Regularities in Data from Factorial Experiments

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This article documents a meta-analysis of 113 data sets from published factorial experiments. The study quantifies regularities observed among factor effects and multifactor interactions. Such regularities are known to be critical to efficient planning and analysis of experiments and to robust design of engineering systems. Three previously observed properties are analyzed: effect sparsity, hierarchy, and heredity. A new regularity is introduced and shown to be statistically significant. It is shown that a preponderance of active two-factor interaction effects are synergistic, meaning that when main effects are used to increase the system response, the interaction provides an additional increase and that when main effects are used to decrease the response, the interactions generally counteract the main effects. © 2006 Wiley Periodicals, Inc. Complexity 11: 32–45, 2006

Key Words: design of experiments; robust design; response surface methodology

1. INTRODUCTION

Researchers in the sciences of complexity seek to discover regularities arising in natural, artificial, and social systems and to identify their underlying mechanisms. The authors have carried out meta-analysis of 113 data sets from published experiments from a wide range of science and engineering disciplines. The goal was to identify and quantify regularities in the experimental data regarding the size of factor effects and interactions among factors. These regularities appear to arise from the interplay of the physical behavior of the systems and the knowledge of the experimenters. Therefore our results should be interesting to a broad range of investigators in complex systems including engineers, statisticians, physicists, cognitive scientists, and social scientists.

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This article is organized as follows: Section 2 presents the motivation for the study and provides some necessary background in the Design of Experiments; Section 3 describes the research methodology; Section 4 gives an example of the analysis using one of our data sets; Section 5 presents the results of the meta-analysis; Section 6 presents an investigation of nonlinear transformation of the responses and its influence on the regularities; and Section 7 presents conclusions and suggestions for future research.

2. MOTIVATION

2.1. What is Design of Experiments and Why Is It Important?

Experimentation is an important activity in design of systems. Most every existing engineering system was shaped by a process of experimentation including preliminary investigation of phenomena, subsystem prototyping, and system verification tests. Major, complex systems typically require thousands of experiments [1]. Consequently, experimentation is a significant driver of development cost and time to market. There is pressure to drive down the resource requirements of experimentation, especially in commercially competitive industries.

The mathematical and scientific discipline of Design of Experiments (DOE) seeks to provide a theoretical basis for experimentation across many domains of inquiry. Commonly articulated goals of DOE include: making scientific investigation more effective and reliable [2]; efficient process and product optimization [3]; and improvement of system robustness to variable or uncertain ambient conditions, internal degradation, manufacturing, or customer use profiles [4–6]. The use of DOE in engineering appears to be rising as it is frequently disseminated through industry "Six Sigma" programs, corporate training courses, and university engineering curricula.

This article relies on several concepts and terms from DOE. To make the discussion clear to a broad audience of investigators in complex systems, the following definitions are provided:

- Response: An output of the system to be measured in an experiment.
- Factor: A variable that is controlled by the experimenter to determine its effect on the response.
- Active factor: A factor that experiments reveal to have a significant effect on the system response.
- Level: The discrete values a factor may take in an experiment.
- Full factorial experiment: An experiment in which every possible combination of factor levels is tested. In a system with k factors, each having two levels, the full factorial experiment is denoted as the 2^k design.
- Main effect: The individual effects of each factor in an experiment [7]. In the 2^k design, the main effect of a factor is computed by averaging of all the responses at each level of that factor and taking the difference.
- Interaction: The failure of a factor to produce the same effect at different levels of another factor [7]. An interaction that can be modeled as arising from the joint effect of two factors is called a two-factor interaction. Similarly, three-factor interactions and higher order interactions may be defined.

2.2. Why Are Regularities in Experimental Data Important?

Based on experience in planning and analyzing many experiments, practitioners and researchers in DOE have identified regularities in the interrelationships among factor effects and interactions. Such regularities are frequently used to justify experimental design and analysis strategies [8]. This section reviews three regularities noted in the DOE literature describing their nature, origins, and influence on theory and practice. These regularities are effect sparsity, hierarchical ordering, and effect heredity.

Effect sparsity refers to the observation that number of relatively important effects in a factorial experiment is generally small [9]. This is sometimes called the *Pareto Principle in Experimental Design,* based on analogy with the observations of the 19th century economist Vilfredo Pareto, who argued that, in all countries and times, the distribution of income and wealth follows a logarithmic pattern resulting in the concentration of resources in the hands of a small number of wealthy individuals.

Effect sparsity appears to be a phenomenon characterizing the knowledge of the experimenters more so than the physical or logical behavior of the system under investigation. Investigating an effect through experimentation requires an allocation of resources—to resolve more effects typically requires more experiments. Therefore, effect sparsity is in some sense an indication of wasted resources. If the important factor effects could be identified during planning, then those effects might be investigated exclusively, resources might be saved, and only significant effects would be revealed in the analysis. But experimenters are not normally able to do this. Effect sparsity is therefore usually evident, but only after the experiment is complete and the data have been analyzed.

Researchers in DOE have devised means by which the sparsity of effects principle can be exploited to seek efficiencies. Many experiments are designed to have projective properties so that when dimensions of the experimental space are collapsed, the resulting experiment will have desired properties. For example, the fractional factorial 2^{3-1} design may be used to estimate the main effects of three factors *A*, *B*, and *C*. As Figure 1 illustrates, if any of the three dimensions associated with the factors is collapsed, the resulting design becomes a full factorial 2^2 experiment in

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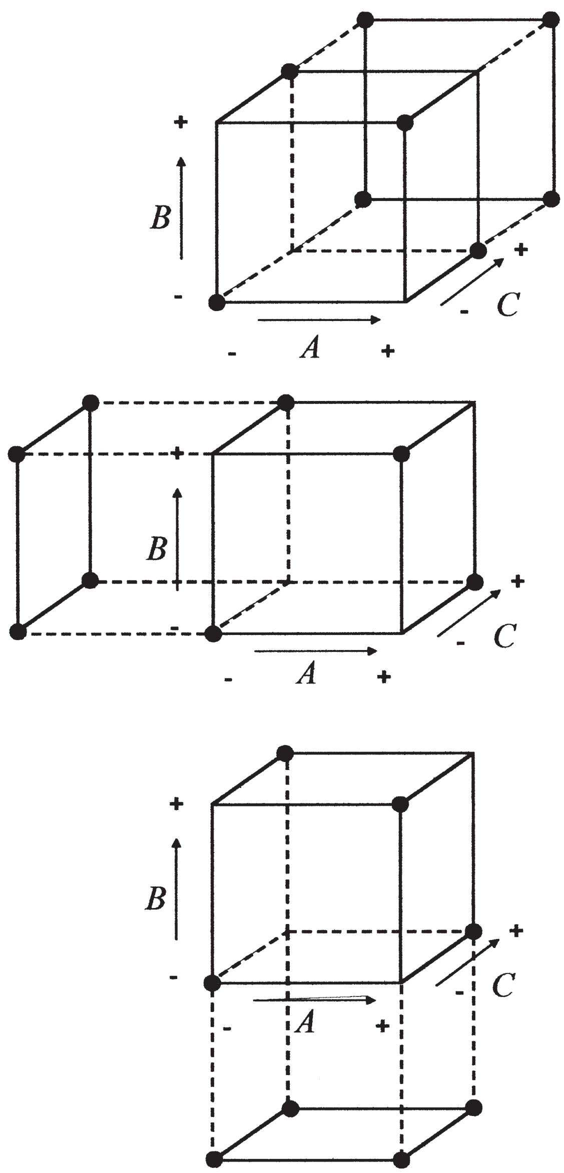
FIGURE 1



The projective property of the fractional factorial 2^{3-1}_{III} design of an experiment.

the two remaining factors. Projection, in effect, removes a factor from the experimental design once it is known to have an insignificant effect on the response. Projective properties of fractional factorial experiments can enable an investigator to carry out a full factorial experiment in the few critical factors in a long list of factors without knowing a priori which of the many factors are the critical few. Similarly, Latin Hypercube Sampling enables an experimenter to sample an n-dimensional space so that, when $n-1$ dimensions collapse, the resulting sampling is uniform in the remaining dimension [10]. Latin Hypercube Sampling has become popular for sampling computer simulations of engineering systems, suggesting that its projective properties provide substantial practical advantages for engineering design. Although effect sparsity is widely accepted as a useful regularity, better quantification seems to be needed. Reliance on effect sparsity has led to strong claims about single array methods of robust design, but field investigation have shown that crossed arrays give better results [11]. Degrees of reliance on effect sparsity may be the root cause of some disagreements about methodology in robust design.

Hierarchical ordering (sometimes referred to as simply "hierarchy") is a term denoting the observation that main effects tend to be larger on average than two-factor interactions, two-factor interactions tend to be larger on average than three-factor interactions, and so on [12]. Effect hierarchy is illustrated in Figure 2 for a system with four factors *A*, *B*, *C*, and *D*. Figure 2 illustrates a case in which hierarchy is not strict—for example, that some interactions (such as the two-factor interaction *AC*) are larger than some main effects (such as the main effect of *B*).

The phenomenon of hierarchical ordering is partly due to the range over which experimenters typically explore factors. In the limit that experimenters explore small changes in factors and to the degree that systems exhibit continuity of responses and their derivatives, linear effects of factors tend to dominate. Therefore, to the extent that hierarchical ordering is common in experimentation, it is due to the fact that many experiments are conducted for the purpose of minor refinement rather than broad-scale exploration.

The phenomenon of hierarchical ordering is also partly determined by the ability of experimenters to transform the inputs and outputs of the system to obtain a parsimonious description of system behavior [13]. For example, it is well known to aeronautical engineers that the lift and drag of wings is more simply described as a function of wing area and aspect ratio than by wing span and chord. Therefore, when conducting experiments to guide wing design, engineers are likely to use the product of span and chord (wing area) and the ratio of span and chord (the aspect ratio) as the independent variables. Therefore, one might say that the experimenters have performed a nonlinear transformation of input variables (span and chord) before conducting the experiments. In addition, after conducting the experiments, further transformations might be conducted on the response variable. In aeronautics, lift and drag are often transformed into a nondimensional lift and drag coefficients by dividing the measured force by dynamic pressure and wing area. It is also common in statistical analysis of data to apply transformations such as a logarithm as part of exploration of the data. A key aspect of hierarchical ordering is its dependence on the perspective and knowledge of the

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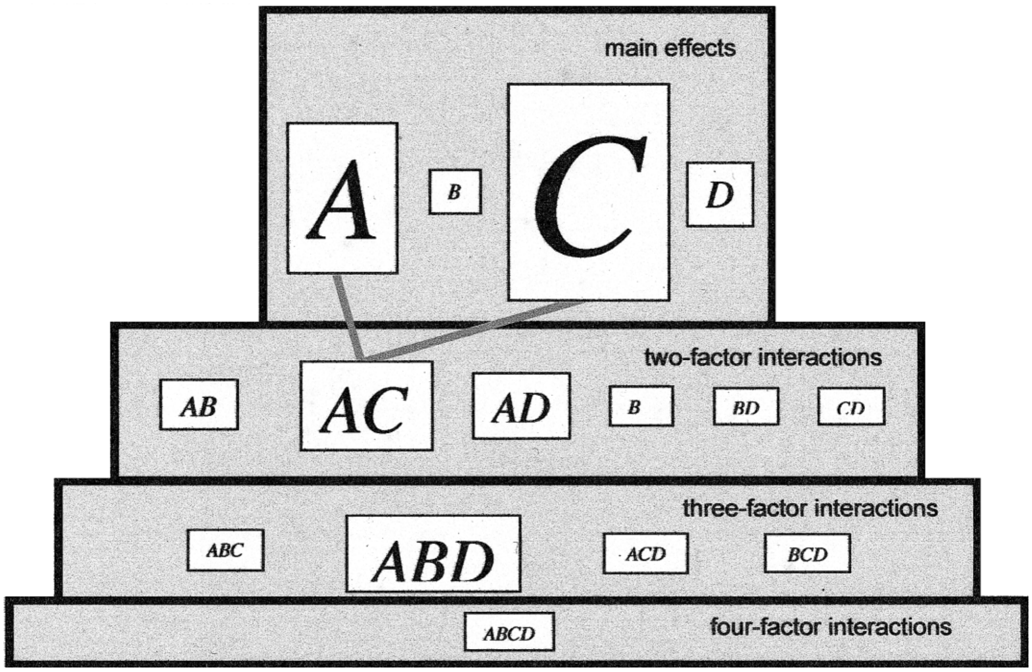
FIGURE 2



The hierarchy and heredity among main effects and interactions in a system with four factors *A*, *B*, *C*, and *D*.

experimenter as well as conventions in reporting data. It is important in assessing regularities in published experimental data that we do not alter the data as it was presented in any ways that affect its hierarchical structure. Section 4 will provide some exploration of this issue.

Effect hierarchy has a substantial effect on the resource requirements for experimentation. A full factorial 2^k experiment allows one to estimate every possible interaction in a system with k two-level factors, but the resource requirements grow exponentially as the number of factors rises. A saturated, resolution *III* fractional factorial design allows one to estimate main effects in a system with k two-level factors with only $k + 1$ experiments, but the analysis may be seriously compromised if there are large interaction effects in the system. Better quantification of effect hierarchy seems to be needed to guide choice between these alternatives and the many other options for experimental planning. For example, the degree to which systems exhibit hierarchy has been shown to strongly determine the effectiveness of robust design methodologies [14]. If such decisions among robust design methods can be based on empirical studies, further efficiencies may be possible.

Effect heredity (sometimes referred to as "inheritance") implies that, in order for an interaction to be significant, at least one of its parent factors should be significant [8]. This regularity can strongly influence sequential, iterative approaches to experimentation. For example, in response surface methodology, high-resolution experiments (e.g., central composite designs) are frequently used with a small number of factors only after screening and gradient-based search bring the response into the neighborhood where interactions among the active factors are likely. Effect heredity can also provide advantages in analyzing data from experiments with complex aliasing patterns, enabling experimenters to identify likely interactions without resorting to high-resolution designs [15].

The effect structures listed above have been identified through long experience by the DOE research community and by practitioners who plan, conduct, and analyze experiments. The effect structures figure prominently in discussion of DOE methods, including their theoretical underpinnings and practical advice on their use. However, effect structures have not been quantified by formal empirical methods. Further, there has been little effort to search for other regularities that may exist in experimental data across many domains. These gaps in the literature motivated the investigation described in the next sections.

3. RESEARCH METHODOLOGY

The present study was performed using a set of 46 published engineering experiments that includes 113 responses in all. A General Linear Model was used to estimate factor effects in each data set and the Lenth method was used to identify active effects. Then, across the set of 113 responses,

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the model parameters and the relevant conditional probabilities were analyzed. Details of the approach are given in the following seven subsections.

3.1. The Set of Experimental Data

We assembled a set of 46 full factorial 2^k experiments published in academic journals or textbooks [16–60]. The experiments come from a variety of fields including biology, chemistry, materials, mechanical engineering, and manufacturing. The reason we used full factorial designs is that we did not want to *assume* the existence of any given effect structure in this investigation, we want to *test* it and *quantify* it. Full factorial experiments allow all the interactions in a system to be estimated. The reason that we used two-level experiments is that they are much more common in the literature than other full factorial experiments and we wanted a large sample size.

Many of the 46 experiments contain several different responses since a single set of treatments may affect many different observable variables. Our set of 46 experiments includes 113 responses in all. Table 1 provides a complete list of these responses. Table 2 summarizes some relevant facts about the overall set. For example, Table 2 reveals that the vast majority of the experiments had either 3 or 4 factors. The number of main effects and interactions are also listed, but this is not based on analysis of the data, but only on the number of effects resolvable by the experimental design. It is notable that the data set includes 569 two-factor interactions and only 383 three-factor interactions because the 54 responses from 2^3 designs each contribute only one potential three-factor interaction. Note that the one response from a 2^7 experiment contributes 35 potential three-factor interactions that represent about 9% of the potential three-factor interactions in the entire set.

All the experimental data in this research were recorded in our database in the form they were originally reported in the literature. No nonlinear transformations were performed before entry into the database nor were nonlinear transformations conducted during the meta-analysis presented in Section 5; therefore the regularities we report in that section are regularities in data as they are presented by experimenters. As it is widely known in the statistics community, nonlinear transformation of the response can sometimes lead to more parsimonious models and reduce active interactions. Therefore, to explore how nonlinear transformations affect regularities, we conducted a follow-up study using the same methods, but performing the analysis of the data after a log transform was applied (these results are in Section 6). This issue of transformation of data is also briefly explored via an example in Section 4.

3.2. The General Linear Model

The General Linear Model (GLM) is frequently used in statistics. The GLM represents the response of a system as a linear combination of functions of the experimental factors. In DOE, the GLM often takes a form of a polynomial. If the experiment uses only two levels of each factor, then an appropriate model should include only selected polynomial terms resulting in the following equation:

$$y(x_1, x_2, \ldots, x_n) = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \sum_{j>i}^{n} \beta_{ij} x_i x_j + \sum_{i=1}^{n} \sum_{j>i}^{n} \sum_{k>j}^{n} \beta_{ijk} x_i x_j x_k + \cdots + \varepsilon. \quad (1)$$

The term β_0 is a constant that represents the mean of the response. The terms β_i quantify the main effects of the factors x_i on the system response. The terms β_{ij} determine the two-factor interactions involving factors x_i and x_j. Similarly, terms β_{ijk} quantify the three-factor interactions. In two-level designs, the input variables are frequently normalized into coded levels of -1 and $+1$. Given this normalization, the sizes of the coefficients β can be compared directly to assess the relative influence of the factor effects.

3.3. The Lenth Method for Effect Analysis

An effect in an experiment is the observed influence of a factor or combination of factors on a response. An effect is sometimes said to be "active" if it is judged to be a significant effect by one of various proposed statistical tests. Among the commonly used test for "active" effects are the Normal Plot (or Half-Normal Plot) method [61], Box-Meyer method [9], and the Lenth method [62]. In this investigation, the Lenth method was selected because it is applicable to unreplicated factorial experiments, because it is computationally simple, and because it can be automated without applying many arbitrary assumptions. In the Lenth method, a plot is made of the numerical values of all effects and a threshold for separating active and inactive effects is calculated based on the standard error of effects. In the first step, a parameter s_0 is formed:

$$s_0 = 1.5 \times median|\beta|, \quad (2)$$

where β includes all estimated effects including main effects and interactions $\beta_1, \beta_2, \ldots, \beta_{12}, \ldots$. Then the pseudo standard error (*PSE*) and margin of error of the effects are defined to be, respectively,

$$PSE = 1.5 \times \underset{|\beta|<2.5 s_0}{median}|\beta| \quad (3)$$

$$\text{Margin of Error} = t_{0.025,df} \times PSE, \quad (4)$$

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TABLE 1

List of the Responses Subjected to Meta-analysis

Engineering System [Ref.]	k^n	Response	Engineering System [Ref.]	k^n	Response
Remediating aqueous heavy metals [16]	2^4	Lead	Finish turning [38]	2^5	Roughness
	2^4	pH	Epitaxial layer growth [8]	2^4	Thickness
	2^4	Lead	Limestone effects [39]	2^3	Surface area
	2^4	pH (alt. method)		2^3	Water demand
Processing of incandescent lamps [17]	2^4	Lumens fluct.		2^3	Init. setting time
	2^4	Power val.		2^3	Final setting time
	2^4	Lumens val.	Cr toxicity and L. nimor [40]	2^{3*}	RGR
	2^4	Life time		2^{3*}	DFR
	2^4	Power fluct.		2^{3*}	Cr in fronds
	2^4	Life fluct.	Wood sanding oper. [41]	2^{4*}	Cherry removal rate
Glass fiber composites [18]	2^3	Stiffness tans.		2^{4*}	Maple removal rate
	2^3	Stiffness		2^{4*}	Oak removal rate
	2^3	Strength		2^{4*}	Pine removal rate
	2^3	Strength trans.		2^{4*}	Cherry surface rough
Solvent extraction of cocaine [19]	2^3	% weight		2^{4*}	Maple surface rough
Plasma spraying of ZrO2 [20]	2^4	Velocity		2^{4*}	Oak surface rough
	2^4	Temp.		2^{4*}	Pine surface rough
	2^4	Size	Grinding of silicon wafers [42]	2^4	Displacement
Post-exp. bake in x-ray mask fab. [21]	2^4	Line width	Concrete mix hot clim. [43]	2^{4*}	Compressive strength
EDM of carbide composites [22]	2^{3*}	Roughness	Color-improved lamps [44]	2^4	Voltage
	2^{3*}	Tool wear		2^4	CCT
	2^{3*}	MRR		2^4	CRI
Polymerization of microspheres [23]	2^3	M_n		2^4	Luminous flux
	2^3	Surf. density	Machinability study [45]	2^4	Tool wear
	2^3	Diameter		2^4	Surface finish
	2^3	M_w	Diffusion welding [46]	2^4	Failure load
	2^3	M_z	Electrocoagulation [47]	2^4	Decolorization
	2^3	Surface density	Fine grinding [48]	2^3	Max grinding force
	2^3	% pepi		2^3	Max motor current
Ball burnishing of an ANSI 1045 [24]	2^4	Roughness		2^3	Grinding cycle time
Abrasive wear of Zi-Al alloy [25]	2^3	Zinc		2^3	Surface roughness
	2^3	Composite	Leaching of manganese [49]	2^4	Mn
Surface morphology of films [26]	2^5	Roughness		2^4	Fe
	2^5	Stress		2^4	Al
MIG process [27]	2^4	Penetration	Aqueous SO2 leaching [50]	2^4	Extraction Mn
	2^4	Reinforce		2^4	Extraction Fe
	2^4	Width	Ident. of radionuclide [51]	2^3	U^{238} extracted
	2^4	Reinforce	Crystal growth [52]	2^4	Experimental scores
Pilot plant filtration rate [28]	2^4	Rate	Yeast b-G [53]	2^3	Observed b-G
Friction measurement machine [29]	2^3	Frict coeff val.	Chl and tetracycline [54]	2^{3*}	CTC
	2^3	Frict coeff fluct.		2^{3*}	TC
Detonation spray process [30]	2^4	Hardness		2^{3*}	pH
	2^4	Roughness	Erosion durability [55]	2^{3*}	Nozzle pressure
	2^4	Porosity	Antifungal antibiotic [56]	2^{3*}	Antifungal antibio. act.
Production of surfactin [31]	2^4	Yield	Xylitol production [57]	2^4	CR
Steam-exp. laser-printed paper [32]	2^5	Brightness		2^4	LDPR
	2^5	Opacity		2^4	Yp/s
	2^5	Light abs.		2^4	Qp
	2^5	Light scatter	Thermal fatigue of PWBs [58]	2^3	Cycles
Hydrosilylation of polypropylene [33]	2^3	Silane	Wire EDM process [59]	2^3	Roughness (μ)
	2^3	Double		2^3	Waviness (μ)
Solid polymer electrolyte cells [34]	2^3	Potential		2^3	Cut spd (μ)
Simulation of earth moving sys. [35]	2^6	Match factor		2^3	Roughness (σ)
	2^6	Production		2^3	Cut spd (σ)
Fractionation of rapeseed lecithin [36]	2^4	Enrichment		2^3	Waviness (σ)
	2^4	Yield	Wet clutch pack [60]	2^7	Drag torque
Deter. of reinforced concrete [37]	2^{3*}	Corros. rate			

*This experiment was not a full factorial design, but contained a full factorial design as a subset. Only the full factorial settings were used in the meta-analysis.

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TABLE 2

A Summary of the Set of 113 Responses and the Potential Effects Therein

Factors	Experiments	Responses	Potential Main Effects	Potential Two-Factor Interactions	Potential Three-Factor Interactions
3	20 (43%)	54 (48%)	162 (40%)	162 (28%)	54 (14%)
4	22 (49%)	51 (45%)	204 (49%)	306 (54%)	204 (54%)
5	2 (4%)	5 (4%)	25 (6%)	50 (9%)	50 (13%)
6	1 (2%)	2 (2%)	12 (3%)	30 (5%)	40 (10%)
7	1 (2%)	1 (1%)	7 (2%)	21 (4%)	35 (9%)
Total	46 (100%)	113 (100%)	410 (100%)	569 (100%)	383 (100%)

where $t_{0.025,df}$ is the 0.975th quantile of the t-distribution and df is the statistical degrees of freedom. Lenth [62] suggests that the degrees of freedom should be one third of the total number of effects.

The margin of error for effects is defined to provide approximately 95% confidence. A more conservative measure, the simultaneous margin of error (*SME*) is also defined as follows:

$$SME = t_{\gamma,df} \times PSE \qquad (5)$$

where

$$\gamma = \frac{(1 + 0.95^{1/m})}{2} \qquad (6)$$

where m is the total number of effects. In the Lenth method, it is common to construct a bar graph showing all effects with reference lines at both the margin of error and at the simultaneous margin of error. In this article, we needed to select one consistent criterion of demarcation between active and inactive effects. We judged it was more appropriate to use the margin of error as the criterion in study of full factorial experiments and that the alternative simultaneous margin of error criterion is more appropriate for screening experiments.

3.4. Method for Quantifying Effect Sparsity

To quantify effect sparsity in the set of data, we used the following procedure:

1. For each experiment, estimate all the main effects and interactions as described in Section 3.2.
2. Apply the Lenth method and label each effect as either active or inactive as described in Section 3.3.
3. Categorize the effects into main effects, two-factor interactions, and three-factor interactions, etc. Calculate the percentage of active effects within each category.
4. Calculate the confidence intervals ($\alpha = 0.05$) for the percentages of potential effects that are active. As some of the active numbers of interactions are very small, we construct exact two-sided confidence intervals based on the binomial distribution.

3.5. Method for Quantifying Hierarchy

To test and quantify effect hierarchy, we compared the size of main effects with that of two-factor interactions, and the size of two-factor interactions with that of three-factor interactions. As the responses in different data sets are in different units, we need to normalize them in order to make comparisons. We choose to make an affine transformation so that the minimum response and maximum response in each experiment were each, respectively, 0 and 100. This normalization was only required in our assessment of hierarchy and did not influence our assessment of other regularities discussed in this article. The following steps summarize the procedure we used to assess hierarchy:

1. Normalize the responses of each experiment by means of an affine transformation so that they all range over the same interval [0, 100].
2. For each experiment, estimate all the main effects and interactions as described in Section 3.2.
3. Use conventional statistical tools such as box-plots to analyze the absolute values of the main effects, two-factor interactions, and three-factor interactions.
4. Calculate the ratio between main effects and two-factor interactions, two-factor interactions and three-factor interactions.

3.6. Method for Quantifying Heredity

To quantify heredity in the set of data, we analyzed probabilities and conditional probabilities of effects being active. Following the definitions and terminology of Chipman et al. [15], we define p as the probability that a main effect is

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active and define a set of conditional probabilities for two factor interactions:

$$p_{00} = \Pr(AB \text{ is active} | \text{neither } A \text{ nor } B \text{ is active}) \quad (7)$$

$$p_{01} = \Pr(AB \text{ is active} | \text{either } A \text{ or } B \text{ is active}) \quad (8)$$

$$p_{11} = \Pr(AB \text{ is active} | \text{both } A \text{ and } B \text{ are active}). \quad (9)$$

Extending the terminology of Chipman et al. [15], we defined conditional probabilities for three-factor interactions as follows:

$$p_{000} = \Pr(ABC \text{ is active} | \text{none of } A, B, C \text{ are active}) \quad (10)$$

$$p_{001} = \Pr(ABC \text{ is active} | \text{one of } A, B, C \text{ is active}) \quad (11)$$

$$p_{011} = \Pr(ABC \text{ is active} | \text{two of } A, B, C \text{ are active}) \quad (12)$$

$$p_{111} = \Pr(ABC \text{ is active} | \text{all of } A, B, C \text{ are active}). \quad (13)$$

On the basis of these definitions, we estimate the conditional probabilities as the frequencies observed in our set of 113 responses and associated factor effects.

3.7. Method for Quantifying Asymmetric Synergistic Interaction Structure

We use the term "asymmetric synergistic interaction structure" (ASIS) to describe the degree to which the signs of main effects provide information about the likely signs of interaction effects. Given the GLM described in Section 2.2, a synergistic two-factor interaction will satisfy the inequality $\beta_i\beta_j\beta_{ij} > 0$ and an antisynergistic two-factor interaction will satisfy the inequality $\beta_i\beta_j\beta_{ij} < 0$. To evaluate the null hypothesis that synergistic two-factor interactions and antisynergistic two-factor interactions are equally likely, we followed these steps:

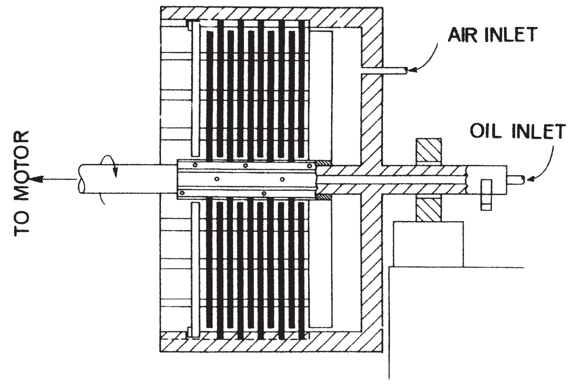
Step 1: For each response

1. Estimate the main effects and interactions for each response as described in Section 3.2.
2. Label each two-factor interaction as either synergistic or antisynergistic according to our definition.

Step 2: Carry out statistics on the set of 113 responses.

1. Calculate the percentage of all two-factor interactions that are synergistic and antisynergistic.
2. Use the Lenth method to discriminate between active effects and inactive effects.
3. Calculate the percentage of active two-factor interactions that are synergistic and antisynergistic.

FIGURE 3



A wet clutch pack (adapted from Lloyd [60]).

4. Calculate the percentage of inactive two-factor interactions that are synergistic and antisynergistic.
5. Calculate 95% confidence intervals for the synergistic and antisynergistic percentages using the binomial distribution.

4. AN ILLUSTRATIVE EXAMPLE FOR A SINGLE DATA SET

Before presenting the meta-analysis of the complete database of 113 responses, it is helpful to observe how the method discussed in Section 3 reveals the effect structures evident in a single data set. Lloyd [60] published a full factorial (2^7) experiment regarding drag torque in disengaged wet clutches. A wet clutch, such as the one depicted in Figure 3, is a device designed to transmit torque from an input shaft that is normally connected to a motor or engine to an output (which in Figure 3 is connected to the outer case). When a wet clutch pack is disengaged, it should transmit no torque and thereby create no load on the motor. In practice, wet clutch packs result in a nonzero drag torque resulting in power losses.

The study in [60] was conducted at Raybestos Manhattan Inc., a designer and manufacturer of clutches and clutch materials. The experiment was designed to assess the influence of various factors on power loss and was likely a part of a long-term effort to make improvements in the design of clutches. The factors in the study were oil flow (A), pack clearance (B), spacer plate flatness (C), friction material grooving (D), oil viscosity (E), friction material (F), and rotation speed (G). Most of these factors are normally under the control of the designer; however, some of these variables such as oil viscosity might vary substantially during operation and therefore were probably included in the study to assess there influence as noise factors. However, for the purpose of the experiment, it must have been the case that

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TABLE 3

The Main Effects from the Clutch Case Study

Effect	Drag Torque (ft lbs)	Active?
A	1.33	Yes
B	−1.55	Yes
C	−1.81	Yes
D	0.067	No
E	2.81	Yes
F	−0.092	No
G	3.01	Yes

all these factors were brought under the control of the experimenter to a substantial degree. Each factor was varied between two levels and the drag torque was measured as the response. The complete results of the full factorial experiment are too lengthy to present here, but the main effects and active two-factor interactions as determined by the Lenth method are presented in Tables 3 and 4. This is slightly different from Lloyd's analysis in the original article because there he simply assumed effects of order 4 or higher were all insignificant.

Every major effect structure under investigation in this study can be observed in this data set:

- Effect sparsity is strongly indicated in the sense that there are 127 effects estimable within this experiment, but only 21 were active, 5 main effects, 9 two-factor interactions, and 7 higher order interactions. Effect sparsity is only weakly indicated by the main effects since 5 out of 7 were active in the study, but is strongly indicated among interactions since only 14 of 122 possible interactions were active.
- Effect hierarchy is strongly indicated because the proportion of potential effects that actually prove to be active is strongly a function of the number of factors involved. Among main effects, 5 of 7 are active. Among two-factor interactions, 9 of 21 are active. Among three-factor interactions, only 7 of 35 are active.
- Effect inheritance is strongly indicated. The four largest two-factor interactions involved two factors both with active main effects. Of the remaining five two-factor interactions, all involved at least one active main effect.
- The hypothesized regularity, ASIS, was strongly evident. Seven of nine active two-factor interactions meet the criterion because the sign of the interaction effect equals the sign of the product of the participating main effects. This example raises an important point about ASIS. Many find the regularity to be surprising because, in their experience, a response becomes increasingly difficult to further improve as successive improvements are made. ASIS is not necessarily inconsistent with this general trend. In this example, to reduce drag torque, the main effects suggest that both oil flow (*A*) and grooving (*D*) should be set to coded levels of −1. However, the significant *AD* interaction would lead to far less reduction of drag torque than one would expect from the linear model. In fact, the interactions will most likely determine the preferred level of *D* rather than the main effect.

TABLE 4

The Active Two-Factor Interactions from the Clutch Case Study

Effect	Drag Torque (ft lbs)	Synergistic?
AD	0.530	Yes
AG	0.964	Yes
BD	−0.520	Yes
BG	−0.830	Yes
CD	0.683	No
CG	−0.695	Yes
DE	0.642	Yes
DG	−0.914	No
EG	1.31	Yes

TABLE 5

The Main Effects from the Clutch Case Study Using a log Transform

Effect	log(Drag Torque)	Active?
A	0.269	Yes
B	−0.350	Yes
C	−0.369	Yes
D	0.040	No
E	0.613	Yes
F	−0.015	No
G	0.529	Yes

Nonlinear transformations of responses can strongly affect regularities in data. To illustrate this, we applied a log transformation to the drag torque of the wet clutch pack and repeated our analysis of the data. The main effects and active two-factor interactions as determined by the Lenth method are presented in Tables 5 and 6. For this particular data set, the log transform failed to improve the hierarchical ordering of the data. The number of active two-factor interactions actually increased from 9 to 12. It is also important to note that in the original data, the synergistic interactions were more numerous, and in the transformed data the synergistic and antisynergistic interactions are equally represented. This motivated an effort to assess the influence of

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TABLE 6

The Active Two-Factor Interactions from the Clutch Case Study Using a log Transform

Effect	log(Drag Torque)	Synergistic?
AD	0.094	Yes
AG	0.159	Yes
BC	−0.072	No
BD	−0.096	Yes
BE	0.108	No
BG	−0.143	Yes
CD	0.182	No
CE	0.071	No
CF	−0.063	No
DE	0.103	Yes
DG	−0.228	No
EG	0.167	Yes

transformations on ASIS through a second meta-analysis reported in Section 6.

5. RESULTS OF META-ANALYSIS OF 133 DATA SETS

The methods described in Section 3 were applied to the set of 113 responses from published experiments (Table 1). Some of the main results of this meta-analysis are summarized in Table 7. The main effects were not very sparse, with more than one third of main effects classified as active. However, only about 7.4% of all possible two-factor interactions were active. The percentage drops steadily as the number of factors participating in the interactions rise. Thus, Table 7 tends to validate both the effect sparsity principle (especially as applied to interactions) and also tends to validate the hierarchical ordering principle. However, this study also supports a caution in applying effect sparsity and hierarchy. For example, if about 2.2% of three-factor interactions are active (as Table 7 indicates), then most experiments with seven factors will contain one or more active three-factor interactions.

Figure 4 depicts a box plot of the absolute values of factor effects for each of three categories: main effects, two-factor interactions, and three-factor interactions. The median of main effect strength is about four times larger than the median strength of two-factor interactions. The median strength of two-factor interactions is more than two times larger than the median strength of three-factor interactions. However, Figure 4 also reveals that many two- and three-factor interactions were observed that were larger than the median main effect. Again, the trends in this study support the principle of hierarchy, but suggest caution in its application.

Table 8 presents the conditional probabilities of observing active effects. This data strongly support the effect heredity principle. Whether the factors participating in an interaction have active main effects strongly determines the likelihood of an active interaction effect. It is noteworthy that, under some conditions, a two-factor interaction is about as likely to be active as a main effect. In addition, it is observed that, under the right conditions, a three-factor interaction can be fairly likely to be active, but still only half as likely as a main effect.

Table 9 presents the results of our investigation into ASIS. First, it is noteworthy that about two-thirds of all two-factor interaction are synergistic. The confidence intervals for that percentage do not include 50%, so we can reject the null hypothesis that the two percentages might be equal. Further, it is of practical significance that the percentage of synergistic effects is much higher among active two-factor interactions than among all two-factor interactions.

6. ADDITIONAL INVESTIGATION OF THE LOG TRANSFORMATION

The analysis in Section 5 is based on the data from experiments as originally published without any nonlinear transformations. However, response transformations are common in analysis of experimental data. For background on good practice, see Wu and Hamada [8] who describe eight commonly used transformations. One motivation for transforming data is variance stabilization. Another is generation of a more parsimonious model with fewer higher order

TABLE 7

Percentage of Potential Effects in 113 Experiments That Were Active as Determined by the Lenth Method

	Main Effects	Two-Factor Interactions	Three-Factor Interactions	Four-Factor Interactions
No. of effects	410	569	383	141
No. of active effects	170	63	26	4
Percentage of effects that were active (%)	41	11	6.8	2.8
Confidence intervals ($\alpha = 0.05$) on the percentage of effects that were active (%)	37–46	9–14	4.5–9.8	0.8–7.1

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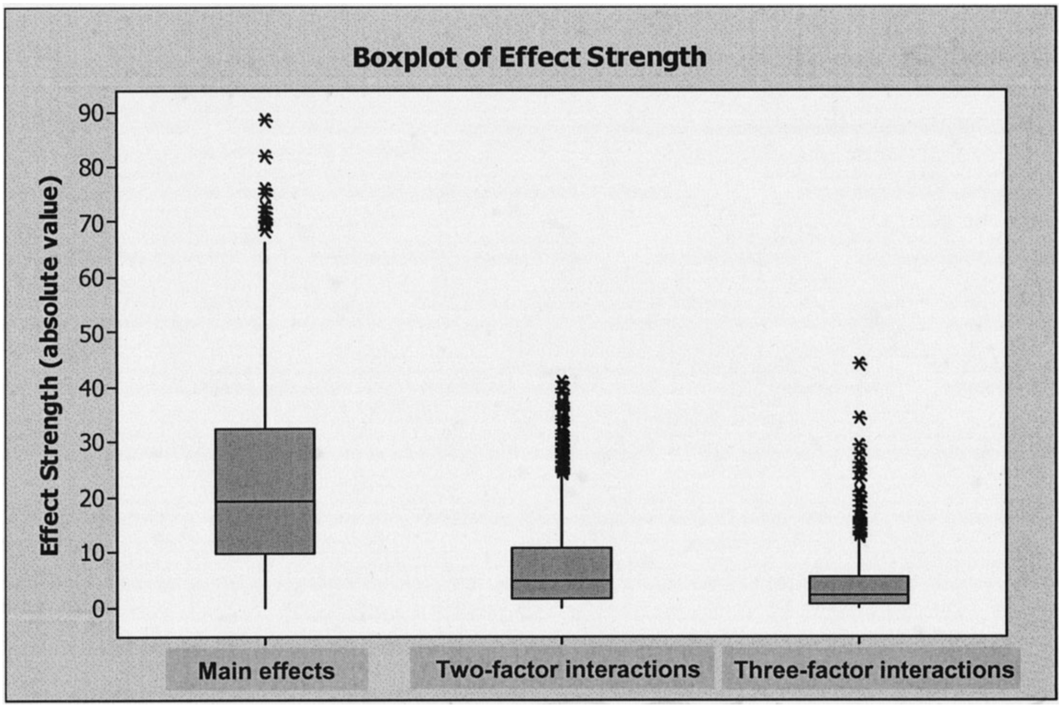
FIGURE 4



Box plot of absolute values for main effects, two-factor interactions, and three-factor interactions.

terms. To provide a rough sense of how such transformations affect the regularities reported here, we focused on just one commonly employed transformation, the logarithm. Of the 107 data sets that could be subject to this transformation (those containing only positive response values), it was found that log transformation resulted in more parsimonious models for 13 responses (meaning that the number of active effects were reduced), whereas the untransformed data produced more parsimonious models in 28 cases. In the other 66 responses, the number of significant effects was unaffected by the use of this transformation. In addition, we observed that in both the full set of 107 transformed responses and in the smaller set of 13 more parsimonious transformed responses, the proportion of synergistic and antisynergistic responses was not significantly different from 50%. An analysis of two-factor interaction synergies on the log transformed data can be found in Table 10. Therefore, we conclude that the newly reported regularity of ASIS is a property of data as they are reported by their experimenters (usually in physical dimensions) and is not generally persistent under nonlinear transformations of the reported data. ASIS is a function of the physical

TABLE 8

The Conditional Probabilities of Observing Active Effects Based Meta-analysis of 113 Experiments

p	p_{11}	p_{01}	p_{00}	p_{111}	p_{011}	p_{001}	p_{000}
41%	33%	4.5%	0.48%	15%	6.7%	3.5%	1.2%

TABLE 9

Synergistic and Antisynergistic Two-Factor Interactions in 113 Experiments

	Synergistic	Antisynergistic	Total
All two-factor interactions			
Number	362	207	569
Percentage (%)	64	36	100
Confidence interval ($\alpha = 0.05$) (%)	60–68	40–32	
Active two-factor interactions			
Number	52	11	63
Percentage (%)	83	17	100
Confidence interval ($\alpha = 0.05$) (%)	71–91	29–9	

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TABLE 10

Synergistic and Antisynergistic Interactions in 107 Experiments Whose Responses Were Transformed Using a Logarithm

	Synergistic	Antisynergistic	Total
Total 107 log-transformed data sets			
All two-factor interactions			
Number	271	268	539
Percentage (%)	50	50	100
C.I. (α = 0.05) (%)	46–55	54–45	
Active two-factor interactions			
Number	31	23	54
Percentage (%)	57	43	100
C.I. (α = 0.05) (%)	43–71	57–29	
13 data sets that became more parsimonious using the log-transform			
All two-factor interactions			
Number	37	41	78
Percentage (%)	47	53	100
C.I. (α = 0.05) (%)	36–59	64–41	
Active two-factor interactions			
Number	2	3	5
Percentage (%)	40	60	100
C.I. (α = 0.05) (%)	5–85	95–15	

systems and whatever transformations experimenters actually use before reporting the data, but may be altered by further transformation.

7. CONCLUSIONS AND FUTURE WORK

The results presented here must be interpreted carefully. It is important to acknowledge the many influences on the data that we subjected to meta-analysis. This investigation was entirely based on two-level full factorial experiments published in journals and textbooks. Full factorial experiments are most likely to be conducted for systems that have already been investigated using less resource intensive means. For example, it is common practice to use a screening experiment before using a higher resolution design. A specific consequence is that all the estimates of percentages of active effects in Table 2 may be inflated. If the screening stage has filtered out several inactive factors, then the experiments with the remaining factors are more likely to exhibit active effects of all kinds. In order to characterize the structure of a larger population of systems on which experiments have been conducted, responses could be selected at random from many engineering domains, and then full factorial experiments might be carried out specifically for the purpose of an extended study of system regularities and analyzed using the methods described here. Such an effort would be resource intensive, but it would guard against potential biases introduced by studying only those systems on which full factorial experiments have already been conducted.

One major outcome of this work is validation and quantification of previously known regularities. All three regularities commonly discussed in the DOE literature (effect sparsity, hierarchy, and heredity) were confirmed as statistically significant. However, many investigators will find that, according to this study, these regularities are not as strong as they previously supposed. Although effect sparsity and hierarchy are statistically significant trends, exceptions to these trends are not unlikely, especially given the large number of opportunities for such exceptions in complex systems. The data presented here suggest that a system with four factors is more likely than not to contain a significant interaction given that $7.4\%\binom{4}{2} + 2.2\%\binom{4}{3} > 50\%$. The data also suggest that a system with a dozen factors is likely to contain around 10 active interactions with roughly equal numbers of two-factor interactions and three-factor interactions since $7.4\%\binom{12}{2} \approx 2.2\%\binom{12}{3} \approx 5$. These observations may be important in robust parameter design. It is known that robust design relies on the existence of some two-factor interactions for its effectiveness. However, some three-factor interactions may interfere with robust design, depending on which method is used. For example, field comparisons of single array methods and crossed array methods have revealed that crossed arrays are more effective. This has led to the conjecture that single arrays rely too strongly on effect sparsity [11]. The meta-analysis in this article suggests that the problem may be more closely related to effect hierarchy. Depending on the number of factors, three-factor interactions may be more numerous than two-factor interactions. Any robust design method that relies on strong assumptions of effect hierarchy is likely to give disappointing results unless some effective steps are taken to reduce the likelihood of these interactions through system design, response definition, or factor transformations.

Another benefit may arise from this study because it quantifies effect heredity. Bayesian methods have been proposed for analyzing data from experiments with complex aliasing patterns [13]. These methods require prior probabilities for the parameters given in Table 8 (p_{11}, p_{01}, and so on). A hypothesis for future investigation is that using the results in Table 8 in concert with Bayesian methods will provide more accurate system models than the same methods using previously published parameter estimates.

Another major outcome of this study is identification and quantification of ASIS—a strong regularity not previously identified in the literature. It was shown that about 80% of active two-factor interactions are synergistic, meaning that $\beta_i\beta_j\beta_{ij} > 0$. The consequences of ASIS for engineering design require further discussion. In cases wherein

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larger responses are preferred, procedures that exploit main effects are likely to enjoy additional increases due to active two-factor interactions even if those interactions have not been located or estimated. By contrast, in cases wherein smaller responses are preferred, procedures that exploit main effects to reduce the response are likely to be penalized by increases due to active two-factor interactions. The discussion of ASIS and its relationship to improvement efforts raises the question of why ASIS was defined as it was in this article. This definition was chosen because it revealed the new, statistically significant regularity in the data set. Other relationships among main effects and interactions were explored and found to be insignificant. However, any regularities associated with improvements rather than increases raise practical and conceptual difficulties. This study was based on meta-analysis of published data sets. If the authors of published data sets do not clearly state whether larger or smaller responses are preferred, how can one define "improvement" for that data set? Further, even if the authors express a preference, might not a different application of the same physical phenomenon reverse that preference? By contrast, regularities associated with the published values reflect regularities in physical phenomena as observed and interpreted by the experimenters. To the extent that such regularities exist and can be confirmed as stable and reliable, they can be helpful in interpreting data.

Some experienced practitioners will find ASIS surprising. It is common for experimenters to report that, if they use experimentation to attain some increases in a response, then any further increase will be harder to attain. We agree that this is the general trend in engineering quality improvements, but how our proposed synergy concept relates to this issue is not so simple. When engineers seek to improve a system, they move toward regions of improvement until locating local maxima or constraints. These maxima and constraints make additional improvements difficult to achieve. Our results are based on meta-analysis of 2^k experiments. It is an interesting question whether such experiments are typically conducted at local maxima or away from them. If 2^k experiments are typically conducted away from local maxima, there are at least two explanations: 1) the maximum has not yet been located, or 2) constraints on the design space are limiting the optimization of that engineering system. Determining the underlying reasons for ASIS is an interesting subject for future research. It is odd that such a strong regularity has not been discussed in either theoretical or practical discourse regarding DOE. The previously known regularities of effect sparsity, hierarchy, and heredity are intellectual cornerstones of DOE and many popular methods provide benefit by exploiting them. Perhaps future research will give rise to new DOE methods that exploit ASIS and thereby reduce resource demands and/or increase effectiveness of engineering experimentation.

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