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Influence of Non-Maxwellian Particles on Dust Acoustic Waves in a Dusty Magnetized Plasma

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Abstract *In this paper an investigation into dust acoustic solitary waves (DASWs) in the presence of superthermal electrons and ions in a magnetized plasma with cold dust grains and trapped electrons is discussed. The dynamic of both electrons and ions is simulated by the generalized Lorentzian (κ) distribution function (DF). The dust grains are cold and their dynamics are studied by hydrodynamic equations. The basic set of fluid equations is reduced to modified Korteweg-de Vries (mKdV) equation using Reductive Perturbation Theory (RPT). Two types of solitary waves, fast and slow dust acoustic soliton (DAS) exist in this plasma. Calculations reveal that compressive solitary structures are possibly propagated in the plasma where dust grains are negatively (or positively) charged. The properties of DASs are also investigated numerically.*

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Key words: dust acoustic soliton, magnetized plasma, superthermal electrons and ions, trapped electrons

1 Introduction

Most plasmas are characterized by the presence of superthermal particles, possibly generated by turbulent acceleration, external sources, or, as in the case of fusion devices, nuclear reactions. Understanding the basic phenomena that determine the superthermal particle dynamics is a key challenge to describe a wide range of plasma system, ranging from magnetically confined plasmas for fusion to space plasmas.[1−3]

Non-Maxwellian velocity distributions have been observed in many astrophysical and space plasmas. Theoretically it has been shown that the velocity distribution function obeys a power law that is emerged as a natural consequence of the presence of superthermal radiation fields in plasma.[4] The observed distribution contains a plentiful supply of superthermal particles, i.e., particles that move faster than the thermal speed. These so-called nonthermal plasmas are found naturally in the magnetospheres of Earth, Mercury, Saturn, Uranus, and in the solar wind.[5−6] The observed distributions of charged particles have been well fitted with a generalized Lorentzian (kappa) distribution.[4−7] The important features of kappa distribution are as follows: (i) At high velocities the distribution obeys an inverse power law. (ii) For all velocities, in the limit that spectral index approaches to large values, the DF approaches to the Maxwellian one. In this sense, the kappa distribution is a generalization of the Maxwellian distribution. Vasyliunas[8] appears to be the first who employed the general form of the kappa distribution and noticed its relation to the Maxwellian distribution. The kappa distribution was later adopted in various physical contexts.[6−7,9−10]

The spacecraft measurements of electron energy spectra have been successfully modeled with kappa distribution. Using this distribution, Summers and Trone[10] have discussed plasma dispersion function theoretically and found the general properties of a dielectric tensor for ion and Langmuir waves. Xiao[11] modeled energetic particles by relativistic kappa-loss-cone distribution function in plasmas and showed that this distribution obeys the power law that is valid not only at the lower energies but also at relativistic energies.

Results from the Voyager 1 and 2 spacecrafts,[12] during their encounters with the magnetosphere of Saturn, it is found that the energy spectra of ions (assumed protons) is like a Maxwellian at low energies ($\leqslant$ 200 keV) and a power law at high energies ($\geqslant$ 200 keV). Krimigis *et al.*,[12] used kappa distribution to fit ion spectral observations in the magnetosphere of Saturn, with typical values of kappa in range 6–8 and thermal energy K_BT approximately in the range 16–28 keV matching the observations extremely well, though with a few exceptions.

In recent years, a non-Maxwellian velocity distribution has found its applications in the field of dusty plasma that is a rapidly growing field because of its vast variety of applications in laboratory, space, and astrophysical plasma environments (e.g., the Earth's ionosphere, asteroid zones, planetary ring, cometary tails, interstellar clouds, etc.). There has been a great deal of interest in understanding different types of collective processes in dusty plasmas.[13−17] It has been shown both theoretically and experimentally that the presence of these extremely massive and highly charged dust grains in a plasma can either modify the behavior of the usual waves and instabilities or

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introduce new eigenmodes. The most well studied of such modes is the so-called "dust acoustic wave" (DAW),[18−20] which arises due to the restoring force provided by the plasma thermal pressure (electrons and ions), while the inertia is due to the dust mass.

One basic problem in all of these studies is the electrostatic charging of the grains that results from various processes. Since, here, the characteristic time scale for dynamical processes (that for dust motion is of the order of tens of milliseconds) is much larger than dust charging time (that is typically of the order of 10^{-6}–10^{-4} s),[21] the dust particle charging process can be considered to take place promptly. Under this condition, the dust charge, with a good accuracy, could be assumed constant.[14,22−25]

This paper aims to investigate the effect of superthermal particles on the nonlinear propagation of dust acoustic waves in presence of a uniform external magnetic field. These waves are evolved into solitary structures when the effect of nonlinearity and dispersion of the plasma are balanced with each other. Solitary wave is called a soliton if it retains its shape after collision with another solitary wave.[26−29] The generalized Lorentzian (κ)-DF is used to model the existence of superthermal electrons and ions. This problem has been recently considered by Baluku *et al.*[30] in a slightly different formalism, without including the effect of trapped electrons and magnetic field. Electron trapping, due to the nonlinear resonant interaction of the DAS and electrons, is included. The resonant interaction only takes place for those electrons that have a velocity close to the DAS velocity.

It is shown that the presence of superthermal particles and trapped electrons has great influence on the nature of magnetized dust acoustic solitons. Moreover, the dependence of the soliton characteristics on relevant physical parameters of the problem is studied.

This paper is organized as follows: In the next section, the basic equations are presented which describe the dusty plasma system and governing equations including superthermal particles. In Sec. 3, a weakly nonlinear analysis is carried out and a modified Korteweg-de Vries (mKdV) equation is derived, in Sec. 4, the numerical results are discussed. Section 5, gives a discussion for the case of positive dust. Last section, contains a brief summary of our investigation of this study.

2 Basic Equation and Formulation

A three component, homogeneous, magnetized dusty plasma is considered which comprised of a mixture of superthermal positive ions, superthermal electrons, and negatively charged dust particles.

The static magnetic field $\vec{B}$ is applied in the z-direction, and propagation vector $\vec{k}$ in the (x,z) plane and the angle between $\vec{k}$ and $\vec{B}$ is θ. For simplicity, it is assumed all the dust grains have the same charge, equal to $q_d = -ez_d$, with positive z_d for negatively charged dust.

Nonlinear dynamics of a low-frequency dust acoustic solitary wave is governed by the following equations:

$$\frac{\partial n_d}{\partial t} + \nabla\cdot(n_d\vec{v}_d) = 0\,, \tag{1}$$

$$\frac{\partial \vec{v}_d}{\partial t} + (\vec{v}_d.\nabla)\cdot\vec{v}_d = \frac{-z_d e}{m_d}\Big(-\nabla\phi + \frac{\vec{v}_d\times\vec{B}}{c}\Big)\,, \tag{2}$$

$$\nabla^2\phi = 4\pi e(n_e + z_d n_d - z_i n_i)\,, \tag{3}$$

n_d, $\vec{v}_d$, m_d, and z_d are the dust density, fluid velocity, mass, and charge state, respectively. ϕ is the self consistent electric potential, c is the speed of light, n_i and n_e refer to ion and electron density, respectively. The charge neutrality at equilibrium requires that

$$n_e^{(0)} + z_d n_d^{(0)} - z_i n_i^{(0)} = 0\,, \tag{4}$$

where $n_e^{(0)}$, $n_i^{(0)}$, and $n_d^{(0)}$ are the unperturbed electron, ion, and dust number density, respectively.

In the dynamical systems, some of electrons are attached to the dust particles to form the dust charged particles, however, some other electrons are bounded back and forth in the potential well, losing energy continuously, and as a result, being ultimately trapped electrons. In this case, the electron density is defined from the Vlasov equation consisting of free and trapped electrons.[15] The distribution function of both the free and trapped electrons was proposed by Gurevich[31] and Shamel[32] for Maxwellian plasma. Accordingly, the following κ DF is introduced:[14,33−34]

$$\begin{aligned} f_{\rm te}(v_{e\|},v_{e\perp}) &= \frac{n_{0e}c_{\kappa_e}}{v_{\rm Te}\sqrt{2\pi}}\Big[1+\beta\frac{m_e v_{e\|}^2-2e\phi}{m_e(2\kappa_e-3)v_{\rm Te}^2}\Big]^{-\kappa}\frac{\delta(v_{e\perp})}{2\pi v_{e\perp}}\,,\quad |v_{e\|}|\leqslant\sqrt{\frac{2e\phi}{m_e}}\,,\\ f_{\rm fe}(v_{e\|},v_{e\perp}) &= \frac{n_{0e}c_{\kappa_e}}{v_{\rm Te}\sqrt{2\pi}}\Big[1+\frac{m_e v_{e\|}^2-2e\phi}{m_e(2\kappa_e-3)v_{\rm Te}^2}\Big]^{-\kappa}\frac{\delta(v_{e\perp})}{2\pi v_{e\perp}}\,,\quad |v_{e\|}|\geqslant\sqrt{\frac{2e\phi}{m_e}}\,, \end{aligned} \tag{5}$$

where $f_{\rm fe}$, $f_{\rm te}$ are the free and trapped electron velocity DF, respectively. n_{0e} is the equilibrium electron density, m_e is an electron mass. $\perp(\|)$ is the sign denotes the perpendicular (parallel) direction to the $\vec{B}$. δ is the Dirac delta function and it is also the indication of temperature anisotropy (i.e. $T_{e\|}\gg T_{e\perp}$).[33,35] By this choice, $T_{e\perp}$ does not appear in the formalism. $v_{\rm Te}=\sqrt{T_{e\|}/m_e}$ is the electron thermal velocity. κ_e is the electron spectral index ($\kappa_e>3/2$), $c_{\kappa_e}=(1/\sqrt{\kappa_e-3/2})[\Gamma(\kappa_e)/\Gamma(\kappa_e-1/2)]$ is the normalization coefficient, Γ is the gamma function. β represents the ratio of the free to the trapped electron temperatures. $\beta=0$ models the plateau structure and $\beta<0$ models the hole structure in the electron DF. Note that the above κ distributions are written so that as $\kappa\to\infty$, their Maxwellian counterparts are obtained.[32]

Integrating the DF over the corresponding velocity range, the number density for the electrons is obtained:[14,34]

$$n_e = \int_0^\infty 2\pi v_{e\perp} \mathrm{d}v_{e\perp} \Big[2\int_{\sqrt{2e\phi/m_e}}^\infty f_{\rm fe}(v_{e\parallel}, v_{e\perp})\mathrm{d}v_{e\parallel} + 2\int_0^{\sqrt{2e\phi/m_e}} f_{\rm te}(v_{e\parallel}, v_{e\perp})\mathrm{d}v_{e\parallel}\Big] . \tag{6}$$

In the weak nonlinear range ($e\phi/T_{e\parallel} < 1$), the Taylor expansion of the Eq. (6), derives the electron density, n_e, as a combination of free and trapped electrons as:[34]

$$\frac{n_e}{n_{0e}} = 1 + a_{k_e}\frac{e\phi}{T_{e\parallel}} - \frac{4}{3}\frac{(1-\beta)}{\sqrt{\pi}} b_{k_e}\Big(\frac{e\phi}{T_{e\parallel}}\Big)^{3/2} , \tag{7}$$

where

$$a_{k_e} = \frac{2\kappa_e - 1}{2\kappa_e - 3} , \tag{8}$$

$$b_{k_e} = \frac{1}{(\kappa_e - 3/2)^{3/2}} \frac{\Gamma(\kappa_e+1)}{\Gamma(\kappa_e - 1/2)} . \tag{9}$$

In a collisionless plasma, the appearance of superthermal ions may have a significant influence on the behavior of DASWs; therefore, the ion DF can then be determined as:[24]

$$f_i(v_{i\parallel}, v_{i\perp}) = \frac{n_{0i}c_{\kappa_i}}{v_{Ti}\sqrt{2\pi}}\Big[1 + \frac{m_i v_{i\parallel}^2 + 2z_i e\phi}{m_i(2\kappa_i - 3)v_{Ti}^2}\Big]^{-\kappa}\frac{\delta(v_{i\perp})}{2\pi v_{i\perp}}, \tag{10}$$

where f_i is the ion velocity DF. n_{0i} is the equilibrium ion density, m_i is an ion mass, $v_{Ti} = \sqrt{T_{i\parallel}/m_i}$ is the ion thermal velocity. κ_i is the ion spectral index ($\kappa_i > 3/2$) and $c_{\kappa_i} = (1/\sqrt{\kappa_i - 3/2})[\Gamma(\kappa_i)/\Gamma(\kappa_i - 1/2)]$ is the normalization coefficient.

The ion density is obtained by integrating the DF over the whole velocity range,[35]

$$n_i = \int_0^\infty 2\pi v_{i\perp}\mathrm{d}v_{i\perp}\int_{-\infty}^\infty f_i(v_{i\parallel}, v_{i\perp})\mathrm{d}v_{i\parallel} . \tag{11}$$

n_i is expanded for small $\phi (e\phi/T_{i\parallel} < 1)$ based on the Taylor series and the expression for ion number density in terms of electrostatic potential is obtained as follows:[24,35]

$$\frac{n_i}{n_{0i}} = 1 - a_{k_i}\frac{z_i e\phi}{T_{i\parallel}} + b_{k_i}\Big(\frac{z_i e\phi}{T_{i\parallel}}\Big)^2 , \tag{12}$$

where,

$$a_{k_i} = \frac{2\kappa_i - 1}{2\kappa_i - 3} , \tag{13}$$

$$b_{k_i} = \frac{4\kappa_i^2 - 1}{2(2\kappa_i - 3)^2} . \tag{14}$$

3 Fast and Slow Compressive Solitary Waves

In order to study dust acoustic soliton, relevant mKdV equation is derived for the present plasma model. For this case, the following normalization is chosen; the local electrostatic potential is normalized to T_e/ez_d, number density to equilibrium number density n_0. Velocity and space variables are normalized to the dust acoustic speed $c_d = (T_e/m_d)^{1/2}$ and electron Debye length $\lambda_{\rm De} = (T_e/4\pi e^2 n_{0e})^{1/2}$, respectively. Also, time variable is normalized to $\lambda_{\rm De}/c_d$.

To simplify the analysis, a new axis ζ in the (x, z) plane is defined and θ is the angle between two axes ζ and z. Then the one-dimensional wave propagating along the ζ-axis is considered. Therefore, the complete set of equations is

$$\frac{\partial n_d}{\partial t} + \sin\theta\frac{\partial(n_d v_{dx})}{\partial \zeta} + \cos\theta\frac{\partial(n_d v_{dz})}{\partial\zeta} = 0 , \tag{15}$$

$$\frac{\partial v_{dx}}{\partial t} + v_{dx}\sin\theta\frac{\partial v_{dx}}{\partial\zeta} + v_{dz}\cos\theta\frac{\partial v_{dx}}{\partial\zeta} = \sin\theta\frac{\partial\phi}{\partial\zeta} - \mu\rho^{1/2}v_{dy} , \tag{16}$$

$$\frac{\partial v_{dz}}{\partial t} + v_{dx}\sin\theta\frac{\partial v_{dz}}{\partial\zeta} + v_{dz}\cos\theta\frac{\partial v_{dz}}{\partial\zeta} = \cos\theta\frac{\partial\phi}{\partial\zeta} , \tag{17}$$

$$\frac{\partial v_{dy}}{\partial t} + v_{dx}\sin\theta\frac{\partial v_{dy}}{\partial\zeta} + v_{dz}\cos\theta\frac{\partial v_{dy}}{\partial\zeta} = \mu\rho^{1/2}v_{dx} , \tag{18}$$

$$\frac{\partial^2\phi}{\partial\zeta^2} = \rho(n_d - 1) + (a_{k_e} + D)\phi - A\phi^{3/2} - E\phi^2 , \tag{19}$$

where,

$$A = \frac{4}{3}\frac{(1-\beta)}{\sqrt{\pi}}\frac{b_{\kappa_e}}{z_d^{1/2}} , \tag{20}$$

$$D = z_i\Big(1 + \frac{\rho}{z_d}\Big)\sigma a_{\kappa_i} , \tag{21}$$

$$E = b_{\kappa_i}\sigma^2\frac{z_i^2}{z_d}\Big(1 + \frac{\rho}{z_d}\Big) . \tag{22}$$

Here,

$$\rho = \frac{z_d^2 n_{0d}}{n_{0e}} , \quad \sigma = \frac{T_e}{T_i} ,$$

$$\mu = \frac{\omega_{\rm Bd}}{\omega_{\rm pd}} \cdot \omega_{\rm pd} = \Big(\frac{4\pi z_d^2 e^2 n_{0d}}{m_d}\Big)^{1/2}$$

is the dust plasma period and $\omega_{\rm Bd} = z_d eB/m_d c$ is the dust cyclotron frequency.

In the unperturbed initial state, $\sum q_j n_{0j} = 0$, and with $\rho = z_d^2 n_{0d}/n_{0e}$ the relation $z_i n_{0i}/n_{0e} = \rho/z_d + 1$, is obtained where $z_i n_{0i}/n_{0e} > 1$ for negatively charged dust particles.

Now reductive perturbation theory (RPT) is used to analyze DASWs with small but finite amplitude in the present plasma, for which the following stretched coordinates are introduced:[13,15,23,34]

$$\xi = \varepsilon^{1/4}(\zeta - \lambda_\nu t) , \tag{23}$$

$$\tau = \varepsilon^{3/4} t , \tag{24}$$

where $\lambda_\nu (\nu = f, s)$ is the unknown phase velocity while it will be determined later. The RPT requires the expansion of dependent quantities. Accordingly, n_d, v_d, and ϕ are introduced in power series based on ε, as follows:

$$n_d = 1 + \varepsilon n_1 + \varepsilon^{3/2} n_2 + \cdots , \tag{25}$$

$$v = \varepsilon v_1 + \varepsilon^{3/2} v_2 + \cdots , \tag{26}$$

$$\phi = \varepsilon\phi_1 + \varepsilon^{3/2}\phi_2 + \cdots\,, \tag{27}$$

where ε is the small parameter.

To obtain the proper μ in the presence of roughly parameters of dusty plasmas, such as $m_d = 3\times10^{-11}$ kg and $c = 3\times10^8$ m/s, $\mu = B/10^{-3}\sqrt{n_{0d}}$. In plasmas which the magnetic field (B) is weak and the density of dust grains (n_{0d}) is dense, $\mu \ll 1$ ($B/\sqrt{n_{0d}} \ll 10^{-3}$) so, $O(\mu) = \varepsilon^{3/4}$. Besides, in plasmas with strong magnetic field (B) and the density of dust particles (n_{0d}) is rare, $\mu \gg 1$ ($B/\sqrt{n_{0d}} \gg 10^{-3}$) and therefore $O(\mu) = \varepsilon^{-1/4}$.[34]

Now substituting Eqs. (23)–(27) into Eqs. (15)–(19), the following mKdV equation for $\mu \ll 1$ is obtained

$$\frac{\partial\phi_1}{\partial\tau} + \frac{\rho^{1/2}}{2(a_{k_e}+D)^{3/2}}\frac{\partial^3\phi_1}{\partial\xi^3} + \frac{3\rho^{1/2}A}{4(a_{k_e}+D)^{3/2}}\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi} = 0\,, \tag{28}$$

where λ_f, the phase velocity, is given as:

$$\lambda_f = \sqrt{\frac{\rho}{a_{k_e}+D}}\,. \tag{29}$$

By the same analysis, an mKdV equation is obtained for $\mu \gg 1$ as follows:

$$\frac{\partial\phi_1}{\partial\tau} + \frac{\rho^{1/2}\cos\theta}{2(a_{k_e}+D)^{3/2}}\frac{\partial^3\phi_1}{\partial\xi^3} + \frac{3\rho^{1/2}A\cos\theta}{4(a_{k_e}+D)^{3/2}}\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi} = 0\,. \tag{30}$$

Also, λ_s, the phase velocity for $\mu \gg 1$, is obtained as:

$$\lambda_s = \sqrt{\frac{\rho}{a_{k_e}+D}}\cos\theta\,. \tag{31}$$

The phase velocities λ_f and λ_s are called as the fast and slow waves respectively, because $\lambda_f \geqslant \lambda_s$.

The stationary solution of Eq. (28) is obtained by assuming $\phi = \phi_f(\xi - u_f\tau)$, where u_f is the constant soliton velocity in the moving frame (the frame which moves with $\lambda_f = \sqrt{\rho/(a_{k_e}+D)}$). Then, by assuming the appropriate boundary condition, namely $\phi_f \to 0$, $\partial\phi_f/\partial\xi \to 0$ and $\partial^2\phi_f/\partial\xi^2 \to 0$, as $|\xi - u_f\tau| \to \infty$, one obtains

$$\phi_f = \phi_{\rm fm}\mathrm{sech}^4\Big[\sqrt{\Big|\frac{(1-\beta)b_{\kappa_e}}{15z_d^{1/2}\sqrt{\pi}}\Big|\sqrt{\phi_{\rm fm}}}(\xi - u_f\tau)\Big]\,, \tag{32}$$

$$u_f = \frac{2\rho^{1/2}}{3(a_{k_e}+D)^{3/2}}\frac{(1-\beta)b_{\kappa_e}}{z_d^{1/2}\sqrt{\pi}}\sqrt{\phi_{\rm fm}}\,, \tag{33}$$

$$\Delta_f = 2\Big(\Big|\frac{15z_d^{1/2}\sqrt{\pi}}{(1-\beta)b_{\kappa_e}}\Big|\frac{1}{\sqrt{\phi_{\rm fm}}}\Big)^{1/2}\cosh^{-1}\sqrt{2}\,, \tag{34}$$

where $\phi_{\rm fm}$ is the maximum amplitude and Δ_f is the width at half maximum of soliton. In laboratory frame the soliton velocity is

$$u_{f,\rm lab} = u_f + \lambda_f\,. \tag{35}$$

The nonlinear evolution of the slow ion-acoustic follows Eq. (30). The stationary solution of equation is similarly obtained by assuming $\phi = \phi_s(\xi - u_s\tau)$, where u_s is the constant soliton velocity in the moving frame (here the frame moves with $\lambda_s = \sqrt{\rho/(a_{k_e}+D)}\cos\theta$). After considering the suitable boundary conditions of $\phi_s \to 0$, $\partial\phi_s/\partial\xi \to 0$, and $\partial^2\phi_s/\partial\xi^2 \to 0$, as $|\xi - u_f\tau| \to \infty$, the following is obtained:

$$\phi_s = \phi_{\rm sm}\mathrm{sech}^4\Big[\sqrt{\Big|\frac{(1-\beta)b_{\kappa_e}}{15z_d^{1/2}\sqrt{\pi}}\Big|\sqrt{\phi_{\rm sm}}}(\xi - u_s\tau)\Big]\,, \tag{36}$$

$$u_s = \frac{2\rho^{1/2}}{3(a_{k_e}+D)^{3/2}}\frac{(1-\beta)b_{\kappa_e}}{z_d^{1/2}\sqrt{\pi}}\cos\theta\sqrt{\phi_{\rm sm}}\,, \tag{37}$$

$$\Delta_s = 2\Big(\Big|\frac{15z_d^{1/2}\sqrt{\pi}}{(1-\beta)b_{\kappa_e}}\Big|\frac{1}{\sqrt{\phi_{\rm sm}}}\Big)^{1/2}\cosh^{-1}\sqrt{2}\,, \tag{38}$$

where $\phi_{\rm sm}$ is the maximum amplitude and Δ_s is the width at half maximum of soliton. The soliton velocity in the laboratory frame is

$$u_{s,\rm lab} = u_s + \lambda_s\,. \tag{39}$$

4 Numerical Result and Discussion

The nonlinear term in mKdV equation is in charge of the steepening effect. Form Fig. 1, the nonlinear term is a decreasing function of κ. It means, for smaller population of superthermal particles (larger κ), the nonlinear term is smaller. Therefore, it is expected that DASs move slower in the moving frame which moves with λ_f. On the other hand, dispersive coefficient in mKdV equation is in charge of broadening effect. This coefficient is an increasing function of κ. So that by increasing of superthermal particles (smaller κ), dispersive coefficient decreases. As a result, the width of DASs should be smaller.[34–35]

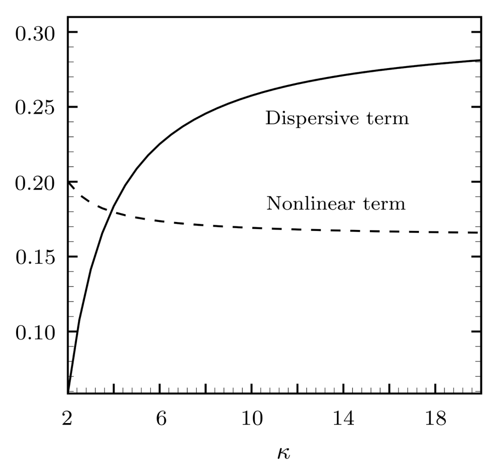

Fig. 1 Nonlinear and dispersive coefficient of mKdV equation versus $\kappa_e = \kappa_i = \kappa$ with $\mu = 10^{-3}$, $z_d = 10$, $\rho = z_d$, $z_i = 1$, and $\beta = -0.5$ for fast mode.

As soliton formation depends on the balance of steepening and broadening effects, the above explanations are base of our physical expectations.

Figures 2–4 depict the fast soliton velocity versus β for the different parameters of plasma.

Figure 2(a) depicts the soliton velocity versus β in the moving frame. Recall, the frame velocity in the case of fast mode is $\lambda_f = \sqrt{\rho/(a_{k_e} + D)}$. As it is seen from the figure, the soliton velocity is increased by increasing the population of superthermal particles. It means that for smaller κ, the soliton velocity is decreased. However, the results in the laboratory frame differ. The reason of this difference is because of the perturbative nature of the model which is used to obtain mKdV equation. In this method, since the higher orders of ε have lower effects the velocity of the moving frame is obtained from a lower order of ε. Therefore, the velocity of the moving frame is much larger than the soliton velocity in the moving frame. So the moving frame velocity plays a significant role. As it is seen from Fig. 2(b), the phase velocity (equal to the moving frame velocity) is a decreasing function of the superthermal particles population (smaller phase velocity for smaller κ). Therefore, it is expected that the soliton velocity, in the laboratory frame, is also a decreasing function of the superthermal particles population.[34]

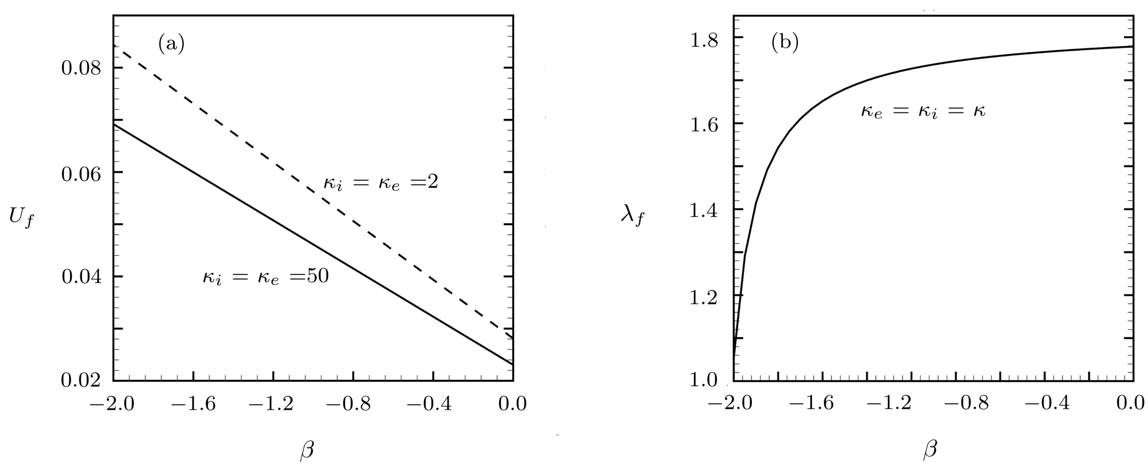

Fig. 2 The fast mode with $\mu = 10^{-3}$ and $\phi_{\rm fm} = 0.1$ for $z_d = 10$, $\rho = z_d$, $z_i = 1$, and $\sigma = 1$. (a) The normalized soliton velocity in the moving frame versus β; (b) The phase velocity versus β for $\kappa_e = \kappa_i = \kappa$.

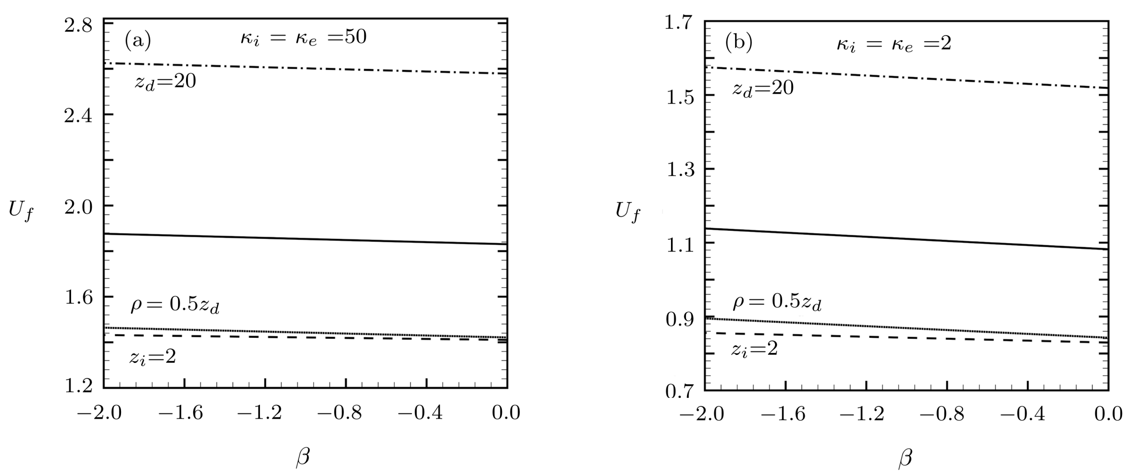

Fig. 3 The normalized soliton velocity of the fast mode with $\mu = 10^{-3}$ and $\phi_{\rm fm} = 0.1$ in the laboratory frame versus β. The solid line is drawn for $z_d = 10$, $\rho = z_d$, $z_i = 1$, and $\sigma = 1$. The other lines indicate the influences of changing different parameters such as z_d, ρ and z_i on the soliton velocity. (a) Maxwellian particles ($\kappa_e = \kappa_i = 50$); (b) Non Maxwellian particles ($\kappa_e = \kappa_i = 2$).

Due to the above discussion, Fig. 3 shows the obtained results of the laboratory frame for soliton velocity. The dependency of the soliton velocity on the trapped electron population does not change because the electron trapping is a nonlinear effect and does not have any counterpart from the lower order.[34]

As it is seen from Fig. 3, obviously in the laboratory frame, the soliton velocity is a decreasing function of population of superthermal particles. That means, for smaller κ, the soliton velocity is smaller. On the other hand, by increasing the population of trapped electrons (more negative β), DASs propagate with larger velocity. The enhanced velocity of solitary structures for larger concentration of trapped electrons is due to the exchange of energy between these low temperature electrons and the DAWs. It appears that the wave picks up the energy from trapped electrons and propagates at larger velocity and, hence, evolves into a solitary structure of higher amplitude, when the concentration of trapped electrons is larger in the plasma.[13]

From these two figures (Figs. 3(a) and 3(b)), one can deduce that for larger number of charge ions ($z_i = 2$) the soliton velocity decreases and also for larger number of charge dust grains ($z_d = 2$) the soliton velocity increases. In agreement with the physical prediction (based on Eqs. (21) and (33)), the changes of number of charge on ions (z_i) and the ratio of the electron to the ion temperature (σ) have the same influence on the behavior of DASWs.

On the other hand, Fig. 3 exhibits the effect of ρ on soliton velocity. It is found from figure that soliton velocity for $\rho = (1/2)z_d$ (the total negative charge of dust grains is half of the total negative charge of electrons) is smaller than $\rho = z_d$ (the total negative charge of electrons and dust grains are equal).

Figure 4 shows that the appearance of superthermal ions deeply modifies the nonlinear features of DASs. These results are the same for slow DASs, too. Therefore, it has been forgone showing them.

Figures 5(a) and 5(b) show the behavior of dust acoustic soliton width respect to β. As it is seen from Eqs. (34) and (38), the width is independent of θ. Therefore, by assuming the same amplitude for both the fast and slow modes, the width can be demonstrated on the same curve. Here $\phi_{\rm fm} = \phi_{\rm sm} = 0.1$. The results can be interpreted in the following manner. By increasing the population of trapped electrons (more negative β) and decreasing the number of charge dust grains (z_d), the width of soliton has a decreasing behavior.

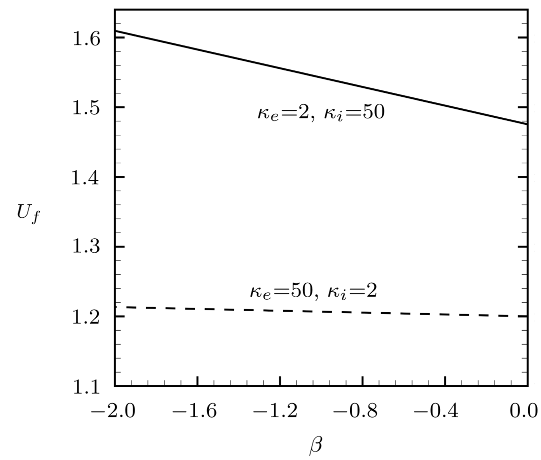

Fig. 4 The normalized soliton velocity of the fast mode with $\mu = 10^{-3}$ and $\phi_{\rm fm} = 0.1$ in the laboratory frame versus β for $z_d = 10$, $\rho = z_d$, $z_i = 1$, $\sigma = 1$, and different κ.

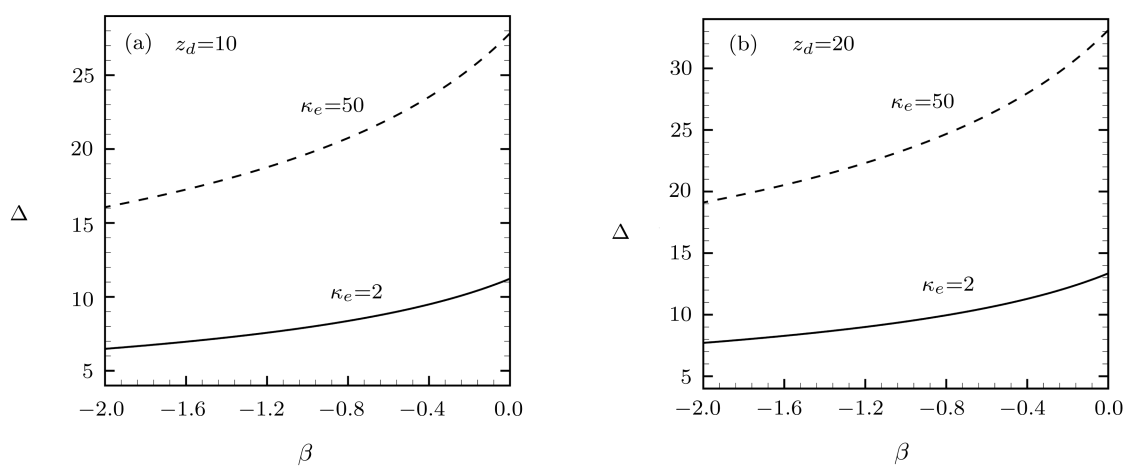

Fig. 5 The normalized soliton width vs. β for different κ_e and $\phi_{\rm fm} = \phi_{\rm sm} = 0.1$. (a) $z_d = 10$; (b) $z_d = 20$.

5 Dust Acoustic Solitons with Positive Dust Grains

What makes dusty plasmas interesting and technologically important is the fact that the dust particles acquire an electric charge in the plasma, typically a negative charge but Fortov[36] has shown that they can be positively charged through three mechanisms which are photoemission in presence of a flux of ultraviolet (UV), thermionic emission induced by radiative heating and secondary emission of electrons from the surface of the dust grains. Recently, positively charged dust grains are found in space plasma environments[37–38] and it is seen that these positively charged grains also play significant role. So, the effect of positive dust in the present plasma will be investigated now.

Using the same analysis, an mKdV equation for fast mode is obtained as follows:

$$\frac{\partial\phi_1}{\partial\tau} + \frac{\rho^{1/2}}{2(a_{k_e} + D')^{3/2}}\frac{\partial^3\phi_1}{\partial\xi^3}$$

$$+\frac{3\rho^{1/2}A}{4(a_{k_e}+D')^{3/2}}\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi}=0\,,\qquad(40)$$

and for slow mode

$$\frac{\partial\phi_1}{\partial\tau}+\frac{\rho^{1/2}\cos\theta}{2(a_{k_e}+D')^{3/2}}\frac{\partial^3\phi_1}{\partial\xi^3}+\frac{3\rho^{1/2}A\cos\theta}{4(a_{k_e}+D')^{3/2}}\phi_1^{1/2}\frac{\partial\phi_1}{\partial\xi}=0\,,\qquad(41)$$

where

$$D'=z_i\Bigl(1-\frac{\rho}{z_d}\Bigr)\sigma a_{\kappa_i}\,,\qquad(42)$$

respectively, where $\rho<z_d$, with $\rho=z_d^2n_{0d}/n_{0e}$ as before.

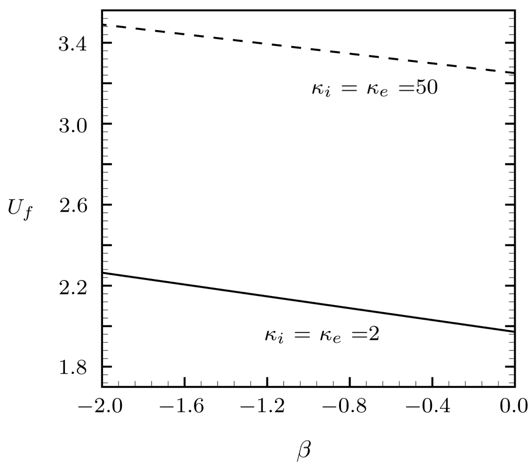

Fig. 6 The normalized soliton velocity of the fast mode with $\mu=10^{-3}$ and $\phi_{\rm fm}=0.1$ in the laboratory frame versus β for $z_d=10$, $z_i=1$, $\sigma=1$, and $\rho=0.5z_d$ (positive dust).

It is obvious from Eqs. (28) and (40) that the presence of positive dust introduces a small correction to mKdV equation. It only changes parameter D, that this parameter determines the soliton velocity.

Figure 6 depicts the soliton velocity of the fast mode with $\mu=10^{-3}$ and $\phi_{\rm fm}=0.1$ in the laboratory frame versus β for $z_d=10$, $z_i=1$, $\sigma=1$ and $\rho=0.5z_d$ (in this case, charge neutrality makes the total charge of ions half of the total charge of electrons). It is shown that the soliton velocity increases in the presence of the dust grains which have been positively charged.

6 Summery

An mKdV equation for the nonlinear propagation dust acoustic waves has been derived. A magnetized plasma with superthermal electrons and ions is considered while dust grains are assumed cold. Moreover, by applying the standard RPT, trapped electrons are investigated. The DF of superthermal electrons and ions was modeled by kappa distribution. Such a plasma is found to support two types of nonlinear DAWs (fast and slow).

The effect of different parameters on behavior of dust acoustic solitary waves is investigated. The DASs are found to attain higher velocity (or amplitude) when the dust grains and trapped electrons are more in number; the same is the case with smaller population of superthermal electrons and ions. Also, they become slower when the number of charge on ions and the ratio of the electron to the ion temperature increase.

The results of this study also reveal that the dust acoustic soliton becomes slimmer when the population of trapped and superthermal electrons is more in number. Also, the effect of charge on dust grains is to enhance the width of these structures.

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