

Reheating in Inflationary Cosmology: Theory and Applications

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Reheating is an important part of inflationary cosmology. It describes the production of Standard Matter particles after the phase of accelerated expansion. We give a review of the reheating process, focusing on an in-depth discussion of the preheating stage which is characterized by exponential particle production due to a parametric resonance or tachyonic instability. We give a brief overview of the thermalization process after preheating and end with a survey of some applications to supersymmetric theories and to other issues in cosmology such as baryogenesis, dark matter and metric preheating.

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The inflationary model [1] has become the current paradigm of early universe cosmology. The first key aspect of the model is a phase of accelerated expansion of space which can explain the overall homogeneity, spatial flatness and large size of the current universe. Microscopic-scale quantum vacuum fluctuations during the phase of acceleration are red-shifted to currently observable scales, and lead to a spectrum of cosmological fluctuations which becomes scale-invariant in the limit in which the expansion rate becomes constant in time [2].

Reheating at the end of the period of accelerated expansion is an important part of inflationary cosmology. Without reheating, inflation would leave behind a universe empty of matter. Reheating occurs through coupling of the inflaton field ϕ , the scalar field generating the accelerated expansion of space, to Standard Model (SM) matter. Such couplings must be present at least via gravitational interactions. However, in many models of inflation there are couplings through the matter sector of the theory directly.

Reheating was initially [3] analyzed using first order perturbation theory and discussed in terms of the decay of an inflaton particle into SM matter particles. As first realized in [4] (see also [5]), such a perturbative analysis may be rather misleading since it does not take into account the coherent nature of the inflaton field. A new view of reheating was then proposed [4] which is based on the quantum mechanical production of matter particles in a classical background inflaton field ¹. As this analysis showed, it is likely that reheating will involve a parametric resonance instability. This proposal was studied more carefully in [8, 9] and then analyzed in detail in [10]. The term “preheating” was coined [8] to describe the initial energy transfer from the inflaton field to matter particles.

¹ See also [6, 7] for other approaches to the out-of-equilibrium dynamics of the inflaton field.

Typically, the state of matter after preheating is highly non-thermal, and thus must be followed by a phase of thermalization.

The first goal of this review article is to present an introduction to the theory of preheating after inflation. In Section 2 of this article, we give a lightning review of inflationary cosmology. Section 3 is the most important section of this review in which we present a comprehensive analysis of preheating. The efficiency of preheating turns out to be rather model-dependent. We first discuss “standard preheating” which will typically occur in simple single-field inflation models like those used in “Chaotic Inflation”. The efficiency of preheating can be much higher if a tachyonic direction develops, which is what occurs in certain small field inflation models and in “Hybrid Inflation”, a model involving two scalar fields. It turns out that while preheating leads to a very rapid start to the process of energy transfer from the inflaton to SM matter, it typically does not drain most of the energy of the inflaton field. This happens in a second stage, a stage characterized by the nonlinear interactions of the fluctuation modes which have been highly excited by the preheating process. According to recent studies [11], this process is turbulent. The initial stage of turbulence (after which the bulk of the energy density is no longer in the inflaton field) is rapid, but the actual thermalization of the decay products takes much longer. These issues are briefly discussed in Section 4. Section 5 focuses on reheating in supersymmetric models. Finally, in Section 6 we give a brief overview of a number of applications of preheating in inflationary cosmology.

II. INFLATION MODELS AND INITIAL CONDITIONS FOR REHEATING

Cosmological inflation [1] is a phase of accelerated expansion of space. In the context of General Relativity as the theory describing space and time, inflation requires scalar field matter. More precisely, the energy-momentum tensor of matter must be dominated by the almost constant potential energy density of the scalar field ϕ .

A scalar field is postulated to exist in the SM of particle physics: the Higgs field used to give elementary fermions their masses. To serve as a Higgs field, its potential energy must have a minimum at a non-trivial field value. The standard example is

$$V(\phi) = \frac{1}{4}\lambda(\phi^2 - \eta^2)^2 \quad (1)$$

where η is the vacuum expectation value of ϕ . It is assumed that at high temperatures the symmetry is restored by finite temperature effects (see e.g. [12] for a review of field theory methods used in inflationary cosmology) and $\phi = 0$. Once the temperature T falls below a critical value T_c , ϕ ceases to be trapped and will start to roll towards one of the lowest energy states $\phi = \pm\eta$.

The SM Higgs must have a coupling constant λ which is set by the gauge coupling constant and cannot be sufficiently small to yield a long time period of slow rolling of ϕ which is required to obtain enough inflation (except possibly if ϕ is non-minimally coupled to gravity [13]).

Hence, scalar field-driven inflation requires us to go beyond the SM of particle physics. Once one makes this step, there are typically many candidate scalar fields which could be the inflaton, in particular in supersymmetric models.

For cosmological studies, the precise nature of the inflaton is often secondary and hence simple toy models are used. “New” inflation [14, 15] maintains the idea that ϕ begins trapped near $\phi = 0$. Inflation takes place during the period when ϕ is undergoing the symmetry-breaking phase transition and slowly rolling towards $\phi = \pm\eta$. The model was based on scalar field dynamics obtained by replacing the potential (1) by a symmetry breaking potential of Coleman-Weinberg [16] form, where the mass term at the field origin is set to zero and symmetry breaking is obtained through quantum corrections. However, new inflation models typically suffer from an initial condition problem [17].

“Chaotic” (or “large-field”) inflation [18] is an alternative scenario. Inflation is triggered by a period of slow-rolling of ϕ unrelated to a symmetry-breaking phase transition. The simplest example occurs in the toy model of a single scalar field with potential

$$V(\phi) = \frac{1}{2}m^2\phi^2 \quad (2)$$

where m is the mass of ϕ (which is of the order $10^{-6}m_{pl}$ if the model is to yield the observed magnitude of cosmological fluctuations [2]). Here, m_{pl} is the Planck mass defined via $m_{pl}^{-2} \equiv G$, G being Newton’s gravitational constant. It is assumed that ϕ starts out at large field values and slowly rolls towards its vacuum state $\phi = 0$. For inflation to be successful, two conditions must be satisfied. Firstly, the energy density must be dominated by the potential energy term, and secondly the acceleration term in the field equation

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi), \quad (3)$$

the Klein-Gordon equation in an expanding background, must be negligible compared to the other two terms. Here, H is the Hubble expansion rate and a prime indicates the derivative with respect to ϕ . Making use of the Friedmann equation

$$H^2 = \frac{8\pi G}{3} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right) \quad (4)$$

it is easy to see that the slow-rolling conditions are only satisfied for super-Planckian field values $|\phi| > m_{pl}$. In the above two equations (3) and (4) we have taken the field configuration to be homogeneous. In the case of chaotic inflation, the homogeneous slow-roll trajectory is

a local attractor in initial condition space [19], even in the presence of linear metric fluctuations [20], and thus this model is free from the initial condition problem of new inflation. In the context of “real” particle physics theories such as supersymmetric models, gravitational effects often steepen the potential for values of $|\phi|$ beyond the Planck mass and therefore prevent slow-roll inflation.

One way to try to avoid this problem but maintain the success of chaotic inflation is to add a second scalar field ψ to the sector of the theory responsible for inflation and to invoke a potential of the form

$$V(\phi, \psi) = \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\psi^2\phi^2 + \frac{1}{4}\lambda(\psi^2 - v^2)^2, \quad (5)$$

where g and λ are dimensionless coupling constants and v is the vacuum expectation value of ψ . For large values of $|\phi|$, the potential in ψ direction has a minimum at $\psi = 0$, whereas for small values of $|\phi|$, $\psi = 0$ becomes an unstable point. The reader can verify that in this model slow-rolling of ϕ does not require super-Planckian field values. This two field model is called “hybrid” inflation [21].

Let us return to the toy model of chaotic inflation with the potential (1). The slow-roll trajectory is given by

$$\dot{\phi} = -\frac{1}{2\sqrt{3}\pi}mm_{pl}, \quad (6)$$

and it is easy to see that the slow-roll conditions break down at the field value

$$\phi_c = \frac{m_{pl}}{2\sqrt{3}\pi}. \quad (7)$$

After the breakdown of slow-rolling, ϕ commences damped oscillatory motion about $\phi = 0$ and the time-averaged equation of state is that of cold matter ($p = 0$ where p denotes pressure). Asymptotically for large times $mt \gg 1$ the solution approaches

$$\phi(t) \rightarrow \frac{m_{pl}}{\sqrt{3\pi}mt}\sin(mt). \quad (8)$$

This scalar field configuration will provide the classical background matter in the reheating phase.

III. INFLATON DECAY

A. Perturbative Decay

Reheating is a key part of inflationary cosmology. It describes the production of SM matter at the end of the period of accelerated expansion when the energy density is stored overwhelmingly in the oscillations of ϕ . Historically, reheating was first treated perturbatively [3].

We assume that the inflaton ϕ is coupled to another scalar field χ . Taking the interaction Lagrangian to be

$$\mathcal{L}_{\text{int}} = -g\sigma\phi\chi^2, \quad (9)$$

where g is a dimensionless coupling constant and σ is a mass scale, then the decay rate of the inflaton into χ particles is given by

$$\Gamma = \frac{g^2\sigma^2}{8\pi m}, \quad (10)$$

where m is the inflaton mass.

In the approach of [3], the energy loss of the inflaton due to the production of χ particles was taken into account by adding a damping term to the inflaton equation of motion which in the case of a homogeneous inflaton field is

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} = -V'(\phi). \quad (11)$$

For small coupling constant, the interaction rate Γ is typically much smaller than the Hubble parameter at the end of inflation. Thus, at the beginning of the phase of inflaton oscillations, the energy loss into particles is initially negligible compared to the energy loss due to the expansion of space. It is only once the Hubble expansion rate decreases to a value comparable to Γ that χ particle production becomes effective. It is the energy density at the time when $H = \Gamma$ which determines how much energy ends up in χ particles and thus determines the “reheating temperature”, the temperature of the SM fields after energy transfer.

$$T_R \sim (\Gamma m_{pl})^{1/2}. \quad (12)$$

Since Γ is proportional to the square of the coupling constant g which is generally very small, perturbative reheating is slow and produces a reheating temperature which can be very low compared to the energy scale at which inflation takes place.

There are two main problems with the perturbative decay analysis described above. First of all, even if the inflaton decay were perturbative, it is not justified to use the heuristic equation (11) since it violates the fluctuation-dissipation theorem: in systems with dissipation, there are always fluctuations, and these are missing in (11). For an improved effective equation of motion see e.g. [22].

The main problem with the perturbative analysis is that it does not take into account the coherent nature of the inflaton field. The inflaton field at the beginning of the period of oscillations is not a superposition of free asymptotic single inflaton states, but rather a coherently oscillating homogeneous field. The large amplitude of oscillation implies that it is well justified to treat the inflaton classically. However, the matter fields can be assumed to start off in their vacuum state (the red-shifting during the period of inflation will remove any matter particles present at the beginning of inflation). Thus, matter fields χ must be treated quantum mechanically. The improved approach to reheating initiated in [4] (see also [5]) is to consider reheating as a quantum production of χ particles in a classical ϕ background.

B. Preheating

We will present the preheating mechanism for the simple toy model with interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}g^2\chi^2\phi^2, \quad (13)$$

where, as before, g is a dimensionless coupling constant². In this subsection we will neglect the expansion of space. Provided that the time period of preheating is small compared to the Hubble expansion time H^{-1} this is a reasonable approximation. In the next subsection we will include the expansion of space explicitly.

The quantum theory of χ particle production in the external classical inflaton background begins by expanding the quantum field $\hat{\chi}$ into creation and annihilation operators \hat{a}_k and \hat{a}_k^\dagger as:

$$\hat{\chi}(t, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\chi_k^*(t) \hat{a}_k e^{i\mathbf{k}\cdot\mathbf{x}} + \chi_k(t) \hat{a}_k^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}} \right), \quad (14)$$

where k is the momentum. If we assume that there are no non-linearities in the χ sector of the theory, then the equation of motion for χ is linear and can be studied simply mode by mode in Fourier space. The mode functions then satisfy the equation

$$\ddot{\chi}_k + (k^2 + m_\chi^2 + g^2\Phi^2 \sin^2(mt)) \chi_k = 0, \quad (15)$$

where Φ is the amplitude of oscillation of ϕ . This is the Mathieu equation which is conventionally written in the form

$$\chi_k'' + (A_k - 2q \cos 2z) \chi_k = 0, \quad (16)$$

where we have introduced the dimensionless time variable $z = mt$ and a prime now denotes the derivative with respect to z . Comparing the coefficients, we see that

$$A_k = \frac{k^2 + m_\chi^2}{m^2} + 2q \quad q = \frac{g^2\Phi^2}{4m^2} \quad (17)$$

The growth of the mode function corresponds to particle production, as in the case of particle production in an external gravitational field [26]. We will return to this point in the next subsection. For now, let us simply state that exponential growth of the mode functions will lead to an exponential growth of the number of χ particles, with the exponent of this growth being twice the corresponding exponent of the mode functions.

It is well known that the Mathieu equation has instabilities for certain ranges of k and leads to exponential growth

$$\chi_k \propto \exp(\mu_k z), \quad (18)$$

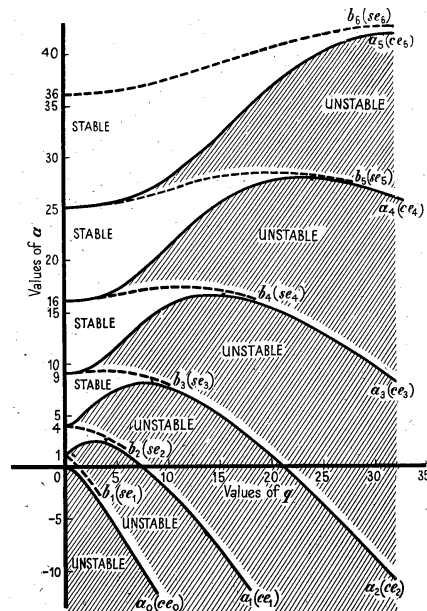


FIG. 1: Instability bands of the Mathieu equation (from [27]). The horizontal axis is the parameter q of (16), the vertical axis is the value of A . The shaded regions are regions in parameter space where there is a parametric resonance instability.

where μ_k is called the Floquet exponent. For small values of q , e.g. $q \ll 1$, resonance occurs in a narrow instability band about $k = m$ (see Figure 1). Hence, in this case we speak of “narrow resonance” (see [27] for in-depth discussions of the Mathieu equation and its generalizations).

The resonance is much more efficient if $q \gg 1$ [8, 10]. In this case, resonance occurs in broad bands. In particular, the bands include all long wavelength modes $k \rightarrow 0$. We then speak of “broad” parametric resonance. A condition for particle production is that the WKB approximation for the evolution of χ is violated. In the WKB approximation, we write: $\chi_k \propto e^{\pm i \int \omega_k dt}$, which is valid as long as the adiabaticity condition

$$\frac{d\omega_k^2}{dt} \leq 2\omega_k^3 \quad (19)$$

is satisfied. In the above, the effective frequency ω_k is given by

$$\omega_k = \sqrt{k^2 + m_\chi^2 + g^2\Phi(t)^2 \sin^2(mt)}, \quad (20)$$

By inserting the effective frequency (20) into the condition (19) and following some algebra, we find that the adiabaticity condition is violated for momenta satisfying

$$k^2 \leq \frac{2}{3\sqrt{3}} gm\Phi - m_\chi^2. \quad (21)$$

For modes with these values of k , the adiabaticity condition breaks down in each oscillation period when ϕ is close to zero. We conclude that the particle number does not increase smoothly, but rather in “bursts”, as was first studied in [10].

² Preheating in a conformally flat scalar field model was analyzed in [23, 24], and in a sine-Gordon potential in [25].

So far, we have studied preheating in a toy model in which Standard Model matter is modeled by a scalar field χ . However, in principle we are interested in the production of SM fermions. Such fermions could be produced after preheating into a scalar field χ which then in turn couples to fermions. In particular, in supersymmetric theories to be discussed in a later section there are many channels for this to happen. However, it turns out that preheating into fermions is also effective, in spite of the fact that the occupation number of any fixed state cannot be greater than one (because of the Pauli exclusion principle). This is discussed in detail in [28, 29].

C. Preheating in an Expanding Background

Provided that the Floquet exponent is not much smaller than unity, the parametric resonance instability leads to an energy transfer from the inflaton to matter particles which is rapid on the scale of the Hubble time. Thus, an analysis neglecting the expansion of the universe is self-consistent. However, as discussed in detail in [10], it is not too difficult to include the expansion of space.

We will first give a qualitative analysis of broad resonance in an expanding background characterized by the cosmological scale factor $a(t)$ which is increasing in time as given by the Friedmann equations. The equation of motion for χ is

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + g^2\Phi(t)^2 \sin^2(mt) \right) \chi_k = 0. \quad (22)$$

The adiabaticity condition is now violated for momenta satisfying:

$$\frac{k^2}{a^2} \leq \frac{2}{3\sqrt{3}} gm\Phi(t) - m_\chi^2. \quad (23)$$

Note that the expansion of space makes broad resonance more effective since more k modes are red-shifted into the instability band as time proceeds. We will see below that the improved analysis yields the same expression for the resonance band except for the exact value of the numerical coefficient of the first term on the r.h.s.. Broad parametric resonance ends when $q \leq 1/4$.

Let us now move on to a quantitative analysis of this problem, building on the comprehensive study of [10]. It proves convenient to eliminate the Hubble friction term in the equation of motion by rescaling the field variable. We consider the variable $X_k(t) = a^{3/2}(t)\chi_k(t)$ in terms of which the equation of motion (22) becomes:

$$\ddot{X}_k + \omega_k^2 X_k = 0 \quad (24)$$

with

$$\omega_k^2 = \frac{k^2}{a^2(t)} + m_\chi^2 + g^2\Phi^2(t) \sin^2(mt) - \frac{9}{4}H^2 - \frac{3}{2}\dot{H}. \quad (25)$$

Note that in the matter-dominated background which we are considering the last two terms on the right-hand side cancel.

The equation of motion (24) represents a harmonic oscillator equation with a time-dependent frequency. The evolution of the solution will be described by the WKB approximation (which entails the absence of particle production) unless the adiabaticity condition is violated. This will happen during short time intervals around the instances $t = t_j$ when $\phi = 0$, as discussed in the previous section. We label the intervals of adiabatic evolution by an integer j . In the j 'th interval (lasting from t_{j-1} to t_j), the adiabatic evolution of X_k is given by

$$X_k^j(t) = \frac{\alpha_k^j}{\sqrt{2\omega_k}} e^{i \int \omega_k dt} + \frac{\beta_k^j}{\sqrt{2\omega_k}} e^{-i \int \omega_k dt}, \quad (26)$$

where the coefficients α_k^j and β_k^j (the ‘‘Bogoliubov coefficients’’) are constant and satisfy the normalization condition $|\alpha_k^j|^2 - |\beta_k^j|^2 = 1$ (derived from the Heisenberg uncertainty principle).

During the brief time periods when ϕ is close to zero, we can use the approximation $\phi^2(t) \simeq \Phi^2 m^2 (t - t_j)^2$. Introducing the new time variable $\tau = g\Phi m(t - t_j)$ and a rescaled momentum $\kappa^2 = \frac{(k^2/a^2) + m_\chi^2}{g\Phi m}$ the equation of motion for X becomes

$$\frac{d^2 X_k}{d\tau^2} + (\kappa^2 + \tau^2) X_k = 0. \quad (27)$$

This equation corresponds to scattering from a parabolic potential.

The non-adiabatic evolution of X_k during the short intervals when ϕ crosses the origin leads to a transformation of the coefficients of Bogoliubov type

$$\begin{aligned} \begin{pmatrix} \alpha_k^{j+1} \\ \beta_k^{j+1} \end{pmatrix} &= \begin{pmatrix} \frac{1}{D_k} & \frac{R_k^*}{D_k^*} e^{-2i\theta_k^j} \\ \frac{R_k}{D_k} e^{2i\theta_k^j} & \frac{1}{D_k^*} \end{pmatrix} \begin{pmatrix} \alpha_k^j \\ \beta_k^j \end{pmatrix} \quad (28) \\ &= \begin{pmatrix} \sqrt{1 + e^{-\pi\kappa^2}} e^{i\varphi_k} & i e^{-\frac{\pi}{2}\kappa^2 - 2i\theta_k^j} \\ -i e^{-\frac{\pi}{2}\kappa^2 + 2i\theta_k^j} & \sqrt{1 + e^{-\pi\kappa^2}} e^{-i\varphi_k} \end{pmatrix} \begin{pmatrix} \alpha_k^j \\ \beta_k^j \end{pmatrix}, \end{aligned}$$

where we defined $\theta_k^j = \int_0^{t_j} dt \omega_k$ which is the phase accumulated at t_j . The reflection and transmission coefficients R_k and D_k are given by

$$R_k = -\frac{i e^{i\varphi_k}}{\sqrt{1 + e^{\pi\kappa^2}}} \quad (29)$$

$$D_k = \frac{e^{-i\varphi_k}}{\sqrt{1 + e^{-\pi\kappa^2}}}, \quad (30)$$

where the phase φ_k is:

$$\varphi_k = \arg \left\{ \Gamma \left(\frac{1 + i\kappa^2}{2} \right) \right\} + \frac{\kappa^2}{2} \left(1 + \ln \frac{2}{\kappa^2} \right). \quad (31)$$

Here Γ stands for the complex Gamma function. In accordance with the general theory of particle production

in external fields (see e.g. [26]), the occupation number of the k 'th mode is

$$n_k = |\beta_k|^2. \quad (32)$$

Making use of the Bogoliubov transformation (28), we obtain the following recursion relation of the particle number:

$$\begin{aligned} n_k^{j+1} &= e^{-\pi\kappa^2} + \left(1 + 2e^{-\pi\kappa^2}\right) n_k^j \\ &+ 2e^{-\frac{\pi}{2}\kappa^2} \sqrt{1 + e^{-\pi\kappa^2}} \sqrt{n_k^j(1 + n_k^j)} \sin \theta_{tot}^j \end{aligned} \quad (33)$$

where $\theta_{tot}^j = 2\theta_k^j - \varphi_k + \arg(\alpha_k^j) - \arg(\beta_k^j)$, we used (32) and $|\alpha_k^j|^2 = 1 + n_k^j$. From (33) it follows that only modes with $\pi\kappa^2 \leq 1$ will grow. Inserting the definition of κ into this condition, we get an expression for the resonance band which reproduces (23) except for the numerical coefficient in the first term on the r.h.s. of the equation which is now $1/\pi$ instead of $2/(3\sqrt{3})$.

We can expand (33) in the limit $n_k \gg 1$ and obtain

$$n_k^{j+1} = \left(1 + 2e^{-\pi\kappa^2} + 2e^{-\frac{\pi}{2}\kappa^2} \sqrt{1 + e^{-\pi\kappa^2}} \sin \theta_{tot}^j\right) n_k^j. \quad (34)$$

The occupation number can only grow if the expression in the bracket is larger than 1. For the fastest growing mode ($k = 0$), $n_k^{j+1} > n_k^j$ if

$$-\frac{\pi}{4} < \theta_{tot}^j < \frac{5\pi}{4}, \quad (35)$$

where we have taken $m_\chi = 0$. Now, since θ_{tot}^j has a rather complicated time-dependence, it could almost be considered as a random variable. Thus, the range of phases (35) implies that the solution grows about 75% of the time.

We can define an effective Floquet exponent μ_k^j for the j 'th interval via

$$n_k^{j+1} = e^{2\pi\mu_k^j} n_k^j. \quad (36)$$

By comparing with (34) we get

$$\mu_k^j = \frac{1}{2\pi} \ln \left(1 + 2e^{-\pi\kappa^2} + 2e^{-\frac{\pi}{2}\kappa^2} \sqrt{1 + e^{-\pi\kappa^2}} \sin \theta_{tot}^j\right). \quad (37)$$

The ‘‘average’’ Floquet exponent μ_k is determined by:

$$\mu_k = \frac{\pi}{m\Delta t} \sum_j \mu_k^j, \quad (38)$$

where Δt is the total duration of the resonance. Making use of (37) we obtain

$$\mu_k \approx \frac{1}{2\pi} \ln 3 - \mathcal{O}(\kappa^2). \quad (39)$$

The fact that the exponent is of the order unity implies that broad parametric resonance in an expanding background is very efficient and can convert a substantial fraction of the inflaton energy density into matter in a time interval small compared to the Hubble time.

The total number density of χ particles is obtained by integrating over all values of k :

$$\begin{aligned} n_\chi(t) &= \frac{1}{(2\pi a)^3} \int_0^\infty d^3k n_k(t) \\ &= \frac{1}{2\pi^2 a^3} \int_0^\infty dk k^2 |\beta_k^0|^2 e^{2m\mu_k t}. \end{aligned} \quad (40)$$

which can be estimated as [30]

$$n_\chi(t) \approx \frac{(gm\Phi_0)^{3/2} |\beta_m^0|^2 e^{2m\mu t}}{16\pi^3 a^3 \sqrt{\frac{m\mu t}{2\pi} + 1}}. \quad (41)$$

This equation determines how fast the energy is drained from the inflaton field, and thus the time interval which it takes before preheating is completed.

D. Termination of Preheating

In the previous analysis of preheating, we have neglected the back-reaction of the produced χ particles on the dynamics of the preheating process. The back-reaction arises at different places. First, the presence of χ particles changes the effective mass of the inflaton oscillations. The rough criterion which we will use below is that this back-reaction effect is negligible as long as the change Δm_ϕ^2 in the square mass of the inflaton is smaller than m^2 .

In the Hartree approximation, the change in the inflaton mass due to χ particles is given by

$$\Delta m_\phi^2 = g^2 \langle \chi^2 \rangle, \quad (42)$$

where the pointed brackets indicate the quantum expectation value. The expectation value of χ^2 is given by

$$\langle \chi^2 \rangle = \frac{1}{2\pi^2 a^3} \int_0^\infty dk k^2 |X_k(t)|^2 \quad (43)$$

Inserting the expansion of the field modes X_k in terms of the Bogoliubov coefficients we find that

$$\Delta m_\chi^2(t) \simeq \frac{gn_\chi(t)}{|\phi(t)|}. \quad (44)$$

It appears that this expression becomes ill-defined when ϕ crosses zero. However, the equation of motion is still well defined at these points. To estimate the strength of back-reaction, we will replace ϕ by its amplitude Φ . Thus, the condition under which this back-reaction effect is negligible is

$$n_\chi(t) \leq \frac{m^2 \Phi(t)}{g}. \quad (45)$$

This is an implicit equation for the time t_1 when back reaction can no longer be neglected. Note that the expression for the number density of χ particles was derived

in (41), and that Φ scales as a function of time as t^{-1} . Because of the exponential growth of n_χ , it is clear that up to logarithmic factors the time interval of preheating is

$$\delta t \sim (\mu m)^{-1}. \quad (46)$$

Thus, since $H \ll m$, this time interval is short compared to the Hubble expansion time, unless the Floquet exponent μ is suppressed.

A second condition must be satisfied in order to be able to neglect the back-reaction of the χ particles on the preheating dynamics: it is the condition that the energy in the χ particles is sub-dominant. Making use of the estimate, $\rho_\chi \sim \langle (\nabla\chi)^2 \rangle \simeq k^2 \langle \chi^2 \rangle$, and inserting the value of $\langle \chi^2 \rangle$ at the time t_1 determined above, we see that ρ_χ is smaller than the potential energy of the inflaton field at the time t_1 as long as the value q at the time t_1 is larger than 1, e.g. $q(t_1) > 1$. This is roughly speaking the same as the condition for the effectiveness of broad resonance.

As it turns out, in many models there is another mechanism which shuts off the resonance before the either of the two conditions mentioned above becomes satisfied. Numerical studies [31–34] have shown that the scattering of χ particles off the inflaton condensate limits the value of the χ modes to a value lower than that which would be obtained from arguments such as the first one above which makes use of the Hartree approximation. For small values of q , the resonant period may be completely absent.

If $q(t_1) \gg 1$ then broad parametric resonance ends when most of the energy is still stored in the inflaton condensate. The further decay must then be analyzed by other techniques, e.g. perturbatively or using numerical simulations. In particular, the time interval of matter-dominated dynamics after inflation could last a long time, leading to a low matter temperature after the matter field excitations have thermalized. On the other hand, if $q(t_1) \sim 1$, then the matter-dominated phase will end with the end of preheating. We still need to study how long it takes for the decay products to thermalize. This topic of thermalization after preheating will be discussed in the next section.

Before moving on to the study of thermalization after preheating, we must discuss some variants of preheating in which the inflaton decay is much more efficient than in the chaotic inflation toy model used so far in this section.

E. Tachyonic Preheating

In the chaotic inflation model we have studied up to this point the effective frequency of the χ oscillations is always positive. If it were negative, then we would obviously get an exponential instability. The simplest way to obtain this instability is to simply change the sign of the coupling term (13) in the interaction Lagrangian. This can be done without giving up stability of the model if

we add quartic potential terms that dominate at large field values but are unimportant during preheating. This model has “negative coupling resonance”, a mechanism that was proposed in [35]. Similar negative coupling instabilities can also occur in models with cubic interaction terms [36–38].

Another simple model in which the effective frequency is negative for a certain time interval is the symmetry breaking potential (1). For small field values, the effective mass of the fluctuations of ϕ is negative and hence a “tachyonic” resonance will occur, as studied in [39, 40]. For small field values, the equation for the fluctuations ϕ_k of ϕ is

$$\ddot{\phi}_k + (k^2 - m^2)\phi_k = 0. \quad (47)$$

Hence, modes with $k < m$ grow with an exponent which approaches $\mu_k = 1$ in the limit $k \rightarrow 0$. Given initial vacuum amplitudes for the modes ϕ_k at the initial time $t = 0$ of the resonance, the field dispersion at a later time t will be given by

$$\langle \delta\phi^2 \rangle = \int_0^m \frac{k dk}{4\pi} e^{2t\sqrt{m^2 - k^2}}. \quad (48)$$

The growth of the fluctuations modes terminates once the dispersion becomes comparable to the symmetry breaking scale.

Tachyonic preheating also occurs in hybrid inflation models like that of (2). In this case, it is the fluctuations of ψ which have tachyonic form and which grow exponentially [39]. Note that reheating in hybrid inflation was first studied in [41] using the tools of broad parametric resonance. Fermion production in this context was discussed in [42]. Fermion production is the context of tachyonic preheating was then analyzed in [43]. The quantum to classical transition of fluctuations in tachyonic preheating was investigated in [44, 45].

Another preheating mechanism which is more effective than the broad resonance process described above arises if the χ particles in the model of (9) are coupled linearly to fermions such that the χ particles created when $\phi \sim 0$ decay after half a ϕ oscillation, thus preventing the χ particles from slowing the decay of ϕ . This mechanism is called “instant preheating” [46].

IV. THERMALIZATION

A. Perturbative Considerations

Neither the perturbative decay of a Bose inflaton condensate (discussed in Section 3.a) nor the preheating mechanism discussed in later subsections of Section 3 produce a thermal spectrum of decay products. For many questions in cosmology it is not sufficient to know when inflation has terminated - it is crucial to know at what temperature the universe first takes on a thermal distribution. Examples are applications of reheating to baryogenesis and to nucleosynthesis constraints. In this section

we first discuss perturbative thermalization. Then we summarize the results of recent non-perturbative studies of inflaton decay. We begin with perturbative considerations.

In a full thermal equilibrium the energy density ρ and the number density n of relativistic particles scale as: $\rho \sim T^4$ and $n \sim T^3$, where T is the temperature of the thermal bath. Thus, in full equilibrium the average particle energy is given by: $\langle E \rangle_{\text{eq}} = (\rho/n)$, which obeys the scaling, $\langle E \rangle_{\text{eq}} \sim \rho^{1/4} \sim T$.

On the other hand, if the inflaton decays perturbatively, then right after the inflaton decay has completed, the energy density of the universe is given by:

$$\rho \approx 3(\Gamma m_{pl})^2, \quad \text{and} \quad \langle E \rangle \approx m \gg \rho^{1/4}, \quad (49)$$

(where m is the inflaton mass). Then, from conservation of energy, the number density of decay products is found to be

$$n \approx \left(\frac{\rho}{m}\right) \ll \rho^{3/4}. \quad (50)$$

Hence, perturbative inflaton decay results in a dilute plasma that contains a small number of very energetic particles.

Reaching full equilibrium requires re-distribution of the energy among different particles, *kinetic equilibrium*, as well as increasing the total number of particles, *chemical equilibrium*. Therefore both number-conserving and number-violating reactions must be involved.

The most important processes for kinetic equilibration are $2 \rightarrow 2$ scatterings with gauge boson exchange in the t -channel. (Scalar exchange in t -channel diagrams are usually suppressed, also vertices that arise from a Yukawa coupling are helicity suppressed.) The cross-section for these scatterings is $\sigma_{2 \rightarrow 2} \sim \alpha^2 |t|^{-1}$, where α is a gauge fine structure constant and the variable t is related to the exchanged energy ΔE and momentum, $\Delta \mathbf{p}$ through $t = \Delta E^2 - |\Delta \mathbf{p}|^2$. Due to an infrared singularity, these scatterings are very efficient even in a dilute plasma [47, 48].

Chemical equilibrium is achieved by changing the number of particles in the reheat plasma. From (50) it follows that in order to reach full equilibrium the total number of particles must *increase* by a factor of n_{eq}/n , where $n \approx \rho/m$ and the equilibrium value is: $n_{\text{eq}} \sim \rho^{3/4}$. This can be a very large number, e.g. $n_{\text{eq}}/n \sim \mathcal{O}(10^3)$. It was recognized in [47, 49] (see also [50, 51]) that the most relevant processes are $2 \rightarrow 3$ scatterings with gauge-boson exchange in the t -channel. The cross-section for emitting a gauge boson whose energy is $E \sim (\Gamma m_{pl})^{1/2}$ (where as in earlier sections of this review Γ is the inflaton decay rate), from the scattering of two fermions (up to a logarithmic ‘‘brenstrahlung’’ factor) is $\sigma_{2 \rightarrow 3} \sim \alpha^3 (\Gamma m_{pl})^{-1}$. When these scattering become efficient, the number of

particles increases very rapidly [52]³ As a result, full thermal equilibrium will be established shortly after that.

Based on the above analysis, one can use the rate for the above inelastic scatterings as the thermalization rate Γ_{th} of the universe. This rate at the time when the inflaton decay completes can be found by using Eqs. (49,50):

$$\Gamma_{\text{th}} \sim \alpha^3 \left(\frac{m_{pl}}{m}\right) \Gamma. \quad (51)$$

For typical values of $\alpha \sim 10^{-2} - 10^{-1}$ and $m \leq 10^{-5} m_{pl}$, we find $\Gamma_{\text{th}} \geq \Gamma$. Therefore the universe reaches full thermal equilibrium immediately after the completion of perturbative inflaton decay.

B. Non-Perturbative Considerations

The purely perturbative considerations of the previous subsection are subject to the same criticisms as the original perturbative analysis of the initial stages of reheating. Hence, we must turn to non-perturbative analyses. Some analytical approaches were pioneered in [6] and [7]. Numerical studies, however, have proved more powerful.

Since the occupation numbers of the excited modes are typically very high after the initial stages of preheating, a classical field theory analysis should be justified. Initial numerical studies were pioneered in [31–34, 53]. Two numerical package to perform such simulations are publicly available [54, 55]. Detailed numerical simulations of tachyonic preheating are given in [40]. Here, we will focus on numerical studies of preheating in models with narrow or broad parametric resonance [11].

The resonant phase of the reheating process described in Section 3 produces either (in the case of narrow resonance) field fluctuations in a narrow interval about the resonant frequency (which is set by the mass of the inflaton field), or else (in the case of broad or tachyonic resonance) field fluctuations at all wavenumbers smaller than the critical value given in (23), whose magnitude is set by the inflaton mass and amplitude.

Once the occupation numbers of the resonant modes become sufficiently large, re-scattering of the fluctuations begins. As first studied in [31–34] this terminates the phase of exponential growth of the occupation numbers. In the case of narrow resonance, new peaks in the spectrum of the number density $n(k)$ develop at harmonic frequencies. Soon, the spectrum shows excitations in a continuum band which reaches to $k = 0$ and has an ultraviolet (UV) cutoff whose value increases with increasing time.

As studied in detail in [11], the evolution of the field fluctuations evolves to a regime of turbulent scaling

³ Decay processes which were considered in [51], are helpful, but in general they cannot increase the number of particles to the required level.

driven by the remnant oscillations of the inflaton condensate. The resulting distribution of fluctuations is characterized by the spectrum

$$n(k) \sim k^{-3/2} \quad (52)$$

which is non-thermal (for a thermal distribution we would have $n(k) \sim k^{-1}$).

The evolution during the phase of turbulence has been shown [11] to be self-similar in the sense that as a function of time the spectrum scales as

$$n(k, \tau) = \tau^{-q} n_0(k\tau^{-p}), \quad (53)$$

where τ is a rescaled time ($\tau = t/t_0$, where t_0 is the time when the turbulent scaling regime begins), n_0 gives the initial distribution of the particles, and p and q are positive rational numbers whose values are determined in numerical simulations. This equation (53) describes the overall growth in the number of fluctuation quanta and at the same time gives the increase of the UV cutoff frequency as a function of time.

The phase of turbulence ends once most of the energy has been drained from the inflaton field. At this time quantum processes take over and lead to the thermalization of the spectrum. In the case of an $\mathcal{O}(N)$ scalar field model, the preheating and thermalization was considered analytically in the large N approximation [56], and applied to thermalization of fermions and gauge fields in hybrid inflation in [57] (see also [58] for a more general study of thermalization of quantum fields in an expanding universe).

Note that the time interval of the resonant preheating phase is given by the inflaton mass m and hence is very short on the Hubble time scale. The period when the initial re-scatterings take place and which ends when the turbulent scaling distribution becomes established is longer than the period of the initial resonance, but not by a large factor [11]. Thus, the time t_0 is of the same order of magnitude as the time when inflation ends. However, the period of driven turbulence is very long, in particular if all of the coupling constants in the field theory model are small. If c_r is the fraction of the energy density ρ_I at the end of inflation which remains in the inflaton at the beginning of the phase of turbulence, then a rough estimate of the time τ_{th} of thermalization is [11]

$$\tau_{\text{th}} \sim \left(\frac{(c_r \rho_I)^{1/4}}{m} \right)^{1/p}. \quad (54)$$

A typical value of p is $p = 1/7$. Given the normalization of m from the observed magnitude of the cosmological fluctuations we find $\tau_{\text{th}} \sim c_r^{7/4} 10^{21}$. Hence, we see that the temperature at which thermal equilibrium finally becomes established is low. The resulting value of the reheating temperature in fact agrees roughly with what is obtained from the perturbative arguments given in the previous subsection.

V. REHEATING IN SUPERSYMMETRIC MODELS

As an important application of the theory of reheating described in the previous sections we consider reheating in supersymmetric models. Supersymmetry (SUSY) introduces new degrees of freedom and new parameters, and a large number of scalar fields that may acquire large VEVs during inflation. These elements can affect various aspects of reheating that we discussed in Sections III and IV. Here we demonstrate some of the possible effects in the context of a well-motivated SUSY model.

A. Inflaton Couplings to Matter Fields

The minimal supersymmetric SM (MSSM) is a well-motivated extension of the SM (for reviews see e.g. [59, 60]). The new fields in the MSSM are scalar partners of leptons and quarks, called sleptons and squarks respectively, and fermionic partners of gauge and Higgs fields called gauginos and Higgsinos respectively. The MSSM superpotential is given by

$$W_{\text{MSSM}} = h_u Q H_u u + h_d Q H_d d + h_e L H_d e + \mu H_u H_d, \quad (55)$$

where H_u, H_d, Q, L, u, d, e in Eq. (55) are chiral superfields representing the two Higgs fields (and their Higgsino partners), left-handed (LH) (s)quark doublets, right-handed (RH) up- and down-type (s)quarks, LH (s)lepton doublets and RH (s)leptons respectively. The dimensionless Yukawa couplings h_u, h_d, h_e are 3×3 matrices in the flavor space, and we have omitted the gauge and flavor indices. The last term is the μ term, which is a SUSY version of the SM Higgs boson mass.

Now we consider inflaton couplings to the MSSM fields. For a gauge singlet inflaton, the only renormalizable coupling occurs through the superpotential term $2g\Phi H_u H_d$, with Φ being the inflaton superfield (for details, see [61]). Taking into account the inflaton superpotential mass term $(m/2)\Phi\Phi$, the *renormalizable part of the scalar potential* that is relevant for the inflaton decay into MSSM scalars is given by:

$$V \supset \frac{1}{2}m^2\phi^2 + g^2\phi^2\chi_1^2 + g^2\phi^2\chi_2^2 + \frac{1}{\sqrt{2}}gm\phi\chi_1^2 - \frac{1}{\sqrt{2}}gm\phi\chi_2^2 \quad (56)$$

where $\chi_{1,2}$ denotes the scalar component of $(H_u \pm H_d)/\sqrt{2}$ superfields, and we have only considered the real parts of the inflaton, ϕ , and $\chi_{1,2}$ fields.

We see that even in the simplest SUSY set up, the scalar potential is more involved than the non-SUSY case given in Eq. (13), which in turn can alter the picture of preheating presented in Section III (see the detailed discussion in Refs. [48, 61]). An interesting feature of Eq. (56) is that both the cubic $\phi\chi^2$ and quartic $\phi^2\chi^2$ interactions appear and SUSY naturally relates their

strengths⁴. Also, the inflaton coupling to fermionic partners of $\chi_{1,2}$ follows naturally from SUSY. The prospects for fermionic preheating will thus be the same as those for the bosonic case.

B. Supersymmetric Flat Directions

A key property of SUSY theories is the presence of flat directions in field space along which the potential identically vanishes (in the limit of unbroken SUSY). Such scalar fields (which are complex) can therefore obtain large VEVs along these special directions at no energy cost. These flat directions, which can be interpreted as a degeneracy of the vacuum state of SUSY theories, arise because of cancellations between fields of opposite charges in the D-term potential. A powerful tool for finding the flat directions has been developed in [62–64] (for reviews see [65, 66]). Flat directions are classified by gauge-invariant monomials $\prod_{i=1}^n X_i$, where X_i are chiral superfields of the model. This ensures that the D-term part of the potential vanishes⁵ along the direction $\langle \chi_1 \rangle = \dots = \langle \chi_n \rangle = \varphi$ (χ_i are scalar components of X_i). This corresponds to a two-dimensional subspace represented by a complex field φ . A flat direction VEV spontaneously breaks gauge symmetries and gives (SUSY conserving) masses to the gauge bosons/gauginos similar to the Higgs mechanism in electroweak symmetry breaking [48, 61, 63, 68, 69]. The induced masses for gauge/gaungino fields are $\sim \alpha^{1/2}|\varphi|$ (we recall that α is a gauge fine structure constant). Similarly, a flat direction VEV induces (SUSY conserving) masses $\sim h|\varphi|$ for those fields that have superpotential couplings to φ (h is a Yukawa coupling). Therefore all fields that are coupled to a flat direction obtain very large masses.

The flat directions are massless if SUSY is exact, but they are lifted when SUSY is broken (which is assumed to happen at a scale of the order of TeV), as a result of which they get a mass $m_\varphi \sim \mathcal{O}(\text{TeV})$. Provided that $m_\varphi \ll H_{\text{inf}}$, H_{inf} being the Hubble expansion rate during inflation, the flat direction can acquire a large VEV by the virtue of quantum jumps during inflation, see the discussion in Refs. [65, 66]. This can dramatically alter the post-inflationary history of the universe as we will see in the next subsections⁶.

C. Perturbative Decay

Consider a flat direction φ that has Yukawa couplings to the inflaton decay products χ . This happens, for example, for MSSM flat directions that are made of squark and/or slepton fields with χ being a MSSM Higgs field, see Eq. (56) (for details, see [61]). This results in the following term in the scalar potential:

$$V \supset h^2 |\varphi|^2 \chi^2, \quad (57)$$

where h denotes a Yukawa coupling. Note that the first generation of leptons and quarks have a Yukawa coupling $\sim \mathcal{O}(10^{-5})$, while the rest of the SM Yukawa couplings are $> 10^{-4}$. Since $|\varphi|$ is virtually frozen while $m_\varphi < H < H_{\text{inf}}$ it is only when $H \simeq m_\varphi$ that the flat direction starts its oscillations. Since the field is complex, typically an elliptical trajectory with an $\mathcal{O}(1)$ eccentricity will result [63]. Hence, $|\varphi|$ will redshift as $|\varphi| \propto H^{-1}$.

While the flat direction has a large amplitude, the induced mass of the inflaton decay products obtained via (57) will lead to the inflaton decay being kinematically forbidden as long as $h|\varphi| \geq m/2$. There are thus two criteria for perturbative inflation decay. First of all, the decay of the inflaton into χ particles must be kinematically allowed which will become possible once the induced χ mass drops below the inflaton mass m . Taking into account the fact that once H falls below the value m_φ the field amplitude of φ decreases linearly in H we find that the kinematic decay becomes possible once $H < \left(\frac{m}{h\varphi_0}\right) m_\varphi$, where φ_0 is the initial VEV of the flat direction. A second condition for perturbative inflaton decay to occur is that $H < \Gamma$, Γ being the rate for perturbative decay of the inflaton. Thus, the inflaton cannot decay until the Hubble rate has decreased to a value H_{dec} given by:

$$H_{\text{dec}} = \min \left[\left(\frac{m}{h\varphi_0} \right) m_\varphi, \Gamma \right]. \quad (58)$$

If φ_0 is sufficiently large, then we can have $H_{\text{dec}} \ll \Gamma$. This happens if $\varphi_0 > h^{-1} \frac{m_\varphi m}{\Gamma}$. Flat directions can therefore significantly delay inflaton decay on purely kinematical grounds.

D. Non-perturbative Decay

In order to understand the preheating dynamics in the presence of flat directions, we consider the governing potential that is obtained from Eqs. (56,57):

$$V = \frac{1}{2} m^2 \phi^2 + g^2 \phi^2 \chi^2 + \frac{g}{\sqrt{2}} m \phi \chi^2 + h^2 |\varphi|^2 \chi^2. \quad (59)$$

As mentioned in the previous section, we generically have $h > 10^{-4}$, and g can be as large as $\sim \mathcal{O}(1)$. After mode decomposition of the field χ , the energy of the mode with

⁴ Note that the cubic term is required for a complete decay of the inflaton field.

⁵ Since the total SM charge of a gauge-invariant monomial is zero by definition, the D-term potential involving only the fields used to build the monomial will also vanish since it is proportional to the sum of the charges.

⁶ The development of large VEVs requires that the flat directions do not obtain positive Hubble-induced supergravity corrections during inflation. This problem can be avoided, for example, by considering non-minimal Kahler potentials.

momentum k , denoted by χ_k , is given by:

$$\omega_k = \left(k^2 + 2g^2\phi^2 + \sqrt{2}gm\phi + 2h^2|\varphi|^2 \right)^{1/2}. \quad (60)$$

Let us freeze the expansion of the universe first. Including the expansion will not change our conclusions.

We will now show that there is also a kinematic blocking of preheating if the initial value of the flat direction fields is large. We consider the most efficient case for preheating, large field inflation, e.g. $\langle\phi\rangle > m_{pl}$. Note that for $g > 10^{-6}$, the inflaton induces a large mass $g\langle\phi\rangle > H_{\text{inf}}$ for χ during inflation. As a result, χ , quickly settles down to the minimum even if it is initially displaced, and remains there. Therefore, φ , does not receive any mass corrections from its coupling to χ during inflation.

As discussed in the previous subsection, in the interval $m_\varphi \leq H \leq m$, the flat direction VEV slides very slowly because of the under damped motion due to large Hubble friction term - it is effectively frozen. Non-perturbative production of χ quanta will occur if there is a non-adiabatic time-variation in the energy, e.g. that $d\omega_k/dt \geq \omega_k^2$. The inflaton oscillations result in a time-varying contribution to ω_k , while the flat direction coupling to χ yields a virtually *constant* piece. This constant piece weakens the non-adiabaticity condition. Indeed time-variation of ω_k will be adiabatic at all times, i.e. $d\omega_k/dt < \omega_k^2$, provided that $h^2|\varphi|^2 > g\Phi m$, where $\Phi \sim \mathcal{O}(m_{pl})$ is the amplitude of the inflaton oscillations. Thus, there will be no resonant production of χ quanta if

$$\varphi_0 > h^{-1} (gm_{pl}m)^{1/2}, \quad (61)$$

Similar arguments lead to a kinematical blocking of fermionic preheating, as the symmetry between bosons and fermions implies similar equations for the momentum excitations, see Eq. (60).

E. Thermalization

The flat direction VEV spontaneously breaks the SM gauge group. The gauge fields of the broken symmetries then acquire a SUSY conserving mass of the order of $\alpha^{1/2}|\varphi|$. This mass provides a physical infrared cut-off for scattering diagrams with gauge boson exchange in the t -channel. Thus the cross-section for inelastic scatterings is now given by $\sigma_{2\rightarrow 3} \sim \alpha^2|\varphi|^{-2}$. For large values of $|\varphi|$ the scattering rate is suppressed, which results in a delayed thermalization. It can be shown that the universe reaches thermal equilibrium when the Hubble expansion rate is (for details see Ref. [48])

$$H_{\text{th}} = \min \left[10\alpha^2 \left(\frac{m_{pl}}{\varphi_0} \right)^2 \frac{m_\varphi^2}{m}, \quad 10\alpha^2 \left(\frac{m_{pl}}{\varphi_0} \right)^2 \left(\frac{\Gamma}{m_\varphi} \right)^{1/2} \frac{m_\varphi^2}{m}, \quad \Gamma \right], \quad (62)$$

where Γ is the rate for perturbative inflaton decay. This yields the following expression for the reheat temperature

$$T_R \sim (H_{\text{th}}m_{pl})^{1/2}. \quad (63)$$

For very large values of φ_0 , thermalization is considerably delayed, e.g. $H_{\text{th}} \ll \Gamma$, and hence $T_R \ll (\Gamma m_{pl})^{1/2}$. This happens for

$$\varphi_0 > 3\alpha \frac{m_\varphi m_{pl}}{(m\Gamma)^{1/2}}. \quad (64)$$

In the above discussions it has been assumed that the flat direction condensate does not decay non-perturbatively. One may think that non-perturbative effects could also result in a fast decay of flat directions similar to preheating [36]. However, there is crucial difference between a rotating and a radially oscillating condensate. The F -term couplings do not lead to resonant particle production from a rotating condensate [70, 71]. It has been shown that D -term couplings of the flat direction can result in non-perturbative particle production [72]. However, unlike the case of a radially oscillating field, the produced quanta have momenta that are less than the mass of the condensate in this case.

The reason is that a rotating condensate has a $U(1)$ global charge⁷ that, in the case of MSSM flat directions, is identified with the baryon and/or lepton number [62, 63, 73]. Now, it is easy to show that the charge per particle in the rotating condensate is of $\mathcal{O}(1)$. Note also that the baryon and lepton number of MSSM fields, whether in the flat direction condensate or produced from the flat direction rotation, is $\pm 1/3$ and ± 1 respectively. Hence preservation of the baryon/lepton number by the D -term interactions, along with the conservation of energy density, implies that for an elliptic trajectory with $\mathcal{O}(1)$ eccentricity the number of produced quanta cannot be much smaller than the number of zero-mode quanta in the rotating condensate [69]. Therefore, the non-perturbative decay of a rotating flat direction does not change the thermalization picture described above.

The reason for a delayed thermalization is due to inducing large masses to the gauge/gauginos fields from the VEV of the flat direction. The induced mass by the plasma is given by $m_{\text{eff}}^2 \sim an/\langle E \rangle$, where $\langle E \rangle$ and n are the average energy and number density of quanta in the plasma respectively. Since they cannot be much different from those in the condensate, the induced masses will also be comparable.

⁷ This charge corresponds to the angular momentum of the rotating condensate in field space.

VI. CONSEQUENCES OF REHEATING/PREHEATING

A. Non-thermal Particle Creation

Reheating and preheating lead to non-thermal particle production, as we have seen in previous sections. In cosmology it is usually assumed that all particles start out in thermal equilibrium at the beginning of the Standard Cosmology phase. However, reheating begins with out-of-equilibrium decay of the inflaton oscillations and, as we discussed, decay products may not reach full thermal equilibrium immediately. During the transition from inflation to the Standard Cosmology various non-thermal processes take place and the assumption of thermal equilibrium of all particles clearly breaks down. In the following, we briefly mention a few applications of non-thermal particle production.

1. Baryogenesis and Leptogenesis:

The first application is to baryo- and leptogenesis. One of the several possible mechanisms to explain the observed asymmetry between baryons and antibaryons is to make use of out-of-equilibrium decay of superheavy Higgs and gauge particles [74]. If reheating were purely perturbative, particles as heavy as the inflaton could be created either in inflaton decay [75] or from scatterings of inflaton decay products [49, 76].

Preheating, however, provides a mechanism to produce a large population of superheavy scalar particles much heavier than the inflaton. In [77] this was studied making use of the same chaotic inflation Lagrangian which we have used in Section 3, with the χ scalars being the superheavy Higgs or gauge fields. A full numerical study [78] showed, however, that large self-interactions may terminate the resonance before it becomes effective.

Another way to generate the observed baryon to entropy ratio is via leptogenesis [79], a scenario in which initially an asymmetry in the lepton number is produced that is then partially converted into baryon asymmetry via SM sphalerons [80]. Preheating after inflation is a way to generate the initial lepton asymmetry. For example [81], preheating can produce a large number density of supermassive RH neutrinos in a model in which the inflaton couples to these neutrinos ψ via the standard fermionic preheating interaction term

$$\mathcal{L}_I = g\phi\bar{\psi}\psi. \quad (65)$$

If hybrid inflation occurs at a scale close to the electroweak scale, then the non-thermal production of particles may provide the out-of-equilibrium condition that is necessary in order to achieve electroweak baryogenesis [82].

2. Dark matter:

Another application of non-thermal particle creation during reheating is to excite dark matter. It is usually assumed that the dark matter particles are thermally distributed. This assumption is implicit in most current analyses of the prospects for dark matter detection in direct and indirect experiments. However, if the dark matter particles couple to the inflaton, then non-thermal production of dark matter during reheating is to be expected. If the dark matter particles have sufficiently strong interactions which allows them to thermalize during reheating, then the signatures of the initial non-thermal distribution will be washed out. However, if the interactions do not permit thermalization after inflation, then the predictions concerning the dark matter distribution will be quite different.

The production of out-of-equilibrium dark matter during preheating was put forwards in [81] and then studied in detail in [76]. In the latter reference, the superheavy dark matter particles produced during reheating were called “Wimpzillas”. Masses of Wimpzillas comparable to the grand unified theory (GUT) scale were considered (see also [83] for a discussion of a purely gravitational production mechanism for Wimpzillas). The dark matter abundance which can be obtained by the preheating channel is very model-dependent, whereas direct gravitational particle production produces dark matter of the required abundance for particle masses of $M_X \sim g^{1/2}10^{15}$ GeV [83].

3. Moduli and Gravitino Production

Preheating could also produce dangerous and unwanted particles [84]. An example are particles with gravitationally suppressed couplings and weak scale masses that arise in many theories beyond the SM. Overproduction of these particles could overclose the universe, if they are stable, or ruin the success of Big Bang nucleosynthesis (BBN) in the case of unstable relics. Here we consider moduli and gravitino production during preheating.

The existence of bosonic and fermionic moduli fields is common in SUSY and superstring theories. Moduli (bosonic modulus, χ , and fermionic modulus, ψ) are typically coupled to the inflaton via non-renormalizable interaction terms such as

$$\mathcal{L} \sim \phi^4 \frac{\chi}{m_{pl}} \quad (\text{bosonic}), \quad \mathcal{L} \sim \frac{\phi^2}{m_{pl}} \bar{\psi}\psi \quad (\text{fermionic}). \quad (66)$$

The production of moduli fields in chaotic and hybrid inflation reheating was analyzed in [85]. It was shown that moduli field can be parametrically amplified (their amplitude remains smaller than that of the inflaton fluctuations).

Another important example is the gravitino, the spin

3/2 partner of the graviton. Gravitinos are produced thermally from scatterings of light particles in the thermal bath. The number density of gravitinos thus produced can be obtained by solving the Boltzmann equation:

$$\dot{n}_X + 3Hn_X \simeq \langle \sigma v \rangle n_l^2, \quad (67)$$

where n_X is the number density of the gravitinos, σ is the production cross section which scales as m_{pl}^{-2} , and $v \sim c$ is the relative velocity of scatterers l whose number density is n_l . The resulting abundance is found to be [86, 87]

$$\frac{n_X}{s} \sim 10^{-2} \frac{T_R}{m_{pl}}, \quad (68)$$

where s is the entropy density and T_R is the reheat temperature of the universe. BBN gives rise to an absolute upper bound $(n_X/s) < 10^{-12}$ (the exact number depends on the gravitino mass and its decay modes), which in turn leads to an upper bound $T_R < 10^9$ GeV (see e.g. [88, 89]).

Gravitino production during preheating was studied in [90]. The gravitino equation of motion is the Rarita-Schwinger equation. Conformal invariance is broken during the reheating phase of inflationary cosmology. The presence of the oscillating inflaton field leads to a periodically varying correction to the effective gravitino mass that results in an instability in the same way that there is an instability for spin 0 and 1/2 particle modes. The exact strength of the instability depends sensitively on the precise SUSY inflationary model one is considering.

Gravitino with helicity $\pm 1/2$ component mainly contain the Goldstino component- the inflatino (superpartner of the inflaton), whose interactions are not suppressed by m_{pl} . One would naturally expect them to be created in large abundance. However, in realistic scenarios, where the scale of inflation is much higher than the scale of SUSY breaking, e.g. $H_{inf} \gg \mathcal{O}(100 \text{ GeV})$, it was argued in [91] and explicitly shown in [92] that the helicity $\pm 1/2$ states that are produced during preheating mainly decay in the form of inflatinos along with the inflaton.

B. Metric Preheating

1. Entropy fluctuations

As we have seen repeatedly in this article, the oscillating inflaton field has potential to lead to preheating of any fields it couples to. The metric itself is no exception. Metric fluctuations come in three types (for details see a review article on the theory of cosmological perturbations, e.g. [93]) - scalar modes, vector modes and tensor modes. In an expanding universe the vector modes are negligible since they decay. The tensor modes represent gravitational waves. The scalar modes are the ‘‘cosmological perturbations’’ which are sourced by matter fluctuations.

It is possible (see e.g. [93]) to choose a coordinate system in which the metric including scalar metric fluctuations is diagonal:

$$ds^2 = a^2(\eta) [(1 + 2\Phi)d\eta^2 - (1 - 2\Psi)\gamma_{ij}dx^i dx^j], \quad (69)$$

where Φ and Ψ are the two scalar metric fluctuation potentials. They are functions of space and time. In the absence of anisotropic stress (e.g. for scalar field matter) the two potentials are in fact equal. In the above, $a(\eta)$ is the scale factor of the background cosmology and η is conformal time defined via $dt = a d\eta$. Also, γ_{ij} is the background metric of the spatial sections after factoring out the cosmological expansion.

In the case of scalar field matter the equation of motion for the metric fluctuation variable Φ reads (see e.g. [93])

$$\Phi'' + 2(\mathcal{H} - \frac{\phi_0''}{\phi_0'})\Phi - \nabla^2\Phi + 2(\mathcal{H}' - \mathcal{H}\frac{\phi_0''}{\phi_0'})\Phi = 0, \quad (70)$$

where a prime indicates the derivative with respect to η and \mathcal{H} is the Hubble expansion rate in conformal time. Also, ϕ_0 denotes the background value of the scalar field. Note that in the above we are assuming that the perturbations are purely adiabatic (the relative density fluctuations in each matter component are the same). If there are entropy fluctuations present, there will be a source term on the right-hand side of (70) which is proportional to the entropy fluctuation.

It appears from (70) that the oscillations of the background inflaton field could induce parametric resonance of the metric fluctuations [94]. From our studies of preheating in Section 3 we would expect the long wavelength modes to be the most sensitive to this instability. However, a careful study [95] (see also [96, 97]) showed that there is no instability of adiabatic metric perturbations during reheating. This can most easily be seen by focusing on a new variable ζ , the curvature fluctuation on constant density hypersurfaces, which is given by

$$\zeta \equiv \frac{2}{3} \frac{H^{-1}\dot{\Phi} + \Phi}{1 + w} + \Phi, \quad (71)$$

where $w = p/\rho$ is the equation of state parameter. In the case of adiabatic fluctuations, ζ on scales larger than the Hubble radius H^{-1} satisfies the equation

$$\dot{\zeta}(1 + w) = 0. \quad (72)$$

In spite of the fact that during the inflaton oscillations $w = -1$ is reached, it can be shown that ζ does not change.

However, for entropy fluctuations the result is rather different [98, 99]. Provided that the entropy field (e.g. the χ field in the chaotic inflation preheating discussion of Section 3 or the field ψ in the hybrid inflation model) undergoes a parametric or tachyonic instability and thus leads to an exponential growth of the entropy fluctuation δS , then the curvature fluctuation ζ will inherit this

exponential growth. This follows since the entropy fluctuations act as a source for ζ - the equation (72) gets replaced by

$$\dot{\zeta} = \frac{\dot{p}}{p + \rho} \delta S. \quad (73)$$

This resonant growth of entropy fluctuations only is important in models in which the entropy fluctuations are not suppressed during inflation. Some recent examples were studied in [100, 101]. The source for this instability need not be the oscillations of the inflaton field. In SUSY models, the decay of flat directions can also induce this instability [102].

2. Gravity waves:

The equation of motion for gravitational waves is similar to that of scalar metric fluctuations (70) except that there is no coupling between the oscillating scalar field and the wave amplitude. Hence, there is no direct preheating of gravitational waves.

Nevertheless, gravitational waves can be produced by secondary processes. As analyzed in [103], gravitational waves can be produced from the interaction of the classical matter waves produced during preheating. This is a re-scattering effect. The induced gravitational wave spectrum is not scale-invariant but has a pronounced peak whose frequency is determined by the scale of inflation. For an inflation energy scale of 10^{15} GeV, the peak of the spectrum is at about 10^8 Hz. In hybrid inflation models, the scale of the peak can be in the kHz range relevant for Advanced LIGO [104]. The amplitude of the peak is a couple of orders of magnitude higher than the scale-invariant background of gravity waves produced directly during inflation.

Recently, several groups [105–108] have performed improved analyses of gravitational wave production during preheating. The formalism of [105, 107, 108] are quite general and were applied e.g. to a $\lambda\phi^4$ model of inflation [108]. In [109], the formalism for the generation of gravitational waves was applied to hybrid inflation, and [106] considered mostly gravitational wave production by the collision of bubbles formed during the tachyonic resonance in hybrid inflation models. The tachyonic instability leads to the formation of bubbles, and the collision of the bubbles is the primary source of gravitational waves. In presence of fermionic couplings, see Eq. 65, the inflaton

can fragment to form non-topological solitons [110], since the fragmentation of the inflaton condensate is inhomogeneous and anisotropic, it leads to large production of gravity waves as shown in [111].

VII. DISCUSSION AND CONCLUSIONS

We have presented an overview of theory and application of reheating in inflationary cosmology. Particular emphasis has been on the preheating mechanisms which in many models leads to rapid energy transfer between the inflaton and regular matter.

Our discussion of applications of reheating has been superficial due to lack of space. We have in fact not discussed a number of issues such as topological defect production during preheating [112–114], magnetic field generation [115, 116], induced non-Gaussianities (see e.g. [117–120]), preheating in theories with non-Standard kinetic terms (see e.g. [121] or extra dimensions (possible excitation of Kaluza-Klein modes), applications to multi-field inflation models [122] and to reheating in brane inflation models [123], and effects of noise on reheating [124, 125] (leading to a new proof [126] of Anderson localization).

It is important to point out that resonant phenomena which are important in reheating can also play a role in other areas of cosmology where there are oscillating scalar fields. One example is in the context of the MSSM where oscillating moduli fields can lead to resonances [102, 127]. The study of resonant effects in early universe cosmology is a rich area of research which has of now barely been touched. Many of the lessons learned in the context of inflationary reheating have more general applicability.

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