Regulated Grammars and Automata

Alexander Meduna and Petr Zemek

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Based on

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Regulated Grammars and Automata
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Outline

- **Part I: An Introduction to the Book**
  - Basic Idea
  - General Info
  - Contents
- **Part II: A Sample: One-Sided Random Context Grammars**
  - Basic Idea
  - Definitions and Examples
  - Generative Power
  - Normal Forms
  - Reduction
  - Other Topics of Investigation

- a grammar or an automaton based upon a finite set of rules R

Example

A context-free grammar with the set of rules R:

$$
R: \quad \begin{aligned}
S &\to ABC \\
A &\to aA \\
B &\to bB \\
C &\to cC \\
A &\to a \\
B &\to b \\
C &\to c
\end{aligned}
$$

Basic Idea

- a grammar or an automaton based upon a finite set of rules R
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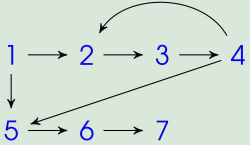
\end{aligned}
$$



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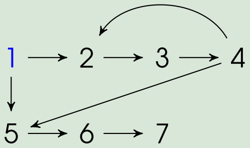
$$

$$S \Rightarrow ABC \quad [1]$$



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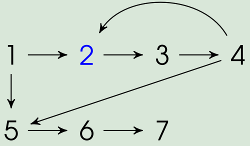

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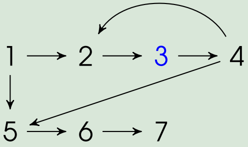

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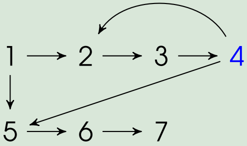

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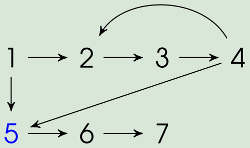

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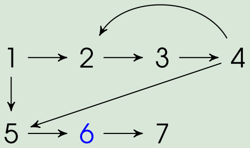

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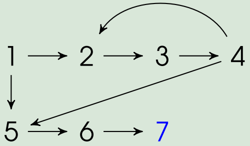

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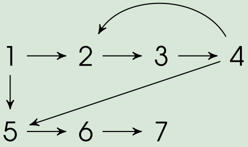

$$L(G) = \{a^n b^n c^n : n \geq 1\}$$

General Info

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Motivation and Subject

- an important trend in formal language theory
- since 1990, no book has been published on the subject although many papers have discussed it

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Purpose

- theoretical: to summarize key results on the subject
- practical: to demonstrate applications of regulated grammars and automata

General Info (Continued)

Focus

- power
- transformation
- reduction

General Info (Continued)

Focus

- power
- transformation
- reduction

Organization

- 9 parts
- 22 chapters

General Info (Continued)

Approach and Features

- theoretically oriented treatment of regulated grammars and automata
- emphasis on algorithms
- intuitive explanation
- many examples
- application perspectives

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- theoretically oriented treatment of regulated grammars and automata
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Book Audience

- computer scientists: professionals, professors, Ph.D. students
- mathematicians
- linguists

Contents (1/4)

Part I Introduction and Terminology

Contents (1/4)

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Contents (2/4)

Contents (3/4)

Contents (3/4)

Contents (3/4)

Contents (4/4)

Contents (4/4)

Part II: A Sample:
One-Sided Random Context Grammars

Basic Idea

- a variant of random context grammars
- $(A \rightarrow x, U, W) \in P$

Basic Idea

- a variant of random context grammars
- $(A \to x, U, W) \in P$
- $P = P_L \cup P_R$

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$$\ldots\ldots \boxed{A} \xrightarrow{\ldots\ldots}$$

Illustration

$(A \rightarrow x, \{B, C\}, \{D\}) \in P_L$

$bBcECbAcD$

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Illustration

$(A \rightarrow x, \{B, C\}, \{D\}) \in P_L$

$$\overleftarrow{bBcECb}\boxed{A}cD \Rightarrow bBcECb\,x\,cD$$

Definitions

Definition

A *one-sided random context grammar* is a quintuple

$$G = (N, T, P_L, P_R, S)$$

where

- N is an alphabet of *nonterminals*;
- T is an alphabet of *terminals* ($N \cap T = \emptyset$);
- P_L and P_R are two finite sets of *rules* of the form

  $$(A \to x, U, W)$$

  where $A \in N$, $x \in (N \cup T)^*$, and $U, W \subseteq N$;
- $S \in N$ is the *starting nonterminal*.

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Definition

If $(A \to x, U, W) \in P_L \cup P_R$ implies that $|x| \geq 1$, then G is *propagating*.

Definitions (Continued)

Definition

The *direct derivation* $\Rightarrow$ is defined as

$$uAv \Rightarrow uxv$$

if and only if

$$(A \rightarrow x, U, W) \in P_L, U \subseteq \text{alph}(u), \text{ and } W \cap \text{alph}(u) = \emptyset$$

or

$$(A \rightarrow x, U, W) \in P_R, U \subseteq \text{alph}(v), \text{ and } W \cap \text{alph}(v) = \emptyset$$

Note: alph(y) denotes the set of all symbols appearing in string y

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Definition

The *language of G* is defined as

$$L(G) = \{w \in T^* : S \Rightarrow^* w\}$$

where $\Rightarrow^*$ is the reflexive-transitive closure of $\Rightarrow$.

Example

Consider the one-sided random context grammar

$$G = (\{S, A, B, \bar{A}, \bar{B}\}, \{a, b, c\}, P_L, P_R, S)$$

where P_L contains

$$(S \rightarrow AB, \emptyset, \emptyset) \qquad (\bar{B} \rightarrow B, \{A\}, \emptyset)$$
$$(B \rightarrow b\bar{B}c, \{\bar{A}\}, \emptyset) \qquad (B \rightarrow \varepsilon, \emptyset, \{A, \bar{A}\})$$

and P_R contains

$$(A \rightarrow a\bar{A}, \{B\}, \emptyset) \qquad (A \rightarrow \varepsilon, \{B\}, \emptyset)$$
$$(\bar{A} \rightarrow A, \{\bar{B}\}, \emptyset)$$

Example (Continued)

Example

P_L: $(S \to AB, \emptyset, \emptyset)$

$(B \to b\bar{B}c, \{\bar{A}\}, \emptyset)$

$(\bar{B} \to B, \{A\}, \emptyset)$

$(B \to \varepsilon, \emptyset, \{A, \bar{A}\})$

P_R: $(A \to a\bar{A}, \{B\}, \emptyset)$

$(\bar{A} \to A, \{\bar{B}\}, \emptyset)$

$(A \to \varepsilon, \{B\}, \emptyset)$

$$S \Rightarrow AB \quad [(S \to AB, \emptyset, \emptyset)]$$

Example (Continued)

Example

P_L: $(S \to AB, \emptyset, \emptyset)$
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$$\begin{array}{lll} S & \Rightarrow & AB \quad [(S \to AB, \emptyset, \emptyset)] \\ & \Rightarrow & a\bar{A}B \quad [(A \to a\bar{A}, \{B\}, \emptyset)] \end{array}$$

Example (Continued)

Example

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$$\begin{array}{llll} S & \Rightarrow & AB & [(S \to AB, \emptyset, \emptyset)] \\ & \Rightarrow & a\bar{A}B & [(A \to a\bar{A}, \{B\}, \emptyset)] \\ & \Rightarrow & a\bar{A}b\bar{B}c & [(B \to b\bar{B}c, \{\bar{A}\}, \emptyset)] \end{array}$$

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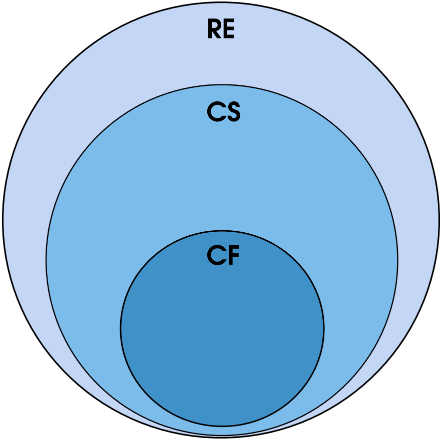
\end{array}
$$

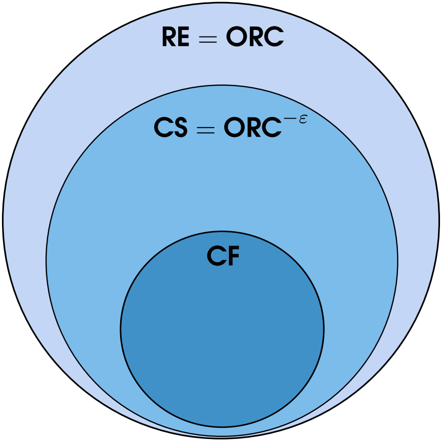
$$L(G) = \{a^nb^nc^n : n \geq 0\}$$

Generative Power



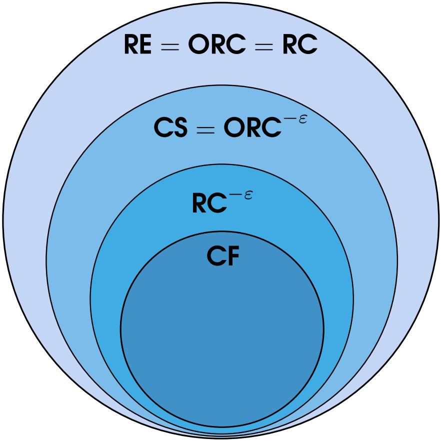
RE the family of recursively enumerable languages
CS the family of context-sensitive languages
CF the family of context-free languages

Generative Power



ORC	the language family generated by one-sided random context grammars
$\mathbf{ORC}^{-\varepsilon}$	the language family generated by propagating one-sided random context grammars

Generative Power



RC	the language family generated by random context grammars
RC$^{-\varepsilon}$	the language family generated by propagating random context grammars

Normal Forms

Results Structure

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$(A \rightarrow x, U, W) \in P_L \cup P_R$ implies that $x \in NN \cup T \cup \{\varepsilon\}$

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For any one-sided random context grammar, there is an equivalent one-sided random context grammar satisfying one of the following normal forms.

Normal Form I

$P_L = P_R$

Normal Form II

$P_L \cap P_R = \emptyset$

Normal Form III

$(A \to x, U, W) \in P_L \cup P_R$ implies that $x \in NN \cup T \cup \{\varepsilon\}$

Normal Form IV

$(A \to x, U, W) \in P_L \cup P_R$ implies that $U = \emptyset$ or $W = \emptyset$

Reduction

- with respect to the total number of nonterminals

Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 10 nonterminals.

- with respect to the total number of nonterminals

Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 10 nonterminals.

- with respect to the number of right random context nonterminals

Definition

If $(A \rightarrow x, U, W) \in P_R$, then A is a *right random context nonterminal.*

- with respect to the total number of nonterminals

Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 10 nonterminals.

- with respect to the number of right random context nonterminals

Definition

If $(A \to x, U, W) \in P_R$, then A is a *right random context nonterminal*.

Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 2 right random context nonterminals.

Reduction (Continued)

- with respect to the number of right random context rules

Definition

If $p \in P_R$, then p is a *right random context rule*.

- with respect to the number of right random context rules

Definition

If $p \in P_R$, then p is a *right random context rule*.

Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 2 right random context rules.

Other Topics of Investigation

- special variants
  - one-sided permitting and forbidding grammars
  - left random context grammars and their variants

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- parsing-related versions
  - LL one-sided random context grammars

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  - one-sided permitting and forbidding grammars
  - left random context grammars and their variants
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  - three types of leftmost derivations
- generalized versions
  - generalized one-sided forbidding grammars
- parsing-related versions
  - LL one-sided random context grammars
- one-sided versions of other grammars
  - left random context ET0L grammars

Discussion