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Regular generalized fuzzy b-separation axioms in fuzzy topology

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Abstract

Regular generalized fuzzy b-closure and regular generalized fuzzy b-interior are stated and their characteristics are examined, also Regular generalized fuzzy b -τ_i separation axioms have been introduced and their interrelations are examined. The characterization of regular generalized fuzzy b –separation axioms are analyzed.

Keywords: rgfbCS; rgfbOS; rgfbCl; rgfbInt; rgfbT$_0$; rgfbT$_{1;}$ rgfbT$_2$; rgfbT$_{2\frac{1}{2}}$ and fuzzy topological spaces X (in short fts).

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1. Introduction

The fundamental theory of fuzzy sets were introduced by Zadeh [16] and Chang [9] studied the theory of fuzzy topology. After this Ghanim.et.al [10] introduced separation axioms, regular spaces and fuzzy normal spaces in fuzzy topology. The theory of regular generalized fuzzy b-closed set (open set) presented by Jenifer et. al [11]. In this study we define rgfb-closure, rgfb-interior and rgfb-separation axioms and their implications are proved. Effectiveness nature of the various concepts of fuzzy separation ideas are carried out. Characterizations are obtained.

2. Preliminary

(X_1, τ),(X_2, σ) (or simply X_1, X_2) states fuzzy topological spaces(in short, fts) in this article.

Definition 2.1[1, 3]: In fts X_1, α be fuzzy set.

(i) If α =IntCl(α)then α is fuzzy regular open(precisely, frOS).
(ii) If α =ClInt(α)then α is fuzzy regular closed (precisely, frCS).
(iii) If $\alpha \leq (\mathrm{IntCl}\alpha) \vee (\mathrm{ClInt}\alpha)$ then α is f b-open set (precisely, fbOS).
(iv) If $\alpha \geq (\mathrm{IntCl}\alpha) \wedge (\mathrm{ClInt}\alpha)$ then α is f b-closed set (precisely, fbCS).

Remark 2.2 [1]: In a fuzzy topological space X, The following implication holds good

$$\boxed{\text{frOS}} \longrightarrow \boxed{\text{f-open}} \longrightarrow \boxed{\text{fbOS}}$$

Figure1. Interrelations between some fuzzy open sets

Definition 2.3[3]: Let α be a fuzzy set in a fts X_1. Then ,
(i) $\mathrm{bCl}(\alpha) = \wedge\{\beta : \beta \text{ is a fbCS}(X_1), \geq \alpha\}$.
(ii) $\mathrm{bInt}(\alpha) = \vee\{\lambda : \lambda \text{ is a fbOS}(X_1), \leq \alpha\}$.

Definition 2.4[11]: In a fts X_1, if bCl(α) $\leq \beta$, at any time when $\alpha \leq \beta$, then fuzzy set α is named as regular generalized fuzzy b-closed (rgfbCS).Where β is fr- open.

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Remark 2.5[11]: In a fts X_1, if 1-α is rgfbCS(X_1) then fuzzy set α is rgfbOS.

Definition 2.6[11]: In a fts X_1, if bInt(α)≥ β , at any time when $\alpha \geq \beta$, then fuzzy set α is named as regular generalized fuzzy b-open (rgfbOS).Where β is fr- closed.

Definition 2.7[13]: Let (X_1, τ), (X_2, σ) be two fuzzy topological spaces. Let f : $X_1 \to X_2$ be mapping,

(i) if $f^{1}(\alpha)$ is rgfbCS(X_1), for each closed fuzzy set α in X_2, then f is said to be regular generalized fuzzy b-continuous (briefly, rgfb-continuous).

(ii) if $f^{1}(\alpha)$ is open fuzzy in X_1, for each rgfbOS α in X_2, then f is called strongly rgfb-continuous.

(iii) if $f^{1}(\alpha)$ is rgfbCS in X_1, for each rgfbCS α in X_2, then f is called rgfb-irresolute.

Definition 2.8[10]: X_1 is a fts which is named as

(i) fuzzy T_0(in short, fT_0) if and only if for each pair of fuzzy singletons p_1 and p_2 with various supports there occurs open fuzzy set U such that either $p_1 \leq U \leq 1- p_2$ or $p_2 \leq U \leq 1-p_1$.

(ii) fuzzy T_1(in short fT_1)if and only if for each pair of fuzzy singletons p_1 and p_2 with
various supports, there occurs open fuzzy sets U and V such that
$p_1 \leq U \leq 1- p_2$ and $p_2 \leq V \leq 1- p_1$.

(iii) fuzzy T_2(in short, fT_2) or f-Hausdorff if and only if for each pair of fuzzy singletons $p_1$and p_2 with various supports ,there occurs open fuzzy sets U and V such that $p_1 \leq U \leq 1- p_2$, $p_2 \leq V \leq 1- p_1$ and $U \leq 1- V$.

(iv) fuzzy $T_{2\frac{1}{2}}$ (in short, $fT_{2\frac{1}{2}}$) or f-Urysohn if and only if for each pair of fuzzy singletons p_1 and p_2 with various supports, there occurs open fuzzy sets U and V such that $p_1 \leq U \leq 1- p_2$, $p_2 \leq V \leq 1- p_1$ and $clU \leq 1-cl\ V$.

3. Regular generalized fuzzy b-closure (rgfbCl) and Regular generalized fuzzy b-Interior (rgfbInt)

Definition 3.1: The regular generalized fuzzy b-closure is denoted and defined by, rgfbCl (α) = Λ { λ : λ is a rgfbCS(X_1), ≥ α }. Where α be a fuzzy set in fts X_1.

Theorem 3.2: Let X_1 be fts, then the properties that follows are occurs for rgfbCl of a set

i. rgfbCl(0) = 0
ii. rgfbCl(1) = 1
iii. rgfbCl(α) is rgfbCS in X_1
iv. rgfbCl[rgfbCl(α)] = rgfbCl(α)

Definition 3.3: Let α and β be fuzzy sets in fuzzy topological space X_1. Then regular generalized fuzzy b-closure of (α V β) and regular generalized fuzzy b-closure of ($\alpha \wedge \beta$) are denoted and defined as follows

i. rgfbCl (α V β) = $\wedge$ { λ : λ is a rgfbCS(X_1) , where $\lambda \geq$ (α V β) }
ii. rgfbCl ($\alpha \wedge \beta$) = $\wedge$ { λ : λ is a rgfbCS(X_1), where $\lambda \geq$ ($\alpha \wedge \beta$) }

Theorem 3.4: Let α and β be fuzzy sets in fts X_1, then the following relations occurs

i. rgfbCl(α) V rgfbCl(β) $\leq$ rgfbCl(α V β)
ii. rgfbCl(α) $\wedge$ rgfbCl(β) $\geq$ rgfbCl($\alpha \wedge \beta$)

Proof: (i) We know that $\alpha \leq$ (α V β) or $\beta \leq$ (α V β)
$\Rightarrow$ rgfbCl (α) $\leq$rgfbCl (α V β) orrgfbCl(β) $\leq$rgfbCl (α V β)
Hence, rgfbCl (α) V rgfbCl (β) $\leq$rgfbCl (α V β).
(ii) We know that $\alpha \geq$ ($\alpha \wedge \beta$) or $\beta \geq$ ($\alpha \wedge \beta$)
$\Rightarrow$ rgfbCl(α) $\geq$rgfbCl ($\alpha \wedge \beta$) orrgfbCl(β) $\geq$rgfbCl ($\alpha \wedge \beta$)
Hence, rgfbCl(α) $\wedge$ rgfbCl(β) $\geq$rgfbCl ($\alpha \wedge \beta$).

Theorem 3.5: α is rgfbCS in a fts X_1, if and only if α=rgfbCl(α).
Proof: Suppose α is rgfbCS. Since $\alpha \leq \alpha$ and $\alpha \in$ { β: β is rgfbCS(X_1) and $\alpha \leq \beta$ }, α is the smallest and contained in β,therefore α=Λ{ β: β is rgfbCS(X_1)and $\alpha \leq \beta$}=rgfbCl(α). Hence, α=rgfbCl (α).
On the other hand, Suppose α=rgfbCl (α), then
α= Λ{ β: β is rgfbCS, $\alpha \leq \beta$ } $\Rightarrow$ $\alpha \in \Lambda$ { β: β is rgfbOS, $\alpha \leq \beta$ }.
Hence, α is rgfbCS.

Definition 3.6: The regular generalized fuzzy b-interior is denoted and defined by, rgfbInt(α) = V { δ: δ is a rgfbOS(X_1), $\leq \alpha$}. Where α be a fuzzy set in fts X_1.

Theorem 3.7: Let X_1 be fts, then the properties that follows are occurs for rgfbInt of a set

i. rgfbInt(0) = 0
ii. rgfbInt(1) = 1
iii. rgfbInt(α) is rgfbOS in X_1
iv. rgfbInt[rgfbInt(α)] = rgfbInt(α) .

Definition 3.8: Let α and β are fuzzy sets in fts X_1. Then regular generalized fuzzy b-interior of ($\alpha \vee \beta$) and regular generalized fuzzy b-interior of ($\alpha \wedge \beta$) are denoted and defined as follows

i. rgfbInt ($\alpha \vee \beta$) = V {δ: δ is a rgfbOS(X_1), where $\delta \leq (\alpha \vee \beta)$ }.
ii. rgfbInt($\alpha \wedge \beta$) = V { δ: δ is a rgfbOS(X_1), where $\delta \leq (\alpha \wedge \beta)$}.

Theorem 3.9: Let α and β are fuzzy sets in fts X_1, then the following relations occurs

i. rgfbInt(α) V rgfbInt(β) $\leq$ rgfbInt($\alpha \vee \beta$)
ii. rgfbInt(α) $\wedge$ rgfbInt(β) $\geq$ rgfbInt($\alpha \wedge \beta$)

Proof: (i) We know that, $\alpha \leq (\alpha \vee \beta)$ or $\beta \leq (\alpha \vee \beta)$
$\Rightarrow$ rgfbInt(α) $\leq$ rgfbInt ($\alpha \vee \beta$) or rgfbInt(β) $\leq$ rgfbInt ($\alpha \vee \beta$)
Hence, rgfbInt(α) V rgfbInt(β) $\leq$ rgfbInt($\alpha \vee \beta$).
(ii)We know that $\alpha \geq (\alpha \wedge \beta)$ or $\beta \geq (\alpha \wedge \beta)$
$\Rightarrow$ rgfbInt(α) $\geq$ rgfbInt ($\alpha \wedge \beta$) or rgfbInt(β) $\geq$ rgfbInt ($\alpha \wedge \beta$)
Hence, rgfbInt(α) $\wedge$ rgfbInt(β) $\geq$ rgfbInt ($\alpha \wedge \beta$).

Theorem 3.10: Let X_1 be fts, α is rgfbOS if and only if α=rgfbInt(α).
Proof: Suppose α is rgfbOS. Since $\alpha \leq \alpha$, $\alpha \in$ { δ: δ is rgfbOS and $\delta \leq \alpha$} Since biggest α contains δ. Therefore, α= V{ δ: δ is rgfbOS $\delta \leq \alpha$} = rgfbInt (α). Hence, α=rgfbInt(α).
On the other hand, Suppose α=rgfbInt (α).Then, α=V{ δ: δ is rgfbOS, $\delta \leq \alpha$} $\Rightarrow \alpha \in$ V { δ: δ is rgfbOS $\delta \leq \alpha$}. Hence, α is rgfbOS.

Theorem 3.11: Let α be a fuzzy set in a fts X_1, in that case following relations holds good

i. rgfbInt(1-α) = 1-rgfbCl(α)
ii. rgfbCl(1-α) = 1- rgfbInt(α)

Proof: (i) Let α be a fuzzy set in fts X_1. Then we have
rgfbCl (α) = Λ { λ : λ is a rgfbCS(X_1), $\geq \alpha$}. Where α be a fuzzy set in fts X_1.
1-rgfbCl (α) = 1- Λ { λ : λ is a rgfbCS(X_1), $\geq \alpha$}.
= V { $1 - \lambda$: λ is a rgfbCS(X_1), $\geq \alpha$}.
= V{ $1 - \lambda$: $1 - \lambda$ is a rgfbOS(X_1), $\leq$ 1-α}.
= rgfbInt (1-α)
Hence, 1-rgfbCl(α) = rgfbInt (1-α).

(ii) Let α be a fuzzy set in fts X_1. Then we have
rgfbInt(α) = V { δ: δ is a rgfbOS(X_1), $\leq \alpha$}. Where α be a fuzzy set in fts X_1.

1-rgfbInt (α) = 1-V { δ : $\delta \leq \alpha$ and δ is rgfbOS (X_1)}
= Λ{1- δ: $\delta \leq \alpha$ and δ is rgfbOS(X_1)}
= Λ {1- δ : 1-$\alpha \leq$ 1- δ and 1- δ is rgfbCS(X_1)}
= rgfbCl (1-α)
Hence 1-rgfbInt (α) = rgfbCl (1-α).

4. rgfb-separation axioms

Definition 4.1: A fts is known as rgfbT_0, that is regular generalized fuzzy bT_0, iff for each pair of fuzzy singletons q_1 and q_2 with various supports, there occurs rgfbOS δ such that either $q_1 \leq \delta \leq 1- q_2$ or $q_2 \leq \delta \leq 1- q_1$.

Theorem 4.2: A fts is rgfbT_0, that is regular generalized fuzzy bT_0, if and only if rgfbCl of crisp fuzzy singletons q_1 and q_2 with various supports are different.
Proof: To prove the necessary condition: Let a fuzzy topological space be rgfbT_0 and two crisp fuzzy singletons be q_1 & q_2 with various supports x_1 & x_2 respectively i.e. $x_1 \neq x_2$. Since fts is rgfbT_0 ,there exist a rgfbOS δ such that, $q_1 \leq \delta \leq 1- q_2 \Rightarrow q_2 \leq 1- \delta$, but $q_2 \leq$ rgfbCl(q_2) $\leq 1- \delta$, where $q_1 \leq$ rgfbCl(q_2) $\Rightarrow q_1 \leq 1- \delta$ where $1- \delta$ is rgfbCS. But, $q_1 \leq$ rgfbCl(q_1). This shows that, rgfbCl(q_1) $\neq$ rgfbCl(q_2).
To prove the sufficiency: Let p_1 & p_2 be fuzzy singletons with various supports x_1 & x_2 respectively, q_1 & q_2 be crisp fuzzy singletons such that $q_1(x_1)=1$, $q_2(x_2)=1$. But, $q_1 \leq$ rgfbCl(q_1) $\Rightarrow$ 1-rgfbCl(q_1) $\leq 1-q_1 \leq 1-p_1$. As each crisp fuzzy singleton is rgfbCS, 1- rgfbCl(q_1) is rgfbOS and $p_2 \leq$ 1- rgfbCl(q_1) $\leq 1-p_1$.This proves, fts is rgfbT_0 space.

Definition 4.3: A fts is known as rgfbT_1,that is regular generalized fuzzy bT_1, iff for each pair of fuzzy singletons q_1 & q_2 with various supports x_1 & x_2 respectively, there occurs rgfbOSs δ_1 & δ_2 such that, $q_1 \leq \delta_1 \leq 1- q_2$ and $q_2 \leq \delta_2 \leq 1- q_1$.

Theorem 4.4: A fts is rgfbT_1, that is regular generalized fuzzy bT_1, if and only if each crisp fuzzy singleton is rgfbCS.
Proof: To prove the necessary condition: Let rgfbT_1 be fts and crisp fuzzy singleton with supports x_0 be q_0 .There occurs, rgfbOSs δ_1 and δ_2 for any fuzzy singleton q with supports x ($\neq x_0$), such that, $q_0 \leq \delta_1 \leq 1- q$ and $q \leq \delta_2 \leq 1- q_0$. Since, it includes each fuzzy set as the collection of fuzzy singletons. So that, $1-q_0 = \underset{q \leq 1-q_0}{V} q = 0$. Thus, $1-q_0$ is rgfbOS. This shows that, q_0 (crisp fuzzy singleton) is rgfbCS.

To prove the sufficiency: Assume p_1 and p_2 be pair of fuzzy singletons with various supports x_1& x_2 .Further on fuzzy singletons with various supports x_1 & x_2 be q_1 & q_2, such that $q_1(x_1) = 1$ and $q_2(x_2)=1$. As each crisp fuzzy singleton is rgfbCS, the fuzzy sets $1-q_1$ & $1-q_2$ are rgfbOSs such that, $p_1\leq 1-q_1\leq 1- p_2$ and $p_2\leq 1-q_2\leq 1- p_1$.This proves, fts is rgfbT_1 space.

Remark 4.5: In a fts X_1, each rgfbT_1 space is rgfbT_0 space.
Proof: It follows the above definition.
The opposite of this theorem is in correct. This is shown as follows –

Example 4.6: Let $X_1=\{a, b\}$,$p_1=\{(a,0),(b,1)\}$ and $p_2=\{(a,0.4),(b,0)\}$ are fuzzy singletons. U= $\{(a, 0.5),(b, 1)\}$ be rgfbOS . Let $\tau= \{ 0,p_1 , p_2 , U,1 \}$. The space is rgfb$T_0$ and it is not rgfbT_1.

Definition 4.7: A fts is known as rgfbT_2 , that is regular generalized fuzzy bT_2 or rgfb-Hausdorff iff, for each pair of fuzzy singletons q_1 & q_2 with various supports x_1 & x_2 respectively, there occurs, rgfbOS δ_1 & δ_2 such that, $q_1\leq \delta_1\leq 1- q_2$, $q_2\leq \delta_2\leq 1- q_1$ and $\delta_1\leq 1- \delta_2$.

Theorem 4.8: A fts is known as rgfbT_2, that is regular generalized fuzzy bT_2 or rgfb-Hausdorff if and only if for each pair of fuzzy singletons q_1 & q_2 with various supports x_1 & x_2 respectively, there occurs an rgfbOS δ_1 such that, $q_1\leq \delta_1\leq$ rgfbCl $\delta_1\leq 1- q_2$.
Proof: To prove the necessary condition: Let rgfbT_1 be fts and fuzzy singletons q_1 & q_2 with various supports .Let δ_1 & δ_2 be rgfbOS such that, $q_1\leq \delta_1\leq 1- q_2$, $q_2\leq \delta_2\leq 1- q_1$ and $\delta_1\leq 1- \delta_2$ where $1- \delta_2$ is rgfbCS. We have by definition, rgfbCl(δ_1)=$\wedge$ {($1- \delta_2$) : ($1- \delta_2$) rgfbCS} where $\delta_1\leq 1- \delta_2$ .Also rgfbCl(δ_1)$\geq \delta_1$.This shows that, $q_1\leq \delta_1\leq$ rgfbCl $(\delta)_1\leq 1- \delta_2\leq 1- q_2 \Rightarrow q_1\leq \delta_1\leq$ rgfbCl $(\delta_1)\leq 1- q_2$.
To prove the sufficiency: Assume q_1 and q_2 are pair of fuzzy singletons with various supports and δ_1 be rgfbOS. Since, $q_1\leq \delta_1\leq$ rgfbCl $(\delta_1)\leq 1- q_2 \Rightarrow q_1\leq \delta_1\leq 1- q_2$. Also $q_1\leq$rgfbCl(δ_1)$\leq 1- q_2\Rightarrow q_2\leq 1-$ rgfbCl $(\delta_1) \leq 1-q_1$. This shows that, 1- rgfbCl (δ_1) is rgfbOS. Also rgfbCl $(\delta_1) \leq$ 1- rgfbCl (δ_2) . This proves that, fts is rgfbT_2 space.

Remark 4.9: In a fts X_1,each rgfbT_2 space is rgfbT_1 space.
Proof: It follows the above definition.
The opposite of this theorem is in correct. This is shown as follows –

Example 4.10: Let $X_1=\{a,b\}$. $q_1=\{(a, 0.2),(b, 0)\}$ and $q_2=\{(a,0), (b,0.4)\}$ are fuzzy singletons, $O_1= \{(a,0.3),(b,0.4)\}$ and $O_2= \{(a,0.8),(b,0.7)\}$ are rgfbOS .Let $\tau = \{ 0, p_1 , p_2 , O_1 , O_2, 1\}$. The space is rgfb$T_1$ and it's not rgfbT_2.

Definition 4.11: A fts is known as rgfbT$_{2\frac{1}{2}}$, that is regular generalized fuzzy bT$_{2\frac{1}{2}}$ or rgfb-Urysohn iff for each pair of fuzzy singletons q_1 & q_2 with various supports x_1 & x_2 respectively, there occurs, rgfbOSs δ_1 & δ_2 such that, $q_1 \leq \delta_1 \leq 1 - q_2$, $q_2 \leq \delta_2 \leq 1 - q_1$ and rgfbCl (δ_1)≤ 1-rgfbCl (δ_2).

Remark 4.12: In a fts X_1,each rgfbT$_{2\frac{1}{2}}$ space is rgfbT$_2$ space.
Proof: It follows from the above definition.
The opposite of this theorem is in correct. This is shown as follows –

Example 4.13: Let $X_1=\{a, b\}$. $q_1=\{(a, 0.1),(b,0)\}$ and $q_2=\{(a,0),(b,0.3)\}$ are fuzzy singletons, $O_1= \{(a,0.2),(b,0.3)\}$ and $O_2= \{(a,0.7),(b,0.5)\}$ are rgfbOSs. Let $\tau =\{ 0, p_1 , p_2 , O_1 , O_2, 1 \}$. The space is rgfbT$_2$ and it's not rgfbT$_{2\frac{1}{2}}$.

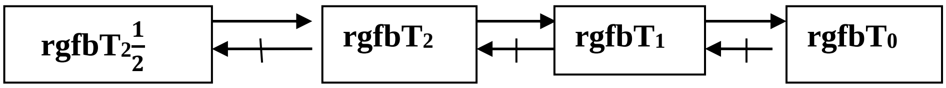

Figure2. From the above definition and examples one can notice that the above chains of implication.

Theorem 4.14: An injective function f: $X_1 \rightarrow X_2$ is rgfb-continuous, and X_2 is fT$_0$, then X_1 is rgfbT$_0$.
Proof: Assume α & β be fuzzy singletons in X_1 with various support then f(α) & f(β) belongs to X_2, As f is injective and f(α)≠ f(β). As X_2 is fT$_0$, there occurs, a open set O in X_2 such that, $f(\alpha) \leq O \leq 1 - f(\beta)$or $f(\beta) \leq O \leq 1 - f(\alpha), \Rightarrow \alpha \leq f^{-1}(O) \leq 1 - \beta$ or $\beta \leq f^{-1}(O) \leq 1 - \alpha$. Since, f: $X_1 \rightarrow X_2$ is rgfb-continuous, $f^{-1}(O)$ is rgfbOS in X_1. This shows that, X_1 is rgfbT$_0$-space[4.1].

Theorem 4.15: An injective function f: $X_1 \rightarrow X_2$ is rgfb-irresolute, and X_2 is rgfbT$_0$, then X_1 is rgfbT$_0$.
Proof: Assume α & β be fuzzy singletons in X_1 with various support. As f is injective $f(\alpha)$ & $f(\beta)$ belongs to X_2 and $f(\alpha) \neq f(\beta)$. As, X_2 is rgfbT$_0$, there occurs rgfbOS O in X_2 so that $f(\alpha) \leq O \leq 1 - f(\beta)$ or $f(\beta) \leq O \leq 1 - f(\alpha) \Rightarrow \alpha \leq f^{-1}(O) \leq 1 - \beta$ or $\beta \leq f^{-1}(O) \leq 1 - \alpha$. As, f is rgfb-irresolute $f^{-1}(O)$ is rgfbOS(X_1). This shows that, X_1 is rgfbT$_0$ space[4.1].

Theorem 4.16: An injective function f: $X_1 \rightarrow X_2$ is strongly rgfb-continuous, and X_2 is rgfbT$_0$, then X_1 is fT$_0$.

Proof: Assume α & β be fuzzy singletons in X_1 with various support. Since f is injective $f(\alpha)$ & $f(\beta)$ belongs to X_2 and $f(\alpha) \neq f(\beta)$. As, X_2 is rgfbT_0, there occurs rgfbOS O in X_2 so that, $f(\alpha) \leq O \leq 1 - f(\beta)$ or $f(\beta) \leq O \leq 1 - f(\alpha), \Rightarrow \alpha \leq f^{-1}(O) \leq 1 - \beta$ or $\beta \leq f^{-1}(O) \leq 1 - \alpha$. Since, f is strongly rgfb-continuous, $f^{-1}(O)$ is fuzzy-open in X_1. This shows that, X_1 is fT_0-space[2.8].

Theorem 4.17: An injective function $f: X_1 \rightarrow X_2$ is rgfb-continuous, and X_2 is fT_1, then X_1 is rgfbT_1.
Proof: Assume α and β be fuzzy singletons in X_1 with various supports.
$f(\alpha)$ and $f(\beta)$ belongs to X_2, Since, f is injective. As, X_2 is fT_1 space hence, by the statement there occurs fuzzy-open sets O_1 & O_2 in X_2 such that, $f(\alpha) \leq O_1 \leq 1 - f(\beta)$ and $f(\beta) \leq O_2 \leq 1 - f(\alpha) \Rightarrow \alpha \leq f^{-1}(O_1) \leq 1 - \beta$ and $\beta \leq f^{-1}(O_2) \leq 1 - \alpha$.
Since, f is rgfb-continuous $f^{-1}(O_1)$ and $f^{-1}(O_2)$ are rgfb-open in X_1. This shows that, X_1 is rgfbT_1 space[4.3].

Theorem 4.18: An injective function $f: X_1 \rightarrow X_2$ is rgfb-irresolute, and X_2 is rgfbT_1, then X_1 is rgfbT_1.
Proof: Assume α & β be fuzzy singletons in with various supports. Since f is injective, $f(\alpha)$ & $f(\beta)$ belongs to X_2. As X_2 is rgfbT_1, there occurs two rgfbOS O_1& O_2 in X_2 so that $f(\alpha) \leq O_1 \leq 1 - f(\beta)$ and $f(\beta) \leq O_2 \leq 1 - f(\alpha) \Rightarrow \alpha \leq f^{-1}(O_1) \leq 1 - \beta \; and \; \beta \leq f^{-1}(O_2) \leq 1 - \alpha$. Since, f is rgfb-irresolute, then $f^{-1}(O_1) \; and \; f^{-1}(O_2)$ are rgfbOS(X_1). This shows that, X_1 is rgfbT_1 space[4.3].

Theorem 4.19: If $f: X_1 \rightarrow X_2$ is strongly rgfb-continuous and X_2 is rgfbT_1, then X_1 is fT_1.
Proof: Assume α & β be fuzzy singletons in X_1 with various supports. Since, f is injective, $f(\alpha)$ &$f(\beta)$ belong to X_2. As, X_2 is rgfbT_1, there occurs two rgfbOSs O_1 and O_2 in X_2 so that, $f(\alpha) \leq O_1 \leq 1 - f(\beta)$ and $f(\beta) \leq O_2 \leq 1 - f(\alpha) \Rightarrow \alpha \leq f^{-1}(O_1) \leq 1 - \beta$ and $\beta \leq f^{-1}(O_2) \leq 1 - \alpha$. Since, f is strongly rgfb-continuous, therefore $f^{-1}(O_1)$ & $f^{-1}(O_2)$ are fuzzy-open in X_1. This shows that, X_1 is fT_1 space[2.8].

Theorem 4.20: An injective function $f: X_1 \rightarrow X_2$ is rgfb-continuous, and X_2 is fT_2, then X_1 is rgfbT_2.
Proof: Assume α & β be fuzzy singletons in X_1 with various supports. Since, f is injective, so $f(\alpha)$ & $f(\beta)$ belongs to X_2 and $f(\alpha) \neq (\beta)$. Since, X_2 is fT_2, therefore there occurs open fuzzy set O in X_2 so that, $f(\alpha) \leq O \leq$

$\mathrm{Cl}(O) \leq 1 - f(\beta) \Rightarrow \alpha \leq f^{-1}(O) \leq f^{-1}[Cl(O)] \leq 1 - \beta$. Since, f is rgfb-continuous $f^{-1}(O)$ is rgfbCS(X_1). Hence, $\alpha \leq f^{-1}(O) \leq f^{-1}[\mathrm{Cl}(O)] \leq f^{-1}[\mathrm{rgfbCl}(O)] \leq \mathrm{rgfbCl}[f^{-1}[(O)] \leq 1 - \beta$. That is, $\alpha \leq f^{-1}(O) \leq \mathrm{rgfbCl}[f^{-1}[(O)] \leq 1 - \beta$. This shows that, X_1 is rgfbT_2[4.7].

Theorem 4.21: An injective function $f : X_1 \to X_2$ is rgfb-irresolute, and X_2 is rgfbT_2. Then, X_1 is rgfbT_2.
Proof: Obvious.

Theorem 4.22: An injective function $f : X_1 \to X_2$ is strongly rgfb-continuous, and X_2 is rgfbT_2. Then, X_1 is fT_2.
Proof: Obvious.

Theorem 4.23: An injective function $f : X_1 \to X_2$ is rgfb-continuous, and X_2 is f$T_{2\frac{1}{2}}$. Then, X_1 is rgfb$T_{2\frac{1}{2}}$.
Proof: Assume α & β be fuzzy singletons in X_1 with various supports. Since, f is injective, then $f(\alpha)$ and $f(\beta)$ belongs to X_2 and $f(\alpha) \neq f(\beta)$. Since, X_2 is f$T_{2\frac{1}{2}}$, then there occurs open fuzzy sets O_1 and O_2 in X_2 such that, $f(\alpha) \leq O_1 \leq 1 - f(\beta)$, $f(\beta) \leq O_2 \leq 1 - f(\alpha)$and $\mathrm{Cl}O_1 \leq 1 - \mathrm{Cl}O_2 \Rightarrow \alpha \leq f^{-1}(O_1) \leq 1 - \beta$,$\beta \leq f^{-1}(O_2) \leq 1 - \alpha$ and $\mathrm{Cl}f^{-1}(O_1) \leq 1 - \mathrm{Cl}f^{-1}(O_2)$.
Since, f is rgfb-continuous $f^{-1}(O_1)$ and $f^{-1}(O_2)$ are rgfbOS(X_1).
$\mathrm{Cl}(f^{-1}(O_1)) \leq \mathrm{rgfbCl}(f^{-1}(O_1))$ and$1 - Cl(f^{-1}(O_2)) \leq 1 - \mathrm{rgfbCl}(f^{-1}(O_2))$. Hence, $\mathrm{rgfbCl}(f^{-1}(O_1)) \leq 1 - \mathrm{rgfbCl}(f^{-1}(O_2))$. This shows that, X_1 is rgfb$T_{2\frac{1}{2}}$[4.11].

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