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Rejecting Mismatches by Correspondence Function

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Abstract A novel method ICF (Identifying point correspondences by Correspondence Function) is proposed for rejecting mismatches from given putative point correspondences. By analyzing the connotation of homography, we introduce a novel concept of correspondence function for two images of a general 3D scene, which captures the relationships between corresponding points by mapping a point in one image to its corresponding point in another. Since the correspondence functions are unknown in real applications, we also study how to estimate them from given putative correspondences, and propose an algorithm IECF (Iteratively Estimate Correspondence Function) based on diagnostic technique and SVM. Then, the proposed ICF method is able to reject the mismatches by checking whether they are consistent with the estimated correspondence functions. Extensive experiments on real images demonstrate the excellent performance of our proposed method. In addition, the ICF is a general method for rejecting mismatches, and it is applicable to images of rigid objects or images of non-rigid objects with unknown deformation.

Keywords Correspondence problem · Point correspondences · Deformable images · Image matching

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1 Introduction

Establishing reliable point correspondences between two images is a fundamental problem in computer vision. For two given images, corresponding points are the projections of a same point in a scene. Many of the computer vision algorithms rely on the successful finding of point correspondences between two images (Hartley and Zisserman 2003; Sonka et al. 1999), for example, stereo vision, motion analysis, object recognition, camera self-calibration, image mosaicking, etc.

In this work, we focus on rejecting mismatches from some given putative point correspondences. The putative correspondences are usually established by matching interest points with local information, for example, the intensity distribution in a small region around interest points, or some kind of local descriptor (Tico et al. 1999; Lowe 2004). However, usually a large proportion of the putative correspondences are mismatches due to viewpoint change, occlusion, local ambiguousness, etc. And the mismatches are usually enough to ruin the traditional estimation methods. Therefore, much of the endeavor in computer vision community is to eliminate or alleviate the undue influence of mismatches.

The existing methods for eliminating or alleviating the undue influence of mismatches can be broadly classified into three types: statistical robust regression methods, resampling methods, and case diagnostic methods. The statistical robust regression methods try to alleviate the undue influence by replacing the sum of squared error criterion with one less influenced by outliers, for example LMedS (Least-Median of Squares) (Rousseeuw 1984) and M-estimators (Huber 1981). In LMedS, the hypothesis is evaluated with the median residual of putative correspondences. This method can handle a large percentage of outliers, but its efficiency is very low. M-estimators are a class of robust methods. They minimize the sum of a symmetric, positive-definite function $\rho(\cdot)$ of residual r_i: $\sum \rho(r_i)$. The M-estimators require a good initial estimation for the parameter(s) to be estimated. The resampling methods, for example, RANSAC (RANdom SAmple Consensus) (Fischler and Bolles 1981) and MLESAC (Torr and Zisserman 2000; Tordoff and Murray 2005), act by trying to get a minimum subset of mismatch-free putative correspondences to estimate a given parametric model. However, the efficiency of these methods will decrease dramatically when mismatch percentage gets high. And they also suffer from the coupling problem of model selection and model estimation. The case diagnostic methods try to eliminate the undue influence of mismatches by checking the influence of putative correspondences on model estimation and rejecting mismatches directly. When there are many mismatches, however, this kind of method suffers from the problems of effectiveness and efficiency. Therefore, the resampling methods or pre-removing the most egregious mismatches by the resampling methods are commonly used when many mismatches are present. Since usually a large proportion of the putative correspondences are mismatches in practice, the existing resampling methods and case diagnostic methods in literature to identify correspondences, directly or indirectly, all suffer from the efficiency problem and the coupling problem among the correspondence identification, model selection and model estimation.

In this work, we introduce a novel concept of correspondence function. The fundamental idea is that there exist two functions to connect the corresponding points between two images by mapping a point in one image to its corresponding point in another. For example, given two images I and I' of a scene, we can define a function f, by which a point $p = (u, v) \in I$ can be mapped to a point $p' = (u', v') \in I'$, where p and p' are a pair of corresponding points between images I and I'. To deal with the exception of uniqueness constraint, another function from I' to I is needed as the complementarity of f. Thus, any correct corresponding points are consistent with at least one of the two functions f and f', where f and f' are called correspondence functions and (f, f') are called a correspondence function pair. Thus, for any putative corresponding points, we can identify them by checking whether they are consistent with the correspondence function pair.[1]

In practice, however, the correspondence functions are unknown for two given images. We further show how to estimate them from given putative correspondences by converting it into a function estimation problem. Compared with the traditional function estimation problem, the difficulty here is that usually a large percentage of the training samples are outliers, which are enough to ruin any traditional function estimation methods. Therefore, a novel algorithm IECF (Iteratively Estimate Correspondence Function) is proposed to cope with it. The IECF is an iterative algorithm, which can gradually eliminate the undue influence of outliers and robustly estimate correspondence function. Compared with the traditional diagnostic methods, the IECF takes a group of putative correspondences as a whole at each time to check whether they have undue influence on the estimation, rather than one by one in the traditional way, and it is shown to be more efficient in estimating correspondence function.

In theory, mismatches can be detected by checking whether or not they are consistent with the estimated correspondence functions. However, since the coordinates of putative corresponding points are usually corrupted by noise, the observed corresponding points usually do not strictly satisfy the equation of correspondence function in practice. Therefore, a measure is introduced to reflect the consistency of a putative correspondence with the estimated correspondence function, and correspondingly a novel algorithm ICF (Identifying point correspondences by Correspondence Function) is proposed to reject mismatches. Experiments show that the ICF is more accurate to reject mismatches as well as to preserve correct point matches than the RANSAC and M-estimators, two mostly used robust techniques in literature. Particularly, the ICF is more computationally efficient than the RANSAC. In addition, experiments show that the ICF is a general method for rejecting mismatches, which is applicable to images of rigid objects or images of non-rigid objects with unknown deformation.

The main contributions of our work include:

(1) The concept of correspondence function is introduced. Correspondence function captures the relationship between corresponding points by mapping a point in one image to its corresponding point in the other.
(2) An algorithm IECF is proposed to robustly estimate the correspondence functions from given putative point correspondences.
(3) An algorithm ICF is proposed to identify correct correspondences and reject mismatches accurately and efficiently by checking whether they are consistent with the estimated correspondence functions.

The remainder of this paper is organized as follows: In Sect. 2, we review the related works and their limitations. Section 3 elaborates the concept of correspondence function. Section 4 presents our algorithm IECF for estimating correspondence functions. A mismatch rejecting algorithm ICF is introduced in Sect. 5. Experimental results are reported in Sect. 6, followed by some concluding remarks in Sect. 7.

[1]Given a pair of corresponding points $p \in I$, $p' \in I'$, and a correspondence function pair $f: I \to I'$, $f': I' \to I$, if (p, p') is consistent with at least one of the two correspondence functions f and f', then we say that (p, p') is consistent with the correspondence function pair (f, f').

2 Related Works, Problems and Focus

In the literature, the undue influence of mismatches is usually tackled by some kind of statistical robust regression methods, resampling methods and case diagnostic methods.

(a) The statistical robust regression methods try to alleviate the undue influence by replacing the traditional sum of squared error criterion with one less influenced by outliers. Two of the most popular statistical robust regression methods are LMedS (Least-Median of Squares) (Rousseeuw 1984) and M-estimators (Huber 1981).

LMedS is a robust method proposed firstly in statistics (Rousseeuw 1984). In this method, the hypothesis is evaluated with the median residual of putative correspondences. Deriche and Zhang et al. (1994, 1995) addressed mismatch-rejecting with LMedS for Fundamental Matrix estimation. Torr and Murray (1993) studied Fundamental Matrix estimation and motion segmentation. LMedS can handle a large percentage of outliers, but the main shortcoming is its low computational efficiency. M-estimators are a type of robust methods. They minimize the sum of a symmetric, positive-definite function $\rho(\cdot)$ of residual r_i:$\sum \rho(r_i)$. Reza and Alireza (2007) studied Fundamental matrix estimation and motion segmentation with M-estimators. However, the M-estimators require a good initial estimation for the parameter(s) to be estimated.

(b) The resampling methods try to eliminate the undue influence by repeatedly generating hypothesis based on the estimation of a parametric model from a minimum number of randomly selected putative correspondences, evaluating it by some means, and finally choosing the hypothesis with the highest score to reject mismatches. Two of the most popular resampling methods are RANSAC (RANdom SAmple Consensus) (Fischler and Bolles 1981), and MLESAC (Torr and Zisserman 2000; Tordoff and Murray 2005).

RANSAC is a robust paradigm originated in vision community. It evaluates the hypotheses with the number of putative correspondences whose residuals are below a given threshold. By rejecting mismatches based on RANSAC, Torr and Murray (1993, 1995) studied Fundamental Matrix estimation and motion segmentation; Torr and Zisserman (1997) investigated the robust estimation of trifocal tensor; Nister (2005) researched pose estimation and live structure from motion. Many of the works to improve RANSAC are reviewed in Subbarao and Meer (2006), Zhang and Kosecka (2006). The MLESAC is a novel paradigm motivated in maximum likelihood estimation framework. In this method, the hypotheses are evaluated with likelihood instead of the number of putative correspondences whose residual are below some given threshold. Torr and Zisserman (2000) studied mismatch-rejecting and its application in estimating fundamental matrix, homography, and quadratic transformations. Tordoff and Murray (2005) speeded up the MLESAC by utilizing the prior probabilities of the correctness of putative correspondences and studied its application in motion estimation.

Main idea behind resampling methods is to get a minimum sample[2] of mismatch-free putative correspondences to estimate one selected parametric model, and reject mismatches by checking their consistency with the estimated model. Such methods can tackle the problems with a high percentage of mismatches in theory. Their major shortcoming is that the computational efficiency will decrease dramatically with the increase of mismatch percentage and the number of model parameters to be estimated. To get at least one mismatch-free sample with probability p, the number N of selected samples should satisfies (Hartley and Zisserman 2003; Rousseeuw and Leroy 1987)

$$N \geq N_p = \lceil \log(1-p)/\log(1-e^s) \rceil, \tag{1}$$

where e is the percentage of correct matches in putative correspondences, and s is the minimal number of putative correspondences necessary to estimate the selected parametric model. Some examples of N_p are presented in Table 1. It is shown that the efficiency of resampling methods is affected strongly by the percentage of correct matches and the number of the model parameters to be estimated.

(c) The case diagnostic methods try to alleviate the undue influence by rejecting mismatches directly before estimating the parametric model of a vision task.

Torr and Murray (1993) proposed a mismatch-rejecting method by extending Cook's squared distance to orthogonal regression and studied its application in motion segmentation by estimating fundamental matrix. This is a single case deletion method. And it is reported that the method is very effective and efficient when there is only a single or a small number of mismatches (Torr and Murray 1993). However, it is vulnerable when there are many mismatches. For L_∞ triangulation, Sim and Hartley (2006) presented a mismatch-rejecting method by repeatedly estimating selected model and throwing away the putative corresponding points with maximal residual. Li (2007) gave a novel algorithm for exactly removing up to k putative correspondences as mismatches by enumerating all local minima of the quasi-convex cost functions up to level k, where k is the estimated upper bound of mismatch number.

The case diagnostic methods are very successful when there is only a single or a small number of mismatches. When there are many mismatches (and usually this is the case in practical vision problem), this kind of methods usually suffer from masking effect (Rousseeuw and Leroy 1987; RahmatullahImon 2005), swamping effect (Barnett and Lewis. 1994) and computational problem (Li 2007;

[2] A minimum sample is a set of minimum number of putative correspondences to estimate one selected parametric model.

Table 1 The least number N_p of samples required to ensure that at least one sample is mismatch-free with probability 0.995, where e is the percentage of correct matches in putative correspondences, and s is the minimal number of putative correspondences that are required to estimate the selected parametric model

$e(\%)$	80	70	60	50	40	30	20	15
$s=2$	6	8	12	19	31	57	130	233
$s=4$	11	20	39	83	205	652	3309	10464
$s=6$	18	43	111	337	1291	7266	82784	465145
$s=8$	29	90	313	1354	8082	80753	2069653	20673207

Liang and Kvalheim 1996). Masking effect is that some mismatches are made invisible by others, and swamping effect is that some correct matches are considered as mismatches. When there are many mismatches, the computational load of this kind method will be very expensive, since they are all based on some greedy search scheme. Therefore, it is a common practice to use the resampling methods (Torr and Murray 1993), or pre-removing the most egregious mismatches by the resampling methods (Sim and Hartley 2006; Li 2007), when many mismatches are present.

In this work, we introduce a novel concept of correspondence function, and propose a model-independent mismatch-rejecting scheme ICF to cope with the above problems.

3 Correspondence Function

Given two images $I : U \times V$ and $I' : U' \times V'$ of a scene, the objective of correspondence problem is to find the relationship between them that connects the image points of a same scene point. If the scene is a plane, there is a one-to-one function relationship between the corresponding points, which can be characterized by homography, and for convenience, we name the function homography-function. And any corresponding point pair is consistent with the homography-function. For two images of a general three-dimensional scene, we will introduce a novel concept "correspondence function" to connect the corresponding points by extending the homography-function, and study its application in rejecting mismatches from given putative correspondences.

If two images I and I' are projected from a general three-dimensional scene, the relationship between the corresponding points may not be one-to-one, it is possible that one point in I corresponds to multiple points in I', and vice versa; On the other hand, function is a many-to-one mapping in mathematics: the output of any input is unique, and multiple inputs can share one output. Therefore, one function is not enough to characterize the relationship between corresponding points in such general circumstances. In this work, we will extend the homography-function in two aspects:

- Extend the linear homography-function to a general vector-valued function.
- Extend the unidirectional mapping, homography-function, to bi-directional mapping. For plane-images, there is a one-to-one relationship between the corresponding points, and one homography-function from I to I' or from I' to I is enough to characterize it. In the general 3-dimensional scene case, we will introduce two functions $f : I \to I'$ and $f' : I' \to I$ to characterize the relationship between corresponding points.

Example 1 For readability, before presenting the concept of correspondence function, we give an example of it, for example, $f : I \to I'$. For a given point $p = (u, v) \in I$, if there exists only one point $p' = (u', v') \in I'$ corresponding to p, then f should be defined $f(u, v) = (u', v')$; Otherwise, if multiple points $S_p = \{(u'_i, v'_i)\} \subset I'$ are corresponding points of p, then f can be defined as $f(u, v) = (u'_{i_0}, v'_{i_0})$, where (u'_{i_0}, v'_{i_0}) is any one selected point from S_p. We have defined a function $f : I \to I'$, and another function $f' : I' \to I$ can be defined similarly.

Remark 1 The above f and f' share the basic characteristics of homography-function: For any corresponding point pair $p = (u, v) \in I \leftrightarrow p' = (u', v') \in I'$, if the corresponding point of p is unique in I', then p and p' are consistent with f according to the definition of this function; Otherwise, if multiple points $S_p = \{(u'_i, v'_i)\} \subset I'$ are corresponding points of p, then p' is a point in S_p, and p and p' are consistent with f'; That is to say, anyone pair of the corresponding points between I and I' is consistent with at least one of the two functions f and f'.

Remark 2 In Remark 1, we assume that there are three kinds of corresponding points between images I and I' of a scene: one-one, one-multiple, multiple-one.[3] This assumption is usually acceptable.

[3] Given a corresponding point pair $p \in I$ and $p' \in I'$, if the corresponding point of p is unique in I' and the corresponding point of p' is also unique in I, then (p, p') is called a one-one type point correspondence; Otherwise, if multiple points in I are corresponding points of p' and the corresponding point of p is unique in I', then (p, p') is called a multiple-one type point correspondence; One-multiple type point correspondence can be defined similarly.

Remark 3 In this work, we assume that $S_p \subset I'$ contains all of the corresponding points of point $p \in I$.

Thus, we can give the definition of correspondence function.

Definition 1 Given two images I and I' of a scene, and a pair of functions

$$f : I \to I', \tag{2}$$

$$f' : I' \to I. \tag{3}$$

If they satisfy that, for any corresponding point pair $p \in I$ and $p' \in I'$, (p, p') is consistent with at least one of the two functions, then f and f' are called a pair of *correspondence function (CF)* of images I and I', and (f, f') are called correspondence function pair.

For any two images I and I' of a scene, Example 1 defines two functions $f : I \to I'$ and $f' : I' \to I$. Based on the analysis in Remark 1, it is known that, for any corresponding point pair $p \in I$ and $p' \in I'$, they are consistent with at least one of the two functions f and f'. That is to say, f and f' meet Definition 1, and they are correspondence functions. Therefore, Example 1 actually gives a method to define correspondence functions between any two images of a scene, and shows the existence of correspondence function.

Conclusion 1 For any image pair I and I' of a scene, there exist correspondence function and correspondence function pair between them.

4 Learn the Correspondence Functions

Based on the discussion in Sect. 3, for any image pair I_1 and I_2 of a scene, there exist a correspondence function pair (f, f') between them (Conclusion 1), and any corresponding point pair between I_1 and I_2 is consistent with at least one of the two correspondence functions (Definition 1). Therefore, if the correspondence functions are known, then we can reject mismatches by checking whether the putative corresponding points are consistent with f or f'.[4]

However, in real applications, the correspondence functions are unknown, and usually only a set of putative point correspondences contaminated with many mismatches is available. In this work, we will study how to estimate the correspondence functions f and f', and further reject mismatches based on the estimated correspondence functions.

[4] Although in theory, we cannot reject all of the mismatches by this method, experiments show that this method is sufficient for pre-removing most of the egregious mismatches.

4.1 Fundamentals: Subspace Projection

In this subsection, we will study how to convert the correspondence function estimation problem into a usual regression problem. *Correspondence functions (CF)* f and f' are two vector-valued functions between a pair of images I and I'. They can be rewritten as

$$f(u, v) = (u', v') = (g_1(u, v), g_2(u, v)), \tag{4}$$

$$f'(u', v') = (u, v) = (g'_1(u', v'), g'_2(u', v')), \tag{5}$$

where g_i and g'_i are usual scalar functions with two variables, $i = 1, 2$. For convenience, g_i and g'_i are called CF component function. Therefore, it is sufficient to discuss how to estimate CF component function from given putative point correspondences, e.g. $g_1(u, v) = u'$.

Given a set of putative corresponding points

$$S = \{(p_i, p'_i) = (u_i, v_i, u'_i, v'_i), i = 1, \dots, n\} \subset I \times I' : U \times V \times U' \times V', \tag{6}$$

we denote its projection in subspace $U \times V \times U'$ by $S_{U \times V \times U'}$, that is,

$$S_{U \times V \times U'} = \{(u, v, u') | (u, v, u', v') \in S\}. \tag{7}$$

For any $(u, v, u') \in S_{U \times V \times U'}$, if the inverseimage[5] of (u, v, u') is a correct match that is consistent with correspondence function f, then (u, v, u') would be consistent with CF component function $g_1(u, v) = u'$. Therefore, if $S_{U \times V \times U'}$ is regarded as a sample set from CF component function g_1, then g_1 can be estimated from $S_{U \times V \times U'}$ by regression methods.

Similarly, we can define

$$S_{U \times V \times V'} = \{(u, v, v') | (u, v, u', v') \in S\}, \tag{8}$$

$$S_{U \times U' \times V'} = \{(u, u', v') | (u, v, u', v') \in S\}, \tag{9}$$

$$S_{V \times U' \times V'} = \{(v, u', v') | (u, v, u', v') \in S\}, \tag{10}$$

and g_2, g'_1 and g'_2 can be estimated from $S_{U \times V \times V'}$, $S_{U \times U' \times V'}$ and $S_{V \times U' \times V'}$ respectively.

In conclusion, from a set of given putative correspondences, correspondence function can be estimated by the SP (Subspace Projection) algorithm in Table 2. Further on estimating correspondence function will be discussed in the next subsection.

[5] Given $(p, p') = (u, v, u', v') \in S$ and its projection $(u, v, u') \in S_{U \times V \times U'}$, we say $(p, p') = (u, v, u', v') \in S$ is the inverseimage of (u, v, u'), and (u, v, u') is the image of $(p, p') = (u, v, u', v')$ in subspace $U \times V \times U'$.

4.2 Refine the Estimation Iteratively

In this subsection, we will discuss the correspondence function f only, f' can be similarly done.

In theory, correspondence function f can be estimated by SP algorithm (Table 2) embedded with anyone nonparametric regression method. In correspondence problem, however, putative correspondences are usually corrupted with noise and the percentage of mismatches is high, sometimes more than 40% or 50%, due to viewpoint change, occlusion, locally ambiguous regions, shadow, noise, etc. And the putative correspondences, which are inconsistent with correspondence function f, are called outliers of f. In real applications, there are two kinds of possible outliers for f: one is mismatches, the other is the correct matches that are inconsistent with f (according to the definition of correspondence function, the correct matches are not necessary to be consistent with both f and f' simultaneously). Therefore, there exist usually many outliers in estimating correspondence function f. Many of the outliers may have undue influence on the estimation of f, and they usually are called influentials in robust statistics. The influentials usually can ruin the traditional regression methods (Rousseeuw and Leroy 1987; Barnett and Lewis. 1994).

Our research shows that the regression method SVM (Support Vector Machine) is very robust against outliers. Even if there is a large percentage of outliers in training data set, SVM still can capture the general trend of the data (Fig. 1). In such circumstances, although the SVM estimation are too coarse to be used for detecting mismatches accurately, it is still enough to help us detect some of the most egregious observations. After deleting them, an improved estimation can be re-estimated. Iteratively using this scheme, we could peel off the outliers that have undue influence on estimation (influentials), and obtain an acceptable estimation. The proposed iterative algorithm is presented in Table 3.

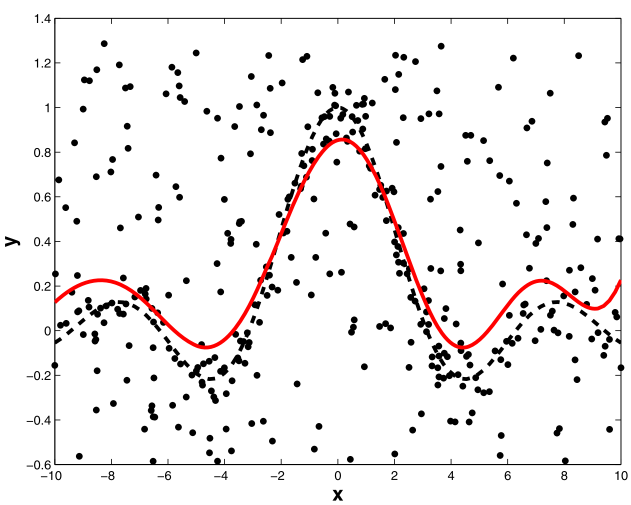

Fig. 1 Robustness of SVM: 150 observations are sampled from $y = \sin(x)/x$ with noise $N(0, 0.1)$, and 225 observations are randomly generated from the area $[-10, 10] \times [-0.6, 1.4]$. The *dotted line* is $y = \sin(x)/x$, the *solid line* is the estimation of SVM

With our proposed IECF algorithm, three questions immediately arise:

(1) How to Select a Suspect Influential Subset

For a given estimation $\hat{f}(u, v)$ of the correspondence function $f(u, v)$, we propose to select the suspect influentials by residual analysis as:

$$S_c = \{(p, p') \,|\, |e_1(p, p')| > \tau\sigma_1, \text{or} \quad |e_2(p, p')| > \tau\sigma_2, (p, p') \in S\}, \tag{11}$$

where $\tau > 0$ is a preset threshold, and $e_i(p, p')$ is the estimation error

$$e_i(p, p') = \hat{g}_i(p, p') - g_i(p, p'), (p, p') \in S, \quad i = 1, 2. \tag{12}$$

Suppose the estimation error e_i follows a Gaussian probability distribution with zero mean and standard deviation σ_i,

Table 3 IECF (Iteratively Estimate Correspondence Function) Algorithm: Iteratively estimate correspondence function f by peeling off outliers gradually. Correspondence function f' can be estimated similarly. SP(SVM): the algorithm SP embedded with SVM regression method in step 2) (Table 2)

Assume S is a set of putative correspondences:

(1) Estimate correspondence function $f(u, v)$ from S by algorithm SP(SVM), and denote the estimation as $\hat{f}(u, v)$.
(2) Choose a subset S_c from S as suspect influentials based on $\hat{f}(u, v)$.
(3) Reestimate $f(u, v)$ from $S_- = S - S_c$, and get $\hat{f}_-(u, v)$.
(4) Determine the influence of S_c by comparing $\hat{f}(u, v)$ and $\hat{f}_-(u, v)$.
(5) If S_c have undue influence on $\hat{f}(u, v)$, then S_c is rejected as influentials, and let $\hat{f}(u, v) = \hat{f}_-(u, v)$, $S = S_-$, go to (2).
(6) If the influence of S_c is appropriate, then it is assumed that there are no more influentials in S and terminate the procedure.

Table 2 SP (Subspace projection) Algorithm: given a set of putative correspondences S, the correspondence function $f(u, v) = (u', v') = (g_1(u, v), g_2(u, v))$ can be estimated from the projections of S. And the correspondence function $f'(u', v')$ can be estimated similarly

(1) Project the putative correspondences S into the subspaces by (7) and (8).
(2) By regression method, the CF component functions g_1 and g_2 are estimated from $S_{U\times V\times U'}$ and $S_{U\times V\times V'}$, respectively.

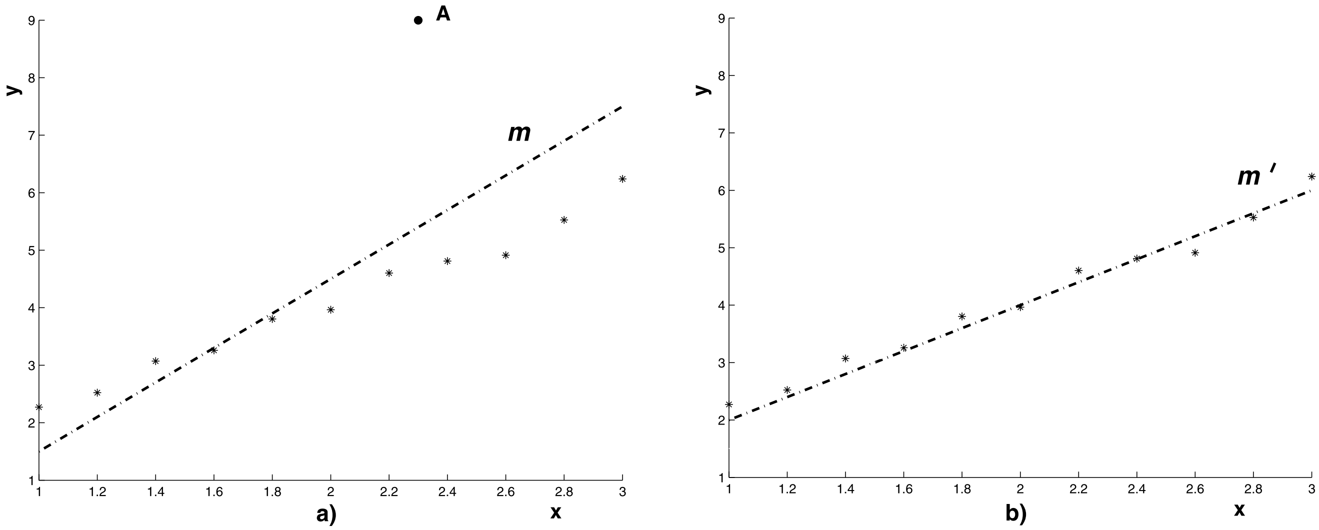

Fig. 2 A sketch map: after deleting some influential observations, the MSE on the effective data set usually becomes smaller. (**a**) A set S that consists of noisy samples from straight line $y = 2x$, with one influential sample A, m is the estimated line from S; (**b**) the data set $S' = S - \{A\}$, m' is the estimated line from S'

then for a given confidence level $0 < \alpha < 1$, we can compute the threshold τ in inequality (11) by the following condition

$$P(|X| \le \tau_\alpha) = \alpha, \tag{13}$$

where X is a random variable following the standard normal distribution $N(0, 1)$. For example, if $\alpha = 0.9544$, then $\tau_\alpha = 2$, and by the inequality (11), we can label the putative correspondences that have undue influence on the estimation, and simultaneously non-influential corresponding points can be preserved with probability 0.9544.

In practice, the standard deviation $\hat{\sigma}_i$ are unknown and the following estimation

$$\hat{\sigma}_i = \sqrt{\sum_{(p,p')\in S} e_i^2(p, p')/n} \tag{14}$$

are adopted, where $\{e_i\}$ are defined in (12).

(2) How to Evaluate the Influence of the Suspect Influential Subset S_c

Upon the assumption that there is sufficient redundancy in S for current estimation, if S no longer contains influential observations, then the estimation of the distribution of residual e_i should not be affected too much by deleting the suspect subset $S_c \subset S$. Furthermore, based on the assumption of zero mean Gaussian distribution, statistical characteristics of estimation error e_i are determined completely by their variances. Therefore, the influence of S_c can be evaluated by comparing $\hat{\sigma}_i^2$ and $\hat{\sigma}_{i-}^2$, where $\hat{\sigma}_i^2$ and $\hat{\sigma}_{i-}^2$ are respectively the estimations of σ_i^2 before and after removing the suspect subset S_c. To offset the disturbance of measurement units, we propose $INFL_i \triangleq (\hat{\sigma}_i^2 - \hat{\sigma}_{i-}^2)/\hat{\sigma}_i^2$ instead of $\hat{\sigma}_i^2 - \hat{\sigma}_{i-}^2$ to be used as evaluation criterion.

(3) How to Terminate the Iterative Procedure

In the algorithm IECF, correspondence function is estimated iteratively, a rule is needed to terminate the iterative procedure.

The MSE (Mean Squared Error) reflects the consistency of a data set with the estimated model. For example in Fig. 2, after deleting the influential observation A, the effective data set $S' = S - \{A\}$ is more consistent with the estimation m' than S with m, and the MSE on S

$$MSE_S = \sum (y_i - m(x_i))^2/size(S) \tag{15}$$

is usually larger than that on S'

$$MSE_{S'} = \sum (y_i - m'(x_i))^2/size(S'). \tag{16}$$

Furthermore, just as shown in Fig. 3, by iteratively diagnosing the influentials, the remaining putative correspondences will become more and more consistent with the estimation of the general trend, and the MSE decreases rapidly and tends to zero. Therefore, we can set a threshold $\varepsilon_{MSE} > 0$, and when the MSE is less than ε_{MSE}, the influencials, if there are still, can be neglected, and terminate the iterative procedure.

The influence of suspect influential subset in two experiments are illustrated in Fig. 4. Most of the influentials are usually identified at the first several iterations, including those observations with top undue influence on estimation. For example, in the experiment of Fig. 5, approximately

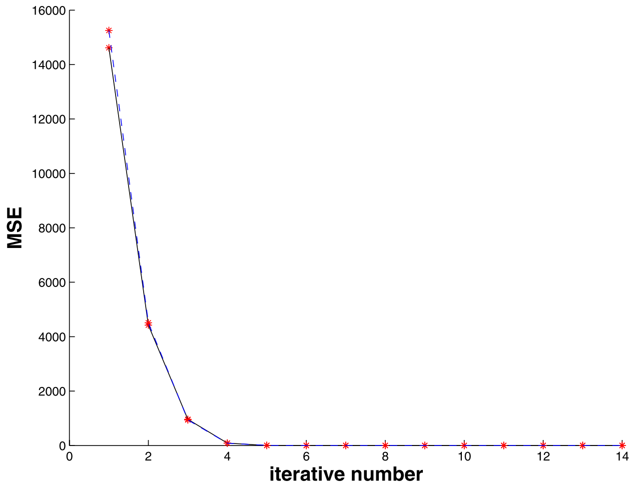

Fig. 3 The changing of MSE when correspondence functions are estimated iteratively in one experiment

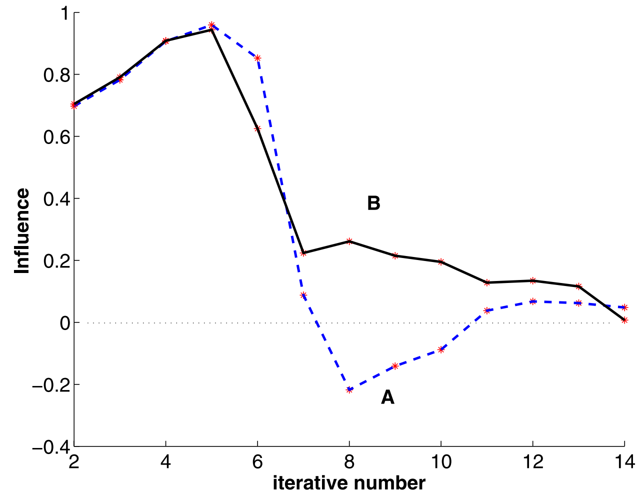

Fig. 4 The changing of influence $INFL_i$ of the suspect influential subset S_c when correspondence functions are estimated iteratively in two experiments

58.79% influentials are detected in the first two iterations, and approximately 96.15% of the influentials are identified in the first four iterations. Therefore, the influence of suspect subset is very high at the beginning. With more iterations, the number of influentials becomes less and less, and the selected putative correspondences are also usually the ones that have less influence on the estimation than those selected in the first several iterations. Hence, the influence tends to vanish when the iterative number become large[6]. In practice, we can use a threshold $\varepsilon_{INFL} > 0$, and terminate the algorithm when $INFL_i \leq \varepsilon_{INFL}$.

In the above identification process, after nearly all of the influentials have been identified, some of the vital observations, to correctly estimate the correspondence functions, may sometimes be wrongly selected into the suspect subsets, then the influence $INFL_i$ may be less than zero, e.g. at the eighth iteration in the experiment A of Fig. 4. And in this case, the iterative process should be terminated.

In all the experiments of this work, the parameters are set as $\varepsilon_{MSE} = 64$, $\varepsilon_{INFL} = 0.3$.

More on IECF: In algorithm IECF, the embedded SVM are implemented by LIBSVM (Chang and Lin 2001) with radial basis kernel $e^{-\gamma(u-v)^2}$. Apart from γ, there are two parameters C and ε in SVM, the interested readers are referred to (Chang and Lin 2001; Vapnik 1998; Smola and Schölkopf 2004). In theory, the optional values of these parameters can be selected by the tool gridregression, which is provided in LIBSVM (Chang and Lin 2001), in each iterative procedure of every experiment. We tested various values for C, ε and γ, and experimental results show that the performance of our proposed scheme is not sensitive to these parameters.[7] The following setting are used in all of our experiments: $C = 512, \gamma = 0.00000009765625, \varepsilon = 0.25$, which are selected by the girdression tool (Chang and Lin 2001) in one experiment, and kept unchanged in all the other experiments.

5 Rejecting Mismatches

In theory, correct corresponding points are consistent with at least one of the two correspondence functions. However, since the coordinates of putative corresponding points are usually corrupted by noise, the observed corresponding points usually do not strictly satisfy the equation of correspondence function $f(u, v) = (u', v')$ or $f'(u', v') = (u, v)$ in practice. Therefore, we need a measure to reflect the consistency of a putative correspondence with correspondence functions.

Suppose $p = (u, v)$ and $p' = (u', v')$ are a pair of corresponding points, let

$$e(p, p') = f(p) - p' = (e_1, e_2), \qquad (17)$$

where e_1 and e_2 are defined in (12). Then, $e(p, p')$ scores the consistency of putative pair (p, p') with correspondence

[6]However, when many influentials are present, diagnostic learning procedure usually suffers from masking effect, which means that some of the influentials with large undue influence are made invisible by others (Rousseeuw and Leroy 1987; RahmatullahImon 2005). The masked influentials will be detected gradually after the barrier influentials are removed. Therefore, the influences usually increase in the first several iterations in case many influentials exist, for example, in the experiment of Fig. 4.

[7]Our experiments also show that the optional values of τ_α in (13) and ξ_ε in (20) for the proposed algorithm will change when we adjust the SVM parameters C, ε and γ.

function $f(u,v) = (u', v')$. However, it is a vector and not convenient to be used for rejecting mismatches directly. Let

$$c(p,p') = e(p,p')D(f)^{-1}e(p,p')^T, \tag{18}$$

where

$$D(f) = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \tag{19}$$

and σ_i^2 are the variances of $e_i, i = 1,2$ respectively. It is noted that the influence of the scale of e_i is eliminated in $c(p,p')$ and it effectively reveals the consistency of (p,p') with correspondence function $f(u,v)$. And then, the mismatches can be rejected by a simple thresholding, $c(p,p') > \xi$, where $\xi > 0$ is a preset threshold.

A remaining problem is how to choose the threshold ξ. Upon the assumption that e_1 and e_2 are independent random variables which obey Gaussian distributions $N(0,\sigma_1^2)$ and $N(0,\sigma_2^2)$ respectively, the measurement $c(p,p')$ would follow $\chi^2(2)$ distribution. And the threshold ξ can be computed by

$$P(Y < \xi_\varepsilon) = \varepsilon, \tag{20}$$

where Y is a random variable following $\chi^2(2)$ distribution, and ε is a given confidence level.

Similarly, a measurement $c'(p,p')$ can be defined to evaluate the consistency of putative pair (p,p') with correspondence function $f'(u',v')$. And the final algorithm is presented in Table 4 to reject mismatches.

Table 4 Algorithm ICF (Identifying point correspondences by Correspondence Function): Identifying correct matches and rejecting mismatches by checking the consistency of putative correspondences with the estimated correspondence functions

(1) Compute threshold $\xi_\varepsilon \geq 0$ by equation (20).
(2) Estimate the correspondence functions $f(u,v)$ and $f'(u',v')$ from putative correspondences by the algorithm IECF in Table 3.
(3) For every putative point pair $(p,p') \in S$, compute the consistency $c(p,p')$ and $c'(p,p')$; if $c(p,p') > \xi_\varepsilon$ or $c'(p,p') > \xi_\varepsilon$, then reject it as a suspect mismatch, otherwise accept as a suspect correct match.

6 Experiments and Discussions

In this section, we test the performance of our proposed ICF and verify the validity of correspondence function on real image pairs. In all our experiments, the putative correspondences are computed from the SIFT keypoints (Lowe 2004) by Nearest Neighbor method. The examinations are done from the following six aspects:

6.1 Identifying Correct Matches and Rejecting Mismatches

In Fig. 5(a), an image pair of a relievo is shown for establishing point correspondences. The putative correspondences are presented in Fig. 5(b), and approximately 45.61% of them are mismatches. By iteratively diagnosing the influentials, the two correspondence functions $\hat{f}(u,v)$ and $\hat{f}'(u',v')$ are estimated by algorithm IECF (Table 3). In the experiment, 98.90% of the mismatches are correctly detected by checking their consistency with the estimated correspondence functions. The percentage of mismatches is dramatically reduced from 45.61% down to 0.95%. The identified suspect correct matches and rejected suspect mismatches are presented in Fig. 5(c) and (d) respectively. More experimental results are presented in Figs. 6, 7, 8 and 9. Especially, in Fig. 7, the experimental results show that the mismatch-rejecting capability of our proposed scheme ICF is not affected by the large view angle; In Fig. 9, there is

Fig. 5 An image pair of a relievo: the mismatch percentage is reduced from 45.61% to 0.95%. (**a**) Original image pair; (**b**) 399 putative correspondences with 182 mismatches, correct match percentage is about 54.39%; (**c**) the identified suspect correct matches, the correct match percentage is increased to about 99.05%; (**d**) the rejected suspect mismatches. Parameters $\tau_\alpha = 1.96$, $\xi_\varepsilon = 10.597$. (For visibility, only 100 randomly selected point pairs are presented in (**b**), (**c**), (**d**))

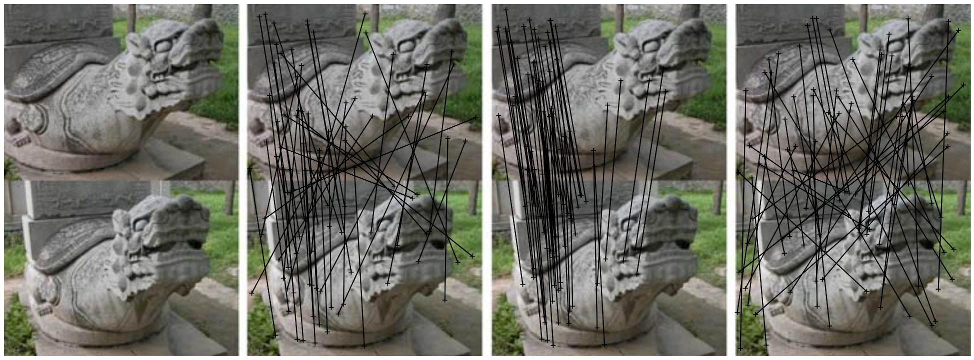

Fig. 6 An image pair of Baxia: the mismatch percentage is reduced from 51.85% to 1.40%. (**a**) Original image pair; (**b**) 1626 putative correspondences with 843 mismatches, correct match percentage is about 48.15%; (**c**) the identified suspect correct matches, the match percentage is increased to about 98.60%; (**d**) the rejected suspect mismatches. Parameters $\tau_\alpha = 1.96, \xi_\varepsilon = 10.597$. (For visibility, only 50 randomly selected point pairs are presented in (**b**), (**c**), (**d**))

Fig. 7 An image pair with a large view angle: the mismatch percentage is reduced from 79.61% to 1.84%. (**a**) Original image pair; (**b**) 2280 putative correspondences with 1815 mismatches, correct match percentage is about 20.39%; (**c**) the identified suspect correct matches, the correct match percentage is increased to about 98.16%; (**d**) the rejected suspect mismatches. Parameters $\tau_\alpha = 1.96, \xi_\varepsilon = 10.597$. (For visibility, only 100 randomly selected point pairs are presented in (**b**), (**c**), (**d**))

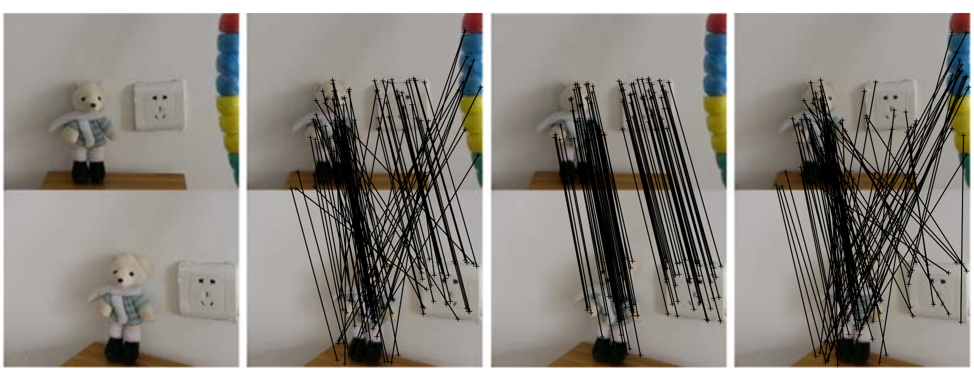

Fig. 8 An image pair of a toy bear: the mismatch percentage is reduced from 44.42% to 0.00%. (**a**) Original image pair; (**b**) 448 putative correspondences with 199 mismatches, correct match percentage is about 55.58%; (**c**) the identified suspect correct matches, the correct match percentage is increased to 100.00%; (**d**) the rejected suspect mismatches. Parameters $\tau_\alpha = 1.96, \xi_\varepsilon = 10.597$. (For visibility, only 100 randomly selected point pairs are presented in (**b**), (**c**), (**d**))

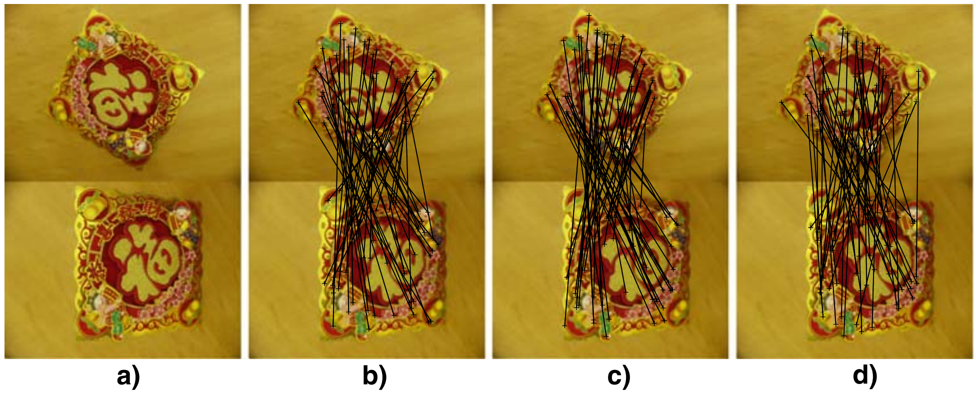

Fig. 9 An image pair of blessing with in-plane rotation, which makes the ordering in the correct matches more complex. In this experiment, the mismatch percentage is reduced from 34.40% to 0.00%. (**a**) Original image pair; (**b**) 846 putative correspondences with 291 mismatches, correct match percentage is about 65.60%; (**c**) the identified suspect correct matches, the correct match percentage is increased to 100%; (**d**) the rejected suspect mismatches. Parameters $\tau_\alpha = 1.96, \xi_\varepsilon = 10.597$. (For visibility, only 50 randomly selected point pairs are presented in (**b**), (**c**), (**d**))

an in-plane rotation between the two images, which makes both the ordering in the correct matches and the form of correspondence functions more complex.

6.2 Robustness: The Performance of ICF on Data Sets with Different Percentage of Mismatches

In this subsection, the robustness of ICF is investigated on putative correspondences with different percentage of correct matches.

Data sets are generated by the following procedure: the initial data set, denoted by S, is the putative correspondences in the experiment of Fig. 5(b), which consists of 217 correct matches, and 182 mismatches. The putative correspondences with different percentage of correct matches are obtained by adding additional mismatches into S. For example, if 144 additional mismatches are added into S, the correct match percentage of the data set will be $217/(399 + 144) \approx 39.96\%$. Here a mismatch is generated by randomly choosing one pixel from each one of the two images in Fig. 5(a) such that they do not satisfy the epipolar constraint. The experimental results are presented in Table 5.

Firstly, it is shown that the results are quite satisfactory, more than 98% of the mismatches are identified correctly although the lowest percentage of correct matches is down to 15.00%. The experiments also show that the capability of rejecting mismatches and identifying correct matches is not weakened by increasing of the mismatch percentage.

Secondly, the experimental results in Table 5 show that the proposed scheme can identify correct matches and reject mismatches both with a high probability. That is to say, the correct matches are usually consistent with at least one of the two correspondence functions, and the mismatches usually largely violate both correspondence function f and f'. Therefore, the underlying theory of our correspondence function is sound.

Table 5 Robustness of ICF. In this experiment, we examined the performance of ICF on putative correspondences with different mismatch percentage. S is the putative correspondences. Parameters $\tau_\alpha = 1.96, \xi_\varepsilon = 10.597$ (%)

Mismatch percentage of S	45.61	60.04	69.99	80.00	85.00
Identified mismatches	98.90	99.39	99.41	99.42	99.35
Identified matches	95.85	94.93	99.08	95.39	97.24

6.3 Necessity of Utilizing Both Correspondence Functions

As discussed in Sects. 4 and 5, the correct matches should be consistent with the correspondence function $f(u, v)$ or $f'(u', v')$ in theory. Therefore, mismatches can be removed by checking their consistency with correspondence functions $f(u, v)$ and $f'(u', v')$. The experimental results in Table 6 show that more than 98% of the mismatches are removed by either one of the two correspondence functions.

Due to the complexity & diversity of practical cases, however, it is inevitable that some samples do not obey the uniqueness constraint. Consequently some of the correct matches are consistent with only one of the two correspondence functions (Table 7), hence both of the two correspondence functions should be used in the mismatch removing process to preserve the correct matches as much as possible.

6.4 Factors Affecting the Efficiency of ICF

Mismatch percentage of the putative correspondences and selection of the parameter τ_α influence the computational efficiency of the proposed scheme.

The experimental results in Table 8 show that although the required computational time of ICF will increase with the increase of mismatch percentage, the computational load

Table 6 Contribution of correspondence function $f(u,v)$ and $f'(u',v')$ to removing mismatches. Percentage of the removed mismatches by $f(u,v)$ and $f'(u',v')$ in the experiments of Table 5 are presented respectively in this table. Parameters $\tau_\alpha = 1.96$, $\xi_\varepsilon = 10.597$ (%)

Mismatch percentage of S	45.61	60.04	69.99	80.00	85.00
Identified by $f(u,v)$	100.00	99.69	100.00	99.77	99.76
Identified by $f'(u',v')$	98.90	99.39	99.41	99.42	99.43

Table 7 Contribution of correspondence functions $f(u,v)$ and $f'(u',v')$ to identifying correct matches. Percentage of the correct matches identified by $f(u,v)$ and $f'(u',v')$ in the experiments of Table 5 are presented respectively in this table. Parameters $\tau_\alpha = 1.96$, $\xi_\varepsilon = 10.597$ (%)

Mismatch percentage of S	45.61	60.04	69.99	80.00	85.00
Identified by $f(u,v)$	80.65	82.03	79.26	83.87	79.72
Identified by $f'(u',v')$	84.79	82.03	88.94	81.11	84.79

Table 8 Efficiency of ICF. The efficiency with different mismatch percentage in the experiments of Table 5 was assessed. n_1 and n_2 are the iterative times in estimating correspondence functions $f(u,v)$ and $f'(u',v')$ respectively. S is the set of putative correspondences. Parameters $\tau_\alpha = 1.96$, $\xi_\varepsilon = 10.597$ (%)

Mismatch percentage of S	45.61	60.04	69.99	80.00	85.00
Iterative times $[n_1, n_2]$	[6, 5]	[8, 7]	[11, 9]	[19, 17]	[37, 30]

Table 9 Efficiency of ICF and influence of parameter τ_α on estimating correspondence functions. Putative correspondence set is the one in the experiment of Table 5 with mismatch percentage 69.99%. Three thresholds τ_α 1.65, 1.96 and 2.24 correspond to three confidence level of 0.9, 0.95 and 0.975 respectively

τ_α	1.65	1.96	2.24
Iterative times	[8, 6]	[11, 9]	[17, 16]

is not significantly affected, especially, when compared with the traditional resampling paradigms (Table 1).

The suspect influential subset is chosen by inequality (11), and therefore the parameter τ_α controls the size of suspect influential subset (Fig. 10). And further, τ_α influences the computational efficiency (Table 9) and accuracy (Table 10) of ICF. For example, in the experiment of Fig. 10, when we set τ_α with a smaller value 1.65, the influentials can be deleted dramatically in the first several iterations and the learning procedure terminates within 8 iterations; however, when τ_α is set with a larger value 2.24, only a small number of influentials can be deleted at each iteration, and it takes 17 iterations to learn the correspondence function $f(u,v)$. Therefore, the smaller τ_α is, the more efficient the proposed scheme is.

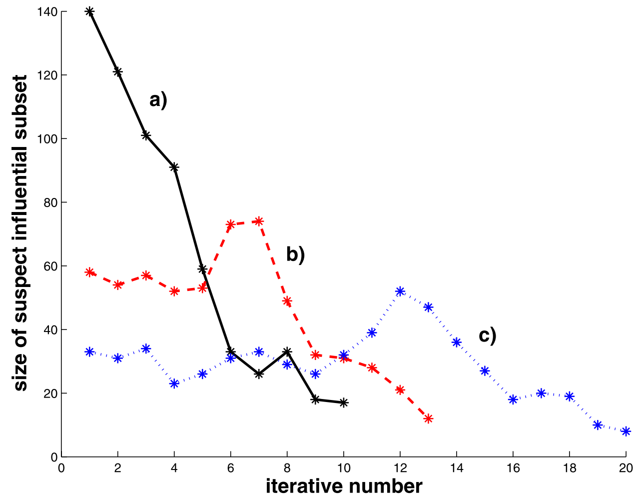

Fig. 10 In this experiment, we investigated how the parameter τ_α controls the size of suspect influential subset, and influences the computational efficiency in estimating correspondence function $f(u,v)$. Putative correspondence set is the one in the experiment of Table 5 with mismatch percentage 69.99%. **(a)** $\tau_\alpha = 1.65$, the correspondence function is estimated within 8 iterations; **(b)** $\tau_\alpha = 1.96$, the learning procedure is finished at the 11th iteration; **(c)** $\tau_\alpha = 2.24$, the learning procedure is finished at the 17th iteration

Table 10 Capability of ICF and influence of parameter τ_α on estimating correspondence functions. Putative correspondence set is the one in the experiment of Table 5 with mismatch percentage 69.99%. Three thresholds τ_α 1.65, 1.96 and 2.24 correspond to three confidence level 0.9, 0.95 and 0.975 respectively (%)

τ_α	1.65	1.96	2.24
Identified matches	88.94	99.08	100.00
Identified mismatches	99.80	99.41	99.21

However, when the value of τ_α is too small, more non-influentials could be mistakenly selected into the suspect subset, which will result in a coarser estimation of correspondence function and less correct matches can be preserved (Table 10). Therefore, a tradeoff should be made between computational efficiency and correct match preservation in practice.

6.5 Comparing ICF with RANSAC and M-estimators

In this subsection, we compared our proposed scheme ICF with the typical methods RANSAC (Hartley and Zisserman 2003; Torr and Murray 1993) and M-estimator (Torr 1995; Zhang 1997). The objective is merely to show that ICF is competent for pre-removing the most egregious outliers, and we do not mean that ICF could replace the RANSAC or M-estimators. In M-estimators, we choose the following weight

Table 11 Comparing ICF with RANSAC on efficiency. The used putative correspondence sets are those in the experiments of Table 5. Second is the used time unit in this table

Mismatch percentage of S (%)	45.61	60.04	69.99	80.00	85.00
ICF	0.703	1.140	1.984	4.906	9.219
RANSAC	1.203	17.468	218.813	8028.531	109541.687

Table 12 Comparing ICF with RANSAC on capability to reject mismatches and identify correct matches. The used putative correspondence sets are those in the experiments of Table 5. CR: correct ratio (%)

Mismatch percentage of S	45.61	60.04	69.99	80.00	85.00
CR after ICF	99.08	99.08	98.62	94.93	89.40
CR after RANSAC	97.70	96.31	95.85	96.31	95.85

Table 13 Accuracy and efficiency of RANSAC with a maximum sampling number constraint 1000. The used putative correspondence sets are those in the experiments of Table 5

Mismatch percentage of S (%)	45.61	60.04	69.99	80.00	85.00
Capability(%)	97.70	95.39	95.39	86.18	75.12
Efficiency (seconds)	1.203	2.531	3.266	4.515	5.922

Table 14 Accuracy and efficiency of M-estimators. The used putative correspondence sets are those in the experiments of Table 5. CR: correct ratio

Mismatch percentage of S(%)	45.61	60.04	69.99	80.00	85.00
Efficiency (seconds)	0.156	0.156	0.157	0.235	0.422
CR after M-estimator (%)	97.24	91.71	83.41	67.74	57.60

function

$$w_i = \begin{cases} 1, & |r_i| \le \sigma \\ \sigma/|r_i|, & \sigma < |r_i| \le 3\sigma \\ 0, & |r_i| > 3\sigma \end{cases} \tag{21}$$

which has been used in computer vision for estimating epipolar geometry (Torr 1995; Olsen 1992; Luong 1992), and σ can be estimated by (Zhang 1998)

$$\hat{\sigma} = 1.4826[1 + 5/(n-p)]median_i|r_i|. \tag{22}$$

Since the coordinates of observed putative corresponding points are usually corrupted by noise, they usually do not strictly satisfy the equation of correspondence functions in ICF, the fundamental matrix in RANSAC and M-estimators. Therefore, a tolerance parameter is needed in the three methods. For convenience, the tolerance parameter in RANSAC is denoted as α_R, α_M in M-estimators and α_I in ICF.

(1) Accuracy and Efficiency

The used putative correspondence sets are still those in the experiments of Table 5, and there are approximately 217 correct matches in each putative set. We choose the values of α_R, α_M and α_I such that approximately 217 point pairs are identified as suspect correct matches in every experiment for comparability.

ICF vs. RANSAC The experimental results in Tables 11 and 12 show that the accuracy of ICF is comparable to that of RANSAC, and the most notable feature of ICF is its efficiency. The ICF is much more efficient than RANSAC in the experiments, especially when mismatch ratio is high. To prevent the efficiency of RANSAC from decreasing excessively, usually a maximum sampling number is preset in the literature. However, this will lead to a serious decline in the accuracy of RANSAC (Table 13).

ICF vs. M-estimators Although the M-estimator technique is robust to noise, it is highly vulnerable to outliers (mismatches) (Zhang 1998), and much more sensitive to mismatch ratio than ICF (Table 14). This is because the M-estimators depend heavily on the initial estimation, which is usually obtained by a linear least squares technique. It is proved that the linear least squares estimation is highly vulnerable to outliers. And this will mislead the M-estimators

Table 15 Comparing ICF with RANSAC and M-estimators on tolerance parameter. The RANSAC is implemented with a maximum sampling constraint 1000. Putative correspondence sets are those in the experiments of Table 5. The parameters $\tau_\alpha = 1.65$, $\xi_\varepsilon = 5.414$. SI: Suspect Inliers. CR: correct ratio

Mismatch percentage of S (%)	45.61	60.04	69.99	80.00	85.00
SI detected by ICF	217	219	220	207	173
CR of ICF (%)	99.08	99.09	98.64	95.65	93.64
SI detected by RANSAC	217	193	201	84	37
CR of RANSAC (%)	97.70	95.53	95.52	88.10	86.49
SI detected by M-estimators	217	143	76	44	24
CR of M-estimators (%)	97.23	97.90	86.84	65.91	50.00

into local minima. The experimental results in Tables 12 and 14 show that the proposed ICF can generally reject the mismatches and preserve the correct matches more accurately than M-estimators.

(2) Tolerance Parameter

Experiments show that the tolerance parameters in RANSAC and M-estimators are more sensitive to mismatch ratio, and more difficult to be set than in ICF. The used putative correspondence sets are those in the experiments of Table 5. To be comparable, the parameters α_R, α_M and α_I are set according to the three benchmark experiments on the putative correspondences with mismatch percentage of 45.61%. Because there are approximately 217 correct matches in each putative correspondence set, we choose $\alpha_R = 0.00006$, $\alpha_M = 0.00008$ and $\alpha_I = 5.414$ such that approximately 217 point pairs can be preserved as suspect correct matches in the benchmark experiments after rejecting mismatches. The experimental results in Table 15 show that the tolerance parameter α_I in ICF is less sensitive to mismatch ratio and easier to be set than that in RANSAC and M-estimators.

To avoid excessively decreasing of efficiency, setting a maximum sampling number is a popular choice in RANSAC. However, this will result in missing the theoretically optimal estimation of RANSAC in case the obtained putative correspondence set is contaminated with a high percentage of mismatches, and make its tolerance parameter sensitive to mismatch ratio (Table 15). Therefore, the tolerance parameter in RANSAC is more difficult to be set than that in ICF in practice.

The basic idea of M-estimators is to enhance the robustness by slowing the going up of the loss function with the absolute error increasing. Thus, the undue influence of outliers can be reduced, to some extent. However, the cumulated undue influence is still too large to be ignored when the number of outliers is large. Therefore, the accuracy of M-estimators will become worse rapidly when the mismatch ratio goes up, and its tolerance parameter is very sensitive to mismatch ratio (Table 15).

6.6 More on ICF and Learning of Correspondence Function

Rejecting Mismatches by ICF Between Images of Non-rigid Object

The followings are two examples in correspondence problem, which are designed by referring to Bartoli (2008), Gay-Bellile et al. (2007). In the images, there exist some deformable objects.

Figure 11: This is an image pair of a napkin, under which there is a plate. The plate is moved from the centre to the edge of the napkin in two images, and this result in a deformation on the observed surface. And on the napkin, there is a spoon, which does a movement independent of the plate. This movable spoon forms occlusions in this correspondence problem.

Figure 12: This is an image pair of a calendar. The calendar is bent downward and upward respectively in two images.

The usual mismatch-rejecting methods RANSAC and M-estimators depend on a parametric model, which is related to a camera model or an object motion model, for example, fundamental matrix. In the above two examples, the deformation models are unknown. Therefore, it is not known how to reject the mismatches by RANSAC or M-estimators in Figs. 11(b) and 12(b).

However, the ICF is a general method based on the non-parametric model correspondence function. The correspondence function captures the essential attribute of correspondence problem—there exists a mapping between corresponding points. Therefore, the correspondence function and ICF method do not depend on any specific camera model or the type of object motion. The experimental results of ICF are presented in Figs. 11(c), (d), 12(c) and (d). In conclusion, the ICF is a general method for rejecting mismatches, which can handle a wide range of images of rigid object and non-rigid object with unknown deformation model.

On Learning of Correspondence Function

As discussed in Sect. 4, correspondence function is a vector-valued function which can be represented by two non-

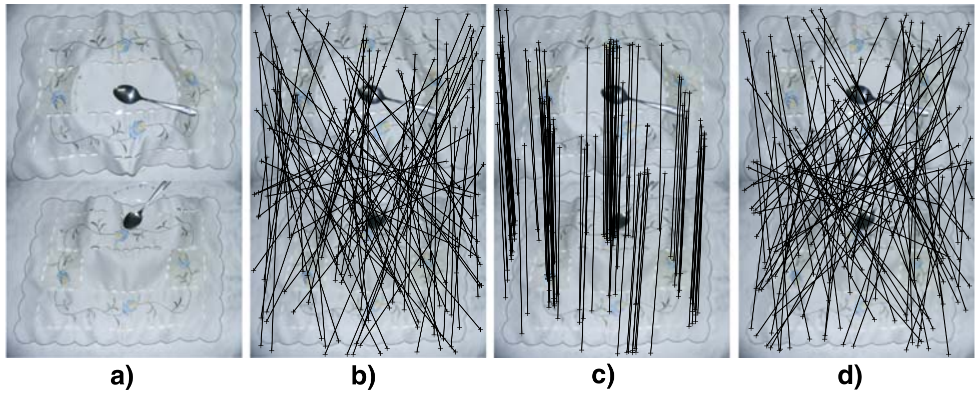

Fig. 11 Deformable image correspondence. This is an image pair of a napkin. (**a**) Original image pair; (**b**) 2594 putative correspondences with many mismatches; (**c**) 255 of them are identified as suspect correct matches; (**d**) the identified suspect mismatches. Parameters $\tau_\alpha = 1.65, \xi_\varepsilon = 10.597$. (For visibility, only 100 randomly selected point pairs are presented in (**b**), (**c**), (**d**))

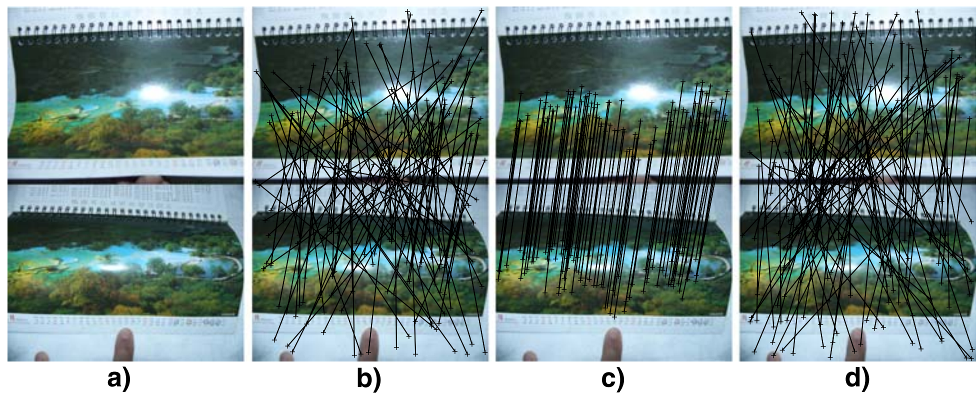

Fig. 12 Deformable image correspondence. This is an image pair of a calendar. (**a**) Original image pair; (**b**) 2796 putative correspondences with many mismatches; (**c**) 672 of them are identified as suspect correct matches; (**d**) the identified suspect mismatches. Parameters $\tau_\alpha = 1.96, \xi_\varepsilon = 10.597$. (For visibility, only 100 randomly selected point pairs are presented in (**b**), (**c**), (**d**))

parameterized binary functions (CF component functions). Although the CF component functions are unknown, the subspace projection of putative correspondences S can be regarded as samples from the CF component function. For example, the correspondence function f can be represented by two CF component functions $u' = g_1(u, v)$ and $v' = g_2(u, v)$, and the subspace projections $S_{U\times V\times U'} = \{(u, v, u')|(u, v, u', v') \in S\}$ and $S_{U\times V\times V'} = \{(u, v, v')|(u, v, u', v') \in S\}$ can be regarded as the observations from them respectively. Therefore, the CF component functions and further the correspondence function can be estimated by a nonparametric regression method in theory.

In order to verify the validity of correspondence function and illustrate its effectiveness in rejecting mismatches, we studied how to estimate correspondence function based on SVM. SVM is a typical nonparametric regression method. To enhance the robustness of SVM against noise and outliers, various loss functions has been designed (Smola and Schölkopf 2004; Smola et al. 1998; Yang et al. 2004). The basic idea of them is to slow the going up of the loss function with absolute error increasing, and reduce the undue influence of outliers on estimation (it is similar to the weighted least squares implementation of M-estimators). The shortcoming of these schemes is that although the designed loss

Table 16 Accuracy of ICF (SVM). Putative correspondence sets are those in the experiments of Table 5. ICF (SVM): Correspondence functions are estimated by SVM instead of IECF in algorithm ICF. CR: correct ratio

Mismatch percentage of S(%)	45.61	60.04	69.99	80.00	85.00
CR after ICF (SVM) (%)	88.94	72.35	60.83	45.62	33.18

functions can reduce the undue influence of outliers to some extent, when the number of outliers is large, their cumulated undue influence usually is still too large to be ignored (Fig. 1, Table 14). For example, if we estimate correspondence function by SVM in algorithm ICF instead of by IECF, the estimation will be more sensitive to the ratio of outliers (Table 16). Therefore, we propose the IECF algorithm to estimate correspondence function based on SVM and diagnostic technique. Compared with SVM, experiments show that the proposed IECF remarkably improve the accuracy of the estimation (Tables 12[8] and 16). Compared with the traditional case deletion diagnostics, the IECF significantly improve the efficiency of the learning procedure:

[8]In the experiments of Table 12, the correspondence function is estimated by IECF algorithm.

in case deletion diagnostics, up to n regressions should be made to detect one influential observation, where n is the number of training data; however, by IECF, many influentials usually can be detected by one regression procedure, for example, in the experiment of Fig. 5, IECF detected 147 influentials in the first two iterations (three regression procedures by SVM) when we estimate correspondence function f; In the experiment on the data of Fig. 1, the IECF detected 115 influentials in the first two iterations.

7 Conclusions

In this work, we introduced a novel concept of correspondence function for rejecting mismatches and presented a novel learning algorithm IECF to robustly estimate correspondence functions from given putative correspondences. Based on the estimated correspondence functions, mismatches are rejected by checking their consistency with the correspondence functions. Extensive experiments on real image pairs demonstrate the good outlier rejection and correct match preserving ability of our method.

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