Economic Theory (2006)
DOI 10.1007/s00199-006-0180-9

SYMPOSIUM

Richard Engelbrecht-Wiggans · Elena Katok

Regret in auctions: theory and evidence

Received: 13 July 2005 / Revised: 23 October 2006

Abstract The sealed-bid first-price auction of a single object in the case of independent privately-known values is the simplest auction setting and understanding it is important for understanding more complex mechanisms. But bidders bid above the risk-neutral Nash equilibrium theory prediction. The reasons for this "over bidding" remain an unsolved puzzle. Several explanations have been offered, including risk aversion, social comparisons, and learning. We present a new explanation based on regret and a model that explains not only the observed over bidding in sealed-bid first-price auctions, but also behavior in several other settings that is inconsistent with risk aversion.

Keywords Auctions · Competitive bidding · Regret · Risk-aversion · Learning · Experimental economics

JEL Classification Numbers D44 · C91

The authors gratefully acknowledge support from the National Science Foundation.

R. Engelbrecht-Wiggans
College of Business, University of Illinois, Urbana-Champaign,
1206 South Sixth Street, Champaign, IL 61820, USA
E-mail: eplus17@uiuc.edu
Tel.: +217-3331088

E. Katok (✉)
Smeal College of Business, Penn State University,
465 Business Building, University Park, PA 16802, USA
E-mail: ekatok@psu.edu
Tel.: +814-8632180
Fax: +814-8632381

1 Introduction and motivation

Why do laboratory subjects with independent privately-known (IPV) values bid higher than the risk-neutral Nash equilibrium (RNNE) prediction in sealed-bid first-price (SBFP) auctions? The fact that they do is not in question. This result has been replicated numerous times by different researchers at different laboratories and under a variety of environments. However, the reason for such overbidding remains unclear.

This question is of more than purely academic interest. For some time now, auction theory has been used in designing real world auctions and exchanges. Important examples include the FCC auctions of spectrum licenses (Rothkopf et al. 1998) electric power auctions, auctions for transportation services (Ledyard et al. 2002), and procurement auctions used for such applications as selling excess inventory and determining prices for newly-introduced products (Keskinocak and Tayur 2001). Since the quality of the resulting auction designs depends on the predictive ability of the theory, we need a good predictive theory.

The single unit SBFP auction is a very simple mechanism, and IPV is the simplest possible setting. Being able to understand bidding behavior in this environment may provide a fundamental building block to understanding more complex situations. A better understanding of what contributes to overbidding in simple SBFP auctions can also provide insights into what might happen in more complex auctions that have some SBFP flavor to them. Examples include the multi-unit uniform-price auctions used by the United States Treasury, descending-bid "Dutch" auctions used in Holland to sell flowers (Katok and Roth 2004), sealed-bid auctions for major construction projects, timber auctions, and procurement auctions in general.

Common explanations for overbidding in SBFP auctions fall roughly into three categories: risk aversion, inter-personal comparisons, and learning. In general, assuming bidders to be risk averse can explain observed behavior in SBFP auctions Cox et al. (1988). However, risk aversion cannot explain the observed laboratory data in several other auction-type settings, such as the third-price auction studied by Kagel and Levin (1993), the random-price auction studied by Cason (1995), and the comparison between behavior in the SBFP auctions and the Becker–Degroot–Marschak (BDM) procedure (Becker et al. 1964) studied by Isaac and James (2000). In his review article, Kagel (1995, p. 525) suggests that "$\cdots$ risk aversion is one element, but far from the only element, generating bidding above the RNNE$\cdots$" in SBFP auctions. In this paper we examine other factors of potential importance.

The second class of explanations for overbidding in SBFP auctions considers inter-personal interactions and comparisons. Isaac and Walker (1985) found that the amount of overbidding relative to the RNNE decreases in four-bidder auctions when the feedback provided at the end of the auction includes all the bids. They suggest that implicit collusion among the bidders may be responsible for the shift. Ockenfels and Selten (2005, p. 156) observe a similar result in two-person auctions. They propose a direction learning explanation, the impulse balance equilibrium, that they interpret "$\cdots$ as a measure of concern for relative standing". Dufwenberg and Gneezy (2002) report a similar shift in common-value auctions. They attribute this to signaling behavior. Morgan et al. (2003) develop a theoretical model of "spiteful bidding" in which "$\cdots$a bidder cares not only about her own surplus in the event she wins the auction, but also about the surplus of her rivals in the event she loses$\cdots$" (p. 1). They predict overbidding in SBFP as well as in the sealed-bid second-price auctions. However, such theories require inter-personal comparisons and, therefore, cannot explain similar behavior by subjects who were bidding against computerized rather than human competitors.

Selten and Buchta (1998) advance a third type of explanation for the observed behavior in SBFP auctions based on learning direction theory (Selten and Stoeker 1986). They note that bid functions tend to be non-linear and unstable over time. Neugebauer and Selten (2006) analyze and test this further. They find that most of their subjects adjust their bids in a way consistent with the learning direction theory. Learning direction theory speaks to the direction in which bids are likely to be adjusted over time, based on feedback. It implies convergence to the impulse balance equilibrium (Ockenfels and Selten 2005; Neugebauer and Selten 2006) – essentially, the learning stops when the propensity to increase a bid balances with the propensity to decrease it. The specific impulse balance point depends on the relative strength of those propensities. However, it does not offer any explanation for initial overbidding.

Engelbrecht-Wiggans (1989) examines the effect of regret on bidding, suggesting that a bidder's utility depends not only on profit, but also on various forms of regret. For example, the winner in a SBFP auction typically pays more than the highest competitors' bid, thereby leaving money on the table (MLOT). Similarly, at the RNNE, the winner in a third price auction sometimes wins at a price higher than his value for the item (see Kagel and Levin 1993); the item is won at an unfavorable price (WAUP). In either case, the winner may regret having bid so high. Alternatively, the winner's price may be below some losing bidder's willingness to pay. In this case, the loser has missed an opportunity to win (MOTW) the object at a favorable price, and may regret having bid so low. When deciding on the bid amount, a bidder may be sensitive not only to the expected profit, but also to the expected amount of each type of regret. Note that the direction in which regret influences bids in a dynamic setting, corresponds to Selten's learning direction theory. The learning direction theory, however, only predicts how bids change from one period to the next. In contrast, regret theory focuses on the equilibrium and on how changing the relative weight on different types of regret affects the equilibrium. Specifically, (Engelbrecht-Wiggans 1989) shows that the more heavily that bidders weight MOTW regret relative to MLOT regret, the more the equilibrium bids in a SBFP auction will exceed RNNE bids – concern for MOTW regret leads to overbidding.

A laboratory experiment that can distinguish between risk aversion and regret would provide a tough test of the regret theory if the regret theory predicts a shift in some treatments, while risk aversion does not. Simultaneously, the experimental setting should control for inter-personal interactions in order to be able to rule out such explanations as collusion, spite, and social utility in general. [1] The Neugebauer and Selten study provides a version of this kind of a test, although their experiment

[1] That is not to imply that those explanations do not have validity, especially in practical settings. However, since our stated goal is to separate the regret and risk aversion explanations, inter-personal interactions would confound the test.

was designed for a different purpose, and we will discuss regret theory in relation to their data in Sect. 3. We also present analysis of our own experimental data that was collected independently and roughly concurrently, and shares some key design characteristics with Neugebauer and Selten. The Neugebauer and Selten experiment, as well as our own, attempts to manipulate directly the MLOT and MOTW regret weights by either displaying or hiding the information needed to compute the amounts of each type of regret at the end of the auction. Both designs control for inter-personal interactions by having one human bidder compete against computerized opponents in all treatments.

Such experiments actually include an auxiliary hypothesis: How people feel about regret can indeed be manipulated through information. Strictly speaking, these experiments test regret theory together with the auxiliary hypothesis. However, this does not decrease the strength of the test. If the shifts predicted by regret theory are observed, then the data is consistent with the regret theory and the auxiliary assumption jointly. If the shifts predicted by the regret theory are not observed, then the implication is that either the regret theory or the auxiliary hypothesis fails to hold.

The next section presents new theory needed to provide theoretical benchmarks and to form specific hypothesis for the proposed setting. As one might expect, the results for how regret affects a bidder's best bid when bidding against a fixed distribution of bids have a similar flavor to those for equilibrium bids. However, since bidding against a fixed distribution of bids is a simpler problem mathematically than equilibrium bidding, we are also able to generalize these results in several directions. We extend the theory to model the a general sealed bid "kth price" auction, which allows us to obtain qualitative predictions for the third price auction studies by Kagel and Levin (1993) and the random price auction studies by Cason (1995). In section three we analyze several different sets of data for which we can obtain quantitative predictions from the regret model. A straight-forward adaptation of the regret model to the BDM procedure allows us to demonstrate how regret theory organizes the Isaac and James (2000) data. We then proceed to summarize data in the Neugebauer and Selten (2006) and our own Engelbrecht-Wiggans and Katok (2006) experiments that manipulate feedback information, and find all treatment effects to be consistent with the regret theory. In section four we summarize the contributions of this paper and further discuss the results.

2 The effect of regret in auctions

2.1 The basic results

Assume that a single unit of some product will be sold using a typical SBFP auction. Each of n bidders makes a bid greater than or equal to the reservation price r. The highest bid wins, and the winning bidder pays his bid.

Consider the problem from the perspective of any one bidder i. Assume that bidder i has a privately-known value v. Let b denote that bidder's bid, and let z denote the maximum of the other $n-1$ bidders' bids. Assume that z is independent of v, has support $[A, B]$ with $r \le A < B$, has a cumulative distribution function $F(z)$, and has a density $\mathrm{d}F(z)$ on $(A, B]$.[2] Also assume that $\mathrm{d}F(z)/F(z)$ is a non-increasing function of z on $[A, B]$. In the experimental settings that we are modeling, one human bidder competes against opponents programmed to bid according to the RNNE, so their bids satisfy all our assumptions.

Assume that bidder i is risk-neutral, but several factors enter (additively) into the utility that she derives from the outcome of the auction. On winning, bidder i realizes a profit (or loss) of $v - b$. Additionally, i may suffer from one of two possible types of regret. MLOT: If i wins, then she regrets leaving an amount of money b-z "on the table" and we assume that her utility suffers by an amount $\alpha_0 + \alpha_1(b - z)$ where $\alpha_1 \geq 0$. MOTW: If i loses and the highest other bid satisfies the inequality $b \leq z \leq v$, then bidder i missed an opportunity to win at a favorable price and we assume that his utility suffers by an amount $\beta_0 + \beta_1(v - z)$ where $\beta_0, \beta_1 \geq 0$.[3] In other auction-type settings, such as the third or the random price auctions, the winner may end up paying more than her price. This creates the possibility for a third type of regret, the regret associated with winning at an unfavorable price (WAUP). We discuss such auctions in Sect. 2.4.

Now, let $U(b; v)$ denote the ex-ante expected utility to a bidder i who has a value v for the object and bids b. Then, for $b < v$,

$$U(b; v) = (v - b)F(b) - \int\limits_{z:z\leq b} [\alpha_o + \alpha_1(b - z)]\mathrm{d}F(z|z \leq b)F(b)$$
$$- \int\limits_{z:b<z\leq v} [\beta_o + \beta_1(v - z)]\mathrm{d}F(z|z \geq b)(1 - F(b)),$$

which simplifies to

$$U(b; v) = (v - b)F(b) - \int\limits_{z:z\leq b} [\alpha_o + \alpha_1(b - z)]\mathrm{d}F(z)$$
$$- \int\limits_{z:b\leq z\leq v} [\beta_o + \beta_1(v - z)]\mathrm{d}F(z), \tag{1}$$

and for $b \geq v$,

$$U(b; v) = (v - b)F(b) - \int\limits_{z:z\leq b} [\alpha_o + \alpha_1(b - z)]\mathrm{d}F(z|z \geq b)(1 - F(b)).$$

[2] In other words, z is a continuous random variable except that it may have positive probability of being equal to A. Thus, we allow for the possibility that there is a positive probability of the others bidding equal to the reservation price r if $A = r$. In general, if Pr $(z = A) > 0$ (or, equivalently, $F(A) > 0$), then interpret $\mathrm{d}F(A)$ to be infinite. For expositional ease, we assume a compact support. However, all the results go through with minor modifications if the support has gaps in it; in this case, for example, we need $\mathrm{d}F(z)/F(z)$ to be a non-decreasing function of z for all z within the support, and in the subsequent theorem all points in a gap may satisfy the required condition of that theorem, in which case the claimed "unique" solution would be the largest z below the gap.

[3] The expressions $\alpha_0 + \alpha_1(b - z)$ and $\beta_0 + \beta_1(b - z)$ may be thought of, and motivated, as the first two terms of the Taylor series expansion of some more general function of $b - z$. Since we allow α_0 to be negative, this constant can include any pleasure derived simply from winning. Note that if α_0 is sufficiently negative, then bidders may want to bid in excess of their value.

which simplifies to

$$U(b; v) = (v - b)F(b) - \int_{z:z \le b} [\alpha_o + \alpha_1(b - z)]\mathrm{d}F(z).$$

Note that each of the regret terms in the above expressions correspond to the amount of the expected regret if it were calculated ex-post to the bidder learning what the next best bid turned out to be. This gives us the following:

Observation 1 (*Utility independent of information*): For fixed parameters $\alpha_0, \alpha_1, \beta_0$ and β_1, the expected utility of bidder i is independent of whether bidder i calculates the expected utility with or without knowing what the next highest bid was, with knowing it only if someone else wins, or with knowing it only if she wins.

Note however, the bidder's parameters may change as the information provided to the bidder changes. For example, changing what information bidder i observes may affect how conscious i is about money left on the table and/or missed opportunities to win and the relative weights that i places on each type of regret. The above observation does not preclude this.

The optimal bid $b^*(v)$ for bidder i – the best reply by i to the other bidders when i has a value of v – may be characterized using the first derivative of the previously derived utility function:

$$\frac{\mathrm{d}U(b; v)}{\mathrm{d}b} = [(-\alpha_o+\beta_o)+(1+\beta_1)(v-b)]\mathrm{d}F(b)-(1+\alpha_1)F(b) \quad \text{for } b < v, \text{ and}$$

$$\frac{dU(b; v)}{db} = [-\alpha_o+(v - b)]dF(b) - (1+\alpha_1)F(b) \quad \text{for } b \ge v. \qquad (2)$$

Dividing both sides of (2) by $1 + \beta_1$ and substituting $\tau \equiv (-\alpha_o + \beta_o)/(1 + \beta_1)$ and $\rho \equiv (1 + \alpha_1)/(1 + \beta_1)$ gives

$$\frac{\mathrm{d}U(b; v)}{\mathrm{d}b}/(1 + \beta_1) = [\tau + (v - b)]\mathrm{d}F(b) - \rho F(b) \quad \text{for } b < v,$$

which leads to the following observation:

Observation 2 (*Actual number of parameters*): For $b^*(v) < v$, $b^*(v)$ depends on the parameters $\alpha_0, \alpha_1, \beta_0$ and β_1 only through the two parameters $\tau \equiv (-\alpha_0 + \beta_0)/(1 + \beta_1)$ and $\rho \equiv (1 + \alpha_1)/(1 + \beta_1)$. Similarly, for $b^*(v) > v$, $b^*(v)$ depends on α_0 and α_1 only. Specifically, note that the model uses only two parameters in the range of values for which $b^*(v) < v$; the model has only two parameters as long as the optimal bid is less than the bidder's value.[4]

Using the above expressions for dU(b; v)/db, we now have the following:

Theorem *If* $\mathrm{d}U(A; v)/\mathrm{d}b \le 0$, *then* $b^*(v) = r$. *If* $\mathrm{d}U(B; v) \ge 0$, *then* $b^*(v) = B$. *Otherwise* $b^*(v)$ *is the (unique)* b^* *between* A *and* B *such that* $\mathrm{d}U(b; v)/\mathrm{d}b > 0$ *for all* $b < b^*$ *and* $\mathrm{d}U(b; v)/\mathrm{d}b < 0$ *for all* $b > b^*$.

[4] If $\tau \equiv -\alpha_0 + \beta_0 = 0$ and $b < v$, then the first order condition would be exactly the same as for a CRRA risk averse bidder (who is insensitive to regret) with risk aversion parameter $r = \rho$; any bidding behavior that might be predicted for a CRRA bidder could also result from a risk neutral but appropriately regret sensitive bidder.

Proof This result follows directly from the following lemma:

Lemma *For each v, $\mathrm{d}U(b;v)/\mathrm{d}b$ has the single crossing property. Specifically, for each v, either* $\mathrm{d}U(b;v)/\mathrm{d}b > 0$ *for all b in (A, B),* $\mathrm{d}U(b;v)/\mathrm{d}b < 0$ *for all b in (A, B), or there exists a unique b^* in (A, B) such that* 1. $\mathrm{d}U(b;v)/\mathrm{d}b^* = 0$, 2. $\mathrm{d}U(b;v)/\mathrm{d}b > 0$ *for all b in (A, b^*), and* 3. $\mathrm{d}U(b;v)/\mathrm{d}b < 0$ *for all b^* in (b^*, B).*

Proof Since $\mathrm{d}U(b;v)/\mathrm{d}b$ has the same sign as

$$\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/F(b) = (1+\beta_1)\left\{[\tau + (v-b)]\mathrm{d}F(b)/F(b) - \rho\right\} \quad \text{for } b < v, \text{ and as}$$

$$\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/F(b) = [-\alpha_o + (v-b)]\mathrm{d}F(b)/F(b) - (1+\alpha_1) \quad \text{for } b \geq v,$$

it suffices to show that $\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/F(b)$ is a decreasing function of b for all b in (A, B). As b increases, $[\tau + (v-b)]$ decreases, and, by assumption, $\mathrm{d}F(b)/F(b)$ does not increase. Therefore, $\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/F(b)$ decreases as b increases when $b < v$. Similarly, $\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/F(b)$ also decreases as b increases when $b \geq v$. Finally, as a result of the restrictions placed on the various parameters and the monotonicity of $F(b)$, we have that for any b_1 and b_2 such that $b_1 < v \leq b_2$,

$$\left[\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/F(b)\right]_{b=b_1} > \left[\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/F(b)\right]_{b=b_2}.$$

Putting these three pieces together gives us the required result that $\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/F(b)$ is a decreasing function of b for all b in (A, B).[5] □

This section concludes with an observation on how regret weights affect the best reply $b^*(v)$:

Observation 3 (*Effects of changing the parameters*):

(1) MLOT regret: As α_0 and/or α_1 increases, $\mathrm{d}U(b;v)/\mathrm{d}b$ decreases and therefore the best reply either stays the same or, in general, also decreases.

(2a) MOTW regret: As β_0 increases, $\mathrm{d}U(b;v)/\mathrm{d}b$ increases and therefore the best reply either stays the same or, in general when $b^*(v) < v$, also increases.

(2b) MOTW regret: As β_1 increases, dU(b; v)/ db increases when $b^*(v) < v$ and stays the same when $b^*(v) > v$. Therefore and the best reply either stays the same or, in general when $b^*(v) < v$, also increases.

2.2 The case of generalized uniform values

We now make some additional assumptions regarding the other bidders. Specifically, assume that the other bidders' values are independent of each other and from the value of bidder i. Also assume that the other bidders' values come from a generalized uniform distribution. In particular, each of the other bidder's values

[5] If $b < v$ then as b increases $\mathrm{d}U(b;v)/\mathrm{d}b$ decreases and by assumption $\mathrm{d}F(b)/F(b)$ does not increase.

has a cumulative probability distribution $G(v) = v^{\lambda}(0 \le v \le 1$ and $\lambda > 0)$; if λ is an integer, then a bidder's value is in effect the largest of λ independent samples from the standard uniform distribution. Finally, assume that each of the other bidders uses the linear bidding strategy $b(v) = A + (B - A)v$. Specifically, for $A \le x \le B$, $F(x) \equiv Pr(z \le x) = \Pr(A + (B - A)v \le x)(n - 1) = \Pr(v \le (x - A)/(B - A))(n - 1) = [(x - A)/(B - A)]^{\lambda(n-1)}$, and $\mathrm{d}F(x) = [\lambda(n-1)/(B-A)][(x-A)/(B-A)]^{\lambda(n-1)-1}$. For convenience, henceforth refer to such bids as being independent, generalized uniform bids on $[A, B]$.

Proposition *(Best reply against generalized uniform bids) If the other bidders make independent, generalized uniform bids on [A, B], then*

$$\begin{aligned}
b^*(v) &= \min\left\{B, \max\left[r, \frac{(-\alpha_o + v)\lambda(n-1) + (1+\alpha_1)A}{\lambda(n-1) + (1+\alpha_1)}\right]\right\} \quad \text{for } v < v^* \\
&\equiv A - \frac{\alpha_o \lambda(n-1)}{1+\alpha_1}, \\
b^*(v) &= \min\left\{B, \max\left[r, \frac{((-\alpha_o + \beta_0) + (1+\beta_1)v)\lambda(n-1) + (1+\alpha_1)A}{(1+\beta_1)\lambda(n-1) + (1+\alpha_1)}\right]\right\} \\
&= \min\left\{B, \max\left[r, \frac{(\tau + v)\lambda(n-1) + \rho A}{\lambda(n-1) + \rho}\right]\right\} \quad \text{for } v > v^{**} \\
&\equiv A + \frac{(-\alpha_0 + \beta_0)\lambda(n-1)}{1+\alpha_1} = A + \tau\lambda(n-1)/\rho, \quad \text{and} \\
b^*(v) &= \min\{B, \max[r, v]\} \quad \text{for } v^* < v < v^{**}.
\end{aligned}$$

Proof Under the assumed conditions, (2) becomes

$$\begin{aligned}
\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/(1+\beta_1) &= [\tau + (v-b)]\left[\frac{\lambda(n-1)}{B-A}\right]^{\lambda(n-1)-1} \\
&\quad -\rho\left(\frac{b-A}{B-A}\right)^{\lambda(n-1)} \quad \text{for } b < v, \text{ and} \\
\frac{\mathrm{d}U(b;v)}{\mathrm{d}b} &= [-\alpha_o + (v-b)]\left[\left(\frac{\lambda(n-1)}{B-A}\right)\left(\frac{b-A}{B-A}\right)\right]^{\lambda(n-1)-1} \\
&\quad -(1+\alpha_1)\left(\frac{b-A}{B-A}\right)^{\lambda(n-1)} \quad \text{for } b > v.
\end{aligned}$$

For $b > A$, this simplifies to

$$\begin{aligned}
\frac{\mathrm{d}U(b;v)}{\mathrm{d}b}/(1+\beta_1) &= [\tau + (v-b)][\lambda(n-1)] - \rho(b-A) \quad \text{for } b < v, \quad \text{and} \\
\frac{\mathrm{d}U(b;v)}{\mathrm{d}b} &= [-\alpha_o + (v-b)]\lambda(n-1) - (1+\alpha_1)(b-A) \quad \text{for } b > v.
\end{aligned}$$

For $b > v$, $\frac{\mathrm{d}U(b;v)}{\mathrm{d}b} = 0$ implies that $b = \frac{[-\alpha_o + v]\lambda(n-1) + (1+\alpha_1)A}{\lambda(n-1) + (1+\alpha_1)}$, and this b will be greater than both A and v if $A + \alpha_o < v < A - \frac{\alpha_o\lambda(n-1)}{1+\alpha_1} \equiv v^*$. Similarly, for

$b < v$, $\frac{dU(b;v)}{db} = 0$ implies that $b = \frac{[\tau+v]\lambda(n-1)+\rho A}{\lambda(n-1)+\rho}$. This b will be less than v if $v > A + \tau\lambda(n-1)/\rho \equiv v^{**}$ (and this b will be greater than A if $v > A - \tau$. Now observe that $v^{**} \geq v^*$, and therefore the claimed result follows from the Theorem. □

Corollary *(CRRA as a special case) If $-\alpha_0 + \beta_0 = 0$, $\rho = (1+\alpha_1)/(1+\beta_1)$ and the other bidders make independent, generalized uniform bids on $[A, B]$, then*

$$b^* = \min\left\{B, \max\left[r, \frac{v\lambda(n-1)+\rho A}{\lambda(n-1)+\rho}\right]\right\} \quad \textit{for } v > A,$$

which is the same as the best reply for a CRRA bidder who has risk parameter ρ.[6]

Corollary *(Independence of others' multiplier): If the other bidders make independent, generalized uniform bids on $[A, B]$, then bidder i's best bid $b^*(v)$ is independent of B for all v such that $b^*(v) < B$.*

This corollary has a practical implication for designing experiments in which a human player bids against computerized rivals. In a typical laboratory auction setting, bidders know the distribution of values, but not the distribution of bids. In order to create an environment that parallels the laboratory auction settings, it may be useful to inform the human bidders about the distribution of values of the computerized rivals (as we do in our implementation) rather than about the distribution of their bids directly (as Neugebauer and Selten do in theirs). As long as the computerized bidders are programmed to bid some fixed fraction of their value (and as long as the best reply is below B, the top of the support of the others' bid distribution), the specific fraction does not affect the best reply.

2.3 General Results for SBFP Auctions

We now examine how regret affects bids more generally. Let $u(v, b, \underline{v-}; \underline{B-})$ denote a bidder's utility when a bidder of type v bids b, and his competitors with types $\underline{v-}$ use the bidding strategies $\underline{B-}$. Note that this utility function is general enough to admit, for example, concerns for social utility (see Bolton and Ockenfels 2000) or spite (Morgan et al. 2003), and the general argument requires neither independent types nor privately-known values; specifically, a bidder's value may be a function not only of his own type v but also of the competitors' types $\underline{v-}$. Let $U(v, b, z; \underline{B-})$ denote the expected value of the utility $u(v, b, \underline{v-}; \underline{B-})$ conditional on v, b, z and $\underline{B-}$, where z denotes the highest bid made by the competitors. Let $F(z; \underline{B-}, v)$ denote the cumulative probability distribution function of z conditional on $\underline{B-}$ and v. Then the expected utility from the auction to a bidder of type v who bids b when the competitors use the strategies $\underline{B-}$ is $\Pi\left(v, b; \underline{B-}\right) \equiv \int U\left(v, b, z; \underline{B-}\right) dF\left(z; \underline{B-}, v\right)$. Finally, define $b^*(v)$ to be the (largest, if there is more than one) optimal bid and note that a necessary condition for a bid of $b^*(v)$ to be an optimal bid is that $\left[d\Pi\left(v, b; \underline{B-}\right)/db\right]_{b=b*(v)} = 0$.

Let $U^{\text{MOTW}}(v, b, z; \underline{B-})$ denote the utility function of a bidder who is more sensitive to MOTW. For each b, we will need to consider three possible regions for $z : z$ such that the bidder loses, such that the bidder just barely loses, and such

[6] See, for example Cox et al. (1988).

that the bidder doesn't lose. Note that whenever the bidder loses (regardless of whether the amount of MOTW is positive or zero), the amount of MOTW is independent of the bidder's bid b. We take being "more sensitive" to MOTW to imply that the utility of barely losing is less than before. And, of course, the utility is unchanged when the bidder wins. Formally, if we define the expected utility of a bidder more sensitive to MOTW as $\Pi^{\text{MOTW}}(v, b; \underline{B-}) \equiv \int U^{\text{MOTW}}(v, b, z; \underline{B-})\mathrm{d}F(z; \underline{B-}, v)$, then U^{MOTW}(v,b,z; B−) must satisfy the following three conditions:

(1a) $\mathrm{d}U^{\text{MOTW}}(v, b, z; \underline{B-})/db = dU(v, b, z; \underline{B-})/\mathrm{d}b$ for all v, b and z such that $z >$b,

(1b) (1b) $\lim_{z\downarrow b}\left[U^{\text{MOTW}}(v, b, z; \underline{B-}) - U(v, b, z; \underline{B-})\right] < 0$ for all v and b such that $z > b$,[7] and

(1c) $U^{\text{MOTW}}(v, b, z; \underline{B-}) = U(v, b, z; \underline{B-})$ for all v, b and z such that $z < b$.[8]

Proposition (Effect of changing sensitivity to MOTW in SBFP auctions) *Conditions* $1a$, $1b$ *and* $1c$ *imply that* $\left[\mathrm{d}\Pi^{\text{MOTW}}(v, b; \underline{B-})/\mathrm{d}b\right]_{b=b^*(v)} > 0$ *whenever* $b^*(v) < v$. *In words, as a bidder becomes more sensitive to the amount of MOTW, a bidder who was bidding strictly below his value can do better by bidding somewhat higher.*

Proof Define $\Delta^{\text{MOTW}}(v, b; \underline{B-}) \equiv \Pi^{\text{MOTW}}(v, b; \underline{B-}) - \Pi(v, b; \underline{B-})$

Then $\frac{\mathrm{d}}{\mathrm{d}b}\Delta^{\text{MOTW}}(v, b; \underline{B-}) = \frac{\mathrm{d}}{\mathrm{d}b}\left[\Pi^{\text{MOTW}}(v, b; \underline{B-}) - \Pi(v, b; \underline{B-})\right]$

Using the condition 1c that the utility is unchanged when the bidder wins:

$$= \frac{\mathrm{d}}{\mathrm{d}b}\int_{z>b} [U^{MOTW}(v, b, z; \underline{B-}) - U(v, b, z; \underline{B-})]\mathrm{d}F(z; \underline{B-}, v)$$

Differentiating inside the integral:

$$= \int_{z>b} \frac{\mathrm{d}}{\mathrm{d}b}[U^{MOTW}(v, b, z; \underline{B-}) - U(v, b, z; \underline{B-})]\mathrm{d}F(z; \underline{B-}, v)$$

$$- \lim_{z\downarrow b}[U^{\text{MOTW}}(v, b, z; \underline{B-}) - U(v, b, z; \underline{B-})]\mathrm{d}F(b; \underline{B-}, v),$$

which must be positive because of conditions 1a and 1b. Therefore, in particular,

$$\left[\mathrm{d}\Pi^{\text{MOTW}}(v, b; \underline{B-})/\mathrm{d}b\right]_{b=b^*(v)} > \left[\mathrm{d}\Pi(v, b; \underline{B-})\mathrm{d}b\right]_{b=b*(v)} = 0.$$

□

So, roughly speaking, the more sensitive a bidder is to MOTW, the higher that bidder should bid.

[7] Since functions such as $U(v, b, z; \underline{B-})$ are discontinuous at $z = b$, we need to specify whether we are coming at the $z = b$ case from the losing or winning side.

[8] For example, in the case of additive linear regret, an increase in β_0 and/or β implies that $U^{\text{MOTW}}(v, b, b; \underline{B-}) < U(v, b, b; \underline{B})$ for all v and b such that $b < v$. Alternatively, still in the case of additive linear regret, if the bid and value are both between zero and one, and if β_0 increases by at least as much as β_1 decreases, then again, $U^*(v, b, b; \underline{B-}) < U(v, b, b; \underline{B})$ for all v and b such that $b < v$. In both cases, conditions 1 and 3 will also be satisfied. So, such a bidder can also be said to be more sensitive to MOTW.

When a bidder is more sensitive to MLOT we again consider three regions for z for each b : z such that the bidder wins, such that the bidder just barely wins, and such that the bidder doesn't win. Now, being "more sensitive" to MLOT means that the utility of winning decreases more rapidly than before as the amount of MLOT increases. Also, the utility of winning is no greater than before when the amount of MLOT is zero. And, of course, the utility is unchanged when the bidder loses. Formally, let $U^{\text{MLOT}}(v, b, z; \underline{B-})$ denote the utility function that is more sensitive to MLOT, and define $\Pi^{\text{MLOT}}(v, b; \underline{B-}) \equiv \int U^{\text{MLOT}}(v, b, z; \underline{B-})\mathrm{d}F(z; \underline{B-}, v)$. Then $U^{MLOT}(v, b, z; \underline{B-})$ must satisfy the following conditions:

(2a) $\mathrm{d}U^{\text{MLOT}}(v, b, z; \underline{B-})/\mathrm{d}(b-z) \leq \mathrm{d}U(v, b, z; \underline{B-})/\mathrm{d}(b-z)$ for all v, b and z such that $z < b$, with the inequality being strict with positive probability for each fixed v and b,

(2b) $\lim_{z \uparrow b}\left[U^{\text{MOTW}}(v, b, z; \underline{B-}) - U(v, b, z; \underline{B-})\right] \leq 0$ for all v and b, and

(2c) $U^{\text{MLOT}}(v, b, z; \underline{B-}) = U(v, b, z; \underline{B-})$ for all v, b and z such that $z > b$.[9]

Proposition *(Effect of changing sensitivity to MLOT in SBFP auctions): Conditions* 2a, 2b *and* 2c *imply that* $\left[\mathrm{d}\Pi^{\text{MLOT}}(v, b; \underline{B-})/\mathrm{d}b\right]_{b=b^*(v)} < 0$. *In words, as a bidder becomes more sensitive to the amount of MLOT, that bidder can do better by bidding less than the amount that was optimal before.*

Proof Similar to before, define

$$\Delta^{\text{MLOT}}(v, b; \underline{B-}) \equiv \Pi^{\text{MLOT}}(v, b; \underline{B-}) - \Pi(v, b; \underline{B-}).$$

Then

$$\frac{\mathrm{d}}{\mathrm{d}b}\Delta^{\text{MLOT}}(v, b; \underline{B-}) = \frac{\mathrm{d}}{\mathrm{d}b}\left[\Pi^{\text{MLOT}}(v, b; \underline{B-}) - \Pi(v, b; \underline{B-})\right]$$

Using the condition 2c that the utility is unchanged when the bidder loses (and assuming zero probability of tied high bids):

$$= \frac{\mathrm{d}}{\mathrm{d}b}\int_{z<b} [U^{\text{MLOT}}(v, b, z; \underline{B-}) - U(v, b, z; \underline{B-})]\mathrm{d}F(z; \underline{B-}, v)$$

Differentiating inside the integral:

$$= \int_{z<b} \frac{\mathrm{d}}{\mathrm{d}b}\left[U^{\text{MLOT}}(v, b, z; \underline{B-}) - U(v, b, z; \underline{B-})\right]\mathrm{d}F(z; \underline{B-}, v)$$
$$+ \lim_{z \uparrow b}\left[U^{\text{MLOT}}(v, b, z; \underline{B-}) - U(v, b, z; \underline{B-})\right]\mathrm{d}F(b; \underline{B-}, v)$$

[9] For example, in the case of additive linear regret, an increase in α_0 implies that $U^{\text{MLOT}}(v, b, b; \underline{B-}) < U(v, b, b; \underline{B})$, while an increase in α_1 implies that $\mathrm{d}U^{\text{MLOT}}(v, b, z; \underline{B-})/\mathrm{d}(b-z) < \mathrm{d}U(v, b, z; \underline{B-})/\mathrm{d}(b-z)$.

Using the chain rule for differentiation and the fact that d $(b-z)$/d b = 1:

$$= \int\limits_{z<b} \frac{\mathrm{d}}{\mathrm{d}(b-z)}[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})]\mathrm{d}F(z;\underline{B-},v)$$
$$+\lim_{z\uparrow b}[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})]\mathrm{d}F(b;\underline{B-},v),$$

which must be negative because of conditions 2a and 2b. Therefore, in particular,

$$[\mathrm{d}\Pi^{\mathrm{MLOT}}(v,b;\underline{B-})/\mathrm{d}b]_{b=b*(v)} < [\mathrm{d}\Pi(v,b;\underline{B-})\mathrm{d}b]_{b=b*(v)} = 0.$$

So, roughly speaking, the more sensitive a bidder is to MLOT, the lower that bidder should bid. □

The above proof suggests that not both conditions 2a and 2b must be satisfied as long as their "sum" goes in the right direction. Specifically, if we add the following condition:

2d $\mathrm{d}F(z;\underline{B-},v)/\mathrm{d}z$ is a non-decreasing function of z on the unit interval (and is zero elsewhere),

then condition 2a may be replaced by the weaker condition

2a' $\left[\frac{\mathrm{d}U^{\mathrm{MLOT}}(v,b,z;\underline{B-})}{\mathrm{d}(b-z)} - \frac{\mathrm{d}U(v,b,z;\underline{B-})}{\mathrm{d}(b-z)}\right] + \lim_{z\uparrow b}\left[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})\right] \geq 0$ for all v, b and z such that $z < b$, with the inequality being strict with positive probability for each fixed v and b. [10]

Proposition (Effect of changing sensitivity to MLOT in SBFP auctions with increasing density functions) *Conditions* 2a′, 2b*, and* 2c *and* 2d *imply that* $[\mathrm{d}\Pi^{\mathrm{MLOT}}(v,b;\underline{B-})/\mathrm{d}b]_{b=b*(v)} < 0$. *In words, if the highest competitors' bid has a non-decreasing density function, then as a bidder becomes more sensitive to MLOT regret, that bidder can do better by bidding less than the amount that was optimal before.*

Proof The proof starts as before, but now continues as follows:

$$\int\limits_{z<b} \frac{\mathrm{d}}{\mathrm{d}(b-z)}[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})]\mathrm{d}F(z;\underline{B-},v)$$
$$+\lim_{z\uparrow b}[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})]\mathrm{d}F(b;\underline{B-},v)$$

[10] To illustrate this last proposition, consider the case of additive linear regret. If the competitors always bid 2/3 of values their values, and their values are distributed uniformly on [0,1], then condition 2d is satisfied. Furthermore, conditions 2a' and 2b will be satisfied if α_0 increases by at least as much as α_1 decreases. Therefore, in this case, the best reply will still decrease so long as α_0 increases by at least as much as α_1 decreases.

Since $\int_{z=0}^{z=1} \mathrm{d}z = 1$ and the second term in the above expression is independent of z:

$$= \int\limits_{z<b} \frac{\mathrm{d}}{\mathrm{d}(b-z)}[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})]\mathrm{d}F(z;\underline{B-},v)$$

$$+ \int\limits_{z=0}^{z=\max\{0,\min\{b,1\}\}} \lim_{z\uparrow b}\left[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}z}F(b;\underline{B-},v)dz$$

$$+ \int\limits_{z=\max\{0,\min\{b,1\}\}}^{z=1} \lim_{z\uparrow b}\left[U^{MLOT}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})\right]\frac{d}{dz}F(b;\underline{B-},v)dz$$

Using conditions 2a′ and 2d:

$$\leq \int\limits_{z<b} \frac{\mathrm{d}}{\mathrm{d}(b-z)}[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})]\mathrm{d}F(z;\underline{B-},v)$$

$$+ \int\limits_{z=0}^{z=\max\{0,\min\{b,1\}\}} \lim_{z\uparrow b}[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})]\mathrm{d}F(z;\underline{B-},v)$$

Combining terms and using the fact that $\mathrm{d}F(z;\underline{B-},v)/dz = 0$ for z outside the unit interval:

$$= \int\limits_{z<b} \left\{\frac{\mathrm{d}}{\mathrm{d}(b-z)}\left[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})\right]\right.$$

$$\left. + \lim_{z\uparrow b}\left[U^{\mathrm{MLOT}}(v,b,z;\underline{B-}) - U(v,b,z;\underline{B-})\right]\right\}\mathrm{d}F(z;\underline{B-},v),$$

which must be negative because of condition 0. Therefore, in particular,

$$\left[\mathrm{d}\Pi^{\mathrm{MLOT}}(v,b;\underline{B-})/\mathrm{d}b\right]_{b=b*(v)} < \left[\mathrm{d}\Pi(v,b;\underline{B-})\mathrm{d}b\right]_{b=b*(v)} = 0.$$

□

So, again, roughly speaking, the more sensitive a bidder is to MLOT, the lower that bidder should bid.

2.4 More general auction settings

We now show how regret provides a qualitative explanation for two sets of observed data known to be inconsistent with risk aversion. Consider a sealed bid kth price auction of a single unit, where, for the rest of this section, $k > 2$. Two specific settings of interest are the Kagel and Levin (1993) sealed-bid third-price auction and the Cason (1995) random-price auction. In the Cason auction, the high bid is compared to a randomly determined price p^*. If the highest of the n subjects' bids exceeds p^*, then the high bidder wins the auction and pays p^*; otherwise, the seller keeps the object. Cason's auction may be viewed as an $(n+1)$st price auction with n bidders and $n+1$ additional bids of p^* submitted by the seller. In general, for kth price auctions, the bidders bid above their values at the RNNE equilibrium, and risk aversion lowers the bids. However, Kagel and Levin observe bids that tend to be below RNNE in auctions with five bidders, but above the RNNE for auctions with ten bidders. Cason also observes that bids tend to be above the RNNE.

In such auctions, the winner's bid does not directly affect the price. However, a bidder may still suffer regret. As before, a losing bidder may regret missing an opportunity to win at a favorable price (MOTW). If the winner bids above his value, then the winner may end up paying more than his value and the winner may well regret WAUP. We now show that, roughly speaking, increasing a bidder's sensitivity to MOTW increases bids, and increasing a bidder's sensitivity to WAUP, decreases bids.

Let $u(v, b, \underline{v-}; \underline{B-})$ denote a bidder's utility when a bidder of type v bids b, and his competitors with types $\underline{v-}$ use the bidding strategies $\underline{B-}$. We now assume that this utility depends on the bidder's bid in a special way. In particular, while the bid b affects whether the bidder wins or loses (which in turn affects utility), we assume that the utility conditional on winning and the utility conditional on losing are independent of b. This assumption rules out first price auctions, but does allow for second- and third-price auctions.

More formally, let $u_w(v, \underline{v-}; \underline{B-})$ denote the bidder's utility if he wins, and let $u_l(v, \underline{v-}; \underline{B-})$ denote the corresponding utility when the bidder loses. Let $F(\underline{v-}; \underline{B-}, v)$ denote the cumulative probability distribution function of $\underline{v-}$ conditional on $\underline{B-}$ and v. Then the expected utility from the auction to a bidder of type v who bids b when the competitors use the strategies $\underline{B-}$ is

$$\Pi\left(v, b; \underline{B-}\right) \equiv \int_{\underline{v-}\text{s.t. a bid of } b \text{ wins}} u_w\left(v, \underline{v-}; \underline{B-}\right)\mathrm{d}F\left(\underline{v-}; \underline{B-}, v\right) + \int_{\underline{v-}\text{s.t. a bid of } b \text{ loses}} u_l\left(v, \underline{v-}; \underline{B-}\right)\mathrm{d}F\left(\underline{v-}; \underline{B-}, v\right).$$

Imagine that our bidder is using the strategy $b^*(v)$ and let $\underline{B-}(\underline{v-})$ denote the set of bids resulting from bidders who have values $\underline{v-}$ and use the strategies $\underline{B-}$. Again, note that a necessary condition for a bid of $b^*(v)$ to be an optimal bid is that $\left[\mathrm{d}\Pi\left(v, b; \underline{B-}\right)/\mathrm{d}b\right]_{b=b*(v)} = 0$. Therefore,

$$\left[\mathrm{d}\Pi\left(v, b; \underline{B-}\right)/\mathrm{d}b\right]_{b=b*(v)} = \int_{\underline{v-}\, s.t.\, \max\{\underline{B-}(\underline{v-})\}=b*(v)} u_w\left(v, \underline{v-}; \underline{B-}\right)\mathrm{d}F \times\left(\underline{v-}; \underline{B-}, v\right) - \int_{\underline{v-}\, s.t.\, \max\{\underline{B-}(\underline{v-})\}=b*(v)} u_l\left(v, \underline{v-}; \underline{B-}\right) \times \mathrm{d}F\left(\underline{v-}; \underline{B-}, v\right) = 0.$$

Let $u_l^{\mathrm{MOTW}}(v, \underline{v-}; \underline{B-})$ and $u_w^{\mathrm{MOTW}}(v, \underline{v-}; \underline{B-})$ denote the utility functions of a bidder who is more sensitive to MOTW regret, and let $\Pi^{\mathrm{MOTW}}\left(v, b; \underline{B-}\right)$ denote this bidder's expected utility. Being more sensitive to MOTW regret means that a losing bidder will experience more regret, thereby decreasing the loser's utility, but that a winner's utility is unchanged. This gives the following:

Proposition (Effect of changing sensitivity to MOTW in k-th price ($k > 1$) auctions) *If* $u_l^{\mathrm{MOTW}}(v, \underline{v-}; \underline{B-}) \le u_l(v, \underline{v-}; \underline{B-})$ *for all* v *and* $\underline{v-}$, *and* $u_w^{\mathrm{MOTW}}(v, \underline{v-}; \underline{B-}) = u_w(v, \underline{v-}; \underline{B-})$ *for all* v *and* $\underline{v-}$, *then* $\left[\mathrm{d}\Pi^{MOTW}\left(v, b; \underline{B-}\right)/\mathrm{d}b\right]_{b=b*(v)} \ge 0.$ *In words, roughly speaking, being more sensitive to MOTW increases bids.*

Similarly, being more sensitive to WAUP regret means that a winning bidder will experience more regret, thereby decreasing the winner's utility, but that a loser's utility is unchanged. This gives the following:

Proposition (Effect of changing sensitivity to WAUP in k-th price ($k > 1$) auctions) *If* $u_l^{\mathrm{WAUP}}(v, \underline{v-B-}) = u_l(v, \underline{v-}; \underline{B-})$ *for all* v *and* $\underline{v-}$, *and* $u_w^{\mathrm{WAUP}}(v, \underline{v-}; \underline{B-}) \le u_w(v, \underline{v-}; \underline{B-})$ *for all* v *and* $\underline{v-}$, *then* $\left[\mathrm{d}\Pi^{\mathrm{WAUP}}\left(v, b; \underline{B-}\right)/\mathrm{d}b\right]_{b=b*(v)} \le 0.$ *In words, roughly speaking, being more sensitive to WAUP decreases bids.*

So far, in calibrating the regret model using the SBFP auction, we found in Engelbrecht-Wiggans and Katok (2006) a consistent empirical regularity: experimental participants tend to put a higher weight on missing opportunities to win than on leaving money on the table. These weights imply bids above the RNNE in the Cason auction, which is consistent with the data and inconsistent with risk aversion.

Now consider the Kagel and Levin third price auction. MOTW regret drives bids upwards from the RNNE while WAUP regret drives them down. Kagel and Levin observe bids below RNNE in auctions with five bidders, but above the RNNE for auctions with ten bidders. A simple simulation of the RNNE in the third price auction shows that the expected loses due to WAUP, conditional on winning, are more than twice as big in auctions with five bidders as in auctions with ten bidders.[11] Therefore, we expect WAUP regret to depress prices more with five bidders than with ten bidders. We know that MOTW regret drives bids up, but WAUP regret drives the bids back down, and the amount by which it drives them back down may be just enough to leave them below the RNNE in auctions with five bidders but not in auctions with ten bidders. This is consistent with the data Kagel and Levin observed.

[11] We will not provide formal derivations here; it is quite straight forward to check that the claims are qualitatively correct.

3 Empirical evidence

3.1 Regret in the BDM game

Isaac and James (2000) estimate the constant relative risk aversion (CRRA) utility function parameter for individual subjects using a SBFP auction and a BDM procedure (Becker et al. 1964) and find the two estimates to be inconsistent. Figure 1 presents a summary of the Isaac and James data, along with the CRRA and regret model predictions.

We constructed our Fig. 1 to exactly match Fig. 1 in Isaac and James (2000), p. 183; each point in the figure corresponds to a single subject, with the X-coordinate representing the CRRA measure for that subject from a SBFP auction, and the Y-coordinate representing the CRRA measure for the same subject based on the BDM procedure. Isaac and James point out that consistent risk aversion across the two environments would imply that the data would fall along the 45 degree line (labeled CRRA prediction in the figure), and if the ranking, but not the numerical values were to be consistent, then the data would fall along a positively-sloped line. In fact, the data falls along a negatively-sloped line, and Isaac and James conclude their paper with a question: "How is it that some people can behave as stable, easily classifiable risk averters one minute, then behave as stable, easily classifiable risk seekers the next (all of this while other people behave as stable, risk neutral decision makers all along)?" (p. 185). In fact, as we explain below, the behavior across the SBFP auction and BDM setting is consistent with the regret model.

Isaac and James (2000) used the BDM procedure to extract truthful valuations, which we will call B, for a lottery. Consider a lottery that pays the prize of π with a probability p, and 0 otherwise. A random number P, with a distribution $F(P)$ is

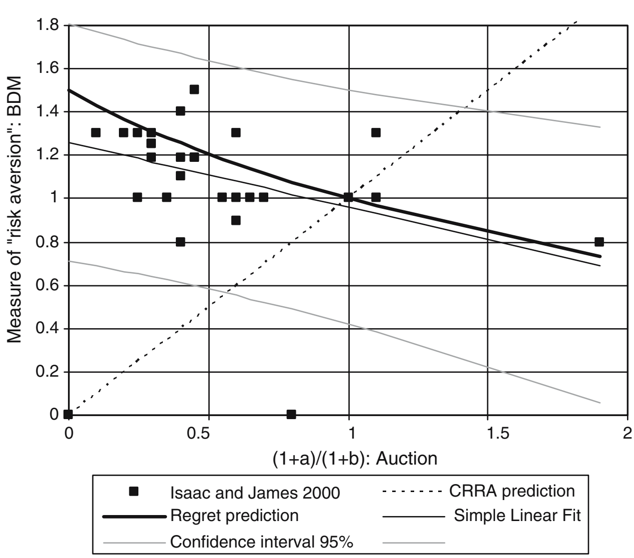

Fig. 1 Summary of the Isaac and James (2000) data and the CRRA and regret model predictions

drawn, and if $B < P$ then the subject effectively sells the lottery and receives P, and otherwise he receives the proceeds of a lottery.

Think of getting the lottery prize π as "winning." Then, ex-post, someone who sold the lottery has given up a chance of winning π and may suffer MOTW regret if the lottery indeed yields the π outcome. We will apply the simplest version of the regret model with a single parameter for each regret type, so in this case the subject's utility suffers by an amount $\beta(\pi - P)$ where $\beta \geq 0$. Someone who keeps the lottery may also suffer a different form of regret if the lottery turns out to be worth zero, in which case the subject's utility suffers by an amount $\alpha\ P$ where $\alpha \geq 0$.

Let $U(B)$ denote the ex-ante expected utility to a regret sensitive (and risk neutral) subject who specifies a cutoff of B. Then,

$$U(B) = \int\limits_{P:P<B} [p\pi + (1-p)(0-\alpha P)]\mathrm{d}F(P) + \int\limits_{P:P\geq B} [P - p\beta(\pi - P)]\mathrm{d}F(P).$$

In the Isaac and James experiment, P is distributed uniformly on [\$0, \$3.99], so $\mathrm{d}F(P) = \mathrm{d}P/3.99$, the lottery is a 50/50 chance of winning \$4 (so $p = 0.5$ and $\pi = 4$). The necessary condition for B^* to be an optimal cutoff is $[\mathrm{d}U(B)/\mathrm{d}B]|_{B=B^*} = 0$. Some straight forward calculations then yield that $B^* = 4/(1+\rho)$ where $\rho \equiv (1+\alpha)/(1+\beta)$.

Recall that this is the same expression for ρ that appeared in our analysis of SBFP auctions, and that it can be interpreted as the CRRA parameter for a bidder in a SBFP auction. Isaac and James estimate ρ for each of their bidders in the SBFP auction. We note that this ρ could be due to regret rather than risk aversion. For each possible ρ, we then calculate the optimal B^* of a bidder who is sensitive to regret (but is risk neutral). We then calculate what risk measure Isaac and James would have ascribed to someone who specified this B^* in the BDM mechanism. In effect, we predict what risk measures Isaac and James would have calculated for bidders who are sensitive to regret rather than risk.

This regret theory predicts the heavy black line in Figure 1 (labeled Regret prediction in the figure). The regret predictions match the aggregate data quite closely. The average slope of the regret prediction line falls inside the 95% confidence interval for the slope of the linear fit line through the data, while the intercept term is only slightly above it.[12] We added a 95% confidence interval for the linear fit in Figure 1, and note that the regret prediction line is well inside that range.

3.2 The effect of feedback information

Neugebauer and Selten (2006) analyze data from an experiment designed to test the learning direction theory in the context of SBFP auctions. They attribute overbidding to the feedback information that is given in a typical laboratory auction

[12] Isaac and James had one subject for whom they estimated a risk parameter of zero in both treatments. Since this is such an extreme risk parameter, we consider this subject to be an outlier and estimate the linear fit without this subject. With this outlier removed, the estimate of the linear fit slope is -0.3, and is statistically significant (p-value – 0.0411). The 95% confidence interval for the slope is -0.58 to -0.01, and the average slope of the regret prediction line is -0.48, which is well inside that confidence interval. The linear fit intercept is 1.26, and the regret model estimate of the BDM ρ for a bidder for whom the auction ρ is 0 is 1.5, which is significantly higher than 1.26. However, note that the regret model estimate is non-linear.

setting, and argue that manipulating feedback information affects behavior through learning by removing ambiguity.

In their experiments, one human player bids against either 3, 4, 5, 6 or 9 computerized rivals. The rivals' bids are independent, and uniformly distributed from 0 to 100. The human bidder has a fixed value of 100 for the duration of the experimental session. Neugebauer and Selten consider three information conditions: (1) neither the first nor the second highest bid is announced (labeled T0); (2) the highest bid is announced but the second is not (T1); (3) both, the highest and the second highest bids are announced (T2). Figure 2 summarizes the resulting data. The T1 and T2 bids are lower than the T0 bids, and Neugebauer and Selten report that the differences are statistically significant.

We show that our regret model can account for these shifts. Ex-post, each bidder knows how much she bid and whether she won. If a winning bidder also sees the second highest bid, then the amount of MLOT becomes quite apparent; if the second highest bid is not revealed, then the amount of MLOT remains ambiguous. Similarly, if a losing bidder also sees the winning bid, then any MOTW become apparent, while they remain ambiguous if the winning bid is not revealed. We suggest that making a type of regret visible to the subject makes it salient and thus increases its weight in the decision-maker's utility function. Therefore, the regret theory predicts lower bids under T1 than T0, and lower bids under T2 than T0 – both shifts Neugebauer and Selten observe in their data.

In Engelbrecht-Wiggans and Katok (2006) we present an experiment designed to test regret theory predictions in the context of SBFP auctions. In that paper we attribute overbidding to bidders placing a higher relative weight on the MOTW than on the MLOT regret, and manipulate feedback information in a way that changes the relative concern over the two types of regret.

Our design in Engelbrecht-Wiggans and Katok (2006) is similar to the Neugebauer and Selten design in that one human bidder competes against a number of computerized rivals. The three main differences are that (1) we keep the number of computerized rivals at 2 throughout; (2) our human bidders have five

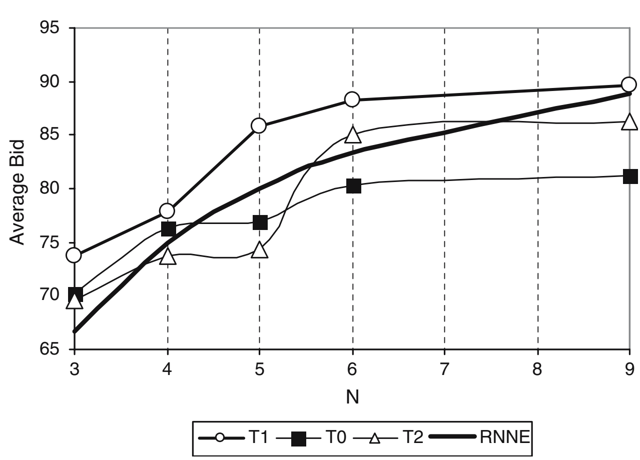

Fig. 2 Average bids and the RNNE benchmark in the Neugebauer and Selten (2006) experiment

different resale values, each repeated 20 times, versus the single resale value in the Neugebauer and Selten study and each bidding decision is used in 10 independent auctions; and (3) in addition to the three Neugebauer and Selten information conditions T0–T2 we also consider the forth condition in which the second highest bid is announced but the highest bid is not. This fourth feedback condition – T3 – completes the 2×2 full factorial design and allows us to test two additional predictions of the regret model.

The first two comparisons come from the implication of the regret model about the effect of the MLOT regret: MLOT regret decreases bids, so average bids in T2 should be below T1 and average bids in T3 should be below T2. The second two comparisons come from the implication of the regret model about the effect of the MOTW regret: MOTW regret increases bids, so average bids in T2 should be above T3 and average bids in T1 should be above T0. In Engelbrecht-Wiggans and Katok (2006) we report that all four predicted shifts are statistically significant (T3 < T0 at 0.1 level only, the rest at 0.05 level) in the second half of the data. We also report that overbidding relative to the RNNE persists in all four information conditions, although its magnitude is quite small in the T3 condition (average bid/value is 0.697, which is not statistically different from the 0.667 RNNE benchmark)

4 Conclusions

The main contribution of this paper is a new theory of optimal bidding in sealed-bid first-price auctions in the IPV setting with fixed opposition. The new theory is based on the notion that bidders try to avoid regret. The theory is flexible enough to admit a variety of ways to model regret. We provide an implicit characterization of the best reply of a bidder who is sensitive to the two types of regret that we consider: winning and paying too much, and missing an opportunity to win at a favorable price.

We examine how regret affects bids in a much more general setting than that of IPV and in several auction-like settings in addition to the sealed-bid first-price. We show that sensitivity to MOTW regret generally increases bids while sensitivity to winning and paying too much generally decreases them. These general results allow us to establish that the data in the sealed-bid third price auctions studied by Kagel and Levin (1993), as well as the data in random-price auctions studies by Cason (1995), that was shown to be inconsistent with risk aversion, is qualitatively consistent with the regret theory.

Additionally, we examine three other data sets in more detail. We find that regret theory organizes the Isaac and James (2000) data well, by explaining why participants who act as if they were highly risk averse in SBFP auctions act as if they were risk seeking when confronted with the BDM procedure, while other participants act as if they were risk neutral in both settings. The treatment effects in two experimental studies that manipulate feedback information, one reported in Neugebauer and Selten (2006) and the other reported in Engelbrecht-Wiggans and Katok (2006) are also consistent with the regret theory. In both studies, providing bidders clear MOTW information causes the average bids to increase and providing clear MLOT information causes average bids to decrease.

Although the CRRA model generally fits the SBFP auction data well, a good model should do more than just fit the data. It should fit the data for the right reason.

To the extent that auction theory is there to guide auction designers, accepting a model that fits some data without clearly understanding why it does may lead to poor designs. For example, risk aversion can not explain shifts due to varying feedback information, but there are at least four independent studies that report that this information makes a difference(Engelbrecht-Wiggans and Katok 2006; Isaac and Walker 1985; Neugebauer and Selten 2006; Ockenfels and Selten 2005). The regret theory explains why feedback information is likely to matter.

Would bidders prefer auction mechanisms that have no regret? For example, ascending bid auctions are regret-free, but in practice we observe many sealed-bid auctions. Ivanova-Stenzel and Salmon (2004) find evidence that when given a choice, bidders do prefer ascending-bid auctions. Engelbrecht-Wiggans and Katok (2005) find a similar result using a different design. Of course sellers may well prefer sealed-bid auctions since with the same number of bidders they generate higher revenues. Sealed bid auctions are also much simpler to conduct, especially for complex objects, and they make it more difficult for bidders to tacitly collude.

References

Becker, G.M., Degroot, M.H., Marschak, J.: Measuring utility by a single-response sequential Method. Behav Sci **9**, 226–232 (1964)

Bolton, G., Ockenfels, A.: ERC: A theory of equity, reciprocity, and competition. Am Econ Rev **90**(1), 166–193 (2000)

Cason, T.N.: An experimental investigation of the seller incentives in the EPA's emission trading auction. Am. Econ Review **85**(4), 905–922 (1995)

Cox, J.C., Smith, V.L., Walker, J.M.: Theory and individual behavior of first price auctions. J Risk Uncertainty **1**, 61–99 (1988)

Dufwenberg, M., Gneezy, U., Information disclosure in auctions: and experiment. J Econ Behav Organ **48**, 431–444 (2002)

Engelbrecht-Wiggans, R.: The effect of regret on optimal bidding in auctions. Manage Sci **35**(6), 685–692 (1989)

Engelbrecht-Wiggans, R., Katok, E.: Experiments on auction valuation and endogenous entry. In: Morgan J. (Ed.) Experimental and Behavior Economics, Advances in Applied Microeconomics, vol. 13, Chap. 7, pp. 171–196 Amsterdam; Elsevier (2005)

Engelbrecht-Wiggans, R., Katok, E.: Regret and Feedback Information in First-Price Sealed-Bid Auctions. Penn State working paper http://lema.smeal.psu.edu/katok/regret_post.pdf (2006)

Isaac, R.M., James, D.: Just who are you calling risk averse? J Risk Uncertainty **20** 177–187 (2000)

Isaac, R.M., Walker, J.M.: Information and conspiracy in sealed bid auctions. J Econ Behav Organ **6**, 139–159 (1985)

Ivanova-Stenzel, R., Salmon, T.C.: Bidder preferences among auction institutions. Econ Enquiry **42**(14), 223–236 (2004)

Kagel, J. H.: Auctions: A Survey of Experimental Research. In: Kagel, J.H. Roth, A.E. (eds.), The Handbook of Experimental Economics, pp 501–585, Princeton: Princeton University Press (1995)

Kagel, J.H., Levin, D.: Independent private value auctions: bidder behavior in first-, second-, and third- price auctions with varying number of bidders. Econ J **103**, 868–879 (1993)

Katok, E., Roth, A.E.: Auctions of homogeneous goods with increasing returns: experimental comparison of alternative "Dutch" auctions. Manag Sci **50**(8), 1044–1063 (2004)

Keskinocak, P., Tayur, S.: Quantitative analysis for internet-enabled supply chains. Interfaces **31**(2), 70–89 (2001)

Ledyard, L.O., Olson, M., Porter, D., Swanson, J.A., Torma, D.P.: The first use of a combined-value auction for transportation services.Interfaces **32**(5), 4–12 (2002)

Morgan, J., Stiglitz, K., Reis, G.: The spite motive and equilibrium behavior in auctions. Contrib Econ Anal Policy **2**(1), article 5 (2003)

Neugebauer, T., Selten, R.: Individual behavior of first-price auctions: the importance of information feedback in computerized experimental markets. Games Econ Behav **54**, 183–204 (2006)
Ockenfels, A., Selten, R.: Impulse balance equilibrium and feedback in first price auctions. Games Econ Behav **51**, 155–179 (2005)
Rothkopf, M.H., Pekeč, A., Harstad, R.M.: Computationally manageable combinational auctions. Manage Sci **44**(8), 1131–1147 (1998)
Selten, R., Buchta, J.: Experimental sealed bid first-price auctions with directly observed bid functions. In: Budescu, D. Erev, I. Zwick, R. (eds.) Games and Human Behavior: Essays in honor of Amnon Rapoport, pp 105–142 Mahwah: Lawrence Erlbaum Associates (1998)
Selten, R., Stoeker, R.: End behavior in sequences of finite prisoners' Dilemma supergames: a learning theory approach. J Econ Behav Organ **7**, 47–70 (1986)