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# Regional Specialization, Urban Hierarchy, and Commuting Costs

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**Abstract**

We consider an economic geography model of a new genre: all firms and workers are mobile and their agglomeration within a city generates rising urban costs through competition on a land market. When commuting costs are low (high), the industry tends to be agglomerated (dispersed). With two sectors, the same tendencies prevail for extreme commuting cost values, but richer patterns arise for intermediate values. When one good is perfectly mobile, the corresponding industry is partially dispersed and the other industry is agglomerated, thus showing regional specialization. When one sector supplies a non-tradeable consumption good, this sector is more agglomerated than the other. The corresponding equilibrium involves an urban hierarchy: a larger array of varieties of each good is produced within the same city.

**Keywords:** interregional mobility, intersectional mobility, agglomeration, commuting costs.

**J.E.L. Classification:** F12, F16, J60, L13, R12.

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# 1 Introduction

The degree of urbanization that characterizes modern economies is such that housing and commuting costs, which we call *urban costs*, stand for a large (in fact, the main) share in consumers' expenditure. In developed countries, they account for more than one third of individual incomes and may even reach one half of them. To take an example, in France the share of housing and transport costs have increased from 23.4 to 42% in consumers' budget from 1960 to 2000 (Rignols, 2002). In addition, both urban economics and empirical evidence show that such costs rapidly increase with city size. For example, French data show that in 2000 urban costs stand for about 45% of individual incomes in large cities, but for 34% in the small ones. Similar data could be presented for other countries and explain why we consider urban costs as one of the main forces pushing toward the geographical dispersion of activities in modern economies.

*The primary purpose of this paper is, therefore, to show how preference for variety on the demand side and increasing returns on the supply side interact with urban costs to shape the space-economy.*[1] Thus, our thought experiment will be on commuting costs (per unit of distance) because they are critical to understand how modern space-economies are organized, especially when trade costs have reached the low level that now prevails. Commuting costs have also vastly decreased during the 20th century, due mainly to the development of rapid transits (Mills and Hamilton, 1994, pp.21-30) and the growing adoption of individual cars (Glaeser and Kahn, 2004), but remain substantial when we account for the value of time spent in commuting.[2] In order to achieve our goal, we use the framework of economic geography but modify it in several important respects with the aim of embodying more relevant forces in the process of intercity trade and city formation.

As in Henderson (1974), we disregard the agricultural sector and assume that all workers and firms are mobile; as in Tabuchi (1998), the dispersion force rests on urban costs that rise with the size of the population established within the same city. Such a setting, which combines the mobility of industry and the existence of urban costs, strikes us as being more suitable to study

[1] Mano and Otsuka (2000) empirically demonstrate that urban costs have acted as a strong dispersion force in the Japanese manufacturing activities.

[2] According to Mokyr (2002), the commuting costs in terms of time in the United States can be roughly estimated at about $356 billion in 2000. However they are measured, the importance of urban costs is not disputable.

modern economies of the 21st century than is the standard core-periphery model developed by Krugman (1991). It is worth stressing here that one of the unsatisfactory aspects of new economic geography is the role ascribed to the agricultural (or immobile) sector; and so for at least two reasons. First, farmers are not allowed to move between regions and sectors, whereas they actually do. Such moves are indeed at the origin of the urbanization of industrialized countries. Second, although this sector acts as a dispersion force, it must be sufficiently large for dispersion to arise as an equilibrium outcome (otherwise there is always agglomeration). All of this makes the core-periphery model somewhat awkward to deal with the fast-growing mobility of production factors.

Another drawback of the core-periphery model is the systematic focus on the industry as a whole.[3] This implies that this model is not able to cope with specific industries displaying different spatial patterns. Yet, economic geography has triggered a large number of empirical studies, surveyed in Head and Mayer (2004), which all deal with several sectors. *The secondary purpose of this paper is to study the location of several industries and to investigate what the prediction of the one-sector model becomes in this context.* Specifically, we consider two industries that differ in the cost of shipping their output and show how such a difference may affect their location.[4]

We obtain results similar to those obtained in the one-sector model when commuting costs are large or small. However, by allowing for several industries to seek location, we also open the door to new and richer spatial configurations for intermediate values of commuting costs, and this is because the industrial composition of regions is now endogenous. Among other things, extending our basic model to two sectors allows us to get rid of one of the "exotica" of the economic geography literature. Specifically, when there are two sectors, we show that the agglomeration process is gradual and continuous. Thus, empirical studies should not search for catastrophic changes in the economic landscape (Davis and Weinstein, 2002). This result, in turn, allows us to study some new issues, such as regional specialization and urban hierarchy, which cannot be properly addressed in a model involving a single

[3] For noticeable exceptions, see Fujita, Krugman and Mori (1999) and Venables (1999).

[4] We want to stress the fact that this modeling strategy, though restrictive, is very much in the tradition of Weber's (1909) and Hoover's (1948) location theory in which different transport costs may explain why firms belonging to different industries obey different locational patterns. Therefore, our paper may be viewed as an attempt to connect "classical" location theory and "new" economic geography.

mobile sector. This is an important point because one of the most striking features of the space-economy is the existence of cities of different sizes showing contrasted internal production patterns. More precisely, we observe large and small cities together with diversified and specialized cities, each producing some goods for local consumption as well as other goods whose consumption is spread over several cities (Henderson, 1988). This problem is very complex, however, because *workers choose the sector in which they work as well as the place where they live.* This in turn implies that the size of each sector, not just the size of each city, is endogenous.

In order to make our model comparable to the existing literature, we begin with the study of the location of one industry. It is worth mentioning right away that our setting leads to results that differ from those obtained in the standard core-periphery model. Even though the general pattern of one industry against declining commuting costs is fairly similar to the one obtained in the standard core-periphery model when transport costs decrease (see Proposition 1), there are striking differences. For example, it appears that product differentiation acts here as a dispersion force. As all consumers are mobile, price competition fosters agglomeration because workers are to be compensated for the high urban costs associated with the emergence of a larger city. Likewise, there is now insufficient agglomeration in that the market outcome sustains dispersion for a range of commuting costs for which agglomeration is socially desirable. All of this suffices to show that using a dispersion force other than the immobile sector may drastically affect the conclusions derived from the standard core-periphery model. As a consequence, *policy recommendations based on standard economic geography models have to be applied with extreme caution.*

In the case of two industries, commodity-specific transport costs may lead to equilibria in which different sectors display different spatial configurations, ranging from full dispersion to full agglomeration, as in the foregoing case. Unfortunately, allowing for both regional and sectoral mobility of workers renders the analysis especially complex and prevents a full analysis of the equilibrium configurations. This leads us to investigate the special, but meaningful, case of an industry with negligible (zero) transport costs, whereas the other faces positive costs. We show that the imperfectly mobile good sector is agglomerated whereas the footloose sector is partially dispersed. To us, this is indication of *regional specialization*, even though full specialization never arises as a stable equilibrium outcome.

In order to gain further insights, we consider another extreme case in which one good is perfectly mobile while the other good is nontradeable. Indeed, despite strong decreases in transport and communication costs, many consumer services are supplied locally often in large cities (Daniels, 1993). In such a context, *the equilibrium involves a Christaller-like central place pattern in that one city supplies more varieties of each good than the other*. In addition, our equilibrium pattern also implies that the large city supplies a relatively larger share of the nontradeable good, which allows it to attract a larger share of the perfectly mobile good industry. This can be viewed as a comparative advantage à la Ricardo, the intensity of which is endogenous instead of being given a priori. Finally, although we assume no technological linkage between sectors, *cities are diversified in that they involve firms belonging to each industry.*

The remainder of the paper is organized as follows. The model is introduced in section 2. Instead of using the Dixit-Stiglitz-iceberg framework, we retain the alternative model developed by Ottaviano, Tabuchi and Thisse (2002) because it leads to analytical results that may be derived with paper and pencil. In section 3, we consider the case of a single sector. The case of two industries is considered in section 4, whereas section 5 concludes.

**Related literature.** Even though the pioneering works of Christaller (1933) and Lösch (1940) are now fairly old and despite the importance of the questions these authors address for trade and development, it is only recently that such issues have attracted attention in the economics profession. This has been accomplished in the context of systems of cities (see Abdel-Rahman (2000) for a detailed survey of the existing literature). However, the corresponding models often fail to stress their trade and factor mobility implications. One prominent exception is the work of Henderson (1974, 1988), who considers a setting in which returns to scale are constant at the firm's level but increasing in the aggregate once they are located together. Workers consume land and face positive commuting costs within each city. Put together, these two forces imply that each city has a positive and finite size. Because of increasing returns, it is efficient for each city to be specialized in the production of a particular good. Accordingly, Henderson ends up with an urban system having cities of different sizes and types according to the good they produce. However, shipping a city's output is costless in his setting, which amounts to treating cities like floating islands. Hence, we do not know where cities are on the map, nor does Henderson provide an explanation for a system involving both diversified and specialized cities.

## 2 The model

Consider an economy formed by a continuum of mobile workers whose mass is 1, by two regions, denoted $H$ and $F$, and by three goods. The first good is homogenous and available as an endowment; it can be shipped costlessly between the two regions and is chosen as the numéraire. The second one is a differentiated good made available under the form of a continuum of varieties. The utility function of a worker defined on these two goods is as follows:[5]

$$\begin{aligned} U(q_0; q(j), j \in [0,N]) &= \alpha \int_0^N q(j)dj - \frac{\beta - \gamma}{2} \int_0^N [q(j)]^2 dj \\ &- \frac{\gamma}{2N} \left[ \int_0^N q(j)dj \right]^2 + q_0 \qquad (1) \end{aligned}$$

in which $N$ stands for the number of varieties of the differentiated good, $q(j)$ for the quantity of variety $j$ and $q_0$ for the quantity of the numéraire.[6] Both $\alpha$ and $\beta$ are positive, whereas $\gamma$ is positive (resp., negative) if varieties are substitutes (resp., complements). For the utility to be quasi-concave, we assume $\beta > \gamma$.

The third good is land (or housing). In order to keep matters simple, we consider the case of a one-dimensional continuous space in which each location has a unit amount of land. Each region has a spatial extension and involves a linear city whose center is given but with a variable size. The city center stands for a central business district (CBD) in which all firms locate once they have chosen to set up in the corresponding region.[7] The two CBDs are two remote points of the location space. Interregional trade flows go from one CBD to the other. While firms are assumed not to consume land, workers when they live in a certain region, are urban residents who

[5]This utility is the one proposed by Vives (1990), which slightly differs from that used by Ottaviano *et al.* (2002). It has been chosen because the coefficients $a$, $b$ and $c$ defined below are independent of the number of varieties supplied by a sector, which is variable in the two-sector model.

[6]The third term in the RHS of (1) is divided by $N$, as in the case where all varieties of the differentiated good are consumed ($q(j) > 0$ for all $j$). When this does not hold, $N$ must be replaced by the measure of the support of the function $q(j)$. The same restriction applies to (6) defined in section 4.

[7]See Fujita and Thisse (2002) for various arguments explaining why firms want to be agglomerated in a CBD.

consume land and commute to the regional CBD in which jobs and varieties of the differentiated good are available. Hence, unlike Krugman (1991) but like Tabuchi (1998), each agglomeration has a spatial extension that imposes commuting and land costs on the corresponding workers.[8] For simplicity, workers consume a fixed lot size normalized to unity, while commuting costs are linear in distance, the commuting cost per unit of distance being given by $\theta > 0$ units of the numéraire. The opportunity cost of land is normalized to zero.[9] Assume that $\lambda_r$ workers live in region $r = H, F$, where $\lambda_r$ denotes the share of workers residing in region $r$ with $\lambda_H + \lambda_F = 1$. At the land market equilibrium, all workers in region $r$ are equally distributed around the $r$-CBD and, since they earn the same wage, reach the same utility level. Furthermore, since each of them consumes one unit of land, the equilibrium land rent at distance $x \leq \lambda_r/2$ from the $r$-CBD is given by

$$R^*(x) = \theta(\lambda_r/2 - x)$$

When the land rents go to absentee landlords, individual urban costs, defined by commuting cost plus land rent at each distance $x$, are given by $\theta\lambda_r/2$. In order to close the model, we assume that land is publicly owned and that the aggregate land rent is equally redistributed among the $r$-city workers. Consequently, the individual urban costs after redistribution are equal to $\theta\lambda_r/4$. Hence, the budget constraint of a worker residing in city $r$ is given by

$$\int_0^N p_r(j)q_r(j)dj + \frac{\theta}{4}\lambda_r + q_0 = \overline{q}_0 + w_r$$

where $p_r(j)$ is the consumer price of variety $j$ in region $r$, $q_r(j)$ the individual demand for variety $j$ of a worker residing in region $r$, and $w_r$ the wage she earns in this region. The initial endowment $\overline{q}_0$ is supposed to be sufficiently large for its equilibrium consumption to be positive for each individual.

Let $a \equiv \alpha/\beta$, $b \equiv 1/(\beta - \gamma)$ and $c \equiv \gamma/[\beta(\beta - \gamma)]$, with $b > c$. The individual demand $q_r(j)$ is given by

$$q_r(j) = a - bp_r(j) + c\frac{P_r}{N}$$

[8]To be precise, Helpman (1998) assumes a fixed supply of housing that has no spatial extension so that his model does not include commuting costs.

[9]It would be interesting to assume that the opportunity cost of land differs between the two regions in order to study the impact of previous and different land uses on the size and industrial composition of cities. Unfortunately, the analysis becomes intractable in the two-sector case.

where the price index $P_r$ in region $r$ is defined as follows:

$$P_r \equiv \int_0^N p_r(k)dk$$

Each variety is supplied by a single firm producing under increasing returns so that $N$ is also the number of firms. The common fixed cost is denoted by $\phi$ whereas the marginal cost is set equal to zero so that the number of firms $N$ is equal to $1/\phi$. Each variety can be shipped at a positive cost of $\tau$ units of the numéraire. Markets are segmented, that is, each firm is able to set a price specific to the market in which its output is sold. This assumption is supported by empirical investigations (Head and Mayer, 2000) and can be given some theoretical justification (Thisse and Vives, 1988). Because firms located in region $r$ are symmetric, they charge the same price in equilibrium. Hence, the profits made by a firm located in region $r = H, F$ may be written as follows:

$$\Pi_r = p_{rr}q_r(p_{rr})\lambda_r + (p_{rs} - \tau)q_s(p_{rs})\lambda_s - \phi w_r$$

where $p_{rr}$ and $p_{rs}$ are respectively the prices quoted by a firm located in $r$ and selling in $r$ and in $s \neq r$.

The labor market clearing condition in region $r$ implies that any change in the population of workers in this region must be accompanied by a corresponding change in the number of firms. As in Krugman (1991), entry and exit are free so that profits are zero in equilibrium. The equilibrium wage $w_r$ in region $r$ is then obtained from the zero-profit condition evaluated at the equilibrium prices. These two assumptions have a major implication for our analysis: it is sufficient to describe the migration of workers because the supply of entrepreneurs in each region is supposed to be large enough for the zero-profit condition to be satisfied regardless of the number of workers.

Prices and wages are determined instantaneously once workers have made their locational decisions. Stated differently, equilibrium prices and wages depend on the interregional distribution of workers ($\lambda_r$). This distribution depends itself on the transport costs $\tau$ and the commuting costs $\theta$. It is a well-documented fact that both types of costs have dramatically decreased since the beginning of the Industrial Revolution (Bairoch, 1985), although the commuting costs are often neglected in the general economics literature. In this paper, we focus instead on commuting costs because transport costs of many commodities are now very low.

# 3 The one-sector economy

## 3.1 Market equilibrium

It is easy to show that the equilibrium prices and wages corresponding to $\lambda_r$ are as follows:

$$\begin{aligned} p_{rr}^* &= \frac{2a + \tau c \lambda_s}{2(2b - c)} \qquad\qquad p_{rs}^* = p_{ss}^* + \frac{\tau}{2} \\ w_r^* &= bN[(p_{rr}^*)^2 \lambda_r + (p_{rs}^* - \tau)^2 \lambda_s] \end{aligned} \tag{2}$$

Thus, equilibrium prices and wages depend on the distribution of firms between regions but not on the total number of firms. Two-way trade occurs for any $\lambda_r$ if

$$\tau < \tau_{trade} \equiv \frac{2a}{2b - c}$$

The welfare of a worker in region $r$ is given by her indirect utility $V_r$ evaluated at the prevailing equilibrium prices and wages, which depend themselves on the distribution $\lambda_r$ of workers. A *spatial equilibrium* thus arises when no worker has a unilateral incentive to move from her location (Nash). More precisely, such an equilibrium arises at $0 < \lambda_r^* < 1$ when $\Delta V(\lambda_r^*) \equiv V_r(\lambda_r^*) - V_s(\lambda_r^*) = 0$, or at $\lambda_r^* = 1$ when $\Delta V(1) \geq 0$, or at $\lambda_r^* = 0$ when $\Delta V(0) \leq 0$.

In general, models of economic geography involves several equilibria. Even though these models are static, stability is used as a refinement to eliminate some of them. Here, we follow a now well-established tradition in migration modeling by assuming that workers are attracted (resp., repelled) by regions having a utility higher (resp., lower) than the average utility. Such a gradual migration process is due to the fact that workers have different moving costs. It is also assumed that the power of attraction of such a region is likely to increase with its size because this makes it more "visible". Formally, this means that we use the replicator (Weibull, 1995; Fujita, Krugman and Venables, 1999):[10]

$$\dot{\lambda}_r = \lambda_r([V_r(\lambda_r) - \lambda_r V_r(\lambda_r) - \lambda_s V_s(\lambda_r)] = \lambda_r (1 - \lambda_r) \Delta V(\lambda_r)$$

[10]Note also that the myopic migration behavior implied by (3) provides a good approximation of foward-looking behavior when individual adjustment costs are not too small (Baldwin, 2001; Ottaviano *et al.*, 2002).

A spatial equilibrium is *asymptotically stable* if, for any marginal deviation of the population distribution from the equilibrium, the equation of motion above brings the distribution of skilled workers back to the original one. When some skilled workers move from one region to the other, we assume that local labor markets adjust instantaneously. Wages are then adjusted for each firm to earn zero profits.

It is then readily verified that

$$\Delta V(\lambda_r) = K\,(1/2 - \lambda_r) \tag{3}$$

where

$$K \equiv \frac{b(6b^2 - 6bc + c^2)\tau^2 - 4ab\,(3b - c)\,\tau + (2b - c)^2\,\phi\theta}{2\,(2b - c)^2\,\phi}$$

is a constant. It follows immediately from (3) that $\lambda_r^* = 1/2$ is always a spatial equilibrium. Furthermore, the symmetric equilibrium is stable if $K > 0$. Otherwise, the industry is agglomerated into a single region, with $\lambda_r^* = 0, 1$ according to the initial distribution.

It is straightforward to compute the corresponding threshold at which the spatial structure changes:

$$\theta^* = \frac{b\tau\left[4a\,(3b - c) - (6b^2 - 6bc + c^2)\tau\right]}{(2b - c)^2\,\phi}$$

It is readily verified that $\theta^* > 0$ as long as $\tau$ does not exceed $\tau_{trade}$. The following result thus holds:

**Proposition 1** *If $\theta > \theta^*$, then the symmetric configuration is the only stable spatial equilibrium; if $\theta < \theta^*$, there are two stable spatial equilibria corresponding to the agglomerated configurations; if $\theta = \theta^*$, then any configuration is a spatial equilibrium.*

This result exhibits the evolutionary process from dispersion to agglomeration as $\theta$ decreases. We have here a setting that bears some resemblance with the core-periphery model developed by Krugman (1991) and by Ottaviano *et al.* (2002) in that *improvements in intraregional* (instead of interregional) *transport technologies induce the spatial transition from dispersion to agglomeration.* In particular, the transition is still of a catastrophic nature.

Next, it is readily verified that

$$\partial\theta^*/\partial c > 0$$

which does not agree with Krugman (1991) and Ottaviano *et al.* (2002). Stated differently, in the absence of an agricultural sector but in the presence of urban costs, the closer the substitutes, the more likely the agglomeration of the industry. Such a seemingly counterintuitive result may be explained as follows. On the one hand, when there is no immobile demand, firms supplying close substitutes have no reason to be dispersed because their demand may be geographically concentrated. That is, *price competition is an agglomeration force when all consumers are mobile.*[11] On the other hand, when there are urban costs, workers–hence firms–always have an incentive to relax congestion (instead of price competition) by moving to the periphery. Accordingly, agglomeration arises provided that firms sell close substitutes because the price competition effect is sufficiently strong to compensate the workers for the high urban costs associated with an agglomeration. By contrast, when substitutes are bad, the price competition effect is too weak to make up for the urban costs, so that dispersion prevails.

The impact of $\phi$ on $\theta^*$ is similar. When $\phi$ decreases, both the agglomeration force (the market size effect) and the dispersion force (the urban costs) are weakened. This is because a new firm needs less workers to set up in the region where the firm is launched. Yet, when fixed costs are sufficiently low, our result shows that the former effect dominates the latter, thus implying that the industry is always agglomerated. Consequently, we may conclude that, *in the absence of an agricultural sector but in the presence of urban costs, agglomeration is more likely when the degree of product differentiation is low and when fixed costs are low.* This is the opposite of what has been obtained in the core-periphery model (Fujita, Krugman and Venables, 1999; Ottaviano *et al.*, 2002) and shows how different the various modeling strategies regarding the dispersion force may be.

Specifically, *the agricultural sector and the urban costs play very different roles as dispersion forces.* The intensity of the dispersion force generated by an immobile demand decreases, as does the agglomeration force, when transport costs decrease. What the core-periphery model shows is that the former declines faster than the latter. By contrast, the dispersion force caused by urban costs is unaffected by a fall in transport costs. Instead, agglomeration is triggered by a fall in commuting costs.

Finally, we know that the sign of $K$ changes at $\theta = \theta^*(\tau)$. It is readily

[11]By contrast, price competition is a dispersion force when demand is immobile (d'Aspremont, Gabszewicz and Thisse, 1979)

verified that $\theta^*(\tau)$ is a concave parabola passing through the origin and intersecting the $\tau$-axis at a positive value that exceeds $\tau_{trade}$. Clearly, the domain of admissible $(\tau, \theta)$-values inside (resp., outside) this parabola is such that the stable equilibrium involves agglomeration (resp., dispersion) because $K < 0$ (resp., $K > 0$). Having said this, we find it useful to reinterpret our results to deal with the case where $\theta$ is constant whereas $\tau$ steadily decreases in order to make our setting comparable to Helpman (1998). When urban costs are sufficiently large, the economy always involves dispersion. However, for lower values of $\theta$, *as $\tau$ steadily decreases from $\tau_{trade}$, the economy moves from agglomeration to dispersion at* $\theta^*(\tau)$, thus confirming the numerical results obtained by Helpman (1998).

## 3.2 Welfare

We now compare the market outcome with the optimal allocation. As usual, in the first best the planner is able to use lump sum transfers (i) in order to assign any number of workers (or, equivalently, of firms) to a specific region and (ii) in order to pay for the loss firms may incur while pricing at marginal cost. Because our setting assumes transferable utility, the planner chooses $\lambda$ in order to maximize the sum of individual indirect utilities:

$$W(\lambda_r) \equiv \lambda_r V_r(\lambda_r) + (1 - \lambda_r)\, V_s(\lambda_r) \tag{4}$$

in which all prices have been set equal to marginal cost:

$$p^o_{rr} = p^o_{ss} = 0 \qquad \text{and} \qquad p^o_{rs} = p^o_{sr} = \tau \tag{5}$$

thus implying that operating profits and, hence, wages are zero. Maximizing (4) subject to (5) yields $\lambda_r = 1/2$ as a candidate for the optimum allocation of workers. Examining the second order condition, we see that $W(\lambda_r)$ is concave (resp., convex) when $\tau$ is larger (resp., smaller) than the threshold

$$\theta^o \equiv \frac{[4a - (2b - c)\,\tau]\,\tau}{\phi}$$

Hence, *the first best optimum involves dispersion for $\theta > \theta^o$ and agglomeration for $\theta < \theta^o$.*

Comparing $\theta^*$ and $\theta^o$ does not lead to straightforward conclusions. Indeed, the sign of

$$\theta^* - \theta^o = \frac{[(b-c)\,(2b^2 - 4bc + c^2)\,\tau - 4a\,(b^2 - 3bc + c^2)]\,\tau}{(2b-c)^2\,\phi}$$

is indeterminate and varies with the parameter values, as in Papageorgiou and Pines (2000). For example, if the varieties are sufficiently differentiated (resp., good substitutes), then $\theta^* < \theta^o$ (resp., $\theta^* > \theta^o$) holds. In the present setting, besides the standard market distortion generated by monopolistic competition, there is something like a "fiscal externality" on the land market because workers underpay for the use of land. Indeed, the (average) urban cost that enters the indirect utility after redistribution of the aggregate land rent is equal to $\theta\lambda_r/4$, whereas the marginal cost is equal to $\theta\lambda_r/2$. This fiscal externality pushes toward more agglomeration because it leads to lower urban costs. As a result, it is hard to compare the market and the social outcomes.

In order to identify the impact of these two distortions, we consider the situation in which the total land rent is not redistributed among workers but goes to absentee landlords. In this case, the fiscal externality problem is fixed because the urban cost is identical to the social cost (see Proposition 5.1 in Fujita (1989)): the average urban cost is now equal to $\theta\lambda_r/2$, i.e. the marginal cost. The equilibrium threshold $\theta^*$ is no longer valid and the new threshold is $\theta^{**} = \theta^*/2$, which is still distorted by monopolistically competitive pricing only but not by the fiscal externality. It is then readily verified that

$$\theta^{**} < \theta^o$$

This inequality implies that, for intermediate values of the trade costs ($\theta^{**} < \theta < \theta^o$), *the market provides insufficient agglomeration*. Thus, we have:

**Proposition 2** *If the aggregate land rent is given to absentee landlords, the market outcome tends to be more dispersed than the socially desirable outcome.*

This is just the opposite of what Ottaviano and Thisse (2002) have obtained. Once workers pay the true marginal urban cost, the only force that pulls apart from the optimum is the home market effect, which is known to lead to excessive agglomeration. By contrast, when the fiscal externality is at work, it may be strong enough to induce the market to provides excessive agglomeration. These results are sufficient to show that *the social desirability of agglomeration may completely vary with the nature of the dispersion force*, thus inviting us to be very careful in policy recommendations.

# 4 The two-sector economy

## 4.1 The model

We now come to the case of two sectors. Assume that all varieties of each good are consumed. The utility (1) may then be extended as follows:

$$U(q_0; q_i(j), j \in [0, N_i], i = 1, 2) = \sum_{i=1}^{2} \left[ \alpha \int_0^{N_i} q_i(j) dj \right.$$
$$\left. - \frac{(\beta - \gamma) N_i}{2(N_1 + N_2)} \int_0^{N_i} [q_i(j)]^2 dj - \frac{\gamma}{2(N_1 + N_2)} \left( \int_0^{N_i} q_i(j) dj \right)^2 \right] + q_0 \quad (6)$$

where $q_i(j)$ is the quantity of variety $j$ of good $i$. In the RHS of (6), the second term in each bracketed term is weighted by the relative size of its sector $N_i/(N_1 + N_2)$ to capture the idea that, everything else being equal, an industry with a small range of varieties has less impact on the consumer well-being than an industry with a large array of varieties.

Since $\beta > \gamma$, (6) encapsulates both a *preference for diversity* between the two goods as well as a *preference for variety* across varieties of the same good. Assume, first, that an individual consumes a given mass of $Q_i$ units of good $i = 1, 2$ and that consumption of good $i$ is uniform and equal to $Q_i/x_i$ on $[0, x_i]$ and zero on $(x_i, N_i]$. Evaluating (6) at this consumption pattern yields

$$\begin{aligned} U &= \sum_{i=1}^{2} \left[ \alpha \int_0^{x_i} \frac{Q_i}{x_i} dj - \frac{(\beta - \gamma) x_i}{2(x_1 + x_2)} \int_0^{x_i} \left( \frac{Q_i}{x_i} \right)^2 dj \right. \\ &\qquad \left. - \frac{\gamma}{2(x_1 + x_2)} \left( \int_0^{x_i} \frac{Q_i}{x_i} dj \right)^2 \right] + q_0 \\ &= \sum_{i=1}^{2} \left[ \alpha Q_i - \frac{\beta Q_i^2}{2(x_1 + x_2)} \right] + q_0 \qquad (7) \end{aligned}$$

which is strictly increasing in $x_i$ for each $i = 1, 2$. Hence, $x_i = N_i$ must hold for each good $i$, that is, each consumer prefers to consume all the varieties of each of the $N$ available goods. Assume now that the total consumption $Q = Q_1 + Q_2$ is fixed. Then, maximizing (7) with respect to $Q_i$ subject to $Q = Q_1 + Q_2$ yields $Q_1 = Q_2 = Q/2$. In words, each good is equally consumed and, hence, each variety of any good is equally consumed.

There are several ways to make the two industries asymmetric: $c_1 \neq c_2$ (different degrees of product differentiation), $\phi_1 \neq \phi_2$ (different levels of fixed cost), and $\tau_1 \neq \tau_2$ (different transport costs). Since we focus on spatial parameters and variables, it is worth investigating the impact of different transport costs with $\tau_1 < \tau_2$.

Let $\lambda_{ir}$ be the share of the labor force established in region $r$ and working for sector $i$. Because the sum of the $\lambda_{ir}$ is one, there are three endogenous variables to be considered: $\lambda_{1H}$, $\lambda_{2H}$ and $\lambda_{1F}$. Since workers can change both places and jobs, the labor market clearing conditions imply that

$$N_i = \frac{\lambda_{iH} + \lambda_{iF}}{\phi} \qquad i = 1, 2$$

so that the number of varieties produced in each sector is now endogenous. An individual working in sector $i$ and residing in region $r$ earns a wage equal to $w_{ir}$ and maximizes (6) under her budget constraint:

$$\int_0^{N_1} p_{1r}(j) q_{1r}(j) dj + \int_0^{N_2} p_{2r}(j) q_{2r}(j) dj + \frac{\theta}{4} (\lambda_{1r} + \lambda_{2r}) + q_0 = w_{ir} + \overline{q}_0$$

where the individual demand for the variety $j$ of good $i = 1, 2$ by a worker located in region $r$ is:

$$q_{ir}(j) = \frac{1}{\phi N_i} \left[ a - b p_{ir}(j) + c \frac{P_{ir}}{N_i} \right] \tag{8}$$

the price index $P_{ir}$ being defined as follows:

$$P_{ir} \equiv \int_0^{N_i} p_{ir}(k) dk$$

It follows from (8) that the demand $q_{ir}(j)$ is affected by $P_{ir}$, which consists of prices of varieties belonging to industry $i$ in the two regions. As a result, the cross-elasticity of demand between any two varieties belonging to different industries is zero, but is positive when they belong to the same industry. This is consistent with the classical definition of an industry given by Triffin (1940). The demand for each variety of good $i$ is negatively affected by the share of the corresponding industry because consumers distribute their consumption over a larger range of varieties.

As seen in the foregoing, one of the main differences with the one-sector model is that the number of varieties $N_i$ produced in industry $i$ is endogenous, being determined by the sectoral mobility of workers. Equilibrium prices and wages (2) are now given by

$$\begin{aligned} p^*_{irr} &= \frac{2a + \tau_i c \lambda_{is}}{2(2b - c)} \qquad\qquad p^*_{irs} = p^*_{iss} + \frac{\tau_i}{2} \\ w^*_{ir} &= \frac{bN}{\lambda_{ir} + \lambda_{is}} \left[ (p^*_{irr})^2 (\lambda_{1r} + \lambda_{2r}) + (p^*_{irs} - \tau_i)^2 (\lambda_{1s} + \lambda_{2s}) \right] \end{aligned} \tag{9}$$

which are independent of the number of firms operating in each sector. However, *equilibrium prices and wages*, hence the indirect utility, *depend on the distribution of the labor force across sectors as well as on the interregional distribution of workers within each industry.* This suffices to show that the analysis is likely to be much harder than in the one-sector model considered above.

The welfare of an individual working in sector $i$ in region $r$ is given by her indirect utility $V_{ir}$ evaluated at the foregoing equilibrium prices and wages. A *spatial-sectoral equilibrium* thus arises when no worker has an incentive to change place and/or to switch job. Formally, this means that $\overline{V}$ exists such that

$$\begin{aligned} V_{ir} &= \overline{V} \quad \text{if} \quad \lambda^*_{ir} > 0 \\ V_{ir} &\leq \overline{V} \quad \text{if} \quad \lambda^*_{ir} = 0 \end{aligned}$$

The value of $w^*_{ir}$ reveals that the expression for $V_{ir}$ is highly nonlinear so that the set of equilibria is very hard to characterize. Yet, because of sectoral mobility, *equilibrium wages must be equal across sectors in each region* so that we may write $w^*_{1r} = w^*_{2r} = w^*_r$ $(r = H, F)$. Note also that the expression of $w^*_{iH}$ shows that goods 1 and 2 are to be supplied in equilibrium because $\lambda_{ir} + \lambda_{is}$ must be positive for $i = 1, 2$. However, we do not know whether or not $\lambda_{ir}$ is uniquely determined in equilibrium.

Another difference with the one-sector model is that there is no natural candidate to model the adjusment process. This is why we propose to study stability through the replicator dynamics, thus treating the choice of a job or of a location in a symmetric way:

$$\dot{\lambda}_{ir} = \lambda_{ir} \left( V_{ir} - \sum_{i=1,2} \sum_{r=H,F} \lambda_{ir} V_{ir} \right) \equiv f_{ir} \tag{10}$$

This means that workers out-migrate (resp., in-migrate) from sector $i$ in region $r$ when her utility $V_{ir}$ is lower (resp., higher) than the intersectional and interregional average utility. Among other things, this implies that intersectional mobility and interregional mobility are determined simultaneously by allowing them to interact in a symmetric manner. In addition, a change in the population of industry $i$-workers in one region is no longer accompanied by a corresponding change in the number of firms in that industry. Finally, all spatial-sectoral equilibria are steady-states of (10). It is, indeed, easy to show that $f_{ir} = 0$ when $V_{ir}$ is equal to the average utility for $i = 1, 2$ and $r = H, F$, whereas $\lambda_{ir} = 0$ when working in sector $i$ in region $r$ yields a utility level lower than the average utility.

By solving $V_{1H} = V_{1F} = V_{2H} = V_{2F}$ simultaneously, we can show that there exists at least one equilibrium given by

$$\begin{pmatrix} \lambda_{1r}^* & \lambda_{1s}^* \\ \lambda_{2r}^* & \lambda_{2s}^* \end{pmatrix} = \begin{pmatrix} \frac{\widehat{\mu}_1}{2} & \frac{\widehat{\mu}_1}{2} \\ \frac{1-\widehat{\mu}_1}{2} & \frac{1-\widehat{\mu}_1}{2} \end{pmatrix} \tag{11}$$

where

$$\widehat{\mu}_1 \equiv \frac{16a^2 - 16a(b-c)\tau_1 + (8b^2 - 12bc + 5c^2)\tau_1^2}{32a^2 - 16a(b-c)(\tau_1 + \tau_2) + (8b^2 - 12bc + 5c^2)(\tau_1^2 + \tau_2^2)} \in [1/2, 1)$$

This equilibrium is interior. However, there are also equilibria involving boundary and interior values, the set of which is much richer than what we get in the one-industry case.

## 4.2 Agglomerated and dispersed equilibria

As seen above, there exists a fully dispersed equilibrium (11) for all $\tau_1 < \tau_2$. We also know that, when both industries are dispersed, the low transport cost industry attracts a larger share of the work force (when $\tau_1 = \tau_2$, we have $\widehat{\mu}_1 = 1/2$). This equilibrium becomes unstable at the symmetry breaking point, which may be obtained by computing the Jacobian of (10) evaluated at (11). More precisely, the characteristic polynomial of the Jacobian of (10) may be written as follows:

$$\varphi(x) = x^3 + C_1(\theta)x^2 + C_2(\theta)x + C_3(\theta)$$

where $x$ stands for an eigenvalue. We know from Routh's theorem that the system is *asymptotically stable* if the following conditions hold (Samuelson, 1947, Appendix B):

$$C_1(\theta) > 0 \qquad C_3(\theta) > 0 \qquad C_1(\theta)C_2(\theta) > C_3(\theta) \tag{12}$$

This yields (at most) three lower bounds on $\theta$. Clearly, the largest bound $\theta_2^*$ is the symmetry breaking point. Indeed, the symmetric equilibrium ceases to be stable when $\theta < \theta_2^*$.

Here too, agglomeration may be an equilibrium

$$\begin{pmatrix} \lambda_{1r}^* & \lambda_{1s}^* \\ \lambda_{2r}^* & \lambda_{2s}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 0 \end{pmatrix} \tag{13}$$

When both industries are agglomerated within the same region, the work force is equally split between the two industries because $\lambda_{1r}^* = \lambda_{2r}^*$ is the unique solution of the equation $V_{1r} = V_{2r}$ in which we have plugged $\lambda_{1s}^* = \lambda_{2s}^* = 0$. This is because the differences in transport costs no longer matter once the two industries are together. This is an equilibrium if and only if the following three conditions

$$V_{1r} \geq V_{1s} \qquad V_{2r} \geq V_{2s} \qquad V_{1r} = V_{2r} \tag{14}$$

hold for $r \neq s$. The stability of this equilibrium is guaranteed because we also have

$$\left.\frac{\partial \left(V_{1r} - V_{2r}\right)_{\lambda_{1r}^*+\lambda_{2r}^*=1}}{\partial \mu_1}\right|_{(\lambda_{1r}^*,\lambda_{2r}^*)=(1/2,1/2)} = -\frac{8a^2 b}{(2b-c)^2 \phi} < 0$$

Let $\theta_1^*$ be the sustain point associated with full agglomeration, which is the unique solution of $V_{1r} = V_{1s}$ evaluated at (13). At $\theta = \theta_1^*$, the three conditions (14) are satisfied because $V_{1r} = V_{1s}$ and $V_{1r} = V_{2r}$ hold when evaluated at (13), whereas $V_{2r} > V_{2s}$ holds because we have

$$V_{2r} - V_{2s}|_{(\lambda_{1r}^*,\lambda_{2r}^*)=(1/2,1/2),\ \theta=\theta_1^*} = \frac{b\left[4a - (2b-c)(\tau_2+\tau_1)\right](\tau_2-\tau_1)}{2(2b-c)\phi} > 0$$

Hence, full agglomeration is a stable equilibrium when commuting costs are sufficiently low ($\theta < \theta_1^*$).

The foregoing discussion may be summarized in the following proposition.

**Proposition 3** *If $\theta > \theta_2^*$, then full dispersion is a stable equilibrium. If $0 < \theta \leq \theta_1^*$, then full agglomeration is a stable equilibrium.*

This is identical to Proposition 1 when $\theta_1^* = \theta_2^*$. In the case of two industries, however, the interval $(\theta_1^*, \theta_2^*)$ is typically nonempty and we expect some form of partial agglomeration to arise as an equilibrium outcome when commuting costs take intermediate values. Unfortunately, due to the strong nonlinearity of the equilibrium conditions, it is beyond our reach to determine the whole set of stable equilibria against decreasing commuting costs. In particular, some preliminary analysis reveals that equilibria vanish whereas others emerge as these costs decrease. This makes the use of numerical analysis fairly problematic. Thus, extending our model (and, since ours is very simple, any economic geography model) to two sectors appears to be a very difficult task. This is why we choose to study two special, but meaningful, cases in which good 1 can be carried at zero cost, whereas good 2 either is shipped at a positive cost lower that does not exceed $\tau_{trade}$ or is nontradeable.

## 4.3 Regional specialization

It is possible to show the existence of a unique stable equilibrium when one good is perfectly tradeable. Formally, we assume from now on that good 1 can be shipped at zero cost from one region to the other: $\tau_1 = 0$. By setting $\tau_1 = 0$ in (9), it is readily verified that the price of good 1 is the same and equal to

$$p^* \equiv \frac{a}{2b - c}$$

regardless of the region where the varieties are sold. Furthermore, profits being zero in each region, it must be that $w_{1r}^* = w_{1s}^*$. The mobility of workers across sectors then ensures that factor price equalization

$$w^* \equiv w_{1H}^* = w_{1F}^* = w_{2H}^* = w_{2F}^* = \frac{b(p^*)^2}{\phi\left(\lambda_{1r}^* + \lambda_{1s}^*\right)}$$

holds in equilibrium. Consequently, whatever the equilibrium worker distribution, *there is no wage differential between regions at the labor market equilibrium* once we allow for the existence of a footloose industry. Note, however, that $w^*$ is not constant because $\lambda_{1r}^* + \lambda_{1s}^*$ changes with $\theta$.

We are now able to provide a full characterization of the equilibrium path. In the absence of good 2, the market equilibrium always involves dispersion because workers seek to minimize urban costs. We will show that the presence of good 2 leads to a richer set of outcome for intermediate values of commuting costs. When $\tau_1 = 0$, the sustain point and the symmetry breaking point are respectively:

$$\theta_1^* = \frac{2b^2 \left[2a - (b-c)\,\tau_2\right] \tau_2}{(2b-c)^2\,\phi} \qquad \theta_2^* = \frac{4b^2 \left[2a - (b-c)\,\tau_2\right]^2}{c\,(2b-c)^2\,\phi}$$

where $\theta_1^* < \theta_2^*$ holds. The symmetry breaking point $\theta_2^*$ is obtained as follows. The stability conditions of the symmetric configuration (12) become

$$\theta - \theta_A > 0 \qquad \theta - \theta_2^* > 0 \qquad (\theta - \theta_A)(\theta - \theta_B) > (\theta - \theta_2^*)$$

where the parameters $\theta_A$ and $\theta_B$ are defined in Appendix 1. Since $\theta_2^* > \theta_A$ and $\theta_2^* > \theta_B$ can be shown to hold, $\theta_2^*$ is the symmetry breaking point. Furthermore, it is easy to see that the equilibrium conditions $V_{1H}^* = V_{1F}^* = V_{2H}^* = V_{2F}^*$ yields a unique interior solution given by (11). So, when the dispersed configuration becomes unstable, it must be that some $\lambda_{ir} = 0$.

In Appendix 2, we show that

$$\lambda_{1r}^* + \lambda_{1s}^* > 1/2 > \lambda_{2r}^* > \lambda_{2s}^* = 0$$

is the unique stable equilibrium when $\theta \in (\theta_1^*, \theta_2^*)$ (see Appendix 2 for the values of $\lambda_{ir}^*$). This means that *the production of good* 2 *is fully concentrated in region* $r$ *whereas good* 1 *is produced in the two regions.* As shipping good 1 is costless, a positive number of firms belonging to sector 1 find it advantageous to be located in region $r$ ($\lambda_{1r}^* > 0$). Indeed, in the case of fully specialized regions à la Henderson (1974) ($\lambda_{1r} = \lambda_{2s} = 0$), all workers in sector 1 would have to pay for shipping good 2 from region $r$. Simultaneously, they would incur higher urban costs because $\lambda_{1s} > 1/2$ as more than half of the labor force works for sector 1. Clearly, such a configuration cannot be an equilibrium. By distributing themselves between the two regions, the workers of sector 1 equalize the benefits made by saving on the transport of good 2 to the additional urban costs they generate by locating together with the workers of sector 2. Furthermore, we also know that getting agglomerated in the core region together with sector 1 ($\lambda_{1s} = 0$) is not an equilibrium because commuting costs, hence urban costs, in region $r$ would be too large. Among other things, the foregoing equilibrium implies that trade in good 2 is unilateral, while trade in good 1 is bilateral.

By solving $V_{1r} = V_{1s}$ and $V_{1r} = V_{2r}$ for $\theta \in (\theta_1^*, \theta_2^*)$, it is readily verified that $\lambda_{1r}^*$ and $\lambda_{2r}^*$ increase when $\theta$ decreases and reaches the value 1/2 at $\theta = \theta_1^*$. Similarly, $\lambda_{1s}^*$ decreases when $\theta$ decreases and reaches the value

0 when $\theta = \theta_1^*$. Hence, region $r$ accommodates an increasing number of workers as commuting costs decrease. This is because the agglomeration forces generated by the transport costs of good 2 is unaffected whereas the dispersion force associated with the urban costs weakens. For a certain value of $\theta$, the majority of sector 1-workers live in region $r$, which becomes a dominant city because we have $\lambda_{1r}^* > \lambda_{1s}^*$ and $\lambda_{2r}^* > \lambda_{2s}^*$. Simultaneously, the number of workers in sector 2 increases because the two industries tend to congregate.

These results are summarized in the following proposition.

**Proposition 4** *Assume that good* 1 *can be shipped as zero costs and good* 2 *at positive costs* $\tau_2$. *If* $\theta \in (\theta_1^*, \theta_2^*)$, *then industry* 2 *is agglomerated. As* $\theta$ *decreases, the share of industry* 1 *collocated with industry* 2 *rises, whereas the share of the labor force working in sector* 1 *decreases.*

The uneven distribution of workers implies the existence of an urban cost differential. In equilibrium, this one is just compensated by the variety price differential. In other words, *some workers choose to live in the larger city in which they bear higher urban costs because they enjoy there the whole range of varieties at lower prices.* This is consistent with the empirical fact that, in cities of different sizes, the price index differential and/or the nominal wage differential is lower than the differential in housing rent (Tabuchi, 2001). Furthermore, the equilibrium distribution of industry 1 now varies continuously with the parameter $\theta$, unlike what we observe in the one-sector model. This is sufficient to show that the existence of a catastrophic change in the economic landscape is an artefact of the one-sector model.

## 4.4 Urban hierarchy

We now deal with another limiting case in which good 1 can be traded at zero costs, whereas good 2 is nontradeable ($\tau_2 > a/(b-c)$ so that even one-way trade is precluded)–think of a business-to-consumer service industry. We will see that *the existence of such a nontradeable consumption good* (other than land) *is sufficient to generate an urban hierarchy.*[12]

Let the total number of varieties available in region $r$ be $N_r \equiv N_1 + \lambda_{2r} N$. Since good 2 cannot be shipped, its price in each region is equal to $p^* \equiv$

[12] This idea has already been put foward by several authors (see Abdel-Rahman (2000) for references).

$a/(2b-c)$, as shown by maximizing $p_{rr}q_r(p_{rr})(\lambda_{1r}+\lambda_{2r})$. As the indirect utility of an individual working in industry $i$ and residing in region $r$ is given by

$$V_{ir}^* = \frac{[a-(b-c)\,p^*]^2}{b-c}N_r^* + w^* - \frac{\theta}{4}(\lambda_{1r}^*+\lambda_{2r}^*) \qquad i=1,2, \quad r=H,F \quad (15)$$

an uneven distribution of workers implies the existence of an urban cost differential (the third term in (15)). In equilibrium, this one is just compensated by the differential in the number of varieties available in each region (the first term in (15)). In other words, workers now choose to live in a larger city in which they bear higher urban costs because they enjoy there a larger variety of differentiated services.

Solving $V_{1H}=V_{1F}=V_{2H}=V_{2F}$ simultaneously, we obtain the following two interior equilibria:[13]

$$\begin{pmatrix} \lambda_{1r}^* & \lambda_{1s}^* \\ \lambda_{2r}^* & \lambda_{2s}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad (16)$$

$$\begin{pmatrix} \lambda_{1r}^* & \lambda_{1s}^* \\ \lambda_{2r}^* & \lambda_{2s}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{4}+\frac{(4-T\theta)\sqrt{3-T\theta}}{4T\theta} & \frac{1}{4}-\frac{(4-T\theta)\sqrt{3-T\theta}}{4T\theta} \\ \frac{1}{4}+\frac{\sqrt{3-T\theta}}{4} & \frac{1}{4}-\frac{\sqrt{3-T\theta}}{4} \end{pmatrix} \quad (17)$$

where

$$T \equiv \frac{(b-c)\phi}{b^2(p^*)^2} \in \left[\frac{2}{\theta}, \frac{3}{\theta}\right]$$

The first equilibrium (16) involves full dispersion. This is always an equilibrium, whatever the value of the commuting cost $\theta$. However, (17) is an equilibrium if and only if $\theta \in [2/T, 3/T]$ because each variable must be real and must take its values in $[0,1]$. This equilibrium can be shown to be stable by applying Routh's theorem to the characteristic polynomial of the Jacobian of (10) evaluated at (17). Note that the fact that $\lambda_{ir}^* + \lambda_{is}^* = 1/2$ holds for $i=1,2$ eases the stability analysis when compared to the case with finite and positive values for $\tau_2$. Furthermore, for all $\theta \in (2/T, 3/T)$, $1/4 < \lambda_{1r}^* < \lambda_{2r}^* < 1/2$ holds, thus implying that the industry producing the nontradeable good is more agglomerated than the other. The two equilibria coincide with full dispersion when $\theta = 3/T$. On the other hand, (17) involves full agglomeration (13) at $\theta = 2/T$. Thus, $\theta = 3/T$ is the symmetry breaking point and $\theta = 2/T$ is the agglomeration sustain point.

[13] The mirror image of the asymmetric equilibrium is disregarded.

Since all possible equilibria have been accounted for, we may conclude that *the labor force is equally split between the two industries despite the fact that workers can shift jobs*. Indeed, when compared to firms producing a costlessly tradeable good, firms producing a nontradeable good have fewer customers, but each individual demand is higher. Because these two effects just cancel out, the labor share is never higher in one sector than in the other.

The foregoing argument may then be summarized as follows:

**Proposition 5** *Assume that good* 1 *can be traded at zero costs whereas good* 2 *is nontradeable. As* $\theta$ *decreases, there is only one stable equilibrium path: (i) if* $\theta \geq 3/T$*, then each industry is fully dispersed; (ii) if* $2/T < \theta < 3/T$*, then both industries are partially agglomerated within the same region and the industry producing the nontradeable good is more agglomerated than the industry producing the costlessly tradeable good; (iii) if* $\theta \leq 2/T$*, then both industries agglomerate in one region.*

The following remarks are in order. First, although varieties of good 1 can be shipped at zero cost, the existence of a nontradeable good allows for a well-defined distribution of industry 1 between regions. More precisely, except for fairly high commuting costs ($\theta \geq 3/T$), *the nontradeable sector acts as a centripetal force that yields partial or full agglomeration of the other industry.* When commuting costs are sufficiently low ($\theta \leq 2/T$), the two industries are located into a single region while the other region is empty. As discussed above, the availability of more differentiated services compensates workers for the higher urban costs they bear within the agglomeration.

Second, *the locational pattern chosen by each of the two industries is in general different*. This is because the footloose industry does not care a priori about its location, whereas the service industry cares about the spatial distribution of its demand. As a result, the former industry tends to be more dispersed than the latter industry. This agrees with empirical facts: within modern cities, the share of the manufacturing industries tends to be smaller than the share of the service sectors.[14] However, one region accommodates

[14]For example, the Tokyo Metropolitan Area consists of Tokyo, Kanagawa, Chiba and Saitama prefectures, whose population share is 26% in 1996. Its employment share is 24% in manufacturing industries, which is slightly less than population. However, the service sectors are more concentrated than population: the employment share is 30% in transport and communications, 32% in wholesale, 33% in restaurants and bars, 35% in finance and insurance, and 40% in real estate.

more than half of each industry. As a result, *the market outcome involves an urban hierarchy* in that, for each good, a larger array of varieties is produced within the same city.

Third, according to export-base theory (Richardson, 1978), a strong export sector, such as low transport costs industries, is a powerful engine of regional development in that it attracts the service sectors. Proposition 5 just says the opposite. When the agglomeration process begins (i.e. when $T$ takes intermediate values), the service sector is always more agglomerated than the export sector. In other words, *it is the agglomeration of the service industry that causes here the agglomeration of the export industry.* This difference in results is due to the fact that the export-base theory relies on intermediate services whereas we focus here on consumer services.

Fourth, and last, we can compute the labor share of industry 1 in each region. Using the equilibrium values (17), we obtain

$$\sqrt{6} - 2 \leq \frac{\lambda_{1r}^*}{\lambda_{1r}^* + \lambda_{2r}^*} \leq 1/2 \leq \frac{\lambda_{1s}^*}{\lambda_{1r}^* + \lambda_{2r}^*} \leq 3/4$$

where the inequalities are strict for all $\theta \in (2/T, 3/T)$. This implies that the large region $r$ has a larger labor share in industry 2 whereas the small region $s$ has a larger labor share in industry 1. This is consistent with Ricardo's *comparative advantage theory* when each region is partially specialized in different industries: the large region has here a comparative advantage in the nontradeables because it has a larger market, whereas the small region has a comparative advantage in terms of urban costs. Another important implication is that (17) implies $\lambda_{2r}^* \geq \lambda_{1r}^* \geq 1/4$: the large region $r$ has a larger share of each industry than the small region $s$, and the large region $r$ has more varieties of both goods than the small region. This agrees with Christaller's (1933) *central place theory* in which large cities have more firms and varieties.

# 5 Concluding remarks

In the advanced societies of our new century, workers are likely to become more and more mobile, while industries will become freer from immobile production factors. Thus, although a model with an immobile sector but without urban costs, such as the standard core-periphery model, is applicable to the time of the Industrial Revolution, a model with no immobile sector but with urban costs, such as the one considered in this paper, seems to be more suitable to our time.

Having said that, we may summarize our main results as follows. In the one-sector model, we obtain the standard evolutionary pattern of economic geography: the economy moves from dispersion to agglomeration as commuting costs decrease, but the impact of the main parameters of the economy on the market outcome differ. In the two-sector model, these two configurations emerge for high and low commuting costs only. For intermediate values, the set of equilibria is much richer in that they typically involve regional specialization and urban hierarchy. Thus, in conducting empirical studies, economists should not take the predictions of the standard core-periphery model at face value.

In this paper, we have chosen to focus on industries that differ only in terms of transport costs. The next line of research to be addressed is the location of several industries which differ in several structural parameters, for example different market sizes as well as different fixed production costs and transport costs. Such an analysis would accomplish what Lösch aimed at doing but did not succeed to do. Similarly, we have used the framework of economic geography that typically involves two regions. Our model should be extended to an arbitrary number of regions in order to build a more general theory of urban systems.

# Appendix

## 1. Definition of the coefficients of $\theta_A$ and $\theta_B$

$$\theta_A \equiv -\frac{b\left[\begin{array}{c} 512a^4 - 128a^3\left(9b-7c\right)\tau_2 + 16a^2\left(76b^2 - 108bc + 39c^2\right)\tau_2^2 \\ -8a\left(80b^3 - 172b^2c + 123bc^2 - 29c^3\right)\tau_2^3 + (10b-7c)(2b-c)(8b^2-12bc+5c^2)\tau_2^4 \end{array}\right]}{2\left(2b-c\right)^2\left[16a^2 - 16a(b-c)\tau_2 + \left(8b^2-12bc+5c^2\right)\tau_2^2\right]\phi}$$

$$\theta_B \equiv \frac{b\tau_2\left[\begin{array}{c} 2048a^5(3b-c) - 512a^4\left(22b^2-27bc+6c^2\right)\tau_2 + 64a^3\left(176b^3-356b^2c+225bc^2-39c^3\right)\tau_2^2 \\ -32a^2\left(208b^4-568b^3c+569b^2c^2-240bc^3+33c^4\right)\tau_2^3 + 4a\left(8b^2-12bc+5c^2\right) \\ \times\left(72b^3-140b^2c+83bc^2-13c^3\right)\tau_2^4 + \left(6b^2-6bc+c^2\right)\left(8b^2-12bc+5c^2\right)^2\tau_2^5 \end{array}\right]}{\left(2b-c\right)^2\phi\left[\begin{array}{c} 512a^4 - 768a^3\left(b-c\right)\tau_2 + 16a^2\left(40b^2-64bc+29c^2\right)\tau_2^2 \\ -32a(b-c)(8b^2-12bc+5c^2)\tau_2^3 + \left(8b^2-12bc+5c^2\right)^2\tau_2^4 \end{array}\right]}$$

## 2. Proof that $\lambda_{1r}^* + \lambda_{1s}^* > 1/2 > \lambda_{2r}^* > \lambda_{2s}^* = 0$ is a stable equilibrium

When $\lambda_{2s}^* = 0$, solving $V_{1r} = V_{1s}$ and $V_{1r} = V_{2r}$ yields

$$\lambda_{1r}^* = \frac{\tau_2\left[2a-\left(b-c\right)\tau_2\right]\left[2b^2\left(b-c\right)\tau_2^2 + \left(2b-c\right)^2\phi\theta\right]\left[2b^2\left(2a-(b-c)\tau_2\right)^2 + \left(b-c\right)\left(2b-c\right)^2\phi\theta\right]}{2\left(2b-c\right)^2\left[2b^2\left(b-c\right)\tau_2^2\left[2a-(b-c)\tau_2\right]^2 + \left(2b-c\right)^2\left[4a^2-2a(b-c)\tau_2+\left(b-c\right)^2\tau_2^2\right]\phi\theta\right]\phi\theta}$$

$$\lambda_{2r}^* = \frac{2b^2\left(b-c\right)\tau_2^2\left[2a-(b-c)\tau_2\right]^2 + \left(2b-c\right)^2\left[2a^2-2a(b-c)\tau_2+\left(b-c\right)^2\tau_2^2\right]\phi\theta}{2b^2\left(b-c\right)\tau_2^2\left[2a-(b-c)\tau_2\right]^2 + \left(2b-c\right)^2\left[4a^2-2a(b-c)\tau_2+\left(b-c\right)^2\tau_2^2\right]\phi\theta}$$

Furthermore, evaluating the eigenvalues of the Jacobian of

$$\begin{cases} \dot{\lambda}_{1r} = \lambda_{1r}\left(V_{1r} - \lambda_{1r}V_{1r} - \lambda_{2r}V_{2r} - \lambda_{1s}V_{1s}\right) \\ \dot{\lambda}_{2r} = \lambda_{2r}\left(V_{2r} - \lambda_{1r}V_{1r} - \lambda_{2r}V_{2r} - \lambda_{1s}V_{1s}\right) \end{cases}$$

with $\lambda_{1r} + \lambda_{2r} + \lambda_{1s} = 1$, shows that this equilibrium is stable. It is also easy to check that

$$\frac{\partial\lambda_{1r}^*}{\partial\theta} < 0 \qquad \frac{\partial\lambda_{2r}^*}{\partial\theta} < 0 \qquad \frac{\partial\lambda_{1s}^*}{\partial\theta} > 0$$

Hence, $\lambda_{1r}^*$ and $\lambda_{2r}^*$ increase when $\theta$ decreases and reaches the value 1/2 at $\theta = \theta_1^*$, whereas $\lambda_{2s}^*$ decreases when $\theta$ decreases and becomes 0 at $\theta = \theta_1^*$.

Finally, when $\lambda_{1s}^* = 0$ solving $V_{1r} = V_{2r}$ and $V_{2r} = V_{2s}$ with $\theta = \theta_2^*$, we obtain a cubic function for $\lambda_{1r}^*$. By examining it, all its solutions are be either greater than 1/2 or complex numbers. Given $\lambda_{1r}^* > 1/2$, it is readily verified that $\lambda_{2r}^*/(\lambda_{2r}^* + \lambda_{2s}^*) \notin [0,1]$, thus implying that there is no equilibrium such that $\lambda_{1s}^* = 0$. Hence, $\lambda_{1r}^* + \lambda_{1s}^* > 1/2 > \lambda_{2r}^* > \lambda_{2s}^* = 0$ is the unique (stable) equilibrium in the interval $(\theta_1^*, \theta_2^*)$. □

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