce a wave to break in a form similar to that at a recreational surfing shoal;

c. produce a spilling to plunging breaker-type;

d. have minimum reflection effects and minimum effects from side walls or beach;

e. have sufficient water depth to minimize scale effects induced by dissipation at the bottom boundary and in the turbulent breaking region; and,

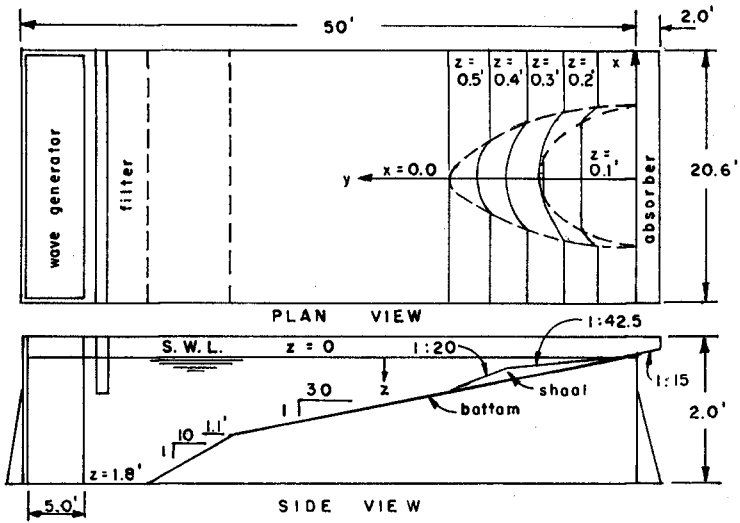f. have the parameters under study be of a scale sufficient to deduce significant conclusions.

PLAN   VIEW

SIDE   VIEW

Figure   Model Basin & Shoal
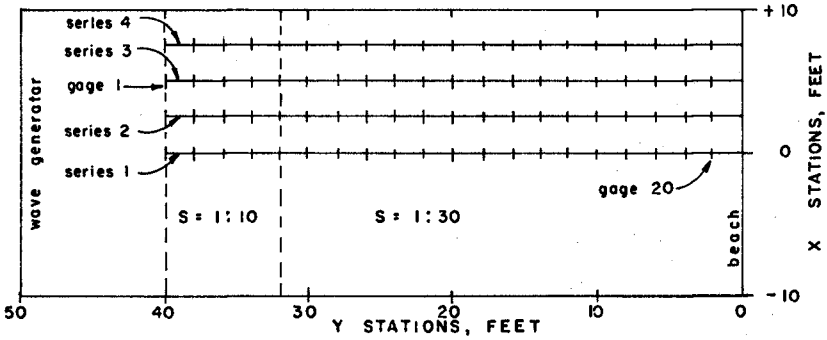

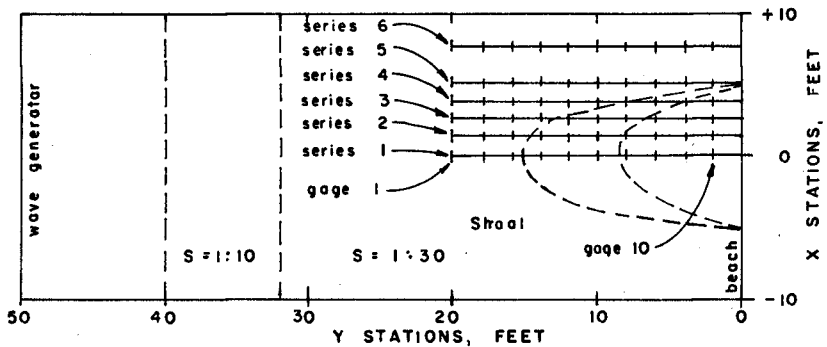Figure   Base Test Series Gage Locations

Figure   Shoal Test Series Gage Locations

The selected shoal shape was a truncated cylinder inclined on a 1:20 slope relative to the horizontal. The cylinder was truncated on the bottom by the 1:30 basin slope and on the top by a 1:42.5 slope. The plan view of a truncated cylinder transcribes an ellipse. The semi-major axis of the 1:30 slope was 15 feet and was oriented parallel to the direction of wave approach. The semi-minor axis was 6.71 feet on a horizontal projection. The shoal and coordinate system are shown in Figure 2. The toe of the shoal starts at station $x = 0$ feet, $y = 15$ feet and intersects the shoreline at station $y = 0$ feet, $x = \pm 5$ feet. The 1:42.5 slope starts at the shoreline to form a plane which also transcribes an ellipse. The 1:42.5 slope intersects the 1:20 slope at $z = 0.222$ feet at station $y = 9.44$ feet, $x = 0$ feet. The bathymetry of the model and shoal for y less than 33 feet is give by three equations:

1. $z_1 = y/30$, for $y < 33$, $/x/ \geq 5.0(1. - y^2/15^2)^{1/2}$

2. $z_2 = (y/20) - (15/20)(1. - x^2/45)^{1/2} + 0.5$, for $y < 15$, $/x/ \leq 5.0(1. - y^2/15^2)^{1/2}$

3. $z_3 = y/42.5$, for $y \leq 9.44$, $/x/ \leq 5.0(1. - y^2/9.44^2)^{1/2}$

The depth, z, is everywhere less than or equal to the z_1 plane and greater than or equal to the z_3 plane. The curved portion of the shoal is described by z_2.

The model basin and shoal were constructed of reinforced, finished concrete. Measurements of the bathymetry after construction indicated that the 1:30 bottom slope was accurate to within ± 0.005 feet and that the test shoal was accurate to within ± 0.010 feet.

Waves were generated by a 20.5-foot-long plunger-type wave generator. The wave period was adjusted on a variable speed crank and the amplitude of the waves was changed by adjusting the stroke of the plunger. A wave filter was placed in front of the generator to reduce high frequency noise produced by the generator. A wave-absorbing beach was placed at the shoreline along $x = 0$ to reduce the entry of reflections into the measurement area. The absorber comprised a 1:15 beach slope covered by two layers of 1/4-inch stone. The stones extended on the 1:30 slope to $y = 1.8$ feet.

Wave heights were measured with double-wire resistance-type wave rods. Signals were displayed on a light beam oscillograph recorder. Wave crest patterns were photographed from a platform located 20 feet above the model.

Two series of tests were conducted using 15 test waves in each series. The base test series was conducted over the constant sloping bottom to calibrate the model and waves. The shoal test series was conducted using the same generator settings as the base tests to monitor the changes in wave response induced by the bathymetry.

The 15 test waves selected for study had periods of 1.16, 1.67, 2.00, and 2.33 seconds. Four wave generator settings were used with each wave period with the exception that the four-inch half-stroke was not used with the 1.16-second period. This latter wave overtopped the

basin walls. The tests covered a range of waves which broke approximate-
ly between station $y = 2$ feet, depth $z = 0.067$ feet, and station $y = 12.5$
feet, $z = 0.42$ feet on the constant slope. The breaker heights ranged
from 0.08 feet to 0.38 feet.

Eighty-one gage locations were used for the base tests. Eighty
gages were located on I-beams along $x = 0$, 2.5, 5.0, and 7.5 feet and at
$y = 2$, 4, 6,...40 feet. Figure 3 shows the locations of the gages during
the base tests. In addition to the evenly spaced gages, a portable gage
was used during a separate run of the group of test waves to measure
breaker heights and locations.

Sixty-one gage locations were monitored in the shoal tests. The
gages were located at stations $x = 0$, 1.25, 2.5, 3.75, 5.0 feet, and 7.5,
and at $y = 2$, 4, 6,...20 feet. Figure 4 shows the location of the gages
for the shoal series of tests. An additional group of test waves was
run with a portable gage to measure the initial breaker height on the
centerline of the shoal at $x = 0$ feet.

Wave rod calibrations were taken prior to and after each group of
test waves had been run. The gage calibrations were slightly non-linear
over a large range of submergence. A second order polynomial was fit to
seven calibration points by the method of least squares to describe the
calibration curve. Calibration on gage 13, located at station $y = 16$
feet on the wave test, was unstable and the results were assumed to be
unreliable.

Test procedures consisted of running the wave generator for approx-
imately one minute while the recording was made. The recording chart
speed was 1/4 inch per second for the series of gages from stations $y =$
22 feet to $y = 40$ feet, and one inch per second for the gages located
from stations $y = 2$ feet to $y = 20$ feet.

Small variations in the generated wave height and tolerances in
basin depths caused the wave to break non-uniformly across the basin.
The variations in wave breaker position induced a current system which
influenced the refraction and the breaking patterns. Even on the
straight beach, the breaking pattern varied considerably when the gen-
erator was allowed to run for an extended period of time. For this reas-
on, measurements were confined to the first five to ten waves, which
were relatively unaffected by the induced currents. In most cases,
little temporal variation in the wave height was evident. The great-
est temporal wave height variations occurred when a gage was located
close to the breaking point. The average wave celerity between suc-
cessive gages was determined by measuring the time differential of
crest passage between gages separated by 2-foot intervals.

The results of this investigation may be viewed in terms of dimen-
sionless parameters. However, lengths and times may be scaled for ap-
plication to coastal design. Scaling of the model was done through use
of the Froude modeling relation. Viewed in terms of a model experiment
of a prototype surf shoal, the length scale selected in the design of
the model was 1:36 and the velocity and time scales were 1:6.

        The laminar dissipation was calculated by the method of Keulegan
(1950) to obtain an estimate of the dissipation in the model prior to
wave breaking. Approximately five percent of the energy from non-break-
ing waves was dissipated by laminar dissipation by the time the wave
reached station $y = 2$ feet. Horikawa and Kou (1962) indicated that wave
height decay in the breaking region was greater in the model than in
the prototype. When applying these data to a prototype situation,
these factors must be taken into account. Wave energy reflecting from
the beach was computed by the Miche (1951) formula. The maximum cal-
culated reflected wave height, H_r, was less than 14 percent of the in-
cident wave.

        The accuracy of the model procedures was estimated from a prelim-
inary test series. The wave rods were spaced 2 inches apart and strung
across the basin at $y = 26$ and 27. Eight test waves were run to deter-
mine the variation in wave height. The measured wave heights were with-
in a ±10 percent deviation at two standard deviations. Wave height var-
iation across the basin introduced an additional ±5 percent error. Wave
celerity was estimated to be within 10 percent error due to tolerance
in positioning the wave gage and in reading the zero phase position on
the wave.

                              RESULTS AND ANALYSIS

Wave Shoaling

        The base test series was conducted to study transformations of
waves propagating from intermediate water depths over a sloping bottom
into the breaking region in shallow water. The primary topics investi-
gated were wave height growth due to shoaling water depths and the in-
fluence of wave height and wave breaking on wave celerity. Measure-
ments were compared with values predicted by application of linear wave
theory commonly used in refraction analysis.

        The characteristics of incident test waves and their breaking pro-
perties measured in the model are summarized in table 1. Figure 5 shows
a typical wave height profile as a function of distance offshore. The
figure indicates the typical scatter and accuracy of the data. The
breaking position was measured by a single gage. An equivalent deep-
water wave height was determined by application of the Airy shoaling
coefficient to the wave measurements at gages located at Stations 26,
28, 30 and 32. Utilizing the calculated deep-water wave height and the
Airy shoaling coefficient, the predicted wave height is shown by the
dashed line in Figure 5. The wave shoaling measured in the model in-
creases much more rapidly than the predicted wave shoaling. This in-
crease is most noticeable just prior to the wave breaking position.
The slope of the measured wave height profile appears to increase sig-
nificantly when $H/d \geq .25$.

        Wave shoaling from each of the tests was plotted in dimensionless
form as a function of d/L_0. The data were smoothed and shoaling curves
are shown in Figure 6 as a function of H_0/L_0. The data are compared
with Airy shoaling theory. The rate of shoaling increases rapidly just
prior to breaking. The low steepness waves, $H_0/L_0 = .002$, break at a

TABLE 1

SUMMARY OF TEST WAVES

| Test No.	Wave Generator Setting		Deep-Water Wave Height & Steepness			Breaking Wave Properties, X = 0.0				Breaking Wave Parameters, Base Test		
	Generator 1/2 Stroke in.	T sec.	H_o' ft.	L_o ft.	H_o'/L_o	Base Tests H_b ft.	Y station ft.	Shoal Tests H_b ft.	Y station ft.	H_b/H_o' measured	H_b/H_o' predicted*	H_b/d_b
1	1	1.17	0.092	6.9	0.01334	.141	4.5	.163	8.0	1.52	1.38	.942
2	1	1.67	0.062	14.2	0.00437	.119	4.0	.139	7.3	1.92	1.81	.893
3	1	2.00	0.045	20.4	0.00224	.105	3.5	.111	6.0	2.29	2.16	.904
4	1	2.33	0.034	27.8	0.00123	.083	3.0	.097	5.0	2.42	2.51	.832
5	2	2.33	0.074	27.8	0.00267	.168	6.0	.156	7.5	2.25	2.05	1.009
6	2	2.00	0.097	20.4	0.00476	.195	5.5	.198	8.3	2.00	1.78	1.066
7	2	1.67	0.125	14.2	0.00885	.236	6.3	.216	9.0	1.88	1.52	1.126
8	2	1.17	0.180	6.9	0.02595	.250	7.5	.230	10.0	1.38	1.16	1.240
9	3	1.17	0.261	6.9	0.03759	.320	9.8	.269	11.0	1.23	1.06	.980
10	3	1.67	0.185	14.2	0.01301	.312	9.0	.279	10.5	1.67	1.38	1.041
11	3	2.00	0.145	20.4	0.00713	.304	7.2	.271	9.4	2.08	1.61	1.267
12	3	2.33	0.112	27.8	0.00403	.245	6.5	.215	9.1	2.19	1.86	1.131
13	4	2.33	0.152	27.8	0.00546	.285	8.5	.264	10.1	1.89	1.72	1.005
14	4	2.00	0.200	20.4	0.00981	.343	9.0	.266	10.0	1.72	1.49	1.144
15	4	1.67	0.247	14.2	0.01738	.379	12.2	.306	12.5	1.53	1.29	.931
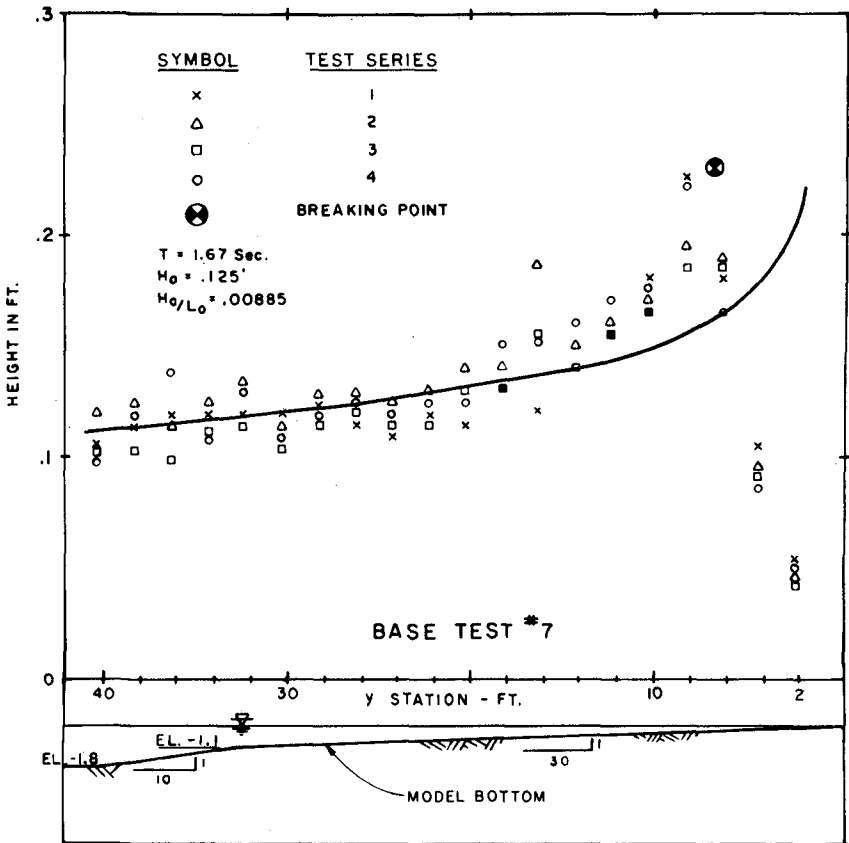
FIGURE 5 - WAVE SHOALING, TEST 7

height 50 percent greater than the Airy shoaling coefficient predicts. The higher initial steepness waves, $H_O/L_O = .03$, break at a height 25 percent greater than Airy theory predicts.

   Figure 6 represents an empirical set of wave shoaling curves for waves propagating over a 1:30 slope. The significance of these results is that the waves in shallow water shoal at a much greater rate than Airy theory predicts. Consequently, application of Airy theory in shallow water predicts a significantly lower wave height than observed in this model. These results are similar to those obtained by Iwagaki and Sakai (1972). The wave shoaling curves given in Figure 7 are compared with the shoaling curves based on cnoidal wave theory developed by Svendsen and Kjaer (1972) and stream function theory by Dean (1974). The comparison shows similar trends, height of shoaling as a function of H_O/L_O. Cnoidal theory predicts a shoaling height less than unity for d/L_O greater than .05. This is not confirmed by observation. Cnoidal theory also appears to predict a slightly greater rate of wave shoaling than measured for H_O/L_O less than 0.01. However, the comparison is not strictly valid since H_O' rather than H_O was measured. Stream function theory compares well with the expected set for $H_O/L_O \geq .01$ with the exception that the published data uses the breaker index $H_b/d_b = .78$ compared to the observed index $= 0.95$. This may be attributable to the effect of bottom slope not accounted for in the theory. For $H_O/L_O < .01$, the stream function shoaling is significantly greater than the observed. This is attributed in part to the estimate of H_O made in the model at the low d/L_O region. The wave had undergone more transformation than predicted by Airy theory. Consequently, the indicated $H_O/L_O = .002$ curve may in fact be representative of some lower H_O/L_O. Conversely, the shoaling curves are theoretical over a constant sloping bottom. The breaker index is equal to 1 for the measured data as drawn in Figure 6. Shoaling from deeper water $d/L_O > .1$ through breaking for $H_O/L_O < .01$ over different bottom slopes requires further research; however, the results of the studies indicate the magnitude of error when Airy theory is invoked. Figures 6 and 7 could be used in design, model, and field studies and in evaluating wave data.

   Comparison of measured breaking height to deep-water height with values predicted by empirical data developed by Koh and Le Méhauté (1966) indicate that predicted values are 10 percent to 15 percent lower than measured. The difference between the observed and predicted values is attributed in part to the method of estimating the deep-water wave height. Rarely in the field or model is the deep-water wave measured and directly related to a shallow water breaking wave. Application of a linear shoaling coefficient to a measured wave height introduces a potential error which increases in magnitude with decreasing d/L_O. The relative breaker height to breaker depth index, B, is also listed in Table 1. The average value of $B = .95$ for this data set. Application of methods developed by Galvin (1968) predicted an average value of $B = .85$.

Wave Celerity

   The average phase velocity, $\bar{c}$, between two wave gages was determined by dividing the elapsed time for passage of the wave crest between gages by the distance between the gages. Figure 8 shows typical average
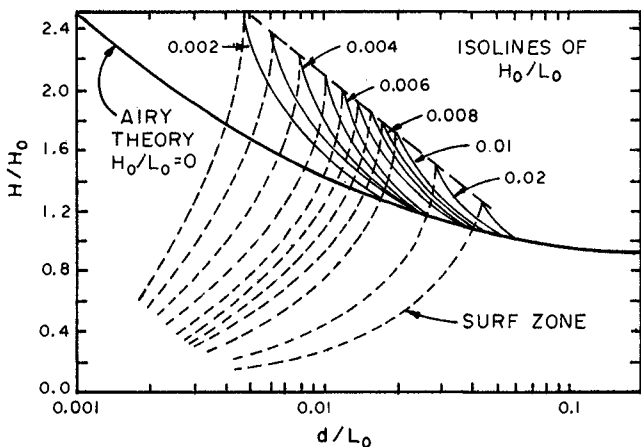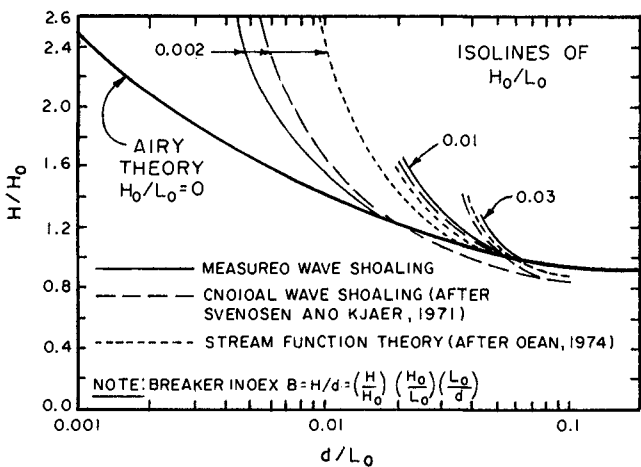
FIGURE 6 – WAVE SHOALING

FIGURE 7

COMPARISON BETWEEN MEASURED
AND THEORETICAL WAVE SHOALING

celerities measured in the model in the base tests. The celerities in-
crease as a function of wave height.

     The measured celerities were compared with celerities commonly used
in refraction analysis by defining a Froude number, $F_r = \bar{c}/\bar{c}_a$,

     where $\bar{c}_a = \dfrac{1}{y_1-y_2} \int_{y_2}^{y_1} c \, dy = \dfrac{1}{y_1-y_2} \int_{y_2}^{y_1} \dfrac{gT}{2\pi} \tanh(dk) dy$,

     where $k = \dfrac{2\pi}{L}$, and y_i is gage i coordinate.

     The Froude numbers are plotted in Figure 8 as a function of H/d.
The results indicated a significant departure from Airy theory which
would have had a $F_r = 1$ in all regions. Several theoretical expressions
were plotted against the data; however, the simple expression below
fits the data through the non-breaking region well:

$$F_r = 1 + 0.25 \frac{H}{d}$$

This expression gives a maximum Froude number of 1.25 at breaking where
H/d = 1. The curve is applicable to the region prior to breaking. Airy
theory predicts celerity approximately 6 percent too low at H/d = .25.
This is also the region where the shoaling curve deviated noticeably
from the Airy theory.

     The Froude number increases with H/d until it exceeds 1.25 at break-
ing. The breaking and broken waves are shown in Figure 8 above $F_r = 1.25$.
The maximum value was $F_r = 1.4$. The higher values appear to occur when
the wave breaks. A discontinuity in wave phase appears to occur during
the plunge of the breaker as the crest curls over the face of the wave.
This is a short-duration occurrence, but accounts for observed appar-
ent divergences in wave form in aerial photos and high Froude numbers
in Figure 8. Data were insufficient to ascertain a reliable expression
for wave celerity in the surf zone. Considering wave decay, scale ef-
fects, and the change of phase during breaking, $c = \sqrt{g(d+H)}$ when H = .78d
or $F_r = 1.33$ gives a reasonable approximation in the surf zone. An analy-
sis was made using wave setup, η, in the surf zone. Setup was included
in the velocity term where $c_b = \sqrt{g(d+\eta)}$; however, the analysis did not re-
duce the scatter of data significantly. After the wave breaks, however,
the application of refraction loses some meaning. The wave crest pat-
terns become the only meaningful relation unless wave decay can be mod-
eled more accurately in the surf zone in regions of converging and diverg-
ing orthogonals.

Shoal Tests – Refraction

     The shoal test series was conducted to study the influence of fi-
nite wave height and breaking on wave refraction. Comparison of wave
crest patterns measured in the model and computed by linear refraction
theory are shown in Figure 9. The comparison shows the wave pattern of
a wave as it propagates over the shoal. The wave crests and initial
surf line are traced over the shoal for test waves 3 and 14. Compari-
son of the theoretical with the measured refraction patterns shows that
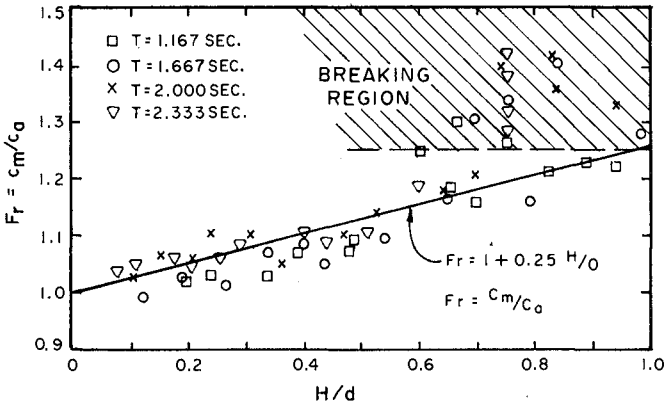the low-height wave represented by test 3 tends to follow the conver-
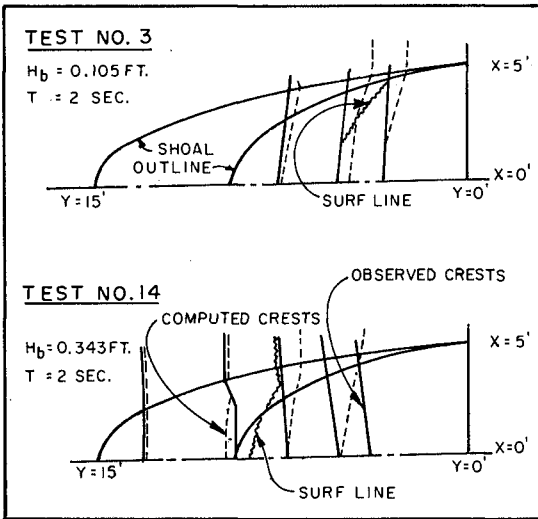
FIGURE 8 — Fr. VS H/d

FIGURE 9 – OBSERVED VS COMPUTED
WAVE CREST PATTERNS

gent patterns predicted by the theoretical analysis. With increase in wave height represented by test 14, the pattern tends to become divergent over the center of the shoal.

Computed refraction coefficients, K_r, range from 0.75 to 2.0. The waves approaching the shoal converge toward the centerline over the curved portion of the shoal. The conventional refraction analysis predicts wave concentration over the shoal centerline. The high refraction coefficients are located over the center of the shoal and the low coefficients along the side of the shoal. The observed wave patterns vary considerably as a function of wave height. The most noticeable feature of the observed patterns is the marked divergence in the vicinity of breaking as shown in the large wave in test 14.

The conclusion drawn from these observations is that finite height and wave breaking have a very significant influence on wave refraction over a shoal. The waves with small heights in Tests 1 through 5 converged toward the center of the shoal and conformed in general with linear refraction patterns up to a short distance seaward of breaking. The larger waves, however, tended to diverge. The combination of orthogonal divergence and diffraction, current, and reflection effects reduced the breaker height to less than that over the planar 1:30 slope for the larger waves. The application to design indicates that refraction over the shoal may not necessarily increase the wave height as would intuitively be expected.

MODIFIED REFRACTION TECHNIQUE

The influences of wave height and wave breaking were show to affect wave refraction patterns in the hydraulic model. Linear refraction procedures were modified to account for the effects of wave height and wave breaking. The basic algorithm utilized the results of the base tests on celerity and shoaling to construct wave fronts at successive time intervals. Chu (1974) and Skovard and Petersen (1976) have developed computer programs utilizing the finite height theories to predict refraction of finite height, but not breaking waves.

The modified refraction theory is briefly described below using test 7 as an example. The incident wave parameters were $H_o = 0.126$ feet and $L_o = 14.22$ feet, resulting in $H_o/L_o = 0.00885$. The wave height was determined as a function of depth from Figure 6, neglecting refraction effects by the equation

$$\frac{H}{d} = (\frac{H}{H_o})(\frac{H_o}{L_o})(\frac{L_o}{d})$$

The wave celerity in the region prior to breaking was determined by the equation

$$c = c_a(1 + 0.25\ H/d)$$

The wave in the breaking region was determined by $c = 1.33\sqrt{gd}$. Breaking was assumed to occur at $H/d = 1$. Refracted wave crests were drawn by the wave crest method and refraction coefficients were determined from or-

thogonals drawn through the crests. The method is tedious and not rec-
ommended for design practice. The wave front method was also employed
using conventional refraction procedures for direct comparison of wave
crest patterns and refraction coefficients shown in Figure 10.

The modified technique, which incorporates wave height and wave
breaking, predicts a smaller degree of orthogonal convergence in the
center of the shoal and a smaller degree of orthogonal divergence along
the sides of the shoal. The refraction coefficients determined by using
the conventional analysis predicts coefficients as high as 2, whereas
the modified procedure yielded a maximum refraction coefficient of 1.25.
Both procedures had coefficients = .75; however, a smaller area had val-
ues less than .75 in using the modified procedure.

The observed refraction patterns have an orthogonal divergence in
the vicinity of the initial breaking region over the center of the shoal.
During breaking, the wave crest slides down the front of the wave. This
is manifested as a greater local wave celerity. This phenomenon was
not modeled in the modified refraction procedure.

FINDINGS

The findings of this investigation illustrate the importance of
considering the effects of finite height and wave breaking on wave
transformation near and in the surf zone. The base test series study-
ing the transformation of waves over a 1:30 slope indicated that the
observed increase in wave height at the breaker point relative to the
deep-water height was 25 percent greater than that predicted by appli-
cation of Airy theory for $H_O/L_O = 0.03$ and 50 percent greater at $H_O/L_O = 0.002$. This result has a significant application when transforming a
shallow-water wave to either a deep-water wave from a gage reading or
transforming the wave to a breaker position. The result also suggests
that model experiments should generate the waves in as deep water as
possible, at least as deep as $H/d < .25$. Simple corrections to Airy
theory have been deduced from the data to include the effects of fi-
nite height and breaking on wave celerity. In addition, during the
breaking process, the lip transforms into a bore resulting in a consi-
derably greater local celerity than the aforementioned correction. The
conclusions above are more applicable to long-period wind-generated
swell of regular form, as found in the Hawaiian Islands, as opposed to
sea or locally generated waves found in the North Sea. Shorter period
waves of greater steepness conform better to conventional analysis.

The model tests on the refraction of waves over a three-dimensional
shoal illustrated the influence of finite height on wave refraction.
The wave orthogonals tend to converge less over a shoal for increasing
wave height due to the effects of finite height on wave celerity. This
factor becomes important near the surf zone. The waves tend to diverge
less over a trough. The influence of finite height and breaking are,
therefore, to reduce the effects of refraction. Most existing computer
programs using linear theory tend to strive for accuracy of bathymetry
interpolation in refracting by the ray equation. Care should be exer-
cised in the application and interpretation of such programs where H/d
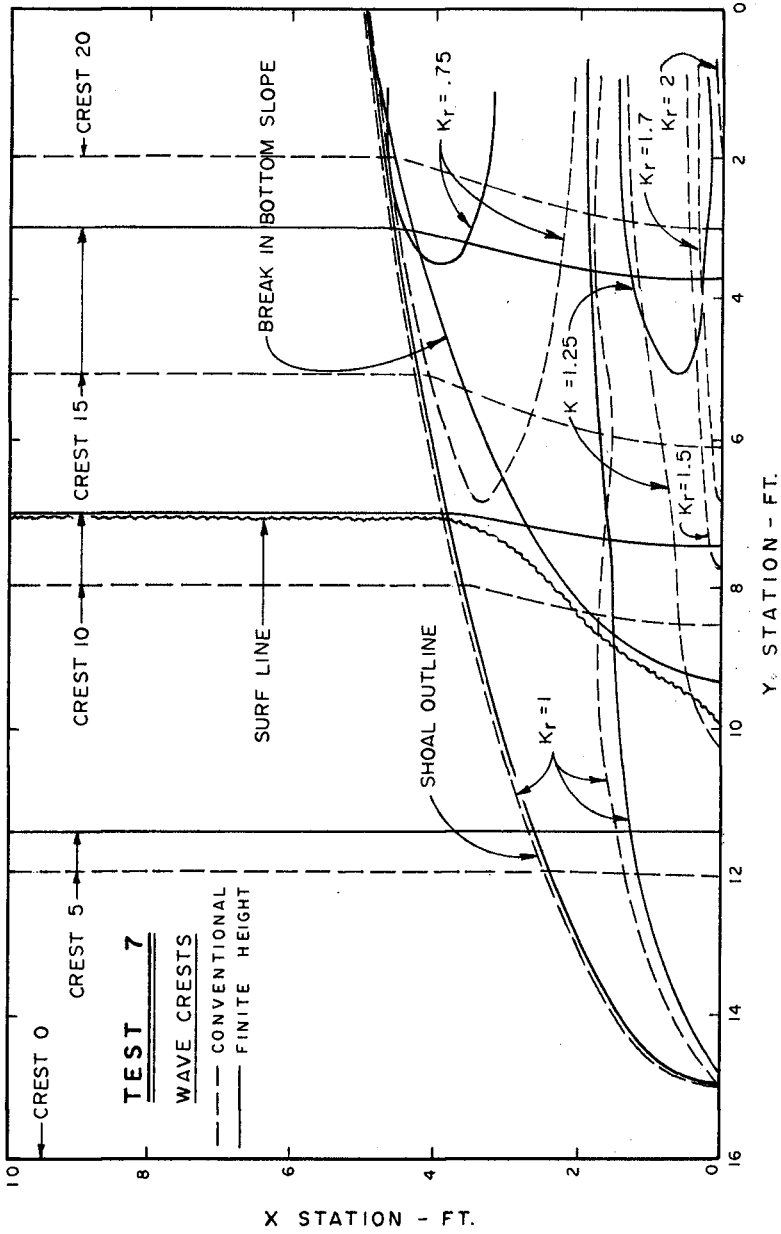exceeds about .25 or when c is increased 6 percent over the Airy celeri-

FIGURE 10 – COMPARISON OF CONVENTIONAL U.S. FINITE HEIGHT REFRACTION PATTERNS

ty. The application of cnoidal theory presented by Skovgaard and Petersen, use of stream function shoaling curves, or a finite height scheme such as presented by Chu appears to be a step in the right direction. However, these schemes are for regular bathymetry. A wave front type method should be developed to more accurately predict finite height along a wave crest.

APPLICATION TO DESIGN PRACTICE

Application of a detailed refraction analysis using finite height procedures or corrections to linear analysis can be a time-consuming chore. Linear refraction and shoaling techniques appear valid for H/d < .25. The recommended procedure is to generally employ conventional refraction analysis using Airy theory and select a few critical cases to study in more detail starting at $H/d = .25$. The study results indicate that in design of a surf shoal similar to the one tested, wave refraction is of secondary importance relative to wave shoaling. The influence of finite height decreases the wave refraction effects considerably over a shoal. This is not a diffraction effect since the wave front pattern which is prescribed by refraction can be accounted for to a large extent by finite height considerations. Effects of wave-induced currents and diffraction are also of importance, but were not studied in this paper.

ACKNOWLEDGEMENT

REFERENCES

Battjes, J. A., "Refraction of Water Waves", J. of Waterways and Harbors Division, ASCE, Vol. 94, WW 4, 1968.

Beitinjani, K. I., and E. F. Brater, "A Study of Refraction of Waves in Prismatic Channels", J. of Waterways and Harbors Division, ASCE, Vol. 91, WW 3, 1965.

Camfield, F. E., and R. L. Street, "The Effects of Bottom Configuration on the Deformation, Breaking, and Run-up of Solitary Waves", Proc. 10th Conf. on Coastal Engineering, ASCE, London, 1968.

Chu, H. L., "Numerical Model for Wave Refraction by Finite Amplitude Wave Theories", Symposium on Modeling Techniques, San Francisco, California, September 1975.

Dean, R. G., "Evaluation and Development of Water Wave Theories for Engineering Application", SR-1, U.S. Army Corps of Engineers, CERC, Ft. Belvoir, Virginia, November 1974.

Galvin, C. J., "Breaker-Type Classification on Three Laboratory Beaches", J. of Geophysical Research, Vol. 73, No. 12, June 1968.

Iwagaki, Y., and T. Sakai, "Shoaling of Finite Amplitude Long Waves on a Beach of Constant Slope", Proc. ASCE Conf. on Coastal Engineering, Vancouver, B.C., 1972.

Keulegan, G. H., "Gradual Damping of Solitary Waves", J. Res. National Bureau of Standards, Vol. 40, page 487, 1950.

Koh, R. C. Y., and B. Le Méhauté, "Wave Shoaling", JGR, Vol. 71, No. 8, 1966.

Miche, R., "Le Pouvoir Reflechissant des Ouvrages Maritimes Exposés à l'Action de la Houle", Annales des Points et Chaussees, Vol. 121, pages 285-319, 1951.

Skovgaard, O., and M. H. Petersen, "Refraction of Cnoidal Waves", Proc. 15th Conf. on Coastal Engineering, Honolulu, Hawaii (Draft), July 1976.

Svendsen, I. A., and O. Brink-Kjaer, "Shoaling of Cnoidal Waves", Proc. 13th Conf. on Coastal Engineering, Vancouver, B.C., 1972.

Walker, J. R., and R. Q. Palmer, "A General Surf Site Concept", Look Lab TR-18, University of Hawaii, Honolulu, Hawaii, September 1971.

Walker, J. R., R. Q. Palmer, and J. K. Kukea, "Recreational Surfing on Hawaiian Reefs", Proc. 13th Conf. on Coastal Engineering, Vancouver, B.C., 1972.

Walker, J. R., "Wave Transformations Over a Sloping Bottom and Over a Three Dimensional Reef", Dissertation, University of Hawaii, Look Lab Miscellaneous Report-75-11, May 1974.

Whalin, R. W., "The Limit of Applicability of Linear Wave Refraction Theory in a Convergent Zone", USA Corps of Engineering Waterways Experiment Station, H-71-3, 1971.

Wiegel, R. L., and A. L. Arnold, "Model Study of Wave Refraction", Beach Erosion Board, TM 103, 1957.