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# REGGE TRAJECTORIES AND THE PRINCIPLE OF MAXIMUM STRENGTH FOR STRONG INTERACTIONS*

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In previous publications, the authors have discussed the possibility that strong interactions "saturate" the unitarity condition; i.e., that they have the maximum possible strength consistent with the unitarity and analyticity of the $S$ matrix.[1] Our earlier discussion was confined to elastic scattering, however, and although the conjectured existence of Regge poles underlay our arguments, we did not at the time of the earlier work appreciate certain essential properties of these poles. We wish here, therefore, to give a general statement of the principle of maximum strength in terms of Regge poles and to explain certain qualitative and quantitative experimental predictions that follow.

In a recent Letter it was proposed that all baryons and mesons (stable or unstable) are associated with Regge poles that move in the complex angular-momentum plane as a function of energy.[2] The trajectory of a particular pole is characterized by a set of internal quantum numbers and by the evenness or oddness of physical $J$ for mesons or $J - \frac{1}{2}$ for baryons; but <u>all</u> $S$-matrix elements, regardless of multiplicity, are supposed to contain any pole whose quantum numbers are appropriate. (The residues of corresponding poles in different $S$-matrix elements will of course differ.) The position $\alpha_i$ of each Regge pole in the $J$ plane is conjectured to be an analytic function of $s = E^2$, and $\mathrm{Re}\alpha(s)$ is supposed to be monotonically increasing for $s < 0$ as well as throughout the (real) positive region of $s$ in which stable and metastable particles occur. The imaginary part of $\alpha_i$ vanishes below the threshold for the lowest energy channel with the quantum numbers in question and is positive definite above this threshold. (Throughout the region of reasonably sharp resonances, we have $\mathrm{Im}\alpha_i \ll 1$.) Stable or metastable particles occur at energies where $\mathrm{Re}\alpha_i$ is equal to a possible physical value of $J$, the half-width of a resonance (metastable particle) being given by

$$\tfrac{1}{2}\Gamma_i = \frac{\mathrm{Im}\alpha_i}{(d\mathrm{Re}\alpha_i/dE)}, \qquad (1)$$

where the right-hand side is evaluated at the resonance energy. All the above conjectures are motivated by the properties of poles in potential-scattering amplitudes—as deduced by Regge.[3] [See note at end of Letter.]

Figure 1 is a plot of the angular momentum of all strongly interacting particles for which spin evidence exists (and which have a baryon number less than two) as a function of mass squared. Each point is supposed to lie on a Regge trajectory, but if the above rules are followed with respect to quantum numbers and slope of trajectory, one concludes that only two particles—the nucleon and the $N_3^*$—could belong to the same trajectory.[4] This circumstance is not surprising if the low-energy slopes of all trajectories are similar in magnitude. The average displacement in $m^2$ between two members of the same family ($\Delta J = 2$) would then be of the order of $100 m_\pi^2$; so the second member of any family—if it exists—will always lie well inside the continuum and be correspondingly difficult to find experimentally. Below, we discuss tentative evidence that $(d\alpha/ds)_{s=0}$ is of the order of $1/(50 m_\pi^2)$ for trajectories other than that to which the nucleon belongs, and a theoretical motivation for such uniformity of slope is provided by Regge's potential-scattering formula,[3]

$$d(\alpha + \tfrac{1}{2})^2/dp^2 = R^2, \qquad (2)$$

where $p$ is the momentum and $R$ an average radius of the bound state. All the baryons and mesons in Fig. 1 are expected to have similar "sizes," and the slope in question corresponds through formula (2) to $R \approx 1/(2m_\pi)$, a plausible order of magnitude.[5]

The principle of maximum strength for strong interactions depends on the assumption that Regge trajectories can be continued to the region $s \leq 0$ and on the result of Froissart that in this region $\alpha_i(s) \leq 1$ for all trajectories.[6] The point is that a given Regge pole gives rise to high-energy amplitudes in "crossed reactions" which are proportional to $E_{\mathrm{lab}}^{\alpha_i(s)}$, where now $s = -\Delta^2$ (the negative square of momentum transfer); and amplitudes that asymptotically increase as a power of energy

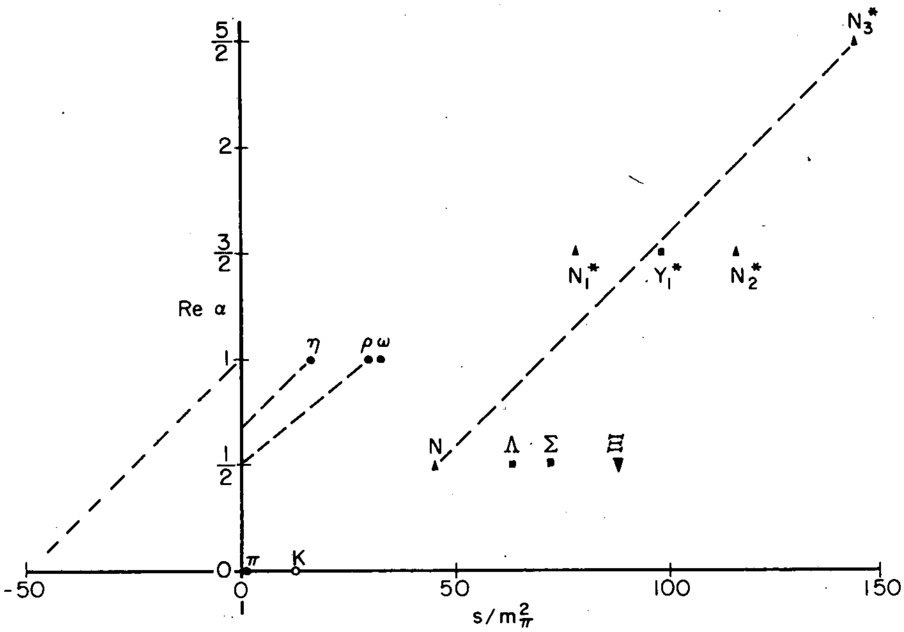


FIG. 1. The spin of particles of baryon number less than two, plotted against the square of their mass in units of $m_\pi^2$. In order to give a rough indication of slopes, the dashed lines connect pairs of points supposedly on the same trajectories, as explained in the text, but a strict linear behavior of the trajectories is not to be inferred.

greater than 1 violate the combined requirements of unitarity and analyticity.[6] From a glance at Fig. 1 it is evident that none of the trajectories associated with known particles is likely to reach the Froissart limit if all slopes are of the order of magnitude $1/(50 m_\pi^2)$. Where then is there evidence for saturation of the unitarity condition?

The evidence, of course, lies in the fact that total cross sections actually appear to approach constants at high energy, implying an imaginary part of forward amplitudes $\propto E_{\rm lab}$; so we have conjectured that a Regge pole with the quantum numbers of the vacuum is responsible—with a trajectory such that $\alpha_{\rm vac}(s=0)=1$.[2] The slope of this vacuum trajectory is expected to be positive at low $s$ (and similar in order of magnitude to the slopes of other trajectories), and it was explained in the previous Letter and is amplified below why it is plausible to have the vacuum trajectory lie above all others.[2] Thus the condition $\alpha_{\rm vac}(s=0)=1$ represents a saturation of Froissart's limit.

Another way of looking at the situation is in terms of the binding forces responsible for the existence of baryons and mesons, all of which are composite in our picture. For quantum numbers where the net forces are weak or repulsive, the Regge trajectories never cross any physical values of $J$, and no particles appear. The stronger the attractive force, the higher in Fig. 1 the corresponding trajectory occurs, and in ordinary potential scattering there is no limit to the "level" of a trajectory if the force strength is unbounded. In the relativistic case, however, unitarity in crossed channels leads to the Froissart limit, which constitutes an upper bound on the level of any trajectory. We believe that further study of crossing conditions will confirm that the strength of forces is in general correlated with simplicity of quantum numbers.[2] If so, the greatest attraction occurs for the quantum numbers of the vacuum, and here the Froissart limit is reached. In this sense, the forces are "as strong as possible."

The empirical association of quantum numbers with the ordering of trajectory levels in Fig. 1 is impressive. Following the "queen" of all trajectories, the vacuum, come four trajectories $(\eta, \rho, \omega, \pi)$ with zero strangeness ($S$) and zero baryon number ($B$). The isotopic spin ($I$) for the "prince consort" trajectory ($\eta$) is not yet definitely known,[7] but $I_\eta=0$ would fit naturally with the circumstance that exchange of these quantum numbers leads to a maximum coherence in high-energy scattering [i.e., the maximum value for $\alpha(s=0)$], next to the quantum numbers of the vacuum. Because $I_\rho=1$, the $\rho$ trajectory should give less coherence, as also should the $\pi$. If the[8] $\omega$ has the same quantum numbers as the $\eta$, it must belong to a "second-rank" trajectory.[9] The next trajectories, $K$ and $K^*$ (if the $K^*$ spin is 1), have $B=0$ and the lowest possible isotopic spin ($I=\frac{1}{2}$) consistent with one unit of strangeness. For the trajectories with $B=1$, there is a definite correlation of level order with strangeness,[10] and although



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the correlation with isotopic spin is not clean, a preference for low $I$ is manifest.

In our first published discussion of the principle of maximum strength, we failed to realize the crucial circumstance that Regge poles move with energy. Mandelstam reminded us of this feature,[11] which invalidates our original conclusion that particles should not occur with angular momentum greater than unity. A modified statement can still be made, however, to the effect that high spin should not occur in conjunction with low mass. From Fig. 1, for example, and our assumption about similarity of slope for all trajectories at low energy, it appears that the best chance for a spin-2 meson lies with the quantum numbers of the vacuum at a mass of the order of $7m_\pi$. If the vacuum trajectory reaches a maximum below $\mathrm{Re}\alpha = 2$, no such particle exists, but an experimental search seems worthwhile.

The first-order deviation from the Pomeranchuk high-energy limits[12] for certain particle combinations should be associated with the $\eta$ trajectory at $s=0$.[13] If the $G$ parity turns out to be (-1), this pole may account for the substantial and slowly decreasing difference between $K^-p$ and $K^+p$ as well as $pp$ and $p\bar{p}$ total cross sections at high energy.[14] This difference will be proportional to $E_{\mathrm{lab}}^{-[1-\alpha_\eta(0)]}$, and since from Fig. 1 we see that $1-\alpha_\eta(0)$ is likely to be a good deal smaller than 1, a slow approach to equality of particle and antiparticle cross sections is easily understandable for the $KN$ and $NN$ combinations. In contrast, as pointed out to us by Udgaonkar,[15] the first-order difference between $\pi^-p$ and $\pi^+p$ total cross sections at high energy (also the $np-pp$ difference if $I_\eta=0$) will be due to the trajectory associated with the $\rho$ meson, where $I=1$ and $G=+1$. Here the difference will be proportional to $E_{\mathrm{lab}}^{-[1-\alpha_\rho(0)]}$ and should die out slightly more rapidly. Detailed and quantitative calculations are now being carried out to see if such a simple mechanism is capable of explaining the facts. A crude fitting of the observed deviations[14,16] from the Pomeranchuk limits suggests that $\alpha_\eta(0)\approx\frac{2}{3}$, while $\alpha_\rho(0)\approx\frac{1}{2}$—numbers that have been used in constructing Fig. 1.

No systematic effort has been made to milk all possible experimental suggestions out of Fig. 1, and readers may well observe significant features that we have overlooked—particularly in connection with strange particles, where our arguments no doubt lead to a prejudice about spin assignments for resonances. For example, $J=\frac{5}{2}$ hyperon resonances with both $I=0$ and $I=1$ are suggested for masses near 13 to $14m_\pi$.

A final remark concerns the slope of the vacuum trajectory. Frautschi, Gell-Mann, and Zachariasen[17] have analyzed recent data of Cocconi et al.[18] on high-energy $pp$ elastic scattering in terms of formula (1) of reference 2 and have deduced therefrom $\alpha_{\mathrm{vac}}(s=-45m_\pi^2)\approx 0.1$. This result is used in Fig. 1, and further supports the notion that where Regge trajectories are vigorously rising, their slopes tend to be of the order $1/(50m_\pi^2)$.

In conclusion we wish to state our belief that all of strong-interaction physics will flow from: (a) the principle of maximal analyticity of the $S$ matrix, in angular as well as linear momenta,[2] (b) the principle of maximum strength, and (c) the conservation of $B$, $S$, and $I$. There should be no arbitrary dimensionless parameters and only one constant with the dimensions of length (or mass) to be added to $h$ and $c$; there are no elementary particles. It seems conceivable to us that principles (b) and (c) above will eventually be shown to have a close relationship to (a), but at present we have no proposals in this direction. That (a), (b), and (c) form the basis for a complete theory of strong interactions has, of course, not been established by this Letter; but we feel greatly encouraged by the above-discussed internal consistency of the experimental facts when viewed in terms of principle (b) together with the notion of Regge-pole trajectories—which we are confident will soon be derived from principle (a).

Note added. We wish to apologize to R. Blankenbecler and M. L. Goldberger for insufficient reference in our previous Letter[2] to their work on Regge poles. In a report of their joint work delivered at the La Jolla Conference on the Theory of Weak and Strong Interactions in June, 1961, Goldberger stressed the existence of a family of particles associated with a given trajectory and the importance of $J$ parity. He also mentioned the occurrence of a given pole in all amplitudes with the same quantum numbers and the possibility of deciding whether particles are elementary by continuation to a crossed channel. We are sincerely sorry for having omitted reference to this talk, which one of us (G.F.C.) heard but did not fully appreciate. We also would like to express belated appreciation for a remark from K. Wilson, Department of Physics, Harvard University, pointing out that the width shrinkage with increasing energy of forward and backward peaks due to Regge poles is only logarithmic.

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[4]We assume that $N_3^*$ is associated with an $F_{5/2}$, $I=\frac{1}{2}$ resonance in $\pi N$ scattering at 900-Mev lab kinetic energy, and that $N_2^*$ is the 600-Mev $D_{3/2}$, $I=\frac{1}{2}$ resonance. For a recent survey of evidence concerning these two resonances, see R. Omnés and G. Valladas, Centre d'Etudes Nucléaires de Saclay Report 61-6 Rev., 1961 (unpublished). The particle we have designated $N_1^*$ is the 33 resonance.

[5]A singularity in the Regge trajectory occurs at each physical threshold, but we do not anticipate an important change in the average slope when crossing a threshold. We expect all trajectories to turn over at sufficiently high energy, and those with weak or repulsive forces rise little, if at all, so the concept of homogeneity of slope must be used with caution. Certainly it can only be used where the binding energies of particles are comparable to $m_\pi$, so that the particle radii are not much larger than $\sim 1/2m_\pi$.

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[9]When the attractive forces for a given set of quantum numbers are sufficiently strong, there may be more than one Regge trajectory that reaches physical $J$ values. A "second-rank" trajectory with the quantum numbers of the vacuum is presumably responsible for the $I=0$ two-pion anomaly observed near $s=4m_\pi^2$ by A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters 7, 35 (1961). Here the trajectory probably reaches its maximum near this value of $s$ and turns over just before reaching $\mathrm{Re}\alpha=0$, producing what is usually called a "virtual state" rather than a resonance. The origin of the phenomenon is still a Regge pole, however. The fact that the highest level trajectories of the second rank are siblings of the "queen" and "prince consort," respectively, gives additional support to the notion that force strength is correlated with simplicity of configuration.

[10]We have assigned $Y_1^*$ a spin of $\frac{3}{2}$ on the basis of tentative evidence recently obtained by R. P. Ely, S. Y. Fung, G. Gidal, Y. L. Pan, W. M. Powell, and H. S. White, Phys. Rev. Letters 7, 461 (1961).

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[12]S. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 725 (1958) [translation: Soviet Phys.–JETP 34(7), 499 (1958)].

[13]This observation has been made independently by M. Gell-Mann and F. Zachariasen, California Institute of Technology (private communication).

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