

**Regression analysis of additive hazards model with
latent variables**

Supplementary Material

Table S1: Results of Simulation 1 – Type (I) $\lambda_0(t)$, $n = 500$, \mathbf{B} is overlapping

Model	CR	Para	Bias	SE	SEE	CP
AH model	20%	β	-0.005	0.366	0.350	0.937
		γ_1	-0.007	0.192	0.192	0.953
		γ_2	0.007	0.190	0.192	0.952
	50%	β	-0.004	0.449	0.443	0.938
		γ_1	0.003	0.249	0.247	0.945
		γ_2	0.012	0.250	0.248	0.955
CFA model		b_{11}	-0.001	0.042	0.043	0.958
		b_{21}	0.001	0.042	0.043	0.967
		b_{31}	0.000	0.038	0.038	0.948
		b_{41}	0.001	0.038	0.038	0.949
		b_{12}	-0.000	0.043	0.043	0.953
		b_{42}	0.002	0.042	0.043	0.956
		b_{52}	0.001	0.037	0.038	0.954
		b_{62}	0.001	0.038	0.038	0.944
		ϕ_{12}	-0.000	0.052	0.053	0.953
		ψ_1	-0.001	0.030	0.029	0.938
		ψ_2	-0.001	0.028	0.028	0.947
		ψ_3	-0.002	0.028	0.028	0.951
		ψ_4	-0.002	0.030	0.029	0.939
		ψ_5	0.000	0.029	0.028	0.951
	ψ_6	-0.001	0.028	0.028	0.952	

Table S2: Results of Simulation 1 – Type (I) $\lambda_0(t)$, $n = 500$, \mathbf{B} is misspecified

CFA model					AH model					
Para.	Bias	SE	SEE	CP	CR	Para.	Bias	SE	SEE	CP
b_{11}	-0.026	0.041	0.058	0.969	20%	β_1	0.000	0.008	0.007	0.946
b_{21}	0.020	0.043	0.055	0.974		β_2	0.000	0.007	0.007	0.947
b_{32}	0.002	0.075	0.084	0.973		β_3	0.001	0.016	0.015	0.936
b_{42}	-0.000	0.080	0.095	0.967		γ_1	-0.000	0.009	0.009	0.939
b_{53}	0.027	0.066	0.115	0.975		γ_2	-0.001	0.009	0.009	0.944
b_{63}	-0.031	0.070	0.138	0.965		γ_3	-0.000	0.008	0.008	0.947
b_{74}	0.033	0.060	0.081	0.873		γ_4	0.001	0.010	0.010	0.965
b_{84}	-0.123	0.101	0.093	0.556	50%	β_1	-0.000	0.010	0.009	0.941
b_{94}	-0.137	0.144	0.196	0.563		β_2	-0.000	0.010	0.009	0.953
						β_3	0.001	0.020	0.019	0.950
						γ_1	0.000	0.011	0.011	0.942
						γ_2	-0.001	0.012	0.011	0.947
						γ_3	-0.000	0.011	0.011	0.957
						γ_4	0.001	0.012	0.012	0.975

Table S3: Results of Simulation 1 – Type (I) $\lambda_0(t)$, $n = 500$, $\epsilon_i \sim N(\mathbf{0}, \mathbf{I})$

Model	CR	Para	Bias	SE	SEE	CP
AH model	20%	β	0.015	0.361	0.355	0.943
		γ_1	0.001	0.233	0.224	0.953
		γ_2	0.010	0.227	0.225	0.943
	50%	β	0.009	0.436	0.447	0.955
		γ_1	-0.002	0.290	0.287	0.963
		γ_2	0.016	0.286	0.287	0.958
CFA model		b_{11}	0.001	0.071	0.070	0.947
		b_{21}	0.001	0.071	0.070	0.946
		b_{31}	-0.000	0.070	0.070	0.950
		b_{42}	0.001	0.071	0.070	0.949
		b_{52}	-0.003	0.070	0.070	0.955
		b_{62}	-0.001	0.068	0.070	0.958
		ϕ_{12}	0.001	0.067	0.066	0.948
		ψ_1	-0.005	0.101	0.100	0.950
		ψ_2	-0.005	0.099	0.100	0.953
		ψ_3	-0.006	0.100	0.100	0.948
		ψ_4	-0.007	0.100	0.100	0.956
		ψ_5	-0.004	0.100	0.100	0.947
	ψ_6	-0.008	0.095	0.100	0.959	

Table S4: Results of Simulation 2 – Type (I) $\lambda_0(t)$, $n = 500$, ξ_i is non-normal

CR	Para	$\xi_i \sim \frac{2}{3}N(\mu_1, \Sigma_1) + \frac{1}{3}N(\mu_2, \Sigma_2)$				$\xi_i \sim \{Gamma(4, 2) - 2\}I_2$				$\xi_i \sim beta(3, 1)I_2$			
		Bias	SE	SEE	CP	Bias	SE	SEE	CP	Bias	SE	SEE	CP
20%	β	0.014	0.348	0.350	0.951	-0.001	0.361	0.351	0.946	0.017	0.346	0.351	0.955
	γ_1	-0.000	0.196	0.195	0.955	0.016	0.192	0.193	0.960	0.008	0.171	0.176	0.961
	γ_2	0.006	0.201	0.196	0.946	0.010	0.213	0.206	0.945	-0.001	0.169	0.168	0.953
50%	β	-0.018	0.444	0.443	0.951	0.012	0.442	0.441	0.950	0.008	0.447	0.443	0.947
	γ_1	0.010	0.252	0.249	0.954	0.009	0.247	0.245	0.954	0.001	0.231	0.229	0.955
	γ_2	-0.000	0.249	0.250	0.952	-0.000	0.267	0.258	0.939	-0.000	0.220	0.219	0.954
	b_{11}	-0.000	0.036	0.038	0.963	0.000	0.043	0.038	0.914	-0.000	0.039	0.038	0.946
	b_{21}	0.001	0.036	0.038	0.958	-0.001	0.045	0.038	0.906	-0.001	0.039	0.038	0.942
	b_{31}	-0.000	0.037	0.038	0.954	-0.000	0.044	0.038	0.904	0.000	0.039	0.038	0.942
	b_{42}	0.003	0.037	0.038	0.962	0.004	0.044	0.038	0.915	0.001	0.039	0.038	0.949
	b_{52}	-0.000	0.036	0.038	0.960	0.002	0.043	0.038	0.919	0.000	0.038	0.038	0.947
	b_{62}	-0.000	0.036	0.038	0.963	0.002	0.044	0.038	0.900	0.002	0.038	0.038	0.948
	ϕ_{12}	0.001	0.047	0.050	0.969	-0.001	0.053	0.052	0.942	-0.000	0.053	0.052	0.934
	ψ_1	0.001	0.031	0.030	0.949	-0.003	0.030	0.030	0.945	-0.001	0.029	0.030	0.953
	ψ_2	-0.001	0.030	0.030	0.941	0.000	0.031	0.030	0.957	0.000	0.029	0.030	0.959
	ψ_3	-0.002	0.031	0.030	0.943	-0.001	0.031	0.030	0.942	-0.001	0.029	0.030	0.957
	ψ_4	-0.001	0.031	0.030	0.946	-0.002	0.029	0.030	0.952	-0.001	0.030	0.030	0.947
	ψ_5	-0.002	0.028	0.030	0.961	-0.001	0.030	0.030	0.961	-0.003	0.029	0.030	0.945
	ψ_6	-0.001	0.030	0.030	0.945	-0.000	0.031	0.030	0.940	-0.001	0.030	0.030	0.946

Table S5: Results of Simulation 2 – Type (I) $\lambda_0(t)$, $n = 500$, ϵ_{ij} is non-normal

		$\epsilon_{ij} \sim \text{Gamma}(5, 4) - 5/4$						$\epsilon_{ij} \sim t_{(3)}$						$\epsilon_{ij} \sim \text{beta}(3, 1)$					
CR	Para	Bias	SE	SEE	CP	Bias	SE	SEE	CP	Bias	SE	SEE	CP	Bias	SE	SEE	CP		
20%	β	-0.017	0.341	0.350	0.956	-0.011	0.346	0.350	0.960	0.018	0.330	0.351	0.965	0.018	0.330	0.351	0.965		
	γ_1	-0.005	0.188	0.188	0.956	-0.000	0.195	0.189	0.937	-0.003	0.194	0.189	0.941	-0.003	0.194	0.189	0.941		
	γ_2	0.005	0.190	0.189	0.947	0.003	0.192	0.188	0.944	0.009	0.189	0.189	0.947	0.009	0.189	0.189	0.947		
50%	β	-0.004	0.451	0.443	0.947	0.005	0.439	0.442	0.962	0.018	0.444	0.442	0.960	0.018	0.444	0.442	0.960		
	γ_1	0.011	0.248	0.243	0.952	-0.003	0.245	0.240	0.945	-0.002	0.245	0.242	0.951	-0.002	0.245	0.242	0.951		
	γ_2	-0.014	0.243	0.243	0.957	0.011	0.239	0.241	0.954	-0.000	0.257	0.243	0.940	-0.000	0.257	0.243	0.940		
	b_{11}	0.002	0.038	0.038	0.953	0.001	0.039	0.038	0.939	0.001	0.039	0.038	0.948	0.001	0.039	0.038	0.948		
	b_{21}	0.002	0.037	0.038	0.957	0.001	0.038	0.038	0.952	0.002	0.039	0.038	0.956	0.002	0.039	0.038	0.956		
	b_{31}	0.002	0.040	0.038	0.938	0.000	0.038	0.038	0.953	-0.000	0.037	0.038	0.960	-0.000	0.037	0.038	0.960		
	b_{42}	-0.000	0.038	0.038	0.952	0.002	0.037	0.038	0.958	0.001	0.039	0.038	0.942	0.001	0.039	0.038	0.942		
	b_{52}	0.001	0.039	0.038	0.940	-0.000	0.038	0.038	0.948	0.002	0.038	0.038	0.943	0.002	0.038	0.038	0.943		
	b_{62}	0.002	0.038	0.038	0.958	0.001	0.037	0.038	0.961	0.000	0.039	0.038	0.942	0.000	0.039	0.038	0.942		
	ϕ_{12}	0.003	0.050	0.050	0.948	0.002	0.050	0.049	0.938	0.002	0.051	0.050	0.947	0.002	0.051	0.050	0.947		
	ψ_1	0.001	0.034	0.030	0.921	-0.005	0.122	0.029	0.531	-0.002	0.031	0.030	0.938	-0.002	0.031	0.030	0.938		
	ψ_2	-0.001	0.033	0.030	0.922	-0.009	0.105	0.029	0.544	-0.001	0.030	0.030	0.948	-0.001	0.030	0.030	0.948		
	ψ_3	-0.002	0.034	0.030	0.909	-0.006	0.147	0.029	0.555	-0.001	0.031	0.030	0.940	-0.001	0.031	0.030	0.940		
	ψ_4	-0.002	0.034	0.030	0.914	-0.000	0.184	0.030	0.544	-0.001	0.031	0.030	0.945	-0.001	0.031	0.030	0.945		
	ψ_5	-0.001	0.034	0.030	0.929	-0.000	0.174	0.030	0.538	0.000	0.030	0.030	0.948	0.000	0.030	0.030	0.948		
	ψ_6	-0.000	0.033	0.030	0.915	-0.001	0.160	0.030	0.544	0.000	0.030	0.030	0.945	0.000	0.030	0.030	0.945		

Table S6: Comparison of the CKD data analysis under different structures of \mathbf{B}

Model	Variable	\mathbf{B} is overlapping		\mathbf{B} is nonoverlapping	
		$b_{81} \neq 0$		$b_{81} = 0$	
		Est	SEE	Est	SEE
CFA	b_{11}	0.9899	0.0192	0.9551	0.0240
	b_{21}	0.8347	0.0184	0.8654	0.0229
	b_{81}	-0.2490	0.0163	–	–
	b_{32}	0.7266	0.0312	0.7143	0.0311
	b_{42}	0.8464	0.0350	0.8611	0.0358
	b_{53}	0.7261	0.0387	0.7331	0.0377
	b_{63}	0.9457	0.0485	0.9368	0.0463
	b_{74}	0.3395	0.0192	0.3413	0.0193
	b_{84}	-0.2535	0.0163	-0.3042	0.0185
	b_{94}	0.9925	0.0437	0.9834	0.0484
AH	Age at enrollment	0.0188	0.0013	0.0191	0.0013
	Duration of diabetes	0.0140	0.0017	0.0140	0.0017
	Sex	-0.0030	0.0022	-0.0032	0.0022
	Obesity	0.0029	0.0012	0.0028	0.0013
	Blood pressure	0.0042	0.0016	0.0038	0.0016
	Glycemia	0.0038	0.0015	0.0040	0.0015
	Lipid	0.0069	0.0017	0.0071	0.0017