REGRESSION ESTIMATES OF DAMAGES IN PRICE-FIXING CASES

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In an antitrust price-fixing case, damages are measured by the difference between the prices paid by the plaintiff purchasers and the prices they would have paid in the absence of defendants' conspiracy. Even small variations in the determination of these differences may become significant because plaintiffs are entitled by statute to receive three times their actual damages.[1] Moreover, in a class action there may be thousands of purchasers whose recoveries are based on an industry-wide overcharge formula.

The difficulty of course lies in estimating the "but for" prices. An obvious idea is to assume that competitive prices during the conspiracy period would have been the same as they were before or after the conspiracy or in interludes of competition within the conspiracy period. Thus, the difference between the conspiratorial price and the actual price from other periods measures the damage. This estimate, however, meets the immediate objection that it is likely to be incorrect because changes in factors affecting price other than the conspiracy would have produced changes in competitive prices if there had been competition during the conspiracy period.

These arguments may be illustrated by *Ohio Valley Electric Corp. v. General Electric Co.*,[2] one of the well-known antitrust cases involving electrical equipment manufacturers. Plaintiffs contended that the purpose and effect of the manufacturers' conspiracy was to keep transaction prices for large steam turbines close to book prices; they proposed to measure damages by the difference between the average 11% discount during the conspiracy period and the average 25.33% discount that prevailed after the conspiracy had been terminated. Defendants replied that economic conditions for the sellers had worsened in the postconspiracy period and that these conditions accounted for the increase in the discount. They pointed to the presence for the first time of effective foreign competition, an increase in the manufacturers' capacity to produce steam turbine generators (which caused an oversupply), a lessening of growth in demand, and a drop in manufacturing costs (which permitted defendants to offer their products at a lower price).

After a bench trial, District Judge Feinberg initially found that these factors did account for some of the increase in the discount in the postconspiracy period. He then confronted the question of how much. Treating the matter as one of

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1. 15 U.S.C. § 15 (Supp. V 1981).
2. 244 F. Supp. 914 (S.D.N.Y. 1965).

"highly subjective" and "inexact" judgment, he concluded that 4.33 percentage points of the 25.33% discount in the postconspiracy period was due to changes in economic circumstances, leaving 21% attributable to the conspiracy. Since the increase in the average discount was approximately 14 percentage points (from approximately 11% to 25.33%), he thus attributed approximately one-third of the increase to changes in economic conditions and two-thirds of the increase to the conspiracy.

The court justified this type of subjective assessment of the discount by citing "similar" judicial problems of fixing a figure for pain and suffering in a negligence case or of selecting the percent of negligence for which the plaintiff was responsible and hence could not recover in a Jones Act case.[3] However, these examples are not analogous. The judgments required in those types of cases are inevitably highly subjective and there is no generally accepted way of translating them into dollars. By contrast, determining the influence on price of various asserted economic forces does not involve subjective values, but estimates made by the objective methods of econometrics. Since these methods are better known to economists and statisticians—and perhaps even to judges and litigants—than they were in 1965 when *Ohio Valley* was decided, it is probable that courts will require a party arguing the effect of collateral circumstances to prove its point with this type of evidence. Thus, econometrics may be used by plaintiffs who contend that the full effect of a conspiracy is concealed by changes in other correlated factors that also affect price, or by defendants who claim that part of a price change should be attributable to other factors.

Several writers have explored the question of whether the statistical technique of multiple regression could appropriately be used for this disentangling purpose.[4] Based on theoretical considerations, the academic verdict is generally favorable. However, practical experience with data sets used in several litigated cases suggests considerable caution in requiring full fledged models as part of a prima facie case. This article proposes to describe the cases in which models have been presented and to discuss the issues they raise.[5] The discussion that follows is not highly technical, but the reader is assumed to be at least generally acquainted with the principles of multiple regression analysis.[6]

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3. See 46 U.S.C. § 688(a) (1982), which incorporates for seamen the provisions of federal statutes modifying common law remedies in cases of injury to railway employees. Among these statutes is 45 U.S.C. § 53 (1982), which provides that the jury may diminish a damage award in favor of an employee in proportion to the amount of negligence attributable to the employee.

4. *See, e.g.*, Fisher, *Multiple Regression in Legal Proceedings,* 80 COLUM. L. REV. 702, 726-29 (1980); Harrison, *The Lost Profits Measure of Damages in Price Enhancement Cases,* 64 MINN. L. REV. 751, 783-87 (1980). J. Leitzinger, *Regression Analysis In Antitrust Cases: Opening The Black Box*, 20 The Philadelphia Lawyer 1 (1983); *see also* A. DAGGETT & FREEDMAN, ECONOMETRICS AND THE LAW: A CASE STUDY IN THE PROOF OF ANTITRUST DAMAGES (Dep't of Statistics, Univ. of Calif., Berkeley, Technical Rep. No. 23, Sept. 1983).

5. Multiple regression analysis has been used in other antitrust contexts that are not discussed. For example, regression models have been used by discontinued dealers to prove lost profits. Shreve Equip., Inc. v. Clay Equip. Corp., No. C75-242A (N.D. Ohio 1978); Coleman Motor Co. v. Chrysler Corp., 376 F. Supp. 546 (W.D. Pa. 1974), *rev'd,* 525 F.2d 1338 (3d Cir. 1975); Rea v. Ford Motor Co., 355 F. Supp. 842 (W.D. Pa. 1973). *See also* Hughes Tool Co. v. Trans World Airlines, Inc., 409 U.S. 363 (1973) (Report of Herbert Brownell, Special Master, at 94, 95).

6. For a description of multiple regression, see articles cited *supra* note 4. *See also* Finkelstein, *Regression*

I
THE CASES

A. Corrugated Containers

In 1977, following the Supreme Court's decision in *United States v. Container Corporation of America*[7] that an exchange of price information constituted an antitrust violation, purchasers of cardboard containers and corrugated sheets brought treble-damage class actions against the manufacturers of these products.[8] The charge was price fixing between 1960 and 1976. A group of defendants settled by putting up a fund of some $300 million. However, three defendants—Alton, Mead, and Westvaco—refused to settle and stood trial before a jury. During the trial, Alton and Westvaco settled for some $6.4 million, leaving Mead as the sole defendant.

On the damage question, plaintiffs' expert, John Beyer, presented two similar multiple regression models.[9] In one study, the monthly average price per square foot of corrugated containers and sheets shipped by members of the Fiber Box Association (FBA) was regressed on (1) price the previous month; (2) change in cost of production from the previous month; (3) level of manufacturing output in industries using corrugated containers (a demand factor); (4) the wholesale price index (WPI) for all commodities (justified as another demand factor, although the reason is unclear); (5) the productive capacity of paperboard plants (a supply factor); and (6) a dummy variable to reflect the period of price controls (a factor which was given the value one in the period of price controls March 1971-March 1974 and zero otherwise). The data were all expressed in the form of index numbers. The period covered by the regression was January 1963 through December 1975 (156 monthly observations); it is during this period that price fixing was deemed to have occurred. The regression equation was thus estimated solely from data during the conspiracy period, and the estimated relationships are those existing during the conspiracy. The fit appears to be very good, the square of the multiple correlation coefficient (R^2) being greater than 0.99. A second model was of the same form and used the same variables, except that the price of containers was drawn from a Bureau of Labor Statistics (BLS) price series (which uses the price of a representative container rather than an average price as in the FBA price series).[10]

The estimated regression equation for the conspiracy period ending December 1975 was then used to estimate prices in the 1978-79 postconspiracy period. These

Models in Administrative Proceedings, 86 HARV. L. REV. 1442 (1973), *reprinted in* M. Finkelstein, QUANTITATIVE METHODS IN LAW ch. 7 (1978).

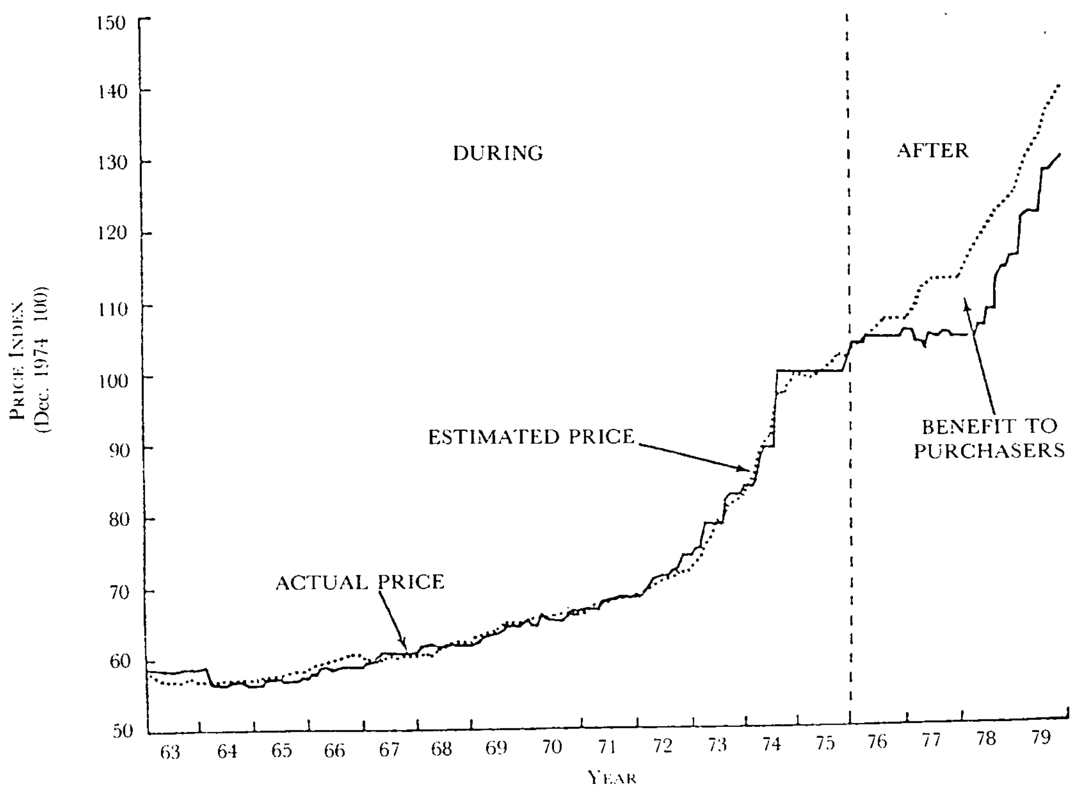
7. 393 U.S. 333 (1969). The civil suits were triggered by a nationwide grand jury investigation of the same conduct; this investigation was conducted in 1975 and 1976.

8. *In re* Corrugated Container Antitrust Litig., 441 F. Supp. 921 (S.D. Tex. 1977). The discussion and analysis of this case are based on the trial transcript, certain computer printout relied on by plaintiffs' expert, John Beyer, workpapers of defendants' expert, Franklin Fisher, interviews with counsel, and the authors' computer reanalysis of data from the first trial.

9. Transcript at 45-121 (July 22, 1980), *In re Corrugated Container Antitrust Litig.*

10. Variables 5 and 6 were subsequently dropped from the BLS model because they were not statistically significant in that model. Transcript at 86-87 (July 22, 1980).

FIGURE 1
CORRUGATED CONTAINERS
(PLAINTIFFS' EXHIBIT)



estimated prices were higher than the actual prices, the average of the difference between the projected and actual prices as a percentage of the projected prices being 7.8% for the BLS price study[11] and 19.1% for the FBA price study.[12] From this, plaintiffs argued that the overcharge in the conspiracy period was at least 7.8%. Figure 1 is plaintiffs' exhibit of the BLS price data in the conspiracy and postconspiracy periods, and the conspiracy prices as projected by the model.

Defendants challenged these estimates on various grounds, but they did not introduce a competing model. The jury found that the overcharge was 5%. The case was then sent to a special master to compute damages by applying a 5% overcharge figure to the purchases by the plaintiffs in the conspiracy period. While the case was pending before the special master, the parties settled with Mead agreeing to pay $45 million over a ten year period.

A second trial followed for certain plaintiffs who had opted out of the first class. Most defendants settled by putting up a fund of some $60 million. How-

11. Plaintiffs' Exhibit No. 4852B, Transcript at 104 (July 22, 1980).
12. Plaintiffs' Exhibit No. 4852A, Transcript at 95-102 (July 22, 1980).

ever, five defendants stood trial before a jury in late 1982 and early 1983. Beyer's model was introduced again, this time with certain technical revisions in the explanatory variables.[13] Among other changes, the variables for the level of manufacturing output and for WPI were refined; a variable for inventory of linerboard was added; and costs were used instead of change in costs. Perhaps more important, the period of collusion was ended a year earlier, this time in December 1974. Thus, the period of transition to a fully competitive market was 1975-78 (for which plaintiff also claimed damages) and the period of full competition was 1979 through August 1981 (the latest period for which data were available).

Using the same technique and relying solely on the FBA price series, Beyer found a remarkable 26% industry-wide overcharge.[14] He translated this by a complex method into an overcharge for each plaintiff. This time, however, Beyer ran into a withering attack from Franklin Fisher, an economist from MIT, who leveled a barrage of criticism at the model and pronounced it "worthless."[15] This article discusses certain of Fisher's criticisms below. Evidently, these criticisms were effective because the jury found that there had been a conspiracy but that it had not affected the prices paid by the opt-out plaintiffs.[16] Hence, there was no recovery in the case.

B. Concrete Pipe

In the 1960's and 1970's, suits alleging price-fixing conspiracies were brought in many states against concrete pipe manufacturers. In one of the first cases to be tried, plaintiffs were the State Highway Department and certain municipalities of New Mexico.[17] Plaintiffs claimed that defendants' conspiracy extracted overcharges on purchases of concrete pipe in New Mexico for the period January 1960 through December 1962. Other defendants having settled, the trial concerned only fifty-two purchases in this period from defendant Martin-Marietta.

Plaintiffs argued that damages could not be computed on the basis of either preconspiracy or postconspiracy prices because Martin-Marietta was in a monopoly position in the preconspiracy period, and lingering effects of the conspiracy maintained pipe prices at noncompetitive levels in the postconspiracy period. According to the plaintiffs, the only proper basis for comparison was the last half of 1960, when a competitor briefly entered the field and prices dropped. Damages for purchases in other periods were computed by comparing actual prices with prices in the competitive period adjusted for changes in the BLS price index for concrete products. This comparison indicated a 20% overcharge.

Defendant argued that plaintiffs' simple comparison of conspiratorial and competitive periods was misleading because it failed to account for such factors as

13. Conversation with John E. Linville, one of the counsel in the case (June 22, 1983).

14. Transcript at 105 (Dec. 15, 1982). The damages estimated from the BLS price series were about half those estimated from the FBA price series.

15. Transcript at 395-444 (Feb. 8, 1983).

16. The jury's decision may also have been affected by the argument that the plaintiffs, who were large, sophisticated purchasers, were less likely to have been overcharged than the run of purchasers.

17. New Mexico v. American Pipe and Constr. Co., Civ. No. 7183 (D.N.M. 1970). The description that follows is from Parker, *Economics in the Courtroom: Proof of Damages In a Price-Fixing Case,* 9 ANTITRUST LAW & ECON. REV. No. 4, at 61 (1977) supplemented with further information from Professor Parker.

transportation costs, quantity, and type of pipe. To reinforce its argument, defendant regressed price per lineal foot on diameter of pipe and number of lineal feet purchased. The equation (in linear logarithmic form) was estimated using data from the nonconspiracy periods (1956-59 and January 1963 through March 1964). Defendant then argued that any price in the conspiratory period within a 95% confidence interval of the regression estimate must be accepted as normal. Using the difference between the actual prices charged and the upper limit of the 95% confidence interval as the measure of the overcharge, defendant obtained a negligible damage figure of $10,811.

Plaintiffs' expert, Alfred Parker of the University of New Mexico, presented a regression model to rebut defendant's regression. Using the conspiratorial period 1960-62, he regressed price per lineal foot of pipe on (1) diameter of pipe; (2) lineal feet of pipe purchased; (3) haul distance in miles; and (4) a dummy variable in which observations in the competitive interlude (the last half of 1960) were assigned a one and all other observations were assigned a zero. There were 259 observations; R^2 was 0.94. The coefficient of the dummy variable was negative and statistically significant at the 1% level. It indicated an average surcharge of $0.725 per lineal foot, or approximately 15.5% over the average price of $4.68 per lineal foot. The jury awarded $150,000, which the court would have trebled and added an award for attorney fees. It is not apparent how the jury reached this figure. The case was subsequently settled for $475,000.

In his analysis, Parker used a "white sale" period of the last half of 1960 when the conspiracy broke up due to the entry of a competitor into the market. This period was more favorable to the plaintiffs as a comparison period than the pre-1960 period would have been; at that time there was no conspiracy but also no competitor and prices were almost as high as those of the conspiracy period. The white sale period ended when the competitor was bought out. Thus, Parker's method questionably attributed the spread between the preconspiracy and competitive price levels to the conspiracy, whereas at least part of the spread may have been attributable to an absence of competition in the market that was disadvantageous to the plaintiff, but not illegal.[18]

C. Broiler Chickens

In *In re Chicken Antitrust Litigation,*[19] plaintiffs attacked the "recommended" price and production program of an association of chicken processors selling ready-to-cook chickens to retailers, wholesalers, and large volume consumers. The organization, known as the National Broiler Marketing Association (NBMA), was formed in 1970 to counteract economic depression in the chicken-processing industry; its members included about half the industry. In an earlier Department of Justice lawsuit against the same defendants, the Department had elected to seek only injunctive and not monetary relief because its experts had concluded that the

18. "If a firm has taken no action to destroy competition it may be unfair to deprive it of the ordinary opportunity to set prices at a profit-maximizing level. Thus, no court has required a lawful monopolist to forfeit to a purchaser three times the increment of its price over that which would prevail in a competitive market." Berkey Photo, Inc. v. Eastman Kodak Co., 603 F.2d 263, 294 (2d Cir. 1979).

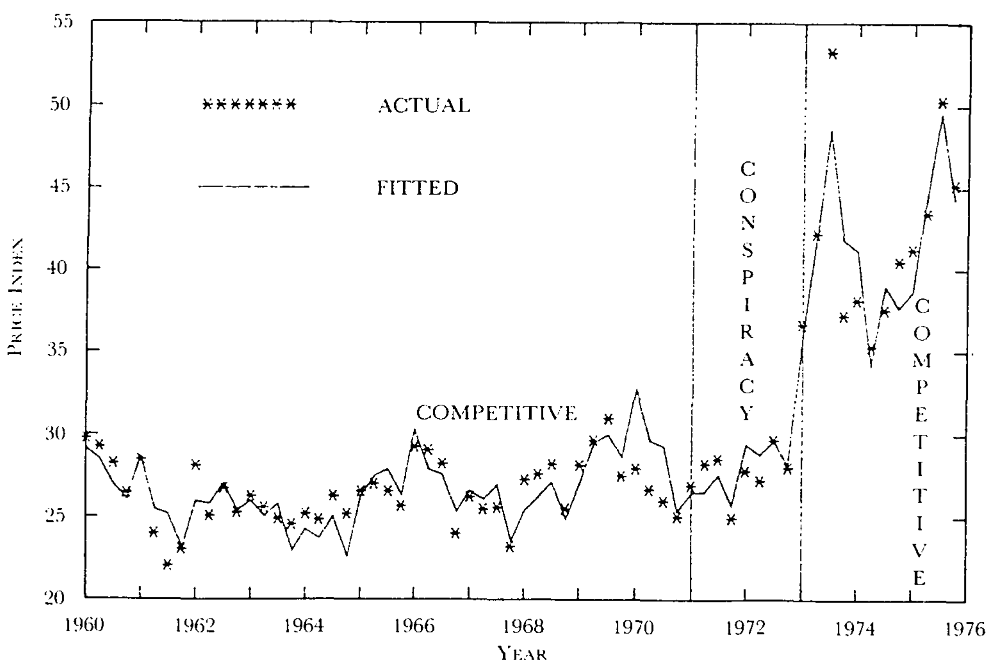
19. 1980-2 Trade Cas. (CCH), ¶ 63,485 (N.D. Ga. 1980) (opinion on fee applications).

NBMA program had not appreciably affected broiler prices.[20]

Edward W. Erickson of North Carolina State University, with the assistance of William Henry of Georgia State University, prepared an econometric analysis for the plaintiffs to show the effect of the NBMA program.[21] The study was presented in the proceeding on fee applications after the case was settled. The study, using USDA data, analyzed the average monthly price of broilers in nine cities for the 192 months between 1960 and 1975, which included the twenty-seven month period between January 1, 1971, and March 31, 1973, when the broiler chicken producers conducted their joint economic program. These prices were regressed on (1) the monthly average prices of beef, pork, and turkey; (2) dummy variables for each quarter of the year (because the demand for broilers is seasonal); (3) the consumer price index; (4) consumer disposable income (BLS data); and (5) the per capita broiler production for each period, in pounds per person, as reported by the USDA. The study included a dummy variable for "all other influences" which took on value one in the conspiracy period (months 133 through 159) and zero otherwise.

Unfortunately for the plaintiffs, the study showed a statistically significant *negative* coeffficient of $1.36 per pound for the dummy variable, indicating that prices

FIGURE 2
BROILER CHICKENS



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20. *Id.* at 76,562.

21. The description that follows used the economic report of E.W. Erickson & W. Henry, Economic Analysis of Potential Damages (Oct. 1979) (unpublished report submitted to the court).

for broilers in the price-fixing period were about 5% *lower* than would have been expected on the basis of the supply and demand factors included in the regression equation model. The fitted values from the estimated regression equation and the actual prices are plotted in Figure 2.

The plaintiffs' experts suggested that this perverse result could be explained by the fact that association members accounted for only about half the broiler producers, and by the possibility that association members had not adhered to the recommended prices and production levels. These reasons would explain a statistically insignificant coefficient, but not a statistically significant negative one.

Another explanation tendered by the experts was that in July 1971 shortly after publication of a book by Ralph Nader and Harrison Wellford charging laxness in USDA meat inspection, there was a much-publicized PCB contamination of broiler feed and broilers; the experts speculated that these two events may have reduced demand for broilers during part of the price-fixing period. Excluding the "PCB period" (August 1971-April 1972) from the time in which the dummy variable was switched on, led to a new estimated coefficient value of 0.43¢ per pound, which was not statistically significant at the .05 level. Since the coefficient was still negative, however, the equation still provided no basis for damages.

To come up with a damage figure, the experts rather tentatively argued that they were justified in considering only months in the price-fixing period with positive residuals, that is, when actual prices exceeded the predicted prices. No deduction was allowed for negative residual months on the theory that the fixing worked only intermittently or was outweighed at times by other factors such as the PCB scandal. To counter the argument that such months reflected simply chance variation, a separate calculation was made in which only those months were counted in which actual prices exceeded predicted prices by more than one standard error. This calculation yielded aggregate damages of $26 million. Defendants subsequently settled for $35 million.

The experts' ad hoc method of estimation was not defensible. By looking only at the positive residuals, a "damage" figure can be generated in any period that is long enough for random variation to generate residuals in excess of one standard error on the positive side. The record contains no indication that the experts were challenged. In its opinion on fee allowances, which followed the settlement, the court referred to their estimate as "reasonable," but it sensibly added that the damage figure was "still a matter of some speculation, despite the extensive testing done by these two experts."[22] The court noted that recent decisions in the circuit had indicated that statistical testing would not sustain the plaintiffs' burden of proof on the issue of damages. It is unclear what was meant by this and the cited cases do not clarify the point.[23] The court may have meant that since the regression estimates provide only an average overcharge with respect to all purchasers, it would not be a sufficient basis for estimating damages with respect to any particular purchaser. If intended, this point would be incorrect, since a computation of

22. *In re* Chicken Antitrust Litig., 1980-2 Trade Cas. (CCH) ¶ 63,485, at 76,562 (N.D. Ga. 1980).
23. *See, e.g.,* Chrysler Credit Corp. v. J. Truett Payne, Inc., 607 F.2d 1133 (5th Cir. 1979).

average damages for a class has been accepted as an appropriate basis for individual damage awards to class members.[24]

D. Plywood

In *In re Plywood Antitrust Litigation,*[25] class actions were brought by plywood purchasers against softwood plywood manufacturers. With respect to the three defendants who stood trial—Georgia Pacific Corporation, Weyerhaeuser Company, and Willamete Industries—the jury found that they and all other manufacturers of southern plywood had engaged in a conspiracy from February 23, 1968, to at least December 31, 1973, by which the prices of southern plywood were inflated. The agreement was that purchasers would be charged "west coast freight" (freight computed as though the product had been shipped from the west coast), which exceeded actual freight charges from southern shipping points (the excess being referred to as "phantom freight"), and would be charged on the basis of "standard weights" to calculate freight even though these standard weights exceeded the actual weights.

Defendants' expert, Peter Steiner from the University of Michigan, argued that in the absence of an agreement to charge phantom freight, southern suppliers would not have reduced their prices because there was sufficient demand for plywood at the higher prices. To sustain that view, he introduced a regression study of plywood prices by himself and a colleague, Daniel Rubinfeld from the University of Michigan, which was designed to show that the market was competitive because prices responded to supply and demand factors.[26] In the study they regressed the price of plywood (expressed in index form) on (1) housing starts by region (a demand factor); (2) income (demand); (3) stumpage price (the price for timber sold from private lands in Louisiana—a supply factor); (4) forest sales (the price for timber sold from publicly owned land in Oregon and Washington); (5) productive capacity of plywood plants (supply); (6) log drying costs (supply); and (7) gluing costs (supply).

The parameters in the model were estimated from data for the period May 1964 through February 1977, which covered the time when the defendants used the west coast freight rates. For that period it appeared to track actual prices quite closely, which Steiner cited as evidence that the market was in fact competitive because prices responded to normal supply and demand factors.

Plaintiffs did not introduce a competing statistical regression model, but rather relied on judgmental expert testimony and cross-examination of the defendants' expert. The jury found that the plaintiffs had been financially injured by the amount of phantom freight and the excess of standard weights over actual weights.

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24. *See, e.g., In re* Sugar Industry Antitrust Litig., 73 F.R.D. 322, 353-55 (E.D. Pa. 1976); *In re* Coordinated Pretrial Proceedings in Antibiotic Antitrust Actions, 1971 Trade Cas. (CCH) ¶ 73,699 (S.D.N.Y. 1971).

25. 376 F. Supp. 1405 (E.D. La. 1974), *aff'd,* 655 F.2d 627 (5th Cir. 1981), *cert. granted sub. nom.* Weyerhaeuser Co. v. Lyman Lamb Co., 456 U.S. 971 (1982).

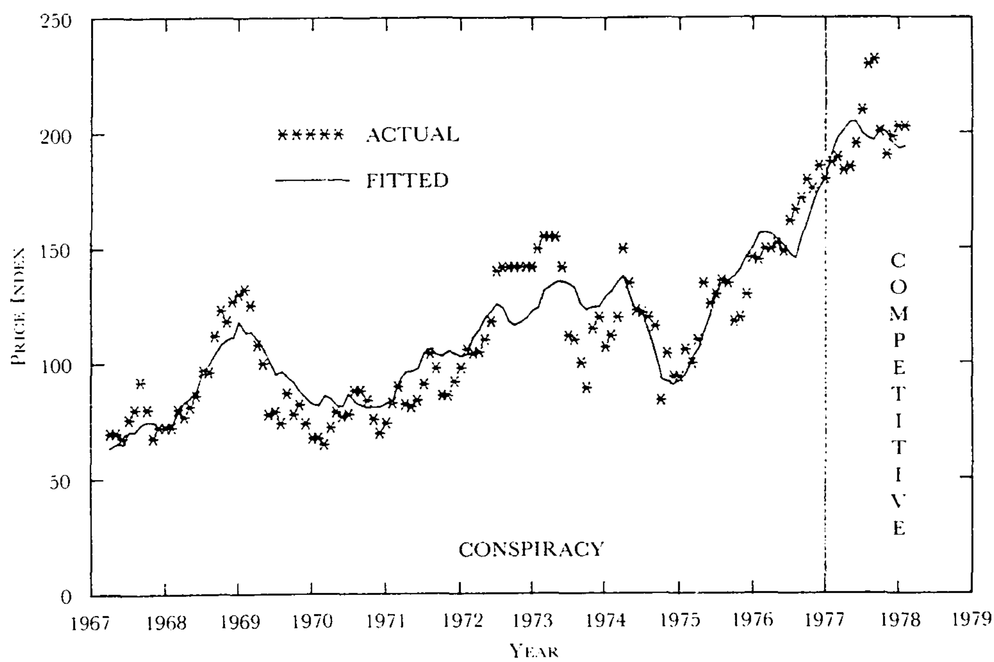
26. Transcript at 2912-42, 2962-65, 2982-3011 (Nov. 8, 1978), *In re Plywood Antitrust Litig.* Details of the study are given in Rubinfeld & Steiner, *Quantitative Methods In Antitrust Litigation,* LAW AND CONTEMP. PROBS., Autumn 1983, at 69. The authors are indebted to Professors Rubinfeld and Steiner for making their manuscript and data available to us.

The court of appeals affirmed,[27] and the Supreme Court subsequently granted certiorari.[28] Before the case could be heard, however, the parties settled with defendants' payment of $165 million.[29]

The model proposed by Steiner does not persuasively support his conclusion. The fact that prices responded to supply and demand factors does not preclude the possibility that they included a component—like phantom freight—that drove the entire level up. Indeed most price equations include both supply and demand factors and a dummy variable for the conspiracy on just the assumption that both may affect price. Moreover, the model did not appear to account very well for turning points even during the fit period and, more significantly, it failed to account for a sharp upward spike in prices that occurred in the postphantom-freight period of 1977-78. For most of that period, it perversely projected phantom-freight prices that were sharply *below* actual prices. The projected and actual prices for the phantom-freight and postphantom-freight periods are shown in Figure 3 below.

This brief summary of cases suggests that regression models frequently will not

FIGURE 3
PLYWOOD



27. *In re* Plywood Antitrust Litig., 655 F.2d 627 (5th Cir. 1981) *cert. granted sub. nom.* Weyerhaeuser Co. v. Lyman Lamb Co., 456 U.S. 971 (1982).

28. *Id.*

29. Wall St. Journal, Jan. 19, 1983, § 1, at 5, col.1.

predict price variations with sufficient accuracy to enable an economist to estimate reliably the relative small effects of a conspiracy. This article has touched on a few of the problems in some of the cases, but in most cases there was no very detailed exploration of the issues raised by the models. Probably the most detailed dissection of a model took place in the second *Corrugated Container* trial when the economists confronted each other. The following discussion looks in greater detail at some of the main issues that are likely to arise when models are introduced, drawing principally on the *Corrugated Container* case, and to a lesser extent on other price-fixing cases. We will also refer to models presented in antidiscrimination class action litigation, because some of the problems are similar and there has been more experience with multiple regression studies in that context than there has been in the price-fixing cases.[30]

II
ISSUES IN MODEL BUILDING

A. Types of Models

The models used in most of the cases consist of a regression equation in which price movement is, in large part, accounted for by certain explanatory factors included in the equation. Price is said to be regressed on these factors, which are its regressors. Typically there are both supply and demand factors in the equation. It is not assumed that the explanatory factors account perfectly for all movements of price, but only that they represent the principal influences so that the regression equation gives the average price. Variations about this average are assumed to be the result of many small accidental forces so that the discrepancies between the actual and regression estimates of average price have a random character. These discrepancies are known as errors when measured with respect to the "true" regression equation, and as residuals when measured with respect to the equation estimated from a sample of data. The randomness of the errors is defined in a precise technical sense that is described later in this article.

When the model is correct, the estimates of the coefficients of the explanatory variables and of the price are unbiased and consistent. Roughly speaking, a method of estimation is unbiased if the average of the estimates over repeated samples is the true value of the parameter being estimated. A method of estimation is consistent if the estimates it generates converge to the true value of the parameter as the sample size increases.

The models used in the cases previously discussed may be divided into two types. In what is known as a "dummy variable" model, the equation is estimated from data covering both conspiratorial and competitive periods and the explanatory factors include a dummy variable to account for the effect of the conspiracy. The dummy variable takes on a value of one for observations in the collusion period and zero otherwise (or vice versa). In a linear equation, the coefficient of the conspiracy dummy variable is the average overcharge due to the conspiracy in the

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30. For a discussion of some of the earlier antidiscrimination class actions, see Finkelstein, *The Judicial Reception of Multiple Regression Studies in Race and Sex Discrimination Cases,* 80 COLUM. L. REV. 737 (1980).

same units in which price is expressed. Plaintiffs' experts in the *Chicken Antitrust* case presented such a model. If price is expressed in natural logarithms, as it frequently is, the coefficient (if not too large, for example, less than 0.25) closely approximates (when multiplied by 100) the percentage overcharge.[31] The coefficient form of equation is the one used in most employment discrimination litigation, where one or more dummy variables may be included for sex and race and the size of the coefficients measures the extent to which the minority group or women have been underpaid relative to Caucasian men.

A dummy variable model assumes that the conspiracy added a certain dollar or percentage amount to price during the conspiracy period and nothing in the competitive period. In fact, the conspiracy might have raised prices in a more complex and varying fashion. The investigator may wish to make no assumption about the effect of the conspiracy, but rather treat it as an unspecified influence that is not explained by the model. In such a case, the determinants of price are the supply and demand factors dictated by economic theory and the effect of the conspiracy is appraised by looking at the differences between the actual and predicted prices, that is, the residuals. For this reason an equation in this form is referred to here as a "residuals model."

Residuals models can be estimated in several ways. One approach is to use data from both conspiracy and competitive periods and to compare the average residual in the two periods. This method can be used to test the null hypothesis that the conspiracy had no effect, but it is not useful to estimate the extent of any overcharge. To use a residuals model for that latter purpose, it should be estimated from data for the conspiracy or competitive periods and the resulting equation should then be used to project what the price would have been in the other period. It is probably preferable to estimate the model from competitive period data and use it to project competitive prices in the conspiracy period, because this approach leads more directly to an estimate of competitive prices in the conspiracy period and the usual supply and demand factors included in the equation are more convincingly relevant in a competitive market.[32] Both these methods were used in the *Corrugated Container* litigation.[33] In the *Concrete Pipe* litigation, plaintiff used a dummy variable model, while defendant used a residuals model.

Which type of model should be preferred? In answering this question it should be noted that the dummy variable and residuals models usually give different measures of the effect of a conspiracy. The reason is that when the conspiracy dummy variable is correlated with other explanatory factors (as it usually will be)

31. More precisely, if a is the coefficient of the conspiracy variable, the overcharge as a proportion of the competitive price is $e^a - 1$, where e is the base of natural logarithms.

32. If separate supply and demand structural equations are being estimated, competitive period data should be used because the assumption that the price is represented by the intersection of the supply and demand curves holds only in a competitive market. *See infra* p. 163. A reduced form equation with both supply and demand factors could be estimated from noncompetitive period data because the conspirators would still look to supply and demand factors in setting price in order to maximize their profit. Fisher, *supra* note 4, at 727-28. If, however, the conspirators do not act with total success to maximize profits (and in most cases they would not) it is unclear what the proper reduced form equation would be.

33. Defendants' expert also computed a dummy variable model which showed a very small but statistically significant overcharge. Transcript at 479-82 (Feb. 8, 1983).

the least-squares regression method will allocate some explanatory weight to each factor; no factor or group of factors is preferred. By contrast, in the residuals model, price movement is first attributed to the supply and demand explanatory factors, and it is only the unexplained residual variation that is assigned to the conspiracy. In effect, the residuals model asks whether the data are consistent with an assumption that the conspiracy had no effect.[34]

From this difference one might fairly argue that the effect of the conspiracy should be measured by the dummy variable model, since it is not tilted against a finding of a conspiracy effect, but rather is neutral as between the conspiracy and other factors. The argument for a neutral method would be strengthened in cases in which there was evidence apart from the conspiracy itself that it had been effective. Of course, defendants may not complain if plaintiffs use a residuals model—as they did in the *Corrugated Container* litigation—because it is less favorable to their case. Nevertheless, the court in at least one employment discrimination case accepted a residuals model introduced by defendants, despite plaintiffs' argument that it was biased against a finding of discrimination.[35]

B. The Choice of Explanatory Variables

The models already examined have displayed a high correlation between projected and actual prices over the period used to fit the equation. This goodness-of-fit was generally cited by the experts as evidence that the models were correct. However, a high correlation in the fit period does not prove that the explanatory factors are in fact good predictors.[36] Nor does the fact that the projection is good when prices are smoothly trending justify a conclusion that the model will accurately predict a break in the trend. In time series models it is not uncommon to find multiple correlation coefficients exceeding 0.9 in equations that perform badly at turning points in a price trend.[37] Moreover, the expert may have adopted the practice of "trying out" different sets of explanatory factors drawn from a large number of candidates to find the best fit. When this is done, the multiple correlation coefficient usually declines markedly in a new data set.

Sometimes defects in the explanatory variables are fairly obvious. One such case occurs when coefficients have the wrong signs. Such misspecification is not uncommon. A costs variable with a negative sign indicates a misspecified equation because it implies that prices would go down as costs go up. However, a perverse

34. Suppose that a residuals model is estimated from all the data (from the conspiratorial and competitive periods combined), and the statistical significance of the conspiracy appraised by comparing the average residuals in the conspiracy and competitive periods. It can be shown that the difference in average residuals equals the conspiracy coefficient times one minus the squared multiple correlation coefficient of the conspiracy dummy variable with the other explanatory factors in the price equation. Thus, as conspiracy becomes more and more confounded (correlated) with other factors, the difference in average residuals becomes smaller relative to the conspiracy coefficient.

35. Sobel v. Yeshiva Univ., 32 Fair Empl. Prac. Cas. (BNA) 154, 169-70 (S.D.N.Y. 1983).

36. When a lagged dependent variable is used as an explanatory variable, the equation is fitted by using the initial actual value of the lagged dependent variable and its subsequent estimated values. This corresponds the fitting technique to the way the equation is subsequently used to make projections. If the explanatory factors are weak predictors, however, the equation may "walk away" from the data, even during the fit period, so that a close correlation in the fit period is better evidence in favor of the equation.

37. For an example from administrative law, see Finkelstein, *supra* note 6, at 1455-62.

sign is not evidence of a misspecified equation if it is not statistically significant (and a zero coefficient would not be perverse), or if the explanatory variable is part of a group of variables that have a high mutual correlation and the sign for the combined group is correct.[38]

But even when all coefficients have the correct sign, the equation cannot be assumed to be correct; its reliability must be tested by using it to make projections on another sample of data. In the *Corrugated Container* case, Fisher performed such a test by reversing Beyer's procedure. He estimated Beyer's model from data for the competitive period and then used the model to project what the competitive price would have been in the collusive period.[39] The projected competitive prices were far above the actual collusive prices—a result indicating that the explanatory factors were not in fact good predictors of price movement.

In the *Chicken Antitrust* litigation, the model itself showed perverse results when it projected competitive prices for the collusive period that were higher than the actual collusive prices.[40] The same indication of inadequacy appears in the *Plywood Antitrust* model, which underpredicts the rise in prices that occurred after the ending of the conspiracy.[41]

A similar but more subtle problem arises when the model is not obviously misspecified. Most explanatory variables are proxies—and sometimes rather crude proxies—for the true quantities of interest. Data for industry-wide costs may only roughly approximate the relevant costs for a group of conspiring producers; a general price index may not reflect the true inflation for a particular industry. In the best of all econometric worlds the proxies are unbiased estimators of the true factors and are highly correlated with them. But even when this is true, the fact that the proxies are imperfect will bias the model's estimates of the conspiracy's effect. This has been called underadjustment bias because the proxies do not fully adjust for the factors they are supposed to represent.

The direction of bias depends on the situation. If the proxies indicate that prices in the postconspiracy period would have been higher than in the conspiracy period (due to an increase in real costs, for example), the projected conspiratorial prices in the postconspiracy period would be lower than they would have been if the proxies had been perfect, and the effect of the conspiracy would be understated. On the other hand, if the proxies indicate that prices in the postconspiracy period would have been lower than in the conspiracy period (due to a weakening

38. For example, in the plywood model, the coefficients of the variables for cost of logs and cost of gluing were negative and statistically significant and thus perverse. Cost of drying, however, had a much larger positive coefficient, and all three cost elements were highly correlated. When the regression was recomputed with the three combined, the coefficient for the combined variable was positive. The fact that the regression was computed with them separately would not generally affect the validity of the estimates of the dependent variable.

39. Fisher's "backcast" was made by projecting competitive prices forward from January 1963. For the lagged price variable, the December 1962 price was used, and each subsequent lagged price was the previously projected price, not the actual price. One might object that the initial price was not securely competitive, but the effect of the starting point wears off quickly so that it becomes essentially irrelevant within a year.

40. *See supra* pp. 151-52.

41. *See supra* p. 154.

in demand, for example), the regression would understate the drop in prices and the effect of the conspiracy would be overstated.

When different proxies point in different directions, the direction of bias cannot be determined. The reason is that the net effect depends on the degree of bias attributable to each of the proxies, and the degree of bias is a function of the correlation between the proxy and the true explanatory factor.[42] But since this correlation is unobservable, the bias cannot be estimated. The problem of underadjustment bias was recently recognized by Judge Goettel in the *Sobel* case as a reason for denying relief to a class of women, even though plaintiffs' multiple regression model included a statistically significant sex coefficient that apart from bias would have indicated discrimination against women.[43]

The potential for bias from underadjustment is a problem for which there is at present no good solution. Whether the problem is serious in the context of price time series is unknown.

C. Lagged Dependent Variables

One type of explanatory variable deserves separate discussion because its use raises special problems. In an econometric model it is not uncommon to find that the dependent variable itself, lagged by a time period, has been included as an explanatory factor. The justification is that (1) the lagged variable picks up other influences on price not specified by the regression equation; or (2) the lagged variable reflects price "stickiness," that is, the slowness with which price responds to changes in explanatory factors. In the *Corrugated Container* case, Beyer justified the use of price the previous month as an explanatory variable by the fact that some prices were determined by annual contracts and thus could not respond immediately to changes in costs and other explanatory factors.[44] He did not explain how this would justify the one-month lag that he used.

One difficulty with a lagged dependent variable is that, if the lag is short, that variable will tend to dominate the equation if its coefficient is estimated from smoothly trending data so that the equation becomes a poor predictor of changes in trend. These problems were evident in the *Corrugated Container* models used by Beyer in both trials. In those models, the dominant explanatory factor was lagged price; no other factor approached it in importance or statistical significance. The result was that the model tended simply to project smooth trends from wherever the fit period was ended. Fisher demonstrated this by showing that the model would have projected collusive prices *below* the competitive prices in the postconspiracy period if the fit period had simply stopped at the end of 1971 instead of the

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42. The effect may be quantified more precisely as follows: If the conspiracy had no effect, the coefficient of a dummy variable for conspiracy in the regression equation (the expected value of which should be zero) would be equal to the difference in average price in the conspiracy and competitive periods times one minus the squared correlation coefficient between the proxy and the true explanatory factor. If the proxy were perfect, the correlation would be one and the conspiracy coefficient would be zero. Conversely, if the proxy were wholly ineffective, the correlation would be zero and the conspiracy coefficient would simply be equal to the difference in average prices. *See* Robbins & Levin, *A Note on the "Underadjustment Phenomenon"*, 1 STATISTICS & PROBABILITY LETTERS 137-39 (1983).

43. Sobel v. Yeshiva Univ., 32 Fair Empl. Prac. Cas. (BNA) 154, 166 (S.D.N.Y. 1983).

44. Transcript at 87 (Dec. 15, 1982).

end of 1974.[45]

One technique for reducing the overprediction of trend is to enlarge the interval between successive price observations. If quarterly instead of monthly prices are used in the *Corrugated Container* data from the first trial, the dominance of the lagged price is reduced and the coefficients of the other explanatory factors are increased in relative importance. However, the estimates of conspiracy effect do not change very much.

Another technique for avoiding an overprediction of trend is a first-difference equation. In such an equation, the dependent variable is the difference in price between two successive periods, and this change in price is regressed on changes in the explanatory factors.[46] In effect, this transformation allows changes in the explanatory factors to account for departures from pure linear trend in the data. When such an equation is estimated from the *Corrugated Container* data in the first trial, the fit of the equation declines from a squared multiple correlation coefficient of 0.99 (which primarily reflects trend) to 0.33, and the estimated average overcharge declines from 7.8% to 4.3%.[47]

Besides overprediction of trend, use of a lagged dependent variable creates the risk of another potential distortion. The most common problem in a time series regression model is serial correlation in the error term; this means that the errors in successive observations are not independent as specified by the theoretical model,[48] but are correlated to some degree. In an equation that does not have a lagged dependent variable, the only consequence of such correlation is to make the usual calculations of standard errors unreliable and the true precision of the equation lower than that indicated by theory. However, tests of statistical significance, which use standard errors, will not be important in most cases and so this defect in and of itself may not be crucial. But, if there is serial correlation of the errors in an equation that includes a lagged dependent variable as an explanatory factor, the resulting estimates of the coefficients of the explanatory variables will become biased and inconsistent in a way that cannot be predicted.[49] This defect goes to the heart of the acceptability of the equation estimates.

A solution to this problem is difficult to achieve. It requires either new explanatory variables that more completely capture the systematic relationships so that what remains for the error term is truely random noise, or various rather complex techniques for estimating the correlation structure of the error term and the estimation of a new equation that takes explicit account of such correlation.[50]

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45. Defendants' Exhibit No. 1352, Transcript at 426 (Feb. 8, 1983), *Corrugated Container*.

46. In his first *Corrugated Container* model, Dr. Beyer inexplicably used change in cost as an explanatory factor while entering all other variables in their original form. This inconsistency was corrected in the second trial.

47. In making these illustrative computations, the period 1963-75 was used to fit the equation and the forecast was made for the period 1977-79. .

48. *See infra* pp. 164-65.

49. J. JOHNSTON, ECONOMETRIC METHODS 211-16 (1963).

50. For a discussion see M.D. INTRILIGATOR, ECONOMETRIC MODELS, TECHNIQUES & APPLICATIONS 159-65 (1978).

D. Tainted Variables

Some explanatory factors may themselves have been influenced by collusion. In the *Ohio Valley* case, the court found that during the period of collusion defendants did not have to face a competitive market and consequently made less of an effort to control costs.[51] To the extent costs are inflated as a by-product of collusive behavior, inclusion of a cost variable to explain price would conceal the full effect of the collusion. While such concealment is an issue to watch, it should not be assumed that all cost data are tainted. A similar objection was made to the cost data in the *Plywood Antitrust* case, but it seems unlikely that plywood gluing or drying costs were inflated as a result of phantom freight or standardized weights. The problem of tainted explanatory factors has arisen in much sharper form in antidiscrimination litigation when, for example, the courts have divided over whether it is proper to include academic rank as an explanatory factor in regression models of faculty salary at academic institutions.[52]

E. Collusive and Competitive Periods

The results of a multiple regression study will generally be significantly influenced by the choice of the collusive and competitive periods. The principal issues have centered on the length of the collusive period and the existence of a transition to full competition.

The regression coefficients of the explanatory variables represent averages over the period used to fit the equation. When this period is long, the estimate of the conspiracy coefficient is open to the objection that, being an average, it does not fairly represent the period immediately adjacent to the competitive period used for comparison. The regressions in both *Corrugated Container* cases were fitted over a long period, which raised a question whether the result would have been different if the regression had been calculated using a shorter period adjacent to the competitive period. Fisher computed regressions for subperiods within the collusive period and concluded (not surprisingly) that the regressions had significantly different coefficients, thus indicating that the relationships did change over time.[53]

A period of collusive activity can be compared with (1) a competitive period prior to the beginning of such activity; (2) a "white sale" period within the collusive period' or (3) a period after the termination of the conspiracy. Each has its difficulties. The preconspiracy period requires a determination of when the con-

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51. Ohio Valley Elec. Corp. v. General Elec. Co., 244 F. Supp. 914.(S.D.N.Y. 1965).

52. *Compare* Sobel v. Yeshiva Univ., 32 Fair Empl. Prac. Cas. (BNA) 154, 166-67 (S.D.N.Y. 1983) (use of rank approved) *with* Melani v. Board of Higher Educ., 561 F. Supp. 769 (S.D.N.Y. 1983) (use of rank disapproved). For a discussion, see Finkelstein, *supra* note 30, at 738-42.

53. Transcript at 429-31 (Feb. 8, 1983), *Corrugated Container*. The method used by Fisher (sometimes referred to as the Chow test) is one commonly used to determine whether there is a statistically significant difference between the coefficients of two linear regressions. The test involves a comparison of the sum of the squared residuals for the two regressions with the sum of the squared residuals for a new regression computed from the pooled data for the two regressions. If the latter is much larger than the former, the hypothesis that the two regressions have identical coefficients is rejected. *See* J. JOHNSTON, *supra* note 49, at 136-38. The finding of a statistically significant difference by means of the Chow test is subject to the caveat discussed *infra* p. 162. Contrary to the intimation in the second *Corrugated Container* trial, the Chow test is not the only method for exploring this question in a time-series context.

spiracy began, which is likely to be in doubt. The white sale period may involve special conditions that did not persist throughout the period of collusive activities. (This difficulty was present in the *Concrete Pipe* litigation where the white sale occurred when a competitive firm entered the market and ended when it was bought out.) More commonly used is the postconspiracy period, since the ending of the conspiracy is usually a fairly dramatic event. In most cases, however, it is argued that there is a transition period in which prices are still affected by the residue of the conspiratorial activities. Whether there is such a transition is a sensitive issue because the allowance of a period of one or two years between the ending of collusion and the beginning of competition will probably change the estimated effects of the collusion considerably, and increase greatly the uncertainty of the econometric projections—which characteristically have a very short reliability life.

One technique for determining whether there is a transition period is to compute separate regressions for successive time periods (for example, successive years) after the termination of the conspiracy. A shift in the coefficients of the regressions as the period of collusion recedes is evidence of a transition, which comes to an end when the coefficients stabilize in the postconspiracy period. Fisher tested the alleged transition period 1975-78 in the second *Corrugated Container* case and found no statistically significant difference between the regression coefficients in this period and in the later period; he relied on this finding to reject Beyer's assertion that there was a transition.[54]

If a statistically significant difference is demonstrated, the existence of a transition period may still be in doubt for various reasons. One statistical reason is that the degree of significance may be overstated. The power of the statistical test to detect a significant difference depends on the number of independent observations used in fitting the regression. If the regression model is misspecified so that the errors are not in fact independent but are correlated, the number of independent residuals will be to some extent smaller than their actual number, and thus findings of statistical significance based on the number of residuals will overstate statistical significance. It is thus easier to disprove the existence of a transition period econometrically than it is to confirm one.

F. Extrapolation

Projection by multiple regression is most reliable when the values of the explanatory factors used for the projection are near the averages for those variables in the data used to fit the equation. As these values depart from their averages, the standard errors of the predicted value increases (although the standard errors of the estimated coefficients of the explanatory variables do not). More significantly, when regression estimates are based on values of explanatory factors that lie near or beyond the range of the data on which the equation is estimated, it becomes problematical whether the estimated regression relationships will continue to hold in the new range. In the *Corrugated Container* cases, the price of containers in the competitive period was much higher than in the collusive period and defendants argued that the demand for corrugated containers declined as users sought other

54. Transcript at 434 (Feb. 8, 1983), *Corrugated Container*.

packaging; the weakening of demand, not the ending of the conspiracy, explained the flattening of price.[55] Cognate problems of extrapolation are likely to arise in most antitrust contexts because the price levels in periods separated by years may differ considerably, even after adjustment for inflation.

G. Structural Equations and Reverse Relationships

A single equation in which price is the dependent variable and supply and demand elements are the explanatory factors is what is known as the "reduced form" of a supply equation and a demand equation. In the separate supply and demand equations, quantity is the dependent variable. Price appears in both as an explanatory factor but otherwise the explanatory factors are different. A technically correct method of estimating the coefficients for price is to solve the two equations for quantity simultaneously using special methods of estimation such as two-stage least squares.[56]

As we have seen, in most cases the parties have not estimated the separate structural equations but contented themselves with the reduced form that combines both supply and demand factors in a single equation. Although this practice is not objectionable per se, it may be desirable also to estimate the supply and demand equations separately. The structural equations disclose the relationships that are concealed by the reduced form, which is essentially a black box; showing them separately may reveal defects in the model. In the data used in the first *Corrugated Container* case, the demand equation when estimated separately had an upward slope, a perverse result indicating a misspecification of the model.[57] This defect was not disclosed because the separate structural equations had not been estimated by either party. The same defect did not appear in the equations used in the second trial.

Estimation of multiple equations becomes necessary if explanatory variables, in addition to influencing the dependent variable, are also influenced by it. When such simultaneous reverse relationships exist, the estimates of the coefficients generated by ordinary least-squares regression become biased and inconsistent.[58]

A point about a possible reverse relationship was made in the second *Corrugated Container* case. Defendants argued that the 26% overcharge was patently ridiculous because a price drop of 26% would have put defendants out of business. Beyer responded that in that event linerboard prices would also have dropped, thus reducing costs.[59] This was a concession of a large reverse influence of price on cost. The same problem arose with respect to the inventory variable, since price was likely to influence inventory as well as inventory influencing price. While these are potent theoretical points, it cannot be assumed that a multistage least-squares approach would produce very different results from ordinary least squares applied to a reduced form equation. The effect of any such difference should be demonstrated.

55. Transcript at 185-93 (Feb. 8, 1983).
56. For a discussion, see R. WONNACOTT & T. WONNACOTT, ECONOMETRICS 172-95 (1970).
57. Conversation with John E. Linville, one of the counsel in the case (June 22, 1983)..
58. M.D. INTRILIGATOR, *supra* note 50, at 389.
59. Transcript at 21-33 (Dec. 16, 1982).

H. Standard Error Computations

In the *Concrete Pipe* and the *Chicken Antitrust* cases, as previously discussed, parties based arguments on the computed values of the standard errors of the regression coefficients.[60] In theory, these standard errors reflect the variation in the regression estimates that would occur if the same regression model were computed from other samples of data generated by the same process. If the average price in the postconspiracy period, for example, lies within two standard deviations of the average predicted values based on the regression equation, most statisticians would decline to say that there was a significant difference between the two. When a conspiracy coefficient model is estimated, a coefficient of the conspiracy dummy variable that is not significantly different from zero indicates that the hypothesis of no effect cannot be rejected. It is not, however, justifiable to argue, as defendants did in the *Concrete Pipe* litigation, that damages are properly measured by the difference between actual prices and the upper (or lower) limit of a 95% confidence interval around the regression estimate. If the model is correct, the best price estimate is the regression estimate and there is no justification for picking another figure with a much lower probability of being correct.

Arguments based on standard errors depend on the theoretical correctness of the calculations, which in turn depend on whether the regression model meets certain specifications. Most models based on time series do not meet those specifications and consequently standard error calculations are, at best, almost always suspect. In general, the true variability of the equation is greater than that indicated by theoretical calculation. Therefore, a statistically insignificant difference between a "projected" competitive and actual conspiratorial price should usually be accepted as evidence that the conspiracy had no measurable impact on price, but the appearance of a difference that is statistically significant should be the occasion for an investigation of the correctness of the model.

The basic requirement of the multiple regression model is that the explanatory factors reflect all major forces bearing on price so that departures from the regression estimate (that is, the errors) are the result of many relatively small influences. This requirement is assumed to give the errors a random character defined by four properties. First, if repeated estimates are made of price, the average of the estimates for any given set of values of the explanatory factors will be equal to the true price, which is another way of saying that the errors sum to zero. In that case the equation is said to be unbiased. Second, the errors are normally distributed about the regression estimate. Third, the variance of the normally distributed errors is constant for all values of price. Fourth, there is no correlation between the errors associated with two different observations of price, so that, for example, an actual price above the regression estimate in one month reveals nothing about whether it will also be above the estimate in another month. Since the errors are unobservable (because the "true" regression relationship is unknown), these properties cannot be directly observed. Instead, the tests must be made on the residuals, which are the differences between the actual prices and their regression

60. *See supra* pp. 149-50, 152.

estimates.[61]

In a regression model of time series data, the most important question to resolve is whether there is serial correlation in the errors; the existence of such correlation frequently is a warning that the model is of the wrong form or that some important explanatory factor has been omitted. The Durbin-Watson test is commonly used to test for such first order serial correlation.[62] In the *Broiler Chicken* regression, the experts did not mention the fact that the Durbin-Watson test suggested serial correlation of the errors, indicating that the model was misspecified.

An even more important diagnostic tool, but one that is frequently ignored, is a plot of the residuals. Standard computer packages facilitate this by generating plots in which the residuals are plotted against the fitted values of the dependent variable or against values of the explanatory factors. From such plots it is relatively easy to spot patterns that suggest the linear model does not fit the data or outlying observations that should be separately investigated. A full justification of a model should include an account of an inspection of such residual plots.[63]

A second important question in time series modeling is whether the variance of the error is constant. This condition will be violated if the entire scale of the equation increases with time, in which case the variance of the error will also increase. The use of undeflated dollars in a period of high inflation can create this defect.[64] To detect departures from a constant variance, the investigator should inspect plots of the residuals to see if they "fan out" along the time line.

Where the errors may not conform to the model, there are alternative ways of proceeding. Probably the best way is to make a direct assessment of variability by computing regressions for subperiods and observing the variation in the estimates. When this is done for the *Corrugated Container* data from the first trial, the results show much greater variation in the regression estimates than that indicated by the standard errors.[65]

I. Time-Series Analysis

The difficulty of constructing econometric models that satisfy theoretical requirements suggests that one might use more general methods that do not

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61. The characteristics of the residuals are not identical to those of the errors. For example, least-squares estimation insures that the algebraic sum of the residuals is always zero, but this in general is not true of the errors in a given sample of data. However, diagnostic tests of the residuals have been devised that shed light on the properties of the errors. *See* D.A. BELSLEY, E.KUH & R.E. WELSCH, REGRESSION DIAGNOSTICS (1980).

62. For a discussion, see M.D. INTRILIGATOR, *supra* note 50, at 159-65. If a lagged dependent variable is included as an explanatory factor, the Durbin-Watson test will tend to show no serial correlation and a different method must be substituted for a correct picture. *Id.* at 164 n.22.

63. *See, e.g.*, S. CHATTERJEE & B. PRICE, REGRESSION ANALYSIS BY EXAMPLE (1977).

64. A nontime series example is the following: a regression of total costs on number of transactions for a large number of New York Stock Exchange firms was used to determine whether there were economies of scale in the brokerage industry. However, the standard error of the coefficient of number of transactions was found to be unreliable because the scale of the entire equation, and with it the variance of error term, grew with the size of the firms. Very different results were obtained when the average cost per transaction was substituted as the dependent variable. Finkelstein, *supra* note 6, at 1463-66.

65. The authors' subperiod tests of the regression model during the conspiracy period showed that the difference between the predicted and actual values as a percentage of predicted values ranged up to 6.06% in the fourth year after the fit.

assume that the explanatory variables included in the equation have accounted for everything except random error. This point of view leads to various methods of time series analysis that have found widespread applications in recent years. The so-called Box-Jenkins methods are among the most popular and successful of these techniques.[66]

Although the estimation of parameters in time series models is a highly technical matter and one that is computationally complex, the basic ideas of the Box-Jenkins approach are not hard to understand. Consider a time series in which the observations (prices, for example) fluctuate about a mean or expected value with a constant variance. Such a series has no trend and is said to be a stationary linear process. Fluctuations of price are caused by independent random influences or "shocks." The shocks are assumed to have some continuing impact that diminishes over time, so that price at any given time reflects the sum of all prior shocks, with the weights usually diminishing (except for seasonality effects) as one moves back in time. Unlike a regression model, which assumes that departures from the regression mean are uncorrelated errors, the linear process for time series described here allows for correlated displacements from the mean in successive observations, the extent of correlation depending on the persistence of the effect of prior shocks. The more persistent those influences are, the greater the correlation between successive observations.

There are three representations of the general linear process that are of practical interest. One is called an autoregressive model, in which current price is expressed as the weighted sum of a finite number of prior prices plus a random shock. The name derives from the fact that the model bears a resemblance to a regression model in which a dependent variable is regressed on independent variables plus an error term. Since price is regressed on prior values of itself, the process is said to be autoregressive. A second type is a moving-average model, in which price is expressed as the weighted sum of a finite number of prior shocks. Although the separate shocks are not directly observable because fluctuations of price reflect the *combined* influence of past shocks, there are methods for estimating the parameters and the shocks. A third type of model combines both autoregressive and moving-average terms in a single equation. The combined approach is frequently used because its flexibility minimizes the number of terms that are needed to express the model.

Since most time series encountered in practice are not stationary, trend must be eliminated from the data before the model can be estimated. This is done by taking first differences, for example, using the differences in price between successive months as the data instead of the values for each month. If first differences still show trend, the data are differenced a second time.

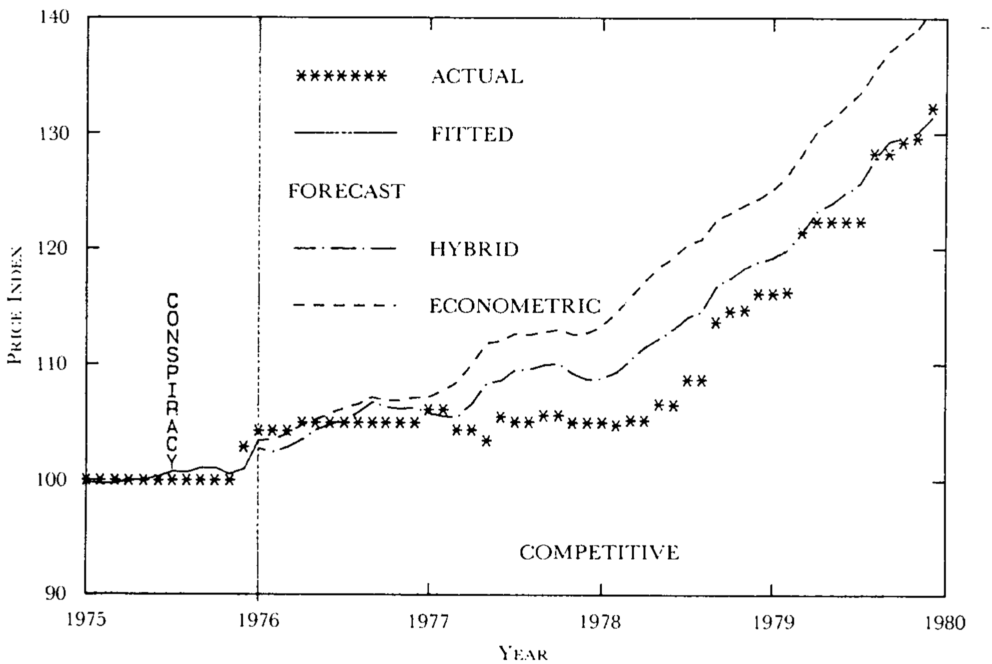
In estimating the model, an investigator must make judgments about the number of autoregressive and moving-average terms to include. There is consider-

66. See G.E.P. BOX & G.M. JENKINS, TIME-SERIES ANALYSIS, FORECASTING AND CONTROL (1970) for a detailed technical explication of the method. For an introductory discussion, see J.P. CLEARY & H. LEVENBACH, THE PROFESSIONAL FORECASTER (1982). *See also* M.G. KENDALL, TIME SERIES (2d ed. 1976). More accessible to the nonspecialist is C. CHATFIELD, THE ANALYSIS OF TIME SERIES (2d ed. 1980).

able latitude here and different choices may lead to widely different results. The weights assigned to the autoregressive and moving-average terms are determined by a nonlinear estimation procedure that minimizes the sum of squared differences between the model projections and the data on which they are based. Although the calculations are complex, computer programs to perform them are readily available to the practitioner. Time series methods have not, to the authors' knowledge, appeared in a litigated case, although they have been used in at least one academic study of overcharges due to price fixing.[67]

A modest use of time series methods is to regard them as complementary to econometric methods. If the errors in a regression model are serially correlated, a time series projection of the residuals, when added to the econometric projection, is likely to produce better forecasts than those generated by the regression model alone, which assumes that future residuals will have an expected value of zero. When this hybrid technique is applied to the data of the first *Corrugated Container* case, the projected overcharge drops from 7.8% to about 4%. The hybrid and regression projections are shown in Figure 4 below. The closer fit of this hybrid

FIGURE 4
CORRUGATED CONTAINERS



67. A time series study of prices in the titanium industry indicated an overcharge of about 1.5%, or $8 million, during a period of collusion from 1970-76, for which the United States brought suit in United States v. RMI Co., Civ. A. No. 78-1108E (W.D. Pa. filed Sept. 28, 1978) and United States v. RMI Co., Civ. A. No. 81-4177 (E.D.N.Y. 1981). *See* Duggan & Narasimham, *Price Fixing In The Titanium Industry: A Time Series Analysis,* in Proceedings, Business and Economic Section, (American Statistical Assoc., 1981).

model to actual prices in the postconspiracy period indicates that part of the overcharge projected by the econometric model was due to its failure to account for the correlation structure of the residuals.

A more aggressive use of time series models would involve their substitution for econometric techniques. As applied to the *Corrugated Container* data, the pure time series approach produces a straight-line projection that is closer to the actual prices than the econometric projection until 1978, but it then becomes increasingly irrelevant as it does not reflect the sharp subsequent upward turn in prices. Which type of estimate is generally superior in the short run has been much debated, and as one would expect there is no clear verdict.[68] In the absence of a consensus, advocates of time series analysis argue that their methods should be preferred because of simplicity and directness. Some econometricians reply that pure time series methods are unuseable because they "explain" nothing.

III

CONCLUSIONS

The nascent excursion of the courts into econometrics suggests two perhaps conflicting observations. The first is the remarkable fact that the judicial system is presiding over the payment (mostly by settlement) of large sums as damages when the econometric evidence of injury is equivocal at best. In *Chicken Antitrust* there was no credible evidence of an overcharge and the settlement exclusive of lawyers' fees was $30 million. In *Plywood Antitrust* the case was settled for $160 million and there was no econometric proof of injury. In the *Corrugated Container* cases the settlements were in excess of $300 million but one jury ultimately found the proof of overcharge unconvincing as to certain large purchasers. Confronted with this record, one is tempted to say that the class action device has loaded things too heavily against the defendants, by making it too risky for them to stand trial.

On the other hand, the introduction of econometric methods may have tilted things too far against the plaintiffs when there is a trial. In the *Ohio Valley* case, the plaintiff proved the overcharge by showing the difference in discount in the conspiracy and competitive periods; that showing sufficed for a prima facie case. It was up to the defendants to prove—if they could—that the difference was due to factors other than the conspiracy. If plaintiffs must now produce an econometric model that accounts for the other factors and isolates the effect of a conspiracy, they will have shouldered an entirely new burden. Given the great difficulties of constructing plausible models, such a burden seems unfair, particularly if there is independent evidence that the conspiracy has had some effect.

This does not mean that a simple-minded comparison of preconspiracy and postconspiracy prices or profits should be sufficient for a prima facie case. Some adjusting factors should be required but not necessarily a full-blown model. A similar type of intermediate approach seems to be emerging in the employment discrimination cases, where the courts expressly have accepted the pragmatic

68. Armstrong, *Forecasting with Econometric Methods: Folklore Versus Fact*, 51 J. BUS. 549 (1978).

notion that a multiple regression study may be sufficient for a prima facie case even if it reflects only some of the factors that plausibly may account for variations in productivity. The parallel in the antitrust context discussed here would be to require only the most basic of adjusting factors in an econometric equation, or even to adopt a pure time series approach. If such a measure shows there was damage, the amount may be treated as determined by a "just and reasonable inference, although the result be only approximate."[69] A full-fledged model would be required only to rebut an inference that would otherwise be drawn from a simple change in price level associated with the beginning or ending of a conspiracy.

How this would work in practice should depend on the context. In the situation that is most persuasive for the plaintiffs, a smoothly trending price series suffers an interruption of trend at the juncture between conspiratorial and competitive periods. Such an interruption was evident in *Corrugated Container*. Assuming that the interruption of trend is consistent with a drop in prices following the shift from conspiracy to competition, plaintiffs' prima facie case should be made by a reasonable projection of prices with the dollars deflated by an appropriate index to account for inflation. The degree of sophistication required at that point might depend on the nonstatistical evidence of the strength of the conspiracy. If defendants contend that factors other than the conspiracy caused the interruption of trend, they should produce a model that persuasively accounts for the differences by making a superior projection; explanations that are unquantified or models designed solely to cast doubt on plaintiffs' projection without being tendered as sufficiently reliable for a finding should not be sufficient.[70] Nevertheless, if defendants do produce a model purporting to demonstrate that the price change was due to factors other than the conspiracy, plaintiffs should be entitled to rebut by showing that the model is defective without producing one of their own.

The same format should be followed, with the parties reversed, when there is no break in the price trend associated with the beginning or ending of the conspiracy, but plaintiffs contend that prices would in fact have changed at that point if other factors had not concealed the effect of the conspiracy. This was the situation in *Chicken Antitrust*. In that event, plaintiffs would need an econometric model to make their prima facie case and defendants would be entitled to rebut simply by demonstrating that plaintiffs' model was not reliable.

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69. Story Parchment Co. v. Paterson Parchment Paper Co., 282 U.S. 555, 563 (1931).

70. Of course, nothing should prevent either party from demonstrating that the class-wide figure determined by regression should not apply to any particular subclass or group of purchasers based on particular evidence with respect to those purchasers.