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Regression Methods in Biostatistics

Linear, Logistic, Survival, and Repeated Measures Models

Second edition



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