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# Relating double field theory to the scalar potential of N = 2 gauged supergravity

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Abstract: The double field theory action in the flux formulation is dimensionally reduced on a Calabi-Yau three-fold equipped with non-vanishing type IIB geometric and non-geometric fluxes. First, we rewrite the metric-dependent reduced DFT action in terms of quantities that can be evaluated without explicitly knowing the metric on the Calabi-Yau manifold. Second, using properties of special geometry we obtain the scalar potential of $N = 2$ gauged supergravity. After an orientifold projection, this potential is consistent with the scalar potential arising from the flux-induced superpotential, plus an additional D-term contribution.

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## Contents



# 1 Introduction

One of the main issues in relating string theory to observable physics is the problem of moduli stabilization. For instance supersymmetric compactifications of string theory on Calabi-Yau (CY) manifolds come with a plethora of massless scalars, so-called moduli. At string tree-level these moduli can be stabilized by turning on fluxes on the Calabi-Yau



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manifold (for reviews see e.g. [1–3]). This procedure is mostly discussed in an effective four-dimensional framework, i.e. one starts with the initial CY geometry and considers the fluxes as off-shell deformations of the theory. In the effective description this leads to the generation of a scalar potential that depends on the moduli fields. The hope is that new classical field theory vacua of this scalar potential reflect new solutions to the true ten-dimensional equations of motion. Finding these solutions concretely is a highly non-trivial step, as it involves going away from the initial CY geometry.

The prime example of the application of flux-induced potentials are type IIB models with non-trivial Neveu-Schwarz-Neveu-Schwarz (NS-NS) and Ramond-Ramond (R-R) three-form fluxes [4, 5]. In this case a no-scale potential involving only the complex structure moduli and the axio-dilaton is obtained. The Kähler moduli remain massless, but can be lifted by subleading perturbative and non-perturbative effects. This is the idea behind the KKLT [6] and LARGE volume scenario [7].

It is known that in order to stabilize also the Kähler moduli at tree-level, one needs to consider additional non-geometric fluxes [8, 9]. For the toroidal case this has been investigated in [10–15] and for generic Calabi-Yau manifolds in [16–22], among others. In particular, it has been shown that the generalized flux-induced scalar potential can be related to the scalar potential of $N = 2$ gauged supergravity [23]. Lately, this kind of flux vacua have been investigated from a string phenomenological point of view, with special emphasis on realizing F-term axion monodromy inflation [24].

From the higher dimensional point of view, it has been argued that non-geometric aspects of string theory can be captured by double field theory (DFT) [25–29]; for reviews see [30–32]. DFT provides a self-consistent framework that features new symmetries such as generalized diffeomorphisms and a manifest global $O(D, D)$ symmetry that close upon invoking the so-called strong constraint. In particular, though derived from string field theory on a torus, DFT is claimed to be background independent. See also [33, 34] for the derivation of a DFT action resulting from string field theory on WZW models.

There exist two formulations of the DFT action, which differ by terms that are either total derivatives or vanish due to the strong constraint. For our purposes it is convenient to use the so-called flux formulation of the DFT action in the form presented in [35]. This is motivated by the scalar potential in gauged supergravity which, as shown in [36], is also related to the early work of W. Siegel [25, 26].

It has been shown that compactifying or Scherk-Schwarz reducing DFT on a toroidal background equipped with constant geometric and non-geometric fluxes gives the scalar potential of half-maximal gauged supergravity in four dimensions [37–39]. The relation of DFT to the scalar potential of $N = 2$ gauged supergravity has however not explicitly been clarified. Clearly, the expectation is that dimensionally reducing DFT on a genuine Calabi-Yau manifold carrying constant geometric and non-geometric fluxes should give the scalar potential of $N = 2$ gauged supergravity. It is the purpose of this paper to fill this gap and explicitly show how the dimensional reduction of DFT can be performed in order to match $N = 2$ gauged supergravity results. This can be considered as the generalization of the computation first performed in [4], where the dimensional reduction of the kinetic terms of the NS-NS and R-R type IIB three-form on a (non-toroidal) CY



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three-fold gives the no-scale scalar potential described in supergravity language by the tree-level Kähler potential for the complex structure/axio-dilaton moduli and the Gukov-Vafa-Witten (GVW) superpotential [40].

The main technical problem is that the action of DFT contains the metric on the CY three-fold, which is not explicitly known. Therefore, one first has to appropriately rewrite the DFT action so that only quantities appear that can be treated without knowing the metric explicitly. For instance, for the simple H-flux case we can write

$$\star\mathcal{L} = -\frac{e^{-2\phi}}{2} d^{10}x\sqrt{-G}\, H_{ijk}\, H_{i'j'k'}\, g^{ii'} g^{jj'} g^{kk'} = -\frac{e^{-2\phi}}{2} H \wedge \star H\,, \tag{1.1}$$

but DFT contains many more terms that are not of this simple type. To perform the dimensional reduction, it is most appropriate to start with the DFT action in the flux formulation. This action essentially contains the kinetic terms of the various geometric and non-geometric fluxes. We will treat the background fluxes as constant parameters that are only subject to their Bianchi identities, which are quadratic constraints for the constant fluxes. The indices for these fluxes are contracted using the constant $O(D,D)$ metric or the generalized metric. The latter contains the background CY metric, hence depending on the complex structure and complexified Kähler moduli. The CY metric is assumed to only depend on the usual coordinates.[1]

This paper is organized as follows: in section 2 we provide a brief review of the main aspects of DFT that are relevant for this paper. As mentioned, we focus on the DFT action in the flux formulation. In section 3, following a step by step procedure, we rewrite the action compactified on a CY in terms of quantities that only contain operations like wedge products, the Hodge-star map and actions of fluxes on $p$-forms. The main result is the generalization of (1.1). We find that all NS-NS terms appearing in the DFT action can be rewritten as

$$\begin{aligned}\star\mathcal{L}_{\mathrm{NS\,NS}} = -e^{-2\phi}\Big[&\frac{1}{2}\,\chi\wedge\star\overline{\chi} \;+\; \frac{1}{2}\,\Psi\wedge\star\overline{\Psi} \\ &-\frac{1}{4}\,(\Omega\wedge\chi)\wedge\star\big(\overline{\Omega}\wedge\overline{\chi}\big) - \frac{1}{4}\,(\Omega\wedge\overline{\chi})\wedge\star\big(\overline{\Omega}\wedge\chi\big)\Big]\,,\end{aligned} \tag{1.2}$$

where $\chi = \mathfrak{D}\, e^{iJ}$ and $\Psi = \mathfrak{D}\,\Omega$. Here $\mathfrak{D}$ denotes a twisted differential that involves all geometric and non-geometric fluxes, and $J$ and $\Omega$ are the Kähler and holomorphic three-form of the CY. In section 4, this action is evaluated after introducing the fluxes on the internal CY manifold as in [16, 18]. We show that the resulting scalar potential takes the same form as the one in [23], which was shown there to be equivalent to the $N=2$ gauged supergravity result. In section 5, we perform an orientifold projection of type IIB with O7-/O3-planes and show that the scalar potential derived from (1.2) in general is a sum of three terms

$$V = V_{\mathrm{F}} + V_D + V_{\text{NS-tad}}\,, \tag{1.3}$$

[1] The same procedure to carry out the dimensional reduction/oxidation of DFT compactified on a torus (orbifold) background was employed in [41, 42].



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where $V_{\rm F}$ is the F-term scalar potential derived from the tree-level Kähler potential and the generalized flux-induced GVW superpotential. $V_D$ is a D-term potential related to the abelian gauge fields arising from the dimensional reduction of the R-R four-form on orientifold even three-cycles of the Calabi-Yau. The last term is the flux induced NS-NS tadpole contribution that will be cancelled by localized sources upon invoking R-R tadpole cancellation. We remark that $V_{\rm F}$ is the scalar potential used in the string phenomenological studies of [19, 20, 24].

## 2 Review of double field theory

In this section, we briefly review the salient features of DFT important for our subsequent discussion. For a more detailed introduction we would like to refer to the reviews [30–32].

### 2.1 Basics of DFT

Double Field Theory is defined on a space with a doubled number of dimensions, where in addition to the standard coordinates $x^i$ one introduces winding coordinates $\tilde{x}_i$. The two types of coordinates can be arranged into a doubled vector of the form $X^I = (\tilde{x}_i, x^i)$, with $i = 1, \ldots, D$. One also introduces an $O(D,D)$-invariant metric as

$$\eta_{IJ} = \begin{pmatrix} 0 & \delta^i{}_j \\ \delta_i{}^j & 0 \end{pmatrix}, \tag{2.1}$$

and combines the dynamical metric and Kalb-Ramond field $G_{ij}$ and $B_{ij}$ into the generalized metric

$$\mathcal{H}_{IJ} = \begin{pmatrix} G^{ij} & -G^{ik}B_{kj} \\ B_{ik}G^{kj} & G_{ij} - B_{ik}G^{kl}B_{lj} \end{pmatrix}. \tag{2.2}$$

For these matrices a coordinates basis with indices $I, J, \ldots$ has been chosen, however, one can also employ a non-holonomic frame

$$\mathcal{H}_{IJ} = E^A{}_I \, S_{AB} \, E^B{}_J \,, \tag{2.3}$$

distinguished by indices $A, B, \ldots$ from the beginning of the alphabet. The diagonal matrix $S_{AB}$ is given by

$$S_{AB} = \begin{pmatrix} s^{ab} & 0 \\ 0 & s_{ab} \end{pmatrix}, \tag{2.4}$$

and $s_{ab}$ denotes the flat $D$-dimensional Minkowski metric, which is related to the curved metric as $G_{ij} = e^a{}_i s_{ab} e^b{}_j$. For the parametrization of the generalized metric shown in (2.2), one can then find that

$$E^A{}_I = \begin{pmatrix} e_a{}^i & -e_a{}^k B_{ki} \\ 0 & e^a{}_i \end{pmatrix}. \tag{2.5}$$

An action for DFT is determined by invoking symmetries: first, one requires the action to be invariant under local diffeomorphisms of the doubled coordinates $X^I$, that is $(\tilde{x}_i, x^i) \to (\tilde{x}_i + \tilde{\xi}_i(X), x^i + \xi^i(X))$. Second, the action should be invariant under a global (or rigid) $O(D,D)$ symmetry. It has been realized that for manifest $O(D,D)$ invariance and for closure of the algebra of infinitesimal diffeomorphisms, one has to impose the so-called strong constraint

$$\partial_i A\, \tilde{\partial}^i B + \tilde{\partial}^i A\, \partial_i B = 0\,, \tag{2.6}$$

where $\tilde{\partial}^i$ denotes the derivative with respect to the winding coordinate $\tilde{x}_i$. There exist two formulations of a DFT action, which differ by terms that are either total derivatives or are vanishing due to the strong constraint (2.6). For our purposes it is convenient to use the so-called flux formulation of DFT, which we review in the following.

### 2.2 The flux formulation of DFT

The flux formulation of DFT has been developed in [37–39] and is, as has been shown in [36], related to earlier work of Siegel [25, 26]. In a frame with flat indices, the action in the NS-NS sector is given by

$$\begin{aligned} \mathcal{S}_{\rm NS\,NS} = \frac{1}{2\kappa_{10}^2} \int d^D X\, e^{-2d} \Big[ & \\ & \mathcal{F}_{ABC}\mathcal{F}_{A'B'C'} \left( \frac{1}{4} S^{AA'} \eta^{BB'} \eta^{CC'} - \frac{1}{12} S^{AA'} S^{BB'} S^{CC'} - \frac{1}{6} \eta^{AA'} \eta^{BB'} \eta^{CC'} \right) \\ & + \mathcal{F}_A \mathcal{F}_{A'} \left( \eta^{AA'} - S^{AA'} \right) \Big]\,, \end{aligned} \tag{2.7}$$

where we used $d^D X = d^D x \wedge d^D \tilde{x}$. The definition of $e^{-2d}$ contains the dilaton $\phi$ and the determinant of the metric $G_{ij}$, and reads $\exp(-2d) = \sqrt{-G}\exp(-2\phi)$. Throughout most parts of the upcoming computation we set $2\kappa_{10}^2 = 1$ and only introduce mass scales $M_{\rm s}$ or $M_{\rm Pl}$ when necessary. The $\mathcal{F}_A$ are expressed as

$$\mathcal{F}_A = \Omega^B{}_{BA} + 2 E_A{}^I \partial_I d\,, \tag{2.8}$$

with the generalized Weitzenböck connection

$$\Omega_{ABC} = E_A{}^I (\partial_I E_B{}^J) E_{CJ}\,. \tag{2.9}$$

The three-index object $\mathcal{F}_{ABC}$ is the anti-symmetrization of $\Omega_{ABC}$, that is[2]

$$\mathcal{F}_{ABC} = 3\, \Omega_{[\underline{ABC}]}\,. \tag{2.10}$$

The Ramond-Ramond sector of DFT has been analyzed in [43–47]. We note that the fields transform in the spinor representation of $O(10,10)$ so that one can expand

$$\mathcal{G} = \sum_n \frac{1}{n!} G^{(n)}_{i_1 \ldots i_n}\, e_{a_1}{}^{i_1} \ldots e_{a_n}{}^{i_n}\, \Gamma^{a_1 \ldots a_n} |0\rangle\,, \tag{2.11}$$

[2] Our convention is that the anti-symmetrization of $n$ indices contains a factor of $1/n!$.

where $\Gamma^{a_1 \cdots a_n}$ defines the totally anti-symmetrized product of $n$ $\Gamma$-matrices. Similarly, we combine the R-R gauge potentials $C^{(2n)}$ into a spinor $\tilde{\mathcal{C}}$. Then, as shown in [38], one can define the R-R field strengths as

$$\mathcal{G} = \not{\nabla} \tilde{\mathcal{C}} \,, \tag{2.12}$$

with the generalized fluxed Dirac operator given by

$$\not{\nabla} = \Gamma^A D_A - \frac{1}{3}\Gamma^A \mathcal{F}_A - \frac{1}{6}\Gamma^{ABC}\mathcal{F}_{ABC} \,. \tag{2.13}$$

For type IIB on a CY, the only relevant R-R form is the three-form field strength, whose action is

$$\mathcal{S}_{\mathrm{RR}} = \frac{1}{2\kappa_{10}^2} \int d^D X \left[ -\frac{1}{12} S^{AA'} S^{BB'} S^{CC'} \mathcal{G}_{ABC} \mathcal{G}_{A'B'C'} \right] . \tag{2.14}$$

### 2.3 Compactification

Taking the point of view that DFT is not only defined on a toroidal background, our aim in this paper is to study compactifications of the DFT action on Calabi-Yau three-folds. In particular, we are interested in the resulting scalar potential for the moduli of the Calabi-Yau that is generated by background fluxes. The fluxes $\mathcal{F}_{ABC}$ and $\mathcal{F}_A$ are treated as small deviations from the Calabi-Yau background. In terms of the vielbeins (2.5), this requirement can be expressed as follows

$$E^A{}_I = \mathring{E}^A{}_I + \overline{E}^A{}_I + \mathcal{O}\left(\overline{E}^2\right) , \qquad \overline{E}^A{}_I \ll 1 \,, \tag{2.15}$$

where $\mathring{E}^A{}_I$ describes the Calabi-Yau background and $\overline{E}^A{}_I$ encodes the fluxes. Using this expansion in (2.9), for (2.8) and (2.10) we obtain up to first order in $\overline{E}$

$$\mathcal{F}_{ABC} = \mathring{\mathcal{F}}_{ABC} + \overline{\mathcal{F}}_{ABC} + \mathcal{O}\left(\overline{E}^2\right) , \qquad \mathcal{F}_A = \mathring{\mathcal{F}}_A + \overline{\mathcal{F}}_A + \mathcal{O}\left(\overline{E}^2\right) . \tag{2.16}$$

Note that $\mathring{\mathcal{F}}_{ABC}$ and $\mathring{\mathcal{F}}_A$ are computed using only the Calabi-Yau generalized vielbein $\mathring{E}^A{}_I$. Since $\mathring{E}^A{}_I$ satisfies the DFT equations of motion, these fluxes do not generate a scalar potential for the moduli of the CY and will be neglected. We consider the action (2.7) up to second order in the deviations $\overline{E}^A{}_I$. Since $\mathcal{F}_{ABC}$ and $\mathcal{F}_A$ start at linear order in $\overline{E}$, it follows that all other quantities appearing in the action are those of the Calabi-Yau background.

In order to define the starting point of our investigation, let us specify the setting considered in this paper:

- We limit our discussion to the internal Calabi-Yau part, and ignore the remaining directions. The exterior derivative $d$ is understood to only contain derivatives with respect to the internal coordinates, and similarly for the Hodge-star operator $\star$.

- For the underlying Calabi-Yau background we impose the strong constraint. In particular, the metric of the Calabi-Yau three-fold is denoted by $g_{ij}$ and only depends on the usual coordinates $x^i$. It appears in (2.5) via the vielbeins $e_a{}^i$ and is in general not known for the Calabi-Yau manifold. However, its complex-structure and Kähler deformations are well understood. Similarly, the $B$-field of the background only depends on coordinates $x^i$, and furthermore satisfies $dB = 0$ on the Calabi-Yau manifold.

- For a Calabi-Yau three-fold there are no non-trivial homological one-cycles, and hence non-trivial fluxes with one index cannot be supported on the CY. For the internal part of the action (2.7) we can therefore set

$$\overline{\mathcal{F}}_A = 0\,. \tag{2.17}$$

- The term $\mathcal{F}_{ABC}\mathcal{F}_{A'B'C'}\eta^{AA'}\eta^{BB'}\eta^{CC'}$ vanishes via Bianchi identities. This will be discussed in section 3.1 below.

- Analogously to Scherk-Schwarz reductions, we consider constant expectation values for the fluxes $\overline{\mathcal{F}}_{IJK} = \mathring{E}^A{}_I\mathring{E}^B{}_J\mathring{E}^C{}_K\overline{\mathcal{F}}_{ABC}$ along the Calabi-Yau three-fold. These are related to the geometric $H$- and $F$-flux, and to the non-geometric $Q$- and $R$-flux as

$$\overline{\mathcal{F}}_{ijk} = H_{ijk}\,, \qquad \overline{\mathcal{F}}^i{}_{jk} = F^i{}_{jk}\,, \qquad \overline{\mathcal{F}}_i{}^{jk} = Q_i{}^{jk}\,, \qquad \overline{\mathcal{F}}^{ijk} = R^{ijk}\,. \tag{2.18}$$

  Again, in analogy to Scherk-Schwarz reductions, for these background fluxes we do not impose the strong constraint, but only the quadratic constraints given by the Bianchi identities, which can be found in equation (3.4).

In the setting explained above, the relevant part of the DFT Lagrangian (2.7) in the NS-NS sector (restricted to a Calabi-Yau three-fold) reduces to [41]

$$\mathcal{L}_{\rm NSNS} = e^{-2\phi}\overline{\mathcal{F}}_{IJK}\overline{\mathcal{F}}_{I'J'K'}\left(\frac{1}{4}\mathcal{H}^{II'}\eta^{JJ'}\eta^{KK'} - \frac{1}{12}\mathcal{H}^{II'}\mathcal{H}^{JJ'}\mathcal{H}^{KK'}\right) + \dots \tag{2.19}$$

Note that the generalized metric $\mathcal{H}$ and the dilaton $\phi$ are that of the Calabi-Yau background, which only depends on the usual coordinates $x^i$. As it will be shown in the remainder of this paper, upon dimensional reduction, it is precisely this part of the DFT action that can be identified with the scalar potential of a gauged supergravity theory. We emphasize that the computations to be performed go through once the quadratic Bianchi identities for the fluxes are imposed. Requiring stronger conditions, such as the strong constraint, only eliminates some of these fluxes.

As explicitly shown in [41], the action (2.19) can be further expanded, for which it turns out to be convenient to introduce the following combinations of fluxes

$$\begin{aligned}
\mathfrak{H}_{ijk} &= H_{ijk} + 3\,F^m{}_{[\underline{ij}}\,B_{m\underline{k}]} + 3\,Q_{[\underline{i}}{}^{mn}B_{m\underline{j}}\,B_{n\underline{k}]} + R^{mnp}B_{m[\underline{i}}\,B_{n\underline{j}}\,B_{p\underline{k}]}\,,\\
\mathfrak{F}^i{}_{jk} &= F^i{}_{jk} + 2\,Q_{[\underline{j}}{}^{mi}\,B_{m\underline{k}]} + R^{mni}B_{m[\underline{j}}\,B_{n\underline{k}]}\,,\\
\mathfrak{Q}_k{}^{ij} &= Q_k{}^{ij} + R^{mij}\,B_{mk}\,,\\
\mathfrak{R}^{ijk} &= R^{ijk}\,.
\end{aligned} \tag{2.20}$$

Let us emphasize that for these fluxes Bianchi identities similar to (3.4) have to be satisfied, which can be checked by explicit computation. We come back to this question below. For the term in (2.19) containing three factors of the metric, we then find

$$\begin{aligned}\mathcal{L}_1 = -\frac{e^{-2\phi}}{12}\bigg(&\mathfrak{H}_{ijk}\mathfrak{H}_{i'j'k'}g^{ii'}g^{jj'}g^{kk'} + 3\mathfrak{F}^i{}_{jk}\mathfrak{F}^{i'}{}_{j'k'}g_{ii'}g^{jj'}g^{kk'} + 3\mathfrak{Q}_i{}^{jk}\mathfrak{Q}_{i'}{}^{j'k'}g^{ii'}g_{jj'}g_{kk'}\\ &+\mathfrak{R}^{ijk}\mathfrak{R}^{i'j'k'}g_{ii'}g_{jj'}g_{kk'}\bigg)\,,\end{aligned} \tag{2.21}$$

whereas for the term in (2.19) with a single factor of the metric we have

$$\mathcal{L}_2 = -\frac{e^{-2\phi}}{2}\Big(\mathfrak{F}^m{}_{ni}\mathfrak{F}^n{}_{mi'}g^{ii'} + \mathfrak{Q}_m{}^{ni}\mathfrak{Q}_n{}^{mi'}g_{ii'} - \mathfrak{H}_{mni}\mathfrak{Q}_{i'}{}^{mn}g^{ii'} - \mathfrak{F}^i{}_{mn}\mathfrak{R}^{mni'}g_{ii'}\Big)\,. \tag{2.22}$$

For the R-R sector of type IIB DFT we can perform a similar analysis. We introduce a three-form in the following way

$$\mathfrak{G}_{ijk} = F^{(3)}_{ijk} - \mathfrak{H}_{ijk}C^{(0)} - 3\,\mathfrak{F}^m{}_{[\underline{ij}}C^{(2)}_{|m|\underline{k}]} + \frac{3}{2}\mathfrak{Q}_{[\underline{i}}{}^{mn}C^{(4)}_{\underline{jk}]mn} + \frac{1}{6}\mathfrak{R}^{mnp}C^{(6)}_{mnpijk}\,, \tag{2.23}$$

where $F^{(3)}$ denotes the background three-form flux in the R-R sector. Up to second order in the fluxes, the Lagrangian (2.14) can then be expressed as

$$\mathcal{L}_{\rm RR} = -\frac{1}{12}\,\mathfrak{G}_{ijk}\,\mathfrak{G}_{i'j'k'}\,g^{ii'}\,g^{jj'}\,g^{kk'}\,. \tag{2.24}$$

Note that again the metric appearing in (2.24) is that of the Calabi-Yau background.

## 3 DFT action on a Calabi-Yau manifold

As we discussed above, in the DFT actions (2.7) and (2.14) the metric $g_{ij}$ appears explicitly. Since the metric of a Calabi-Yau three-fold is in general not known, a dimensional reduction is not straightforward. However, by rewriting the various terms and expressing them via known objects, the problem becomes tractable. In the following, we explain this approach in simple cases with only one type of flux turned on, and later give the general result.

### 3.1 Fluxes as operators

For our subsequent discussion, it will turn out to be convenient to interpret the geometric $H$- and $F$-flux, as well as the non-geometric $Q$- and $R$-flux, as operators acting on $p$-forms. In particular, we have [10–12]

$$\begin{aligned} H\wedge: &\quad p\text{-form} \to (p+3)\text{-form}\,,\\ F\circ: &\quad p\text{-form} \to (p+1)\text{-form}\,,\\ Q\bullet: &\quad p\text{-form} \to (p-1)\text{-form}\,,\\ R\llcorner: &\quad p\text{-form} \to (p-3)\text{-form}\,. \end{aligned} \tag{3.1}$$

Employing a local basis $\{dx^i\}$ and the contraction $\iota_i$ satisfying $\iota_i dx^j = \delta_i^j$, this mapping can be implemented by

$$\begin{aligned} H\wedge &= \frac{1}{3!} H_{ijk} dx^i \wedge dx^j \wedge dx^k \,, \\ F\circ &= \frac{1}{2!} F^k{}_{ij} dx^i \wedge dx^j \wedge \iota_k \,, \\ Q\bullet &= \frac{1}{2!} Q_i{}^{jk} dx^i \wedge \iota_j \wedge \iota_k \,, \\ R\llcorner &= \frac{1}{3!} R^{ijk} \iota_i \wedge \iota_j \wedge \iota_k \,. \end{aligned} \tag{3.2}$$

Note that our convention for a $p$-form $\eta$ is such that $\eta = \frac{1}{p!}\eta_{i_1\ldots i_p} dx^{i_1} \wedge \ldots \wedge dx^{i_p}$. Furthermore, the above fluxes can be combined with the exterior derivative $d$ into a so-called twisted differential $\mathcal{D}$

$$\mathcal{D} = d - H\wedge - F\circ - Q\bullet - R\llcorner \,. \tag{3.3}$$

Requiring this operator to be nilpotent, in particular that $\mathcal{D}^2 = 0$, leads to a set of constraints on the fluxes (2.18). These constraints correspond to Bianchi identities, and take the form

$$\begin{aligned} 0 &= H_{m[\underline{ab}} F^m{}_{\underline{cd}]} \,, \\ 0 &= F^m{}_{[\underline{bc}} F^d{}_{\underline{a}]m} + H_{m[\underline{ab}} Q_{\underline{c}]}{}^{md} \,, \\ 0 &= F^m{}_{[\underline{ab}]} Q_m{}^{[\underline{cd}]} - 4\, F^{[\underline{c}}{}_{m[\underline{a}} Q_{\underline{b}]}{}^{\underline{d}]m} + H_{mab} R^{mcd} \,, \\ 0 &= Q_m{}^{[\underline{bc}} Q_d{}^{\underline{a}]m} + R^{m[\underline{ab}} F^{\underline{c}]}{}_{md} \,, \\ 0 &= R^{m[\underline{ab}} Q_m{}^{\underline{cd}]} \,, \\ 0 &= R^{mn[\underline{a}} F^{\underline{b}]}{}_{mn} \,, \\ 0 &= R^{amn} H_{bmn} - F^a{}_{mn} Q_b{}^{mn} \,, \\ 0 &= Q_{[\underline{a}}{}^{mn} H_{\underline{b}]mn} \,, \\ 0 &= R^{mnl} H_{mnl} \,. \end{aligned} \tag{3.4}$$

We will further impose

$$F^i{}_{ij} = 0 \,, \qquad Q_i{}^{ij} = 0 \,, \tag{3.5}$$

which are standard in the literature [8, 9]. Moreover, on a CY three-fold there are no homologically non-trivial one- and five-cycles, so that it is justified to require that all combinations leaving effectively one free-index are trivial. The five first identities in the upper block were originally obtained both by T-duality and from Jacobi identities of a flux algebra [8, 9]. By virtue of (3.5) the three identities in the second block follow by taking appropriate index contractions in the middle identities of the upper block. We are then left with the last identity $R^{mnl}H_{mnl}=0$, which in turn implies $F^{l}{}_{mn}Q_{l}{}^{mn}=0$. Similar results have been reported in [22]. Let us emphasize that these two identities imply that $\mathcal{F}_{ABC}\mathcal{F}_{A'B'C'}\eta^{AA'}\eta^{BB'}\eta^{CC'}=0$. Furthermore, we note that in orientifold theories $R^{mnl}H_{mnl}=0$ is automatically satisfied, as $H$ and $R$ are of opposite parity with respect to the $\mathbb{Z}_2$ orientifold projection $\Omega_{\mathrm{P}}(-1)^{F_{\mathrm{L}}}$. This will be discussed below.

### 3.2 Lessons from one type of flux

Let us now consider situations with vanishing $B$-field, and only one type of non-trivial flux on the Calabi-Yau manifold. More involved cases are discussed in the subsequent sections.

**Pure $H$-flux.** We begin by turning on only $H$-flux in the Lagrangians (2.21) and (2.22). For the NS-NS sector $\mathcal{L}_{\mathrm{NS\,NS}}=\mathcal{L}_1+\mathcal{L}_2$ we obtain

$$\star\,\mathcal{L}_{\mathrm{NS\,NS}}=-\frac{e^{-2\phi}}{12}\,H_{ijk}\,H_{i'j'k'}\,g^{ii'}g^{jj'}g^{kk'}\star 1=-\frac{e^{-2\phi}}{2}\,H\wedge\star H\,, \tag{3.6}$$

where $\star 1=\sqrt{g}\,d^6x$ is a convenient way to write the six-dimensional volume form of the Calabi-Yau manifold. Let us note that the Hodge-star operator for a three-form on a Calabi-Yau three-fold can be evaluated using special geometry, for which the explicit form of the metric is not needed. For the next case we follow a similar strategy.

**Pure $F$-flux.** We now turn to a more complicated situation and consider pure $F$-flux in the case of vanishing $B$-field. The Lagrangians in the NS-NS sector (2.21) and (2.22) then take the form

$$\mathcal{L}_{\mathrm{NS\,NS}}=-\frac{e^{-2\phi}}{4}\Big(F^{i}{}_{jk}\,F^{i'}{}_{j'k'}\,g_{ii'}g^{jj'}g^{kk'}+2\,F^{m}{}_{ni}\,F^{n}{}_{mi'}\,g^{ii'}\Big)\,. \tag{3.7}$$

**Part 1.** For the first term, we define a three-form $\Xi_3=-\mathcal{D}J=F\circ J$, where $J$ denotes the Kähler form of the Calabi-Yau three-fold. Using our conventions (3.2), we determine the components of $\Xi_3$ as

$$\Xi_{ijk}=F^{m}{}_{ij}\,J_{mk}+\text{cyclic}\,. \tag{3.8}$$

We then consider the analogue of the kinetic term for the $H$-flux and compute

$$\frac{1}{2}\,\Xi_3\wedge\star\Xi_3=\left[\frac{1}{4}\,F^{m}{}_{ij}\,F^{m'}{}_{i'j'}\,g_{mm'}\,g^{ii'}\,g^{jj'}-\frac{1}{2}\,F^{m}{}_{ij}\,F^{m'}{}_{i'j'}\,I^{j'}{}_{m}\,I^{j}{}_{m'}\,g^{ii'}\right]\star 1\,, \tag{3.9}$$

where $I_i{}^j=J_{im}g^{mj}$ is the complex structure of the Calabi-Yau three-fold, which satisfies $I_i{}^mI_m{}^j=-\delta_i{}^j$. Note that the first term in (3.9) agrees with the first term in (3.7). For the second term in (3.9) we switch to a complex coordinate basis and compute

$$-\frac{1}{2}\,F^{m}{}_{ij}\,F^{m'}{}_{i'j'}\,I^{j'}{}_{m}\,I^{j}{}_{m'}\,g^{ii'}=\Big(F^{c}{}_{ab}\,F^{b}{}_{\overline{a}c}+F^{\overline{c}}{}_{a\overline{b}}\,F^{\overline{b}}{}_{\overline{a}\overline{c}}-F^{\overline{c}}{}_{ab}\,F^{b}{}_{\overline{a}\overline{c}}-F^{c}{}_{a\overline{b}}\,F^{\overline{b}}{}_{\overline{a}c}\Big)\,g^{a\overline{a}}\,. \tag{3.10}$$

Next, we note that using the second Bianchi identity in (3.4) (for vanishing $H$- and $Q$-flux) as $F^{k}{}_{a\overline{b}}\,F^{\overline{b}}{}_{\overline{a}k}+\text{cyclic}=0$, we can show that

$$g^{a\overline{a}}F^{c}{}_{a\overline{b}}\,F^{\overline{b}}{}_{\overline{a}c}=g^{a\overline{a}}F^{\overline{c}}{}_{ab}\,F^{b}{}_{\overline{a}\overline{c}}\,. \tag{3.11}$$

We use this relation in (3.10) and obtain

$$-\frac{1}{2}F^m{}_{ij}\,F^{m'}{}_{i'j'}\,I^{j'}{}_m\,I^j{}_{m'}\,g^{ii'}=\Big(F^c{}_{ab}\,F^b{}_{\overline{a}c}+F^{\overline{c}}{}_{a\overline{b}}\,F^{\overline{b}}{}_{\overline{a}\overline{c}}-2\,F^{\overline{c}}{}_{ab}\,F^b{}_{\overline{a}\overline{c}}\Big)\,g^{a\overline{a}}\,. \tag{3.12}$$

**Part 2.** Equation (3.9) together with (3.12) only partially reproduce the Lagrangian (3.7). Let us therefore consider a second term given by $\Xi_6=\Omega\wedge\Xi_3=-\Omega\wedge(\mathcal{D}J)$, where $\Omega$ is the holomorphic three-form of a Calabi-Yau three-fold. In components, $\Xi_6$ reads

$$\Xi_{ijklmn}=20\,\Omega_{[\underline{ijk}}\,\Xi_{\underline{lmn}]}\,, \tag{3.13}$$

and the corresponding kinetic term can be evaluated using the relations shown in appendix A.1. After a somewhat tedious but straightforward computation, we find

$$-\frac{1}{2}\,\Xi_6\wedge\star\overline{\Xi}_6=-2\left[F^{\overline{c}}{}_{ab}\,F^c{}_{\overline{a}\overline{b}}\,g_{c\overline{c}}\,g^{a\overline{a}}\,g^{b\overline{b}}-2\,F^{\overline{c}}{}_{ab}\,F^b{}_{\overline{a}\overline{c}}\,g^{a\overline{a}}\right]\star 1\,. \tag{3.14}$$

**Part 3.** One of the terms in (3.14) has the required form to complete the matching with the Lagrangian (3.7). However, also an additional term was generated. In order to cancel this new term, we consider $\Xi_4=-\mathcal{D}\Omega=F\circ\Omega$. Note that $\Xi_4$ is a $(2,2)$-form, since there are no cohomological $(3,1)$-forms on a Calabi-Yau three-fold. In components, we find

$$\Xi_{ijkl}=6\,\Omega_{[\underline{ijm}}\,F^m{}_{\underline{kl}]}\,, \tag{3.15}$$

and the kinetic term can again be evaluated using the identities given in appendix A.1. In particular, we have

$$\frac{1}{2}\,\Xi_4\wedge\star\overline{\Xi}_4=2\,F^{\overline{c}}{}_{ab}\,F^c{}_{\overline{a}\overline{b}}\,g_{c\overline{c}}\,g^{a\overline{a}}\,g^{b\overline{b}}\;\star 1\,. \tag{3.16}$$

**Combining the individual terms.** We can now combine equation (3.9) and (3.12), together with (3.14) and (3.16). Since the prefactors have been chosen in an appropriate way, with the help of the Bianchi identity (3.11) we obtain

$$\star\mathcal{L}_{\mathrm{NS\,NS}}=-e^{-2\phi}\left[\frac{1}{2}\,\Xi_3\wedge\star\Xi_3+\frac{1}{2}\,\Xi_4\wedge\star\overline{\Xi}_4-\frac{1}{2}\,\Xi_6\wedge\star\overline{\Xi}_6\right]. \tag{3.17}$$

Substituting the definitions for $\Xi_3$, $\Xi_4$ and $\Xi_6$ given above, we arrive at

$$\begin{aligned}\star\mathcal{L}_{\mathrm{NS\,NS}}=-e^{-2\phi}\bigg[&\frac{1}{2}\,(F\circ J)\wedge\star(F\circ J)+\frac{1}{2}\,(F\circ\Omega)\wedge\star(F\circ\overline{\Omega})\\&-\frac{1}{2}\,(\Omega\wedge F\circ J)\wedge\star(\overline{\Omega}\wedge F\circ J)\bigg].\end{aligned} \tag{3.18}$$

Let us emphasize that this expression does not contain the metric of the Calabi-Yau manifold explicitly but depends only on $J$ and $\Omega$. It can therefore be evaluated using special geometry. We will come back to this point in section 4.



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**Pure $Q$-flux.** The case of pure $Q$-flux is similar to the situation with pure $F$-flux. We do not present a detailed derivation here, but only want to mention one important technical step in the computation. In particular, since there are no one-forms on a Calabi-Yau manifold, we have $Q \bullet J = 0$ (in cohomology). This implies that

$$\left(Q \bullet \frac{1}{2} J^2\right)_{ijk} = Q_i{}^{mn} J_{jm} J_{kn} + \text{cyclic}\,. \tag{3.19}$$

With all other fluxes set to zero the only Bianchi identity in (3.4) that survives is $Q_m{}^{[ij} \cdot Q_k{}^{l]m} = 0$. In a complex basis it implies in particular

$$g_{a\bar{a}} Q_{\bar{b}}{}^{ac}\, Q_c{}^{\bar{a}\bar{b}} = g_{a\bar{a}} Q_b{}^{a\bar{c}}\, Q_{\bar{c}}{}^{\bar{a}b}\,. \tag{3.20}$$

Using this identity, the relation (3.19), as well as properties shown in appendix A.1, we then obtain

$$\begin{aligned} \star\mathcal{L}_{\text{NS NS}} = -e^{-2\phi} \Bigg[ & \frac{1}{2}\left(Q \bullet \frac{1}{2}J^2\right) \wedge \star \left(Q \bullet \frac{1}{2}J^2\right) + \frac{1}{2}\,(Q \bullet \Omega) \wedge \star\, (Q \bullet \overline{\Omega}) \\ & -\frac{1}{2}\left(\Omega \wedge Q \bullet \frac{1}{2}J^2\right) \wedge \star \left(\overline{\Omega} \wedge Q \bullet \frac{1}{2}J^2\right)\Bigg]\,. \end{aligned} \tag{3.21}$$

Note that this expression is completely analogous to the pure $F$-flux result (3.18).

**Pure $R$-flux.** Finally, we analyze the case of pure $R$-flux. To do so, we first note that the volume form of a Calabi-Yau three-fold can be expressed using the Kähler form $J$ as

$$\sqrt{g}\, d^6x = \frac{1}{6!}\, \epsilon_{i_1 \ldots i_6}\, dx^{i_1} \wedge \ldots \wedge dx^{i_6} = \frac{1}{3!} J^3\,, \tag{3.22}$$

where $\epsilon_{i_1\ldots i_6}$ is the totally anti-symmetric tensor with $\epsilon_{123456} = \sqrt{g}$. Recalling the definition of $R\llcorner$ shown in (3.2), we can compute

$$R\llcorner \left(\frac{1}{3!}\, J^3\right) = -\frac{1}{3!\,3!}\, R^{ijk}\, \epsilon_{ijkpqr} dx^p \wedge dx^q \wedge dx^r\,. \tag{3.23}$$

Next, with the relation $\epsilon^{m_1 m_2 m_3 i_1 i_2 i_3} \epsilon_{m_1 m_2 m_3 j_1 j_2 j_3} = 3!\,3!\, \delta^{[i_1}_{j_1}\, \delta^{i_2}_{j_2}\, \delta^{i_3]}_{j_3}$ (valid for a manifold with Euclidean signature), we compute

$$R\llcorner \left(\frac{1}{3!}\, J^3\right) \wedge \star R\llcorner \left(\frac{1}{3!}\, J^3\right) = \frac{1}{3!}\, R^{ijk}\, R^{i'j'k'}\, g_{ii'} g_{jj'} g_{kk'} \star 1\,. \tag{3.24}$$

For the Lagrangians (2.21) and (2.22) we therefore obtain

$$\star\mathcal{L}_{\text{NS NS}} = -\frac{e^{-2\phi}}{2}\, R\llcorner \left(\frac{1}{3!}\, J^3\right) \wedge \star R\llcorner \left(\frac{1}{3!}\, J^3\right)\,. \tag{3.25}$$

For future use we also note the following relation, which can be verified using for instance equation (A.1):

$$(R\llcorner\Omega) \wedge \star\, (R\llcorner\overline{\Omega}) - \left(\Omega \wedge R\llcorner \frac{1}{3!} J^3\right) \wedge \star \left(\overline{\Omega} \wedge R\llcorner \frac{1}{3!} J^3\right) = 0\,. \tag{3.26}$$



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### 3.3 General result

In the last section we have shown how the DFT Lagrangians (2.21) and (2.22) can be rewritten, such that only the Kähler form $J$ and the holomorphic three-form $\Omega$ appear explicitly. We assumed a vanishing $B$-field, and have considered only one type of flux being present. In this section, we now allow for all types of fluxes $H$, $F$, $Q$ and $R$ being present simultaneously (subject to Bianchi identities).

Motivated by our previous results, summarized in equations (3.6), (3.18), (3.21), (3.25) and (3.26), we define the three-form

$$\chi = -H - F \circ (iJ) - Q \bullet \left(\frac{(iJ)^2}{2!}\right) - R \llcorner \left(\frac{(iJ)^3}{3!}\right) . \tag{3.27}$$

Noting then that the Kähler form $J$ is closed under $d$, we can write

$$\chi = \mathcal{D}\left(e^{iJ}\right) . \tag{3.28}$$

Similarly, recalling that the holomorphic three-form is $d$-closed, we define a multi-form of even degree as

$$\Psi = -H \wedge \Omega - F \circ \Omega - Q \bullet \Omega - R \llcorner \Omega = \mathcal{D}\Omega . \tag{3.29}$$

We now propose the following form of the DFT Lagrangian in the NS-NS sector on a Calabi-Yau three-fold with vanishing $B$-field

$$\begin{aligned} \star\mathcal{L}_{\mathrm{NS\,NS}} = -e^{-2\phi}\Big[&\frac{1}{2}\chi \wedge \star\overline{\chi} + \frac{1}{2}\Psi \wedge \star\overline{\Psi} \\ &- \frac{1}{4}(\Omega \wedge \chi) \wedge \star(\overline{\Omega} \wedge \overline{\chi}) - \frac{1}{4}(\Omega \wedge \overline{\chi}) \wedge \star(\overline{\Omega} \wedge \chi)\Big] . \end{aligned} \tag{3.30}$$

In appendix B, we show that this Lagrangian indeed corresponds to the full Lagrangian (2.19) (for vanishing $B$-field).

### 3.4 Including the $B$-field

Let us now include a non-vanishing $B$-field, which however satisfies $dB = 0$ on the Calabi-Yau manifold. The computations are completely analogous to section 3.2, provided we substitute

$$H \to \mathfrak{H} , \qquad F \to \mathfrak{F} , \qquad Q \to \mathfrak{Q} , \qquad R \to \mathfrak{R} , \tag{3.31}$$

where the flux orbits have been defined in (2.20). This implies that the Lagrangian (3.30) is the correct expression even with $B$-field, but with the twisted differential $\mathcal{D}$ in (3.28) and (3.29) replaced by

$$\mathcal{D} \to \mathfrak{D} = d - \mathfrak{H} \wedge - \mathfrak{F} \circ - \mathfrak{Q} \bullet - \mathfrak{R} \llcorner . \tag{3.32}$$



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Next, we note that using the (local) form of the flux operators (3.2), we can check that $\mathfrak{D}$ can be expressed in terms of $\mathcal{D}$ as

$$\mathfrak{D} = e^{-B}\mathcal{D}\,e^{B} - \frac{1}{2}\Big(\mathfrak{Q}_i{}^{mn}B_{mn}\,dx^i + \mathfrak{R}^{imn}B_{mn}\,\iota_i\Big)\,. \tag{3.33}$$

On a Calabi-Yau manifold, the last two terms may be locally defined, but not globally. This is due to the absence of non-trivial one-forms (in cohomology), and therefore we can discard them in the following. However, in general these terms combine with the fluxes (2.8) into new flux orbits, analogous to (2.20).

Employing then the relation (3.33) on a Calabi-Yau manifold, we can conclude that the rewritten Lagrangian for non-vanishing $B$-field is also given by (3.30), that is

$$\begin{aligned}\star\mathcal{L}_{\mathrm{NS\,NS}} = -e^{-2\phi}\bigg[&\frac{1}{2}\,\chi\wedge\star\overline{\chi} \;+\; \frac{1}{2}\,\Psi\wedge\star\overline{\Psi}\\ &-\frac{1}{4}\left(\Omega\wedge\chi\right)\wedge\star\left(\overline{\Omega}\wedge\overline{\chi}\right) - \frac{1}{4}\left(\Omega\wedge\overline{\chi}\right)\wedge\star\left(\overline{\Omega}\wedge\chi\right)\bigg]\,,\end{aligned} \tag{3.34}$$

together with

$$\begin{aligned}\chi &= -\mathfrak{H} - \mathfrak{F}\circ(iJ) - \mathfrak{Q}\bullet\left(\frac{(iJ)\wedge(iJ)}{2}\right) - \mathfrak{R}\llcorner\left(\frac{(iJ)\wedge(iJ)\wedge(iJ)}{6}\right)\\ &= \mathfrak{D}\,e^{iJ}\\ &= e^{-B}\mathcal{D}\left(e^{B+iJ}\right)\,,\end{aligned} \tag{3.35}$$

and

$$\begin{aligned}\Psi &= -\mathfrak{H}\wedge\Omega - \mathfrak{F}\circ\Omega - \mathfrak{Q}\bullet\Omega - \mathfrak{R}\llcorner\Omega\\ &= \mathfrak{D}\,\Omega\\ &= e^{-B}\mathcal{D}\left(e^{B}\,\Omega\right)\,.\end{aligned} \tag{3.36}$$

### 3.5 The Ramond-Ramond sector

The R-R sector (2.24) of the DFT action is much simpler to rewrite. Let us first introduce an even multi-form of R-R potentials $C^{(2n)}$ as

$$\mathcal{C} = C^{(0)} + C^{(2)} + C^{(4)} + C^{(6)} + C^{(8)} + C^{(10)}\,. \tag{3.37}$$

The individual components are not all independent, but are subject to duality relations. Furthermore, our convention is that the forms $C^{(2n)}$ are closed on the Calabi-Yau three-fold, and the only non-vanishing flux is $F^{(3)}$, corresponding to the R-R two-form. With the help of the operators (3.2), we can express the flux shown in (2.23) as

$$\begin{aligned}\mathfrak{G} &= F^{(3)} - \mathfrak{H}\wedge C^{(0)} - \mathfrak{F}\circ C^{(2)} - \mathfrak{Q}\bullet C^{(4)} - \mathfrak{R}\llcorner C^{(6)}\\ &= F^{(3)} + \mathfrak{D}\,\mathcal{C}\\ &= F^{(3)} + e^{-B}\mathcal{D}\left(e^{B}\,\mathcal{C}\right)\,.\end{aligned} \tag{3.38}$$

The DFT action in the Ramond-Ramond sector (2.24) can then be written as

$$\star\mathcal{L}_{\mathrm{RR}} = -\frac{1}{2}\,\mathfrak{G}\wedge\star\mathfrak{G}\,. \tag{3.39}$$



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## 4 Relation to $N = 2$ gauged supergravity

In this section, we evaluate the DFT actions (3.34) and (3.39) on a Calabi-Yau three-fold. We show that the resulting scalar potential in four dimensions is that of $N = 2$ gauged supergravity. As we have emphasized before, the rewritten DFT actions (3.34) and (3.39) no longer depend on the metric explicitly, but only on the Kähler form $J$ and the holomorphic three-form $\Omega$. We can therefore employ special geometry to carry out the dimensional reduction.

### 4.1 Generalities

Let us first introduce some notation, and recall relations in special geometry. For more details and derivations, we would like to refer the reader for instance to [48].

**Odd cohomology.** In the following, we consider a Calabi-Yau three-fold $\mathcal{X}$, and denote a symplectic basis for the third cohomology by

$$\{\alpha_\Lambda, \beta^\Lambda\} \in H^3(\mathcal{X})\,, \qquad \Lambda = 0, \ldots, h^{2,1}\,. \tag{4.1}$$

This basis can be chosen such that the only non-vanishing pairings satisfy

$$\int_{\mathcal{X}} \alpha_\Lambda \wedge \beta^\Sigma = \delta_\Lambda{}^\Sigma\,. \tag{4.2}$$

The holomorphic three-form $\Omega$ can be expanded in the basis (4.1) as

$$\Omega = X^\Lambda \alpha_\Lambda - F_\Lambda \beta^\Lambda, \tag{4.3}$$

where the periods $X^\Lambda$ and $F_\Lambda$ are functions of the complex-structure moduli $\mathcal{U}^m$, with $i = m, \ldots, h^{2,1}$. The periods $F_\Lambda$ can be determined from a holomorphic prepotential $F$ as $F_\Lambda = \partial F/\partial X^\Lambda$, and using $F_{\Lambda\Sigma} = \partial F_\Lambda/\partial X^\Sigma$ we define the so-called period matrix as

$$\mathcal{N}_{\Lambda\Sigma} = \overline{F}_{\Lambda\Sigma} + 2i\,\frac{\mathrm{Im}(F_{\Lambda\Gamma})X^\Gamma\,\mathrm{Im}(F_{\Sigma\Delta})X^\Delta}{X^\Gamma\,\mathrm{Im}(F_{\Gamma\Delta})X^\Delta}\,. \tag{4.4}$$

This matrix can be used to determine

$$\begin{aligned}
\int_{\mathcal{X}} \alpha_\Lambda \wedge \star\alpha_\Sigma &= -\,(\mathrm{Im}\mathcal{N})_{\Lambda\Sigma} - \Big[(\mathrm{Re}\mathcal{N})\,(\mathrm{Im}\mathcal{N})^{-1}\,(\mathrm{Re}\mathcal{N})\Big]_{\Lambda\Sigma}\,,\\
\int_{\mathcal{X}} \alpha_\Lambda \wedge \star\beta^\Sigma &= -\,\Big[(\mathrm{Re}\mathcal{N})\,(\mathrm{Im}\mathcal{N})^{-1}\Big]_\Lambda{}^\Sigma\,,\\
\int_{\mathcal{X}} \beta^\Lambda \wedge \star\beta^\Sigma &= -\,\Big[(\mathrm{Im}\mathcal{N})^{-1}\Big]^{\Lambda\Sigma}\,.
\end{aligned} \tag{4.5}$$

For later convenience, we also define

$$\begin{aligned}
\mathcal{M}_1 &= \begin{pmatrix} \mathbb{1} & \mathrm{Re}\mathcal{N} \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} -\mathrm{Im}\mathcal{N} & 0 \\ 0 & -\mathrm{Im}\mathcal{N}^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ \mathrm{Re}\mathcal{N} & \mathbb{1} \end{pmatrix}\\
&= \int_{\mathcal{X}} \begin{pmatrix} \alpha_\Lambda \wedge \star\alpha_\Sigma & \alpha_\Lambda \wedge \star\beta^\Sigma \\ \beta^\Lambda \wedge \star\alpha_\Sigma & \beta^\Lambda \wedge \star\beta^\Sigma \end{pmatrix}.
\end{aligned} \tag{4.6}$$



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**Even cohomology.** For the $(1,1)$- and $(2,2)$-cohomology of $\mathcal{X}$ we introduce bases of the form

$$\begin{aligned} \{\omega_{\mathsf{A}}\} &\in H^{1,1}(\mathcal{X})\,, \\ \{\sigma^{\mathsf{A}}\} &\in H^{2,2}(\mathcal{X})\,, \end{aligned} \qquad \mathsf{A}=1,\ldots,h^{1,1}\,. \tag{4.7}$$

For later convenience, we can group these two- and four-forms together with the zero- and six-form of the Calabi-Yau three-fold. In particular, we write

$$\begin{aligned} \{\omega_A\} &= \left\{\frac{\sqrt{g}}{\mathcal{V}}dx^6,\,\omega_{\mathsf{A}}\right\}\,, \\ \{\sigma^A\} &= \left\{1,\,\sigma^{\mathsf{A}}\right\}\,, \end{aligned} \qquad A=0,\ldots,h^{1,1}\,, \tag{4.8}$$

where $\mathcal{V}=\int_{\mathcal{X}}\sqrt{g}d^6x$ is the volume of the Calabi-Yau three-fold $\mathcal{X}$. These two bases can be chosen such that

$$\int_{\mathcal{X}}\omega_A\wedge\sigma^B=\delta_A{}^B\,. \tag{4.9}$$

The triple intersection numbers corresponding to the bases (4.7) are given by

$$\kappa_{\mathsf{ABC}}=\int_{\mathcal{X}}\omega_{\mathsf{A}}\wedge\omega_{\mathsf{B}}\wedge\omega_{\mathsf{C}}\,. \tag{4.10}$$

The Kähler form $J$ of the Calabi-Yau three-fold $\mathcal{X}$ and the Kalb-Ramond field $B$ are expanded in the basis $\{\omega_{\mathsf{A}}\}$ in the following way

$$J=t^{\mathsf{A}}\omega_{\mathsf{A}}\,,\qquad B=b^{\mathsf{A}}\omega_{\mathsf{A}}\,, \tag{4.11}$$

which can be combined into a complex field $\mathcal{J}$ as

$$\mathcal{J}=B+iJ=\left(b^{\mathsf{A}}+it^{\mathsf{A}}\right)\omega_{\mathsf{A}}=\mathcal{J}^{\mathsf{A}}\omega_{\mathsf{A}}\,. \tag{4.12}$$

***B*-twisted Hodge-star operator and Mukai pairings.** For later convenience, let us define the so-called Mukai pairing between forms $\rho$ and $\nu$. It is given by

$$\langle\rho,\nu\rangle=\left[\rho\wedge\lambda(\nu)\right]_{\text{top}}\,, \tag{4.13}$$

where the projection operator $\lambda$ acts on $2n$-forms as $\lambda(\rho^{(2n)})=(-1)^n\rho^{(2n)}$ and on $(2n-1)$-forms as $\lambda(\rho^{(2n-1)})=(-1)^n\rho^{(2n-1)}$. Furthermore, we define a $B$-twisted Hodge-star operator acting on forms $\rho$ as [17, 21, 49]

$$\star_B\rho=e^{+B}\wedge\star\lambda\left(e^{-B}\rho\right)\,. \tag{4.14}$$

For three-forms $\alpha_\Lambda$, we then find for instance that

$$\left\langle\alpha_\Lambda,\star_B\alpha_\Sigma\right\rangle=\left(\alpha_\Lambda\wedge e^{-B}\right)\wedge\star\left(\alpha_\Sigma\wedge e^{-B}\right)\,, \tag{4.15}$$

and similarly for the others. Since for three-forms on a Calabi-Yau three-fold the factor $e^{-B}$ gives no contribution, we can express the matrix (4.6) also in the following way

$$\mathcal{M}_1 = +\int_{\mathcal{X}} \begin{pmatrix} \langle \alpha_\Lambda, \star_B \alpha_\Sigma \rangle & \langle \alpha_\Lambda, \star_B \beta^\Sigma \rangle \\ \langle \beta^\Lambda, \star_B \alpha_\Sigma \rangle & \langle \beta^\Lambda, \star_B \beta^\Sigma \rangle \end{pmatrix} . \tag{4.16}$$

The analogue of in (4.16) for the even co-homology takes a very similar form. In particular, we have

$$\mathcal{M}_2 = -\int_{\mathcal{X}} \begin{pmatrix} \langle \omega_A, \star_B \omega_B \rangle & \langle \omega_A, \star_B \sigma^B \rangle \\ \langle \sigma^A, \star_B \omega_B \rangle & \langle \sigma^A, \star_B \sigma^B \rangle \end{pmatrix} , \tag{4.17}$$

where for instance

$$\langle \omega_A, \star_B \omega_B \rangle = -\left(\omega_A \wedge e^{-B}\right) \wedge \star \left(\omega_B \wedge e^{-B}\right) . \tag{4.18}$$

Note that both $\mathcal{M}_1$ and $\mathcal{M}_2$ are positive definite matrices.

**Fluxes.** The action of fluxes on the cohomology *in a local basis* has been given in (3.2). However, for a Calabi-Yau manifold this can be made more specific. Similarly to [18], we define

$$\begin{aligned} \mathcal{D}\alpha_\Lambda &= q_\Lambda{}^A \omega_A + f_{\Lambda A} \sigma^A , & \mathcal{D}\beta^\Lambda &= \tilde{q}^{\Lambda A} \omega_A + \tilde{f}^\Lambda{}_A \sigma^A , \\ \mathcal{D}\omega_A &= -\tilde{f}^\Lambda{}_A \alpha_\Lambda + f_{\Lambda A} \beta^\Lambda , & \mathcal{D}\sigma^A &= \tilde{q}^{\Lambda A} \alpha_\Lambda - q_\Lambda{}^A \beta^\Lambda . \end{aligned} \tag{4.19}$$

Here, $f_{\Lambda \mathsf{A}}$ and $\tilde{f}^\Lambda{}_\mathsf{A}$ denote the geometric $F$-fluxes, while $q_\Lambda{}^\mathsf{A}$ and $\tilde{q}^{\Lambda \mathsf{A}}$ are the non-geometric $Q$-fluxes. Furthermore, we use the following convention for the $H$- and $R$-flux

$$\begin{aligned} f_{\Lambda 0} &= r_\Lambda , & \tilde{f}^\Lambda{}_0 &= \tilde{r}^\Lambda , \\ q_\Lambda{}^0 &= h_\Lambda , & \tilde{q}^{\Lambda 0} &= \tilde{h}^\Lambda . \end{aligned} \tag{4.20}$$

Let us also note that the $H$-flux from section 3 is related to the flux parameters as $H = -\tilde{h}^\Lambda \alpha_\Lambda + h_\Lambda \beta^\Lambda$. For later convenience, we also define a $(2h^{2,1}+2) \times (2h^{1,1}+2)$ matrix as follows

$$\mathcal{O} = \begin{pmatrix} -\tilde{f}^\Lambda{}_A & \tilde{q}^{\Lambda A} \\ f_{\Lambda A} & -q_\Lambda{}^A \end{pmatrix} . \tag{4.21}$$

### 4.2 Evaluating the action

Next, we evaluate the action derived in the previous section for a Calabi-Yau three-fold. The action for the NS-NS sector has been shown in (3.34), and for the R-R sector in equation (3.39).

**NS-NS sector — Part 1.** We begin with the NS-NS sector, and focus on the three-form $\chi$ defined in (3.35). First, we expand in the basis (4.7)

$$e^{B+iJ} = e^{\mathcal{J}} = 1 + \mathcal{J}^\mathsf{A} \omega_\mathsf{A} + \frac{1}{2}\left[\kappa_{\mathsf{ABC}} \mathcal{J}^\mathsf{B} \mathcal{J}^\mathsf{C}\right] \sigma^\mathsf{A} + \frac{1}{6}\left[\kappa_{\mathsf{ABC}} \mathcal{J}^\mathsf{A} \mathcal{J}^\mathsf{B} \mathcal{J}^\mathsf{C}\right] \omega_0 . \tag{4.22}$$



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Using then the combined basis $\{\omega_A, \sigma^A\}$, we can define a complex $(2h^{1,1}+2)$-dimensional vector $V_1$ in the following way

$$V_1 = \begin{pmatrix} \frac{1}{6}\kappa_{\mathsf{ABC}}\mathcal{J}^{\mathsf{A}}\mathcal{J}^{\mathsf{B}}\mathcal{J}^{\mathsf{C}} \\ \mathcal{J}^{\mathsf{A}} \\ 1 \\ \frac{1}{2}\kappa_{\mathsf{ABC}}\mathcal{J}^{\mathsf{B}}\mathcal{J}^{\mathsf{C}} \end{pmatrix}, \tag{4.23}$$

and employing matrix multiplication we observe that $(\omega_A\ \sigma^A)\cdot V_1 = e^{\mathcal{J}}$. Next, we note that the last line in (4.19) can be expressed using (4.21) as

$$\mathcal{D}\begin{pmatrix}\omega_A \\ \sigma^A\end{pmatrix} = \mathcal{O}^T\begin{pmatrix}\alpha_\Lambda \\ \beta^\Lambda\end{pmatrix}. \tag{4.24}$$

We then evaluate

$$\chi = e^{-B}\mathcal{D}\,e^{\mathcal{J}} = e^{-B}\left(\alpha_\Lambda\ \beta^\Lambda\right)\cdot\mathcal{O}\cdot V_1\,, \tag{4.25}$$

and together with the matrix $\mathcal{M}_1$ given in equation (4.16), we have

$$\int_{\mathcal{X}} \chi\wedge\star\overline{\chi} = V_1^T\cdot\mathcal{O}^T\cdot\mathcal{M}_1\cdot\mathcal{O}\cdot V_1\,. \tag{4.26}$$

**NS-NS sector — Part 2.** A very similar route can be followed for the even multi-form $\Psi$ defined in equation (3.36). We introduce a $(2h^{2,1}+2)$-dimensional vector as

$$V_2 = \begin{pmatrix} X^\Lambda \\ -F_\Lambda \end{pmatrix}, \tag{4.27}$$

and with the basis of three-forms given in (4.1) we can write

$$\Omega = \left(\alpha_\Lambda\ \beta^\Lambda\right)\cdot V_2\,. \tag{4.28}$$

Analogous to (4.24), we observe that using the matrix $\mathcal{O}$ defined in (4.21) we have

$$\mathcal{D}\begin{pmatrix}\alpha_\Lambda \\ \beta^\Lambda\end{pmatrix} = -\tilde{\mathcal{O}}\begin{pmatrix}\omega_A \\ \sigma^A\end{pmatrix}, \qquad \tilde{\mathcal{O}} = C\cdot\mathcal{O}\cdot C^T\,, \tag{4.29}$$

where we introduced a matrix $C$ defined as

$$C = \begin{pmatrix} 0 & +\mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}. \tag{4.30}$$

As it will be clear from the context, the dimensions of this symplectic structure are either $(2h^{1,1}+2)\times(2h^{1,1}+2)$ or $(2h^{2,1}+2)\times(2h^{2,1}+2)$. We then obtain

$$\Psi = e^{-B}\mathcal{D}\left(e^B\,\Omega\right) = -e^{-B}\left(\omega_A\ \sigma^A\right)\cdot\tilde{\mathcal{O}}^T\cdot V_2\,, \tag{4.31}$$

and with the help of (4.17) we evaluate

$$\int_{\mathcal{X}} \Psi\wedge\star\overline{\Psi} = V_2^T\cdot\tilde{\mathcal{O}}\cdot\mathcal{M}_2\cdot\tilde{\mathcal{O}}^T\cdot\overline{V}_2\,. \tag{4.32}$$



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**NS-NS sector — Part 3.** Let us now consider the second line in the NS-NS action (3.34). We first note that for two six-forms $\rho_1$ and $\rho_2$ and with $\mathcal{V}$ the volume of the Calabi-Yau three-fold $\mathcal{X}$ the following relation holds

$$\int_{\mathcal{X}} \rho_1 \wedge \star \rho_2 = \frac{1}{\mathcal{V}} \int_{\mathcal{X}} \rho_1 \times \int_{\mathcal{X}} \rho_2 \,. \tag{4.33}$$

Using (4.2) and the matrix $C$ defined in (4.30), let us then determine for instance

$$\int_{\mathcal{X}} \Omega \wedge \chi = V_2^T \cdot C \cdot \mathcal{O} \cdot V_1 \,. \tag{4.34}$$

The various other combinations are obtained analogously, and we can combine these results in the following way

$$\begin{aligned} &\int_{\mathcal{X}} \Big[ (\Omega \wedge \chi) \wedge \star (\overline{\Omega} \wedge \overline{\chi}) + (\Omega \wedge \overline{\chi}) \wedge \star (\overline{\Omega} \wedge \chi) \Big] \\ &\qquad = \frac{1}{\mathcal{V}}\, V_2^T \cdot C \cdot \mathcal{O} \cdot \Big( V_1 \times \overline{V}_1^T + \overline{V}_1 \times V_1^T \Big) \cdot \mathcal{O}^T \cdot C^T \cdot \overline{V}_2 \,. \end{aligned} \tag{4.35}$$

**R-R sector.** We finally turn to the Ramond-Ramond sector. The corresponding rewritten action is shown in equation (3.39). We expand the R-R three-form flux $F^{(3)}$ in the basis (4.1), and define a corresponding $(2h^{2,1}+2)$-dimensional vector as

$$F^{(3)} = -\tilde{\mathsf{F}}^{\Lambda} \alpha_{\Lambda} + \mathsf{F}_{\Lambda} \beta^{\Lambda} \Rightarrow \mathsf{F}^{(3)} = \begin{pmatrix} -\tilde{\mathsf{F}}^{\Lambda} \\ \mathsf{F}_{\Lambda} \end{pmatrix} . \tag{4.36}$$

For the $B$-twisted R-R potentials, we expand the relevant contributions in the basis of even forms (4.8) as

$$e^B \mathcal{C} = \mathsf{C}^{(0)} + \mathsf{C}^{(2)\mathsf{A}} \omega_{\mathsf{A}} + \mathsf{C}^{(4)}{}_{\mathsf{A}} \sigma^{\mathsf{A}} + \mathsf{C}^{(6)} \omega_0 \,, \tag{4.37}$$

which defines a $(2h^{1,1}+2)$-dimensional vector $\mathsf{C}$

$$\mathsf{C} = \begin{pmatrix} \mathsf{C}^{(6)} \\ \mathsf{C}^{(2)\mathsf{A}} \\ \mathsf{C}^{(0)} \\ \mathsf{C}^{(4)}{}_{\mathsf{A}} \end{pmatrix} . \tag{4.38}$$

For the three-form flux given in equation (3.38), we recall (4.24) and determine

$$\mathfrak{G} = \big( \alpha_{\Lambda} \;\; \beta^{\Lambda} \big) \cdot (\mathsf{F} + \mathcal{O} \cdot \mathsf{C}) \,. \tag{4.39}$$

Employing finally the matrix $\mathcal{M}_1$ given in (4.6), we arrive at

$$\int_{\mathcal{X}} \mathfrak{G} \wedge \star \mathfrak{G} = \big( \mathsf{F}^T + \mathsf{C}^T \cdot \mathcal{O}^T \big) \cdot \mathcal{M}_1 \cdot (\mathsf{F} + \mathcal{O} \cdot \mathsf{C}) \,. \tag{4.40}$$

**Final result.** We can now combine the above results and obtain the scalar potential originating from evaluating the DFT actions (3.34) and (3.39) on a Calabi-Yau three-fold. Including the appropriate pre-factors, we find from the above expressions

$$\begin{aligned} V = \frac{1}{2}\left(\mathsf{F}^T + \mathsf{C}^T\cdot\mathcal{O}^T\right)\cdot\mathcal{M}_1\cdot\left(\mathsf{F}+\mathcal{O}\cdot\mathsf{C}\right) \\ + \frac{e^{-2\phi}}{2}\, V_1^T\cdot\mathcal{O}^T\cdot\mathcal{M}_1\cdot\mathcal{O}\cdot V_1 \\ + \frac{e^{-2\phi}}{2}\, V_2^T\cdot\tilde{\mathcal{O}}\cdot\mathcal{M}_2\cdot\tilde{\mathcal{O}}^T\cdot\overline{V}_2 \\ - \frac{e^{-2\phi}}{4\mathcal{V}}\, V_2^T\cdot C\cdot\mathcal{O}\cdot\left(V_1\times\overline{V}_1^T+\overline{V}_1\times V_1^T\right)\cdot\mathcal{O}^T\cdot C^T\cdot\overline{V}_2\,. \end{aligned} \tag{4.41}$$

This scalar potential can be brought into the form given in equation (9) in [23] (see also [52]), which was shown to agree with the scalar potential of $N=2$ gauged supergravity. To see that, we first rescale $V_{1,2}\to\sqrt{8\mathcal{V}}\,V_{1,2}$ and note that the potential (4.41) is multiplied by $M_{\rm s}^4$, where $M_{\rm Pl}^4 = M_{\rm s}^4\,\mathcal{V}^2\,e^{-4\phi}$. Introducing then $\Phi=\frac{1}{2}e^{-2\phi}\mathcal{V}$, we can write (4.41) as

$$\begin{aligned} V\to V' = \frac{M_{\rm Pl}^4}{8\,\Phi^2}\left(\mathsf{F}^T + \mathsf{C}^T\cdot\mathcal{O}^T\right)\cdot\mathcal{M}_1\cdot\left(\mathsf{F}+\mathcal{O}\cdot\mathsf{C}\right) \\ + \frac{2\,M_{\rm Pl}^4}{\Phi}\, V_1^T\cdot\mathcal{O}^T\cdot\mathcal{M}_1\cdot\mathcal{O}\cdot V_1 \\ + \frac{2\,M_{\rm Pl}^4}{\Phi}\, V_2^T\cdot\tilde{\mathcal{O}}\cdot\mathcal{M}_2\cdot\tilde{\mathcal{O}}^T\cdot\overline{V}_2 \\ - \frac{8\,M_{\rm Pl}^4}{\Phi}\, V_2^T\cdot C\cdot\mathcal{O}\cdot\left(V_1\times\overline{V}_1^T+\overline{V}_1\times V_1^T\right)\cdot\mathcal{O}^T\cdot C^T\cdot\overline{V}_2\,. \end{aligned} \tag{4.42}$$

Thus, we have succeeded in relating DFT on Calabi-Yau three-folds to the scalar potential of $N=2$ gauged supergravity. This is a quite satisfying result in that DFT not only provides the higher dimensional origin of $N=4$, but also of $N=2$ gauged supergravity.

## 5 Relation to type IIB orientifolds

In this section, we show how the scalar potential (4.41) can be expressed within the $N=1$ supergravity framework. Since in section 4 the four-dimensional theory preserved $N=2$ supersymmetry, we therefore perform an orientifold projection. We choose this projection such that it leads to orientifold three- and seven-planes. Related computations have appeared for instance in [16–18, 23]. For completeness, here we present the full derivation of the scalar F- and D-term potentials, and provide explicit expressions for the case of type IIB orientifolds.

### 5.1 Generalities

We begin our discussion by introducing the notation and conventions to be employed below, and by recalling some well-known properties of type IIB orientifold compactifications on Calabi-Yau three-folds [50].

**Cohomology.** The orientifold projection we perform is $\Omega_{\mathrm{P}}(-1)^{F_{\mathrm{L}}}\sigma$, where $\Omega_{\mathrm{P}}$ denotes the world-sheet parity operator and $F_{\mathrm{L}}$ is the left-moving fermion number. The holomorphic involution $\sigma : \mathcal{X} \to \mathcal{X}$ acts on the Kähler form $J$ and the holomorphic $(3,0)$-form $\Omega$ of the Calabi-Yau three-fold $\mathcal{X}$ as

$$\sigma^* : J \to J\,, \qquad \sigma^* : \Omega \to -\Omega\,, \tag{5.1}$$

and the fixed loci of this involution correspond to O7- and O3-planes. This holomorphic involution splits the cohomology into even and odd parts. This means in particular that

$$H^{p,q}(\mathcal{X}) = H^{p,q}_{+}(\mathcal{X}) \oplus H^{p,q}_{-}(\mathcal{X})\,, \qquad h^{p,q} = h^{p,q}_{+} + h^{p,q}_{-}\,. \tag{5.2}$$

Note that constants as well as the volume form $\sqrt{g}\, d^6x$ on $\mathcal{X}$ are always even under the involution. For the other bases introduced in section 4.1 we employ the following notation

$$\begin{aligned}
\{\omega_\alpha\} &\in H^{1,1}_{+}(\mathcal{X}) & \alpha &= 1,\ldots,h^{1,1}_{+}, & \{\omega_a\} &\in H^{1,1}_{-}(\mathcal{X}) & a &= 1,\ldots,h^{1,1}_{-},\\
\{\sigma^\alpha\} &\in H^{2,2}_{+}(\mathcal{X}) & \alpha &= 1,\ldots,h^{1,1}_{+}, & \{\sigma^a\} &\in H^{2,2}_{-}(\mathcal{X}) & a &= 1,\ldots,h^{1,1}_{-},\\
\{\alpha_{\hat\lambda},\beta^{\hat\lambda}\} &\in H^{3}_{+}(\mathcal{X}) & \hat\lambda &= 1,\ldots,h^{2,1}_{+}, & \{\alpha_\lambda,\beta^\lambda\} &\in H^{3}_{-}(\mathcal{X}) & \lambda &= 0,\ldots,h^{2,1}_{-}.
\end{aligned} \tag{5.3}$$

**Moduli.** The fields of the ten-dimensional theory transform under the combined world-sheet parity and left-moving fermion number in the following way

$$\Omega_{\mathrm{P}}(-1)^{F_{\mathrm{L}}} = \begin{cases} g,\ \phi,\ C^{(0)},\ C^{(4)} & \text{even}\,,\\ B,\ C^{(2)} & \text{odd}\,. \end{cases} \tag{5.4}$$

Together with (5.1), it then follows that the holomorphic three-form $\Omega$ is expanded in the odd cohomology $H^3_{-}(\mathcal{X})$

$$\Omega = X^\lambda \alpha_\lambda - F_\lambda \beta^\lambda\,. \tag{5.5}$$

Note that the complex-structure moduli $\mathcal{U}^\mu$ with $\mu = 1,\ldots,h^{2,1}_{-}$ are encoded in the holomorphic three-form. The Kähler form $J$ and the components of the ten-dimensional form fields along the six-dimensional space $\mathcal{X}$ can be expanded as

$$J = t^\alpha \omega_\alpha\,, \qquad B = b^a \omega_a\,, \qquad C^{(2)} = c^a \omega_a\,, \qquad C^{(4)} = \rho_\alpha \sigma^\alpha\,, \tag{5.6}$$

where the components $t^\alpha$ of the Kähler form are in string frame. Quantities in Einstein frame will be denoted by a hat, and the transition between string and Einstein frame is achieved by

$$\hat t^\alpha = e^{-\phi/2}\, t^\alpha\,. \tag{5.7}$$

Apart from the complex structure moduli, the remaining moduli fields in the effective four-dimensional theory after compactification are the following [50]

$$\begin{aligned}
\tau &= C^{(0)} + i\, e^{-\phi}\,,\\
G^a &= c^a + \tau\, b^a\,,\\
T_\alpha &= -\frac{i}{2}\,\kappa_{\alpha\beta\gamma}\,\hat t^\beta \hat t^\gamma + \rho_\alpha + \frac{1}{2}\kappa_{\alpha ab}\, c^a b^b - \frac{i}{4}\, e^{\phi}\,\kappa_{\alpha ab}\, G^a (G-\overline{G})^b\,,
\end{aligned} \tag{5.8}$$

where $\kappa_{\alpha\beta\gamma}$ and $\kappa_{\alpha ab}$ are the triple intersection numbers defined in (4.10). Using the sum of even R-R potentials $\mathcal{C}$ defined in (3.37), these moduli can be encoded in a complex and even multi-form $\Phi_c^{\rm ev}$ as follows [17]

$$\begin{aligned}\Phi_c^{\rm ev} &= e^B \mathcal{C} + i\, e^{-\phi}\, \mathrm{Re}\left(e^{B+iJ}\right) \\ &= \tau + G^a \omega_a + T_\alpha \sigma^\alpha \,.\end{aligned} \tag{5.9}$$

**Fluxes.** For the R-R three-form flux $F^{(3)}$ and the various geometric and non-geometric NS-NS fluxes, we observe the following behavior under the combined world-sheet parity and left-moving fermion-number transformation. In particular, we have

$$\Omega_{\rm P}(-1)^{F_{\rm L}} = \begin{cases} F,\, R & \text{even}\,, \\ H,\, Q,\, F^{(3)} & \text{odd}\,. \end{cases} \tag{5.10}$$

Including the holomorphic involution $\sigma$ defined in (5.1), recalling (5.3), and employing the same notation as at the end of section 4.1, we can deduce the non-vanishing flux components as follows

$$\begin{aligned} F^{(3)} :& \qquad \mathsf{F}_\lambda \,, \tilde{\mathsf{F}}^\lambda \,, \\ H :& \qquad h_\lambda \,, \tilde{h}^\lambda \,, \\ F :& \qquad f_{\hat\lambda \alpha} \,, \tilde{f}^{\hat\lambda}{}_{\alpha} \,, f_{\lambda a} \,, \tilde{f}^{\lambda}{}_{a} \,, \\ Q :& \qquad q_{\hat\lambda}{}^{a} \,, \tilde{q}^{\hat\lambda a} \,, q_{\lambda}{}^{\alpha} \,, \tilde{q}^{\lambda\alpha} \,, \\ R :& \qquad r_{\hat\lambda} \,, \tilde{r}^{\hat\lambda} \,. \end{aligned} \tag{5.11}$$

### 5.2 F-term potential

In this section, we show how after the orientifold projection (part of) the scalar potential (4.41) can be expressed in terms of an F-term potential in an $N = 1$ supergravity language.

**General form.** The Kähler potential for the moduli of type IIB orientifolds with O3- and O7-planes takes the following general form [50]

$$\mathcal{K} = -\log\bigl[-i(\tau-\overline{\tau})\bigr] - 2\log\hat{\mathcal{V}} - \log\left[\, i\int_{\mathcal{X}} \Omega\wedge\overline{\Omega}\,\right] , \tag{5.12}$$

where $\hat{\mathcal{V}} = \frac{1}{3!}\,\kappa_{\alpha\beta\gamma}\hat{t}^\alpha\hat{t}^\beta\hat{t}^\gamma$ denotes the volume of the Calabi-Yau three-fold in Einstein frame. The superpotential in the presence of R-R three-form flux $F^{(3)}$ and general NS-NS fluxes can be written as [8] (see also [10–12, 19, 21, 24, 51])

$$W = \int_{\mathcal{X}} \Bigl(F^{(3)} + \mathcal{D}\Phi_c^{\rm ev}\Bigr)\wedge\Omega\,. \tag{5.13}$$

The resulting F-term potential is expressed via the Kähler-covariant derivative $D_I W = \partial_I W + \mathcal{K}_I W$, where $\partial_I$ denotes the derivative with respect to the scalar fields mentioned above and where $\mathcal{K}_I = \partial_I\mathcal{K}$. With $G^{I\overline{J}}$ the inverse of the Kähler metric $G_{I\overline{J}} = \partial_I\partial_{\overline{J}}\mathcal{K}$, we have

$$V_{\rm F} = e^{\mathcal{K}}\left[\, G^{I\overline{J}} D_I W\, D_{\overline{J}}\overline{W} - 3|W|^2\,\right] . \tag{5.14}$$

When using the Kähler potential (5.12), the scalar F-term potential can be simplified. For that purpose, let us split the appearing sums into a sum over complex-structure moduli $U^\mu$, and a sum over $i = \{\tau, G^a, T_\alpha\}$. Employing the no-scale property of (5.12), that is [50]

$$G^{i\bar{j}}\,\mathcal{K}_i\,\mathcal{K}_{\bar{j}} = 4\,, \tag{5.15}$$

and defining $\mathcal{K}^i = G^{i\bar{j}}\partial_{\bar{j}}\mathcal{K}$, we obtain

$$V_{\rm F} = e^{\mathcal{K}}\Big[G^{U\overline{U}}D_U W D_{\overline{U}}\overline{W} + G^{i\bar{j}}\,\partial_i W\,\partial_{\bar{j}}\overline{W} + \big(\mathcal{K}^i\partial_i W\,\overline{W} + \text{c.c.}\big) + |W|^2\Big]\,. \tag{5.16}$$

**Rewriting part 1.** We now consider each line in (5.16) separately and bring them into a form suitable for comparison with the general expression given at the end of section 3. We start with the complex-structure moduli in the first line. For ease of notation we define

$$\mathcal{A} = F^{(3)} + \mathcal{D}\Phi_c^{\rm ev} = \Big[F^{(3)} + \mathcal{D}\left(e^B\mathcal{C}\right)\Big] + i\Big[e^{-\phi}\,\mathcal{D}\,\mathrm{Re}\,\big(e^{B+iJ}\big)\Big], \tag{5.17}$$

for the superpotential (5.13). Let us observe that the real and imaginary part of $\mathcal{A}$ correspond to the three-forms (3.38) and (3.35), respectively. In particular, taking into account (5.11) and recalling that five-forms on a Calabi-Yau three-fold are trivial in cohomology, we have

$$\mathcal{A} = \check{\mathfrak{G}} + i\,e^{-\phi}\,\mathrm{Re}\,\check{\chi}\,, \tag{5.18}$$

where the check indicates the quantities after the orientifold projection. Using then the relations given in (A.10) and (A.11), we can write for the first line in (5.16)

$$e^{\mathcal{K}}G^{U\overline{U}}D_U W D_{\overline{U}}\overline{W} = \frac{e^\phi}{4\hat{\mathcal{V}}^2}\left[\int_{\mathcal{X}}\mathcal{A}\wedge\star\overline{\mathcal{A}} + i\int_{\mathcal{X}}\mathcal{A}\wedge\overline{\mathcal{A}}\right] - e^{\mathcal{K}}\left|\int_{\mathcal{X}}\mathcal{A}\wedge\overline{\Omega}\right|^2. \tag{5.19}$$

Using (5.18), the first term on the right-hand side of (5.19) can be written out as follows

$$\frac{e^\phi}{4\hat{\mathcal{V}}^2}\int_{\mathcal{X}}\mathcal{A}\wedge\star\overline{\mathcal{A}} = \frac{e^\phi}{4\hat{\mathcal{V}}^2}\left[\int_{\mathcal{X}}\check{\mathfrak{G}}\wedge\star\check{\mathfrak{G}} + e^{-2\phi}\int_{\mathcal{X}}(\mathrm{Re}\,\check{\chi})\wedge\star(\mathrm{Re}\,\check{\chi})\right]. \tag{5.20}$$

The second term in (5.19) contributes to various D$p$-brane tadpoles and has to be canceled by local sources. Employing the relation shown in equation (A.13), we find

$$\begin{aligned}\frac{e^\phi}{4\hat{\mathcal{V}}^2}\,i\int_{\mathcal{X}}\mathcal{A}\wedge\overline{\mathcal{A}} &= +\frac{1}{2\hat{\mathcal{V}}^2}\int_{\mathcal{X}}F^{(3)}\wedge\mathcal{D}\,\mathrm{Re}\,\big(e^{B+iJ}\big)\\ &= -\frac{e^\phi}{2\hat{\mathcal{V}}^2}\int_{\mathcal{X}}\Big[(\mathrm{Im}\,\tau) - (\mathrm{Im}\,G^a)\omega_a + (\mathrm{Im}\,T_\alpha)\sigma^\alpha\Big]\wedge\mathcal{D}F^{(3)}\,.\end{aligned} \tag{5.21}$$

The third term on the right-hand side of (5.19) will be addressed below.



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**Rewriting part 2.** For the second line in (5.16) we recall that $\Phi_c^{\rm ev}$ in the superpotential (5.13) is given by (5.9). We can therefore compute

$$\partial_i W = \int_{\mathcal{X}} \mathcal{D}\left(\partial_i \Phi_c^{\rm ev}\right) \wedge \Omega = \int_{\mathcal{X}} \mathcal{D} \begin{pmatrix} 1 \\ \omega_a \\ \sigma^\alpha \end{pmatrix} \wedge \Omega\,, \tag{5.22}$$

where $i = \tau, G^a, T_\alpha$. Using then the relations shown in equation (A.13) of the appendix, we obtain

$$\partial_i W = \int_{\mathcal{X}} \begin{pmatrix} 1 \\ -\omega_a \\ \sigma^\alpha \end{pmatrix} \wedge \mathcal{D}\Omega = \begin{pmatrix} (\mathcal{D}\Omega)^0 \\ -(\mathcal{D}\Omega)_a \\ (\mathcal{D}\Omega)^\alpha \end{pmatrix}\,, \tag{5.23}$$

where, taking into account (5.11), we expanded $\mathcal{D}\Omega$ in the basis (4.8) as

$$\mathcal{D}\Omega = (\mathcal{D}\Omega)^0 \omega_0 + (\mathcal{D}\Omega)_a \sigma^a + (\mathcal{D}\Omega)^\alpha \omega_\alpha\,. \tag{5.24}$$

Let us now evaluate the second line in (5.16). Using the formula for the inverse Kähler metric $G^{i\bar{j}}$ given in (A.18), we obtain

$$e^{\mathcal{K}}\, G^{i\bar{j}}\, \partial_i W\, \partial_{\bar{j}} \overline{W} = e^{\mathcal{K}}\, \frac{4\mathcal{V}}{e^{2\phi}} \int_{\mathcal{X}} \left[e^{-B}\mathcal{D}\Omega\right] \wedge \star \left[e^{-B}\mathcal{D}\overline{\Omega}\right]\,, \tag{5.25}$$

where $\mathcal{V}$ (without the hat) denotes the volume of $\mathcal{X}$ in string frame. By comparing with (3.36) and noting that in cohomology there are no five-forms on a Calabi-Yau three-fold, we can identify $e^{-B}\mathcal{D}\Omega = \check{\Psi}$. Furthermore, for the scalar potential evaluated at a particular point in field space, we can use the relation (A.1). We then find that

$$e^{\mathcal{K}}\, G^{i\bar{j}}\, \partial_i W\, \partial_{\bar{j}} \overline{W} = \frac{e^{-\phi}}{4\hat{\mathcal{V}}^2} \int_{\mathcal{X}} \check{\Psi} \wedge \star \overline{\check{\Psi}}\,. \tag{5.26}$$

**Rewriting part 3.** Next, we discuss the third line in equation (5.16). With the help of the Kähler metric computed from the Kähler potential (5.12), and after a somewhat tedious but straightforward computation, we find

$$\mathcal{K}^\tau = -(\tau - \overline{\tau})\,, \qquad \mathcal{K}^{G^a} = -(G - \overline{G})^a\,, \qquad \mathcal{K}^{T_\alpha} = -(T - \overline{T})_\alpha\,, \tag{5.27}$$

where as before $\mathcal{K}^i = G^{i\bar{j}}\partial_{\bar{j}}\mathcal{K}$. For the derivatives of $\Phi_c^{\rm ev}$ defined in (5.9) with respect to the moduli, we then determine

$$\mathcal{K}^i \partial_i \Phi_c^{\rm ev} = -\Phi_c^{\rm ev} + \overline{\Phi}_c^{\rm ev}\,. \tag{5.28}$$

Employing the short-hand notation (5.17) for the superpotential (5.13), we find

$$\mathcal{K}^i \partial_i W = -\int_{\mathcal{X}} \mathcal{A} \wedge \Omega + \int_{\mathcal{X}} \overline{\mathcal{A}} \wedge \Omega\,. \tag{5.29}$$



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Coming back to the potential (5.16), using (5.29), and re-arranging terms, we obtain for the third line

$$e^{\mathcal{K}}\Big[\,\big(\mathcal{K}^i\partial_i W\,\overline{W}+\text{c.c.}\big)+|W|^2\,\Big]=e^{\mathcal{K}}\left|\int_{\mathcal{X}}\mathcal{A}\wedge\overline{\Omega}\right|^2-e^{\mathcal{K}}\left|\int_{\mathcal{X}}(\mathcal{A}-\overline{\mathcal{A}})\wedge\overline{\Omega}\right|^2. \tag{5.30}$$

The first term on the right-hand side will be cancelled by the last term in (5.19). For the second term we recall (5.18), (4.33) and (A.1), and determine

$$\begin{aligned}-e^{\mathcal{K}}\left|\int_{\mathcal{X}}(\mathcal{A}-\overline{\mathcal{A}})\wedge\overline{\Omega}\right|^2&=-\frac{e^{-\phi}}{4\hat{\mathcal{V}}^2}\int_{\mathcal{X}}\Big[(\operatorname{Re}\check{\chi})\wedge\Omega\Big]\wedge\star\Big[(\operatorname{Re}\check{\chi})\wedge\overline{\Omega}\Big]\\&=-\frac{e^{-\phi}}{8\hat{\mathcal{V}}^2}\int_{\mathcal{X}}\Big[(\Omega\wedge\check{\chi})\wedge\star(\overline{\Omega}\wedge\overline{\check{\chi}})+(\Omega\wedge\overline{\check{\chi}})\wedge\star(\overline{\Omega}\wedge\check{\chi})\Big].\end{aligned} \tag{5.31}$$

In the last step we noted that due to (5.11) we have $(\operatorname{Im}\check{\chi})\in H^3_+(\mathcal{X})$ whereas $\Omega\in H^3_-(\mathcal{X})$, and therefore $\int(\operatorname{Im}\check{\chi})\wedge\Omega=0$.

**Combining the results.** We finally combine the individual results obtained above to obtain the full scalar F-term potential. In particular, we can rewrite (5.16) as

$$\begin{aligned}V_{\rm F}=\frac{M_{\rm Pl}^4\,e^{\phi}}{2\hat{\mathcal{V}}^2}\int_{\mathcal{X}}\bigg(&e^{-2\phi}\bigg[\frac{1}{2}(\operatorname{Re}\check{\chi})\wedge\star(\operatorname{Re}\check{\chi})+\frac{1}{2}\check{\Psi}\wedge\star\overline{\check{\Psi}}\\&\qquad-\frac{1}{4}(\Omega\wedge\check{\chi})\wedge\star(\overline{\Omega}\wedge\overline{\check{\chi}})-\frac{1}{4}(\Omega\wedge\overline{\check{\chi}})\wedge\star(\overline{\Omega}\wedge\check{\chi})\bigg]\\&+\frac{1}{2}\check{\mathfrak{G}}\wedge\star\check{\mathfrak{G}}-\Big[(\operatorname{Im}\tau)-(\operatorname{Im}G^a)\omega_a+(\operatorname{Im}T_\alpha)\sigma^\alpha\Big]\wedge\mathcal{D}F^{(3)}\bigg).\end{aligned} \tag{5.32}$$

Taking into account that the prefactor is proportional to $M_s^4$, the first two lines match with the orientifold projected DFT actions (3.34) and (3.39) in the NS-NS and R-R sector. Note, however, that only the real part of $\check{\chi}$ appears; the imaginary part is contained in a D-term, which we discuss in the next section. The third line in (5.32) corresponds to tadpole terms, which have to be cancelled by local sources.

### 5.3 D-term potential

We now want to consider the imaginary part of $\check{\chi}$, which does not appear in the scalar F-term potential (5.32). As mentioned before, we have $(\operatorname{Im}\check{\chi})\in H^3_+(\mathcal{X})$ and therefore the only contribution in the DFT Lagrangian (3.34) relevant here comes from

$$\star\mathcal{L}_{H^3_+}=-\frac{1}{2}\,e^{-2\phi}\,(\operatorname{Im}\check{\chi})\wedge\star(\operatorname{Im}\check{\chi})\,. \tag{5.33}$$

Using the definition (3.35) as well as (5.11), we can evaluate $\operatorname{Im}\check{\chi}$ as

$$\operatorname{Im}\check{\chi}=\begin{pmatrix}\alpha_{\hat{\lambda}} & \beta^{\hat{\lambda}}\end{pmatrix}\cdot\begin{pmatrix}\tilde{\mathsf{D}}^{\hat{\lambda}}\\ \mathsf{D}_{\hat{\lambda}}\end{pmatrix}, \tag{5.34}$$

where we defined

$$\begin{aligned}\tilde{\mathsf{D}}^{\hat\lambda} &= \tilde{r}^{\hat\lambda}\left(\mathcal{V} - \frac{1}{2}\kappa_{\alpha ab}t^\alpha b^a b^b\right) + \tilde{q}^{\hat\lambda a}\,\kappa_{a\alpha b}\,t^\alpha b^b - \tilde{f}^{\hat\lambda}{}_\alpha\,t^\alpha\,,\\ \mathsf{D}_{\hat\lambda} &= -r_{\hat\lambda}\left(\mathcal{V} - \frac{1}{2}\kappa_{\alpha ab}t^\alpha b^a b^b\right) - q_{\hat\lambda}{}^a\,\kappa_{a\alpha b}\,t^\alpha b^b + f_{\hat\lambda\alpha}\,t^\alpha\,.\end{aligned} \tag{5.35}$$

Similarly as in section 4.2, we can now evaluate (5.33). We find

$$\begin{aligned}\star\mathcal{L}_{H_+^3} &= -\frac{1}{2}\,e^{-2\phi}\begin{pmatrix}\tilde{\mathsf{D}}\\ \mathsf{D}\end{pmatrix}^T\cdot\check{\mathcal{M}}_1\cdot\begin{pmatrix}\tilde{\mathsf{D}}\\ \mathsf{D}\end{pmatrix}\\ &= \frac{1}{2}\,e^{-2\phi}\Big[(\mathsf{D}_{\hat\lambda} + \mathrm{Re}\,\mathcal{N}_{\hat\lambda\hat\kappa}\,\tilde{\mathsf{D}}^{\hat\kappa})\,(\mathrm{Im}\,\mathcal{N}^{-1})^{\hat\lambda\hat\sigma}\,(\mathsf{D}_{\hat\sigma} + \mathrm{Re}\,\mathcal{N}_{\hat\sigma\hat\rho}\,\tilde{\mathsf{D}}^{\hat\rho}) + \tilde{\mathsf{D}}^{\hat\lambda}\,\mathrm{Im}\,\mathcal{N}_{\hat\lambda\hat\sigma}\,\tilde{\mathsf{D}}^{\hat\sigma}\Big]\,,\end{aligned} \tag{5.36}$$

where $\mathcal{M}_1$ has been defined in (4.6), and the check indicates the restriction to indices $\hat\lambda = 1,\ldots,h_+^{2,1}$. Note that (5.36) corresponds to a positive semi-definite scalar potential in four dimensions.

Let us now check that this scalar potential can be understood as a D-term from the $N = 1$ supergravity point of view. We will follow the discussion first presented in [22] (see also [42]). To begin, let us recall that in the absence of a Fayet-Iliopolous term, $\xi_a = i\delta_a W/W$, the D-term of an abelian gauge field $A^a$ in supergravity is given by

$$D_a = i\sum_i(\partial_i\mathcal{K})\,\delta_a\phi_i\,, \tag{5.37}$$

where $\delta_a\phi_i$ is the variation of the chiral superfield $\phi_i$ under a gauge transformation $A^a \to A^a + d\Lambda^a$, and $\mathcal{K}$ denotes again the Kähler potential. The corresponding D-term potential reads

$$V_D = M_{\rm Pl}^4\left[(\mathrm{Re}\,f)^{-1}\right]^{ab}D_a\,D_b\,, \tag{5.38}$$

with $\mathrm{Re}\,f_{ab}$ the real part of the gauge kinetic function for the gauge fields. In our case, the gauge fields of interest originate from the R-R four-form $C^{(4)}$ via a dimensional reduction on three-cycles of the Calabi-Yau three-fold. Let us therefore expand

$$C^{(4)} = A^{\hat\lambda}\alpha_{\hat\lambda} + \tilde{A}_{\hat\lambda}\beta^{\hat\lambda} + \ldots\,, \qquad \hat\lambda = 1,\ldots,h_+^{2,1}\,, \tag{5.39}$$

where the ellipsis denote terms of different degree in the internal manifold not of importance here. The gauge transformations of $A_{\hat\lambda}$ and $\tilde{A}^{\hat\lambda}$ have their origin in a higher-dimensional gauge symmetry. In particular, note that the DFT Lagrangian (3.39) is invariant under

$$\mathcal{C} \to \mathcal{C} + \mathfrak{D}\Lambda\,, \tag{5.40}$$

with $\mathcal{C}$ the sum of even R-R potentials (3.37), $\mathfrak{D}$ was defined in (3.33), and $\Lambda$ is a sum of odd forms. In order to obtain the gauge transformation $A^{\hat\lambda} \to A^{\hat\lambda} + d\Lambda^{\hat\lambda}$ and $\tilde{A}_{\hat\lambda} \to \tilde{A}_{\hat\lambda} - d\tilde{\Lambda}_{\hat\lambda}$ in four dimensions, we therefore have to choose the gauge parameter $\Lambda$ as

$$e^B\mathcal{C} \;\to\; e^B\mathcal{C} + \mathcal{D}\Big(\Lambda^{\hat\lambda}\alpha_{\hat\lambda} - \tilde{\Lambda}_{\hat\lambda}\beta^{\hat\lambda}\Big)\,. \tag{5.41}$$

In turn, this gauge transformation implies variations of the chiral superfields $\phi \in \{\tau, G^a, T_\alpha\}$. Indeed, using (5.9) together with (4.19) and (5.11) we find that

$$\begin{aligned}
\tau &\to \tau + \left( r_{\hat\lambda} \Lambda^{\hat\lambda} - \tilde r^{\hat\lambda} \tilde\Lambda_{\hat\lambda} \right) , \\
G^a &\to G^a + \left( q_{\hat\lambda}{}^a \Lambda^{\hat\lambda} - \tilde q^{\hat\lambda a} \tilde\Lambda_{\hat\lambda} \right) , \\
T_\alpha &\to T_\alpha + \left( f_{\hat\lambda\alpha} \Lambda^{\hat\lambda} - \tilde f^{\hat\lambda}{}_\alpha \tilde\Lambda_{\hat\lambda} \right) .
\end{aligned} \tag{5.42}$$

Note that due to the nilpotency of $\mathcal{D}$, the superpotential is invariant under transformations of the form (5.40) and thus no Fayet-Iliopolous parameter is generated.

In order to evaluate (5.37), let us also determine the derivatives of the Kähler potential (5.12) with respect to the moduli fields (5.8). As in the previous section, we perform the computation in Einstein frame, and then transform the result to string frame. We find

$$\begin{aligned}
&\partial_\tau K = \frac{i e^\phi}{2\mathcal{V}} \left( \mathcal{V} - \frac{1}{2} \kappa_{abc} t^a b^b b^c \right) \\
&\partial_{T_\alpha} K = -\frac{i e^\phi}{2\mathcal{V}} t^\alpha , \qquad \partial_{G^a} K = \frac{i e^\phi}{2\mathcal{V}} \kappa_{a\beta c} t^\beta b^c .
\end{aligned} \tag{5.43}$$

Using these results and the transformations of the moduli fields under gauge transformations (5.42), we can compute the D-terms (5.37) as follows

$$\begin{aligned}
\tilde D^{\hat\lambda} &= \frac{e^\phi}{2\mathcal{V}} \left[ \tilde r^{\hat\lambda} \left( \mathcal{V} - \frac{1}{2} \kappa_{\alpha a b} t^\alpha b^a b^b \right) + \tilde q^{\hat\lambda a} \kappa_{a\alpha b} t^\alpha b^b - \tilde f^{\hat\lambda}{}_\alpha t^\alpha \right] , \\
D_{\hat\lambda} &= \frac{e^\phi}{2\mathcal{V}} \left[ - r_{\hat\lambda} \left( \mathcal{V} - \frac{1}{2} \kappa_{\alpha a b} t^\alpha b^a b^b \right) - q_{\hat\lambda}{}^a \kappa_{a\alpha b} t^\alpha b^b + f_{\hat\lambda\alpha} t^\alpha \right] .
\end{aligned} \tag{5.44}$$

We observe that up to an overall factor, these D-terms agree with the expressions (5.35) obtained from a reduction of the DFT action (5.33). We furthermore note that the Ramond-Ramond four-form potential $C^{(4)}$ is self-dual in ten dimensions. The two sets of gauge fields $A^{\hat\lambda}$ and $\tilde A_{\hat\lambda}$ in (5.39) are therefore not independent, and in the following we choose to eliminate $\tilde A_{\hat\lambda}$ in favor of $A^{\hat\lambda}$. Also, as argued in [22], as long as the fluxes are integer-valued one can rotate them by an $\mathrm{Sp}(h^{2,1}_+, \mathbb{Z})$ transformation into a basis where $\tilde r^{\hat\lambda} = \tilde q^{\hat\lambda a} = \tilde f^{\hat\lambda}{}_\alpha = 0$. This implies that the D-term $\tilde D^{\hat\lambda}$ vanishes.[3]

Let us finally turn to the D-term potential (5.38). The gauge kinetic function for the gauge fields $A^{\hat\lambda}$ is given by the imaginary part of the matrix (4.4) [50], properly restricted to indices $\hat\lambda = 1, \ldots, h^{2,1}_+$

$$f_{\hat\lambda\hat\sigma} = -\frac{i}{2} \bar{\mathcal{N}}_{\hat\lambda\hat\sigma} . \tag{5.45}$$

[3]In this basis the Bianchi identities connecting the fluxes in the D-terms are trivially satisfied. There are further Bianchi identities which mix the flux parameters in the superpotential (5.13) with those in $D_{\hat\lambda}$ in (5.44).

Furthermore, $\mathrm{Im}\,\mathcal{N}$ is meant to only depend on the complex structure moduli $U^\mu$ surviving the orientifold projection. For the D-term potential we therefore obtain

$$\begin{aligned} V_D &= M_{\rm Pl}^4 \left[-2\,(\mathrm{Im}\,\mathcal{N})^{-1}\right]^{\hat\lambda\hat\sigma} D_{\hat\lambda}\, D_{\hat\sigma} \\ &= -\frac{M_{\rm Pl}^4\, e^{2\phi}}{2\,\mathcal{V}^2} \left[(\mathrm{Im}\,\mathcal{N})^{-1}\right]^{\hat\lambda\hat\sigma} \mathsf{D}_{\hat\lambda}\, \mathsf{D}_{\hat\sigma}\,, \end{aligned} \tag{5.46}$$

where $\mathsf{D}_{\hat\lambda}$ was defined in (5.35). Expressing then again the Planck mass in terms of the string scale via $M_{\rm Pl}^4 = M_{\rm s}^4\, \mathcal{V}^2\, e^{-4\phi}$ and noting that the potential appears as $-V$ in the Lagrangian $\mathcal{L}$, we see that the D-term potential (5.46) agrees with the DFT result (5.36), after $\tilde{\mathsf{D}}^{\hat\lambda}$ has been set to zero. We therefore conclude that the scalar potential resulting from the dimensional reduction of DFT for $h_+^{2,1} > 0$ also correctly reproduces the expected D-term potential.

## 6 Conclusions

In this paper we have performed the dimensional reduction of the DFT action in its flux formulation on a Calabi-Yau three-fold with non-trivial constant fluxes turned on. The main initial obstacle that the DFT action contained explicitly the unknown metric on the CY could be overcome by rewriting all contributions to the action in terms of the Kähler form, holomorphic three-form, and operations that could be further evaluated on the CY using special geometry. The induced scalar potential agrees with that of $N = 2$ gauged supergravity. Up to additional D-terms, a further orientifold projection to $N = 1$ leads to the potential derived from the generalized Gukov-Vafa-Witten superpotential containing the non-geometric fluxes. This nicely confirms the consistency of the whole approach.

Our results put the generalized flux-induced scalar potential on firmer grounds, thereby lending further support to its use in tree-level moduli stabilization applied to string phenomenology and cosmology. It is known that, with all types of fluxes turned on, there does not exist a dilute flux limit so that it is not straightforward to argue for a consistent higher dimensional uplift of the solutions found in the four-dimensional field theory model. However, in view of the now established DFT origin of the four-dimensional potential, the fate of these vacua is closely related to the claim that DFT, though not an effective low-energy theory, might be a consistent truncation of full string (field) theory.

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## A Useful relations on a Calabi-Yau three-fold

In this appendix, we collect some technical relations concerning Calabi-Yau three-folds, which are important for the computations in the main part of the paper.

### A.1 Normalization and primitivity

Since a Calabi-Yau manifold is a complex Kähler manifold, it is useful to work in a complex basis with indices $a$ and $\overline{a}$. The hermitian metric then has non-vanishing components $g_{a\bar{b}}$, whereas the almost complex structure reads $I^a{}_b = i\,\delta^a{}_b$ and $I^{\overline{a}}{}_{\overline{b}} = -i\,\delta^{\overline{a}}{}_{\overline{b}}$. The Kähler form $J_{ij} = g_{im}\, I^m{}_j$ in complex coordinates is given by $J_{a\bar{b}} = i\, g_{a\bar{b}}$. For the holomorphic three-form on a Calabi-Yau three-fold, we employ the normalization

$$\frac{i}{8}\,\Omega \wedge \overline{\Omega} = \frac{1}{6}\,J^3 \,. \tag{A.1}$$

Using (A.1), one can show the following useful relations

$$\begin{aligned} \Omega_{abc}\,\overline{\Omega}_{\bar{a}\bar{b}\bar{c}}\,g^{c\bar{c}} &= 8\left(g_{a\bar{a}}\,g_{b\bar{b}} - g_{a\bar{b}}\,g_{b\bar{a}}\right) , \\ \Omega_{abc}\,\overline{\Omega}_{\bar{a}\bar{b}\bar{c}}\,g^{b\bar{b}}\,g^{c\bar{c}} &= 16\,g_{a\bar{a}} \,, \\ \Omega_{abc}\,\overline{\Omega}_{\bar{a}\bar{b}\bar{c}}\,g^{a\bar{a}}\,g^{b\bar{b}}\,g^{c\bar{c}} &= 48 \,. \end{aligned} \tag{A.2}$$

Since on a Calabi-Yau three-fold there are no homologically non-trivial one- and five-cycles, we can assume that all combinations leaving effectively one free-index are trivial. This includes e.g.

$$H \wedge J = 0\,, \qquad Q \bullet J = 0\,, \qquad R \llcorner (J \wedge J) = 0\,, \tag{A.3}$$

as well as the conditions (3.5). Note that (A.3) can be considered as generalized *primitivity* constraints on the fluxes.

### A.2 Relations regarding complex-structure moduli

In this section, we derive some formulas important for section 5.2. We begin by noting that a complex basis of $(2,1)$-forms $\chi_\mu$ with $\mu = 1, \ldots, h^{2,1}_{-}$ is given by

$$D_{U^\mu}\Omega = \chi_\mu \,, \tag{A.4}$$

with $D_U$ the Kähler covariant derivative defined below (5.13). In a similar fashion, a basis of $(1,2)$-forms $\overline{\chi}_{\overline{\mu}}$ can be introduced. The Kähler metric for the complex-structure moduli derived from (5.12) is expressed as

$$G_{\mu\overline{\nu}} = -\frac{\int \chi_\mu \wedge \overline{\chi}_{\overline{\nu}}}{\int \Omega \wedge \overline{\Omega}} \,. \tag{A.5}$$

Next, we observe that on a Calabi-Yau three-fold the holomorphic $(3,0)$-form $\Omega$ and the $(2,1)$-forms $\chi_\mu$ introduced in equation (A.4), and their complex conjugates form a basis of



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the third cohomology. An arbitrary complex three-form $A$ can therefore be expanded in the following way

$$A = a^0\,\Omega + a^\mu\,\chi_\mu + \tilde{a}^\mu\,\overline{\chi}_{\overline{\mu}} + \tilde{a}^0\,\overline{\Omega}\,. \tag{A.6}$$

Using the Kähler metric (A.5), the coefficients in this expansion can be determined as

$$\begin{aligned} a^0 &= +\frac{\int A\wedge\overline{\Omega}}{\int \Omega\wedge\overline{\Omega}}\,, & a^\mu &= -\frac{\int A\wedge\overline{\chi}_{\overline{\nu}}}{\int\Omega\wedge\overline{\Omega}}\,G^{\overline{\nu}\mu}\,, \\ \tilde{a}^0 &= -\frac{\int A\wedge\Omega}{\int\Omega\wedge\overline{\Omega}}\,, & \tilde{a}^{\overline{\mu}} &= -\frac{\int A\wedge\chi_\nu}{\int\Omega\wedge\overline{\Omega}}\,G^{\nu\overline{\mu}}\,. \end{aligned} \tag{A.7}$$

Furthermore, we note that the Hodge-star operator acting on $\Omega$ and $\chi_\mu$ gives

$$\star\Omega = -i\,\Omega\,, \qquad \star\chi_\mu = +i\,\chi_\mu\,. \tag{A.8}$$

Using the above relations, for two different complex three-forms $A$ and $B$ we can then compute

$$\int A\wedge\star\overline{B} = i\int\Omega\wedge\overline{\Omega}\times\Big[a^0\,\overline{b}^0 + \tilde{a}^0\,\overline{\tilde{b}}^0 + a^\mu\,G_{\mu\overline{\nu}}\overline{b}^\nu + \overline{\tilde{b}}^\mu\,G_{\mu\overline{\nu}}\,\tilde{a}^{\overline{\nu}}\Big]\,. \tag{A.9}$$

Employing (A.7) and defining $\mathcal{K}_{\rm cs} = -\log\big[i\int\Omega\wedge\overline{\Omega}\big]$, we arrive at

$$\begin{aligned} \int A\wedge\star\overline{B} = e^{\mathcal{K}_{\rm cs}}\Big[&G^{\mu\overline{\nu}}D_{U^\mu}\Big(\int A\wedge\Omega\Big)\,D_{\overline{U}^{\overline{\nu}}}\Big(\int\overline{B}\wedge\overline{\Omega}\Big) \\ &+G^{\mu\overline{\nu}}D_{U^\mu}\Big(\int\overline{B}\wedge\Omega\Big)\,D_{\overline{U}^{\overline{\nu}}}\Big(\int A\wedge\overline{\Omega}\Big) \\ &+\Big(\int A\wedge\overline{\Omega}\Big)\Big(\int\overline{B}\wedge\Omega\Big)+\Big(\int A\wedge\Omega\Big)\Big(\int\overline{B}\wedge\overline{\Omega}\Big)\Big]\,. \end{aligned} \tag{A.10}$$

Similarly, we determine for the wedge product of two three-forms $A$ and $B$

$$\begin{aligned} \int A\wedge\overline{B} = -i\,e^{\mathcal{K}_{\rm cs}}\Big[&G^{\mu\overline{\nu}}D_{U^\mu}\Big(\int A\wedge\Omega\Big)\,D_{\overline{U}^{\overline{\nu}}}\Big(\int\overline{B}\wedge\overline{\Omega}\Big) \\ &-G^{\mu\overline{\nu}}D_{U^\mu}\Big(\int\overline{B}\wedge\Omega\Big)\,D_{\overline{U}^{\overline{\nu}}}\Big(\int A\wedge\overline{\Omega}\Big)+\Big(\int A\wedge\overline{\Omega}\Big)\Big(\int\overline{B}\wedge\Omega\Big) \\ &-\Big(\int A\wedge\Omega\Big)\Big(\int\overline{B}\wedge\overline{\Omega}\Big)\Big]\,. \end{aligned} \tag{A.11}$$

### A.3 Relations regarding $\mathcal{D}$

We now derive relations for the twisted differential $\mathcal{D}$, which was defined via (4.19). Let us consider a closed three-form $\mathsf{A}$ with $d\mathsf{A} = 0$, and expand $\mathsf{A}$ the basis (4.1) as

$$\mathsf{A} = \mathsf{A}^\Lambda\alpha_\Lambda + \mathsf{A}_\Lambda\beta^\Lambda\,. \tag{A.12}$$



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Using the definitions (4.19), we can then show by explicit computation that

$$\begin{aligned} \int \mathcal{D}\omega_A \wedge \mathsf{A} &= -\int \omega_A \wedge \mathcal{D}\mathsf{A}\,, \\ \int \mathcal{D}\sigma^A \wedge \mathsf{A} &= +\int \sigma^A \wedge \mathcal{D}\mathsf{A}\,. \end{aligned} \tag{A.13}$$

Let us also consider an even, $d$-closed multi-form $\mathsf{B}$, which can be expanded in the basis (4.8) as

$$\mathsf{B} = \mathsf{B}^A \omega_A + \mathsf{B}_A \sigma^A\,. \tag{A.14}$$

For a Calabi-Yau three-fold with the action of $\mathcal{D}$ given by (4.19), it follows that $\mathcal{D}\mathsf{B}$ is a three-form. Setting then $\mathsf{A} = \mathcal{D}\mathsf{B}$ and using the Bianchi identities $\mathcal{D}^2 = 0$, it follows that

$$\int \mathcal{D}\mathsf{B} \wedge \mathcal{D}\omega_A = 0\,, \qquad \int \mathcal{D}\mathsf{B} \wedge \mathcal{D}\sigma^A = 0\,. \tag{A.15}$$

### A.4 Kähler metric and inverse

We now discuss the Kähler metric $G_{i\bar{j}}$ for the moduli $\tau$, $G^a$ and $T_\alpha$, which were defined in (5.8). From [17] we know that this metric can be expressed as

$$G_{i\bar{j}} = \frac{e^{2\phi}}{4\mathcal{V}} \int \big[\nu_i \wedge e^{+B}\big] \wedge \star \big[\nu_j \wedge e^{+B}\big]\,, \tag{A.16}$$

where $i, j = \tau, G^a, T_\alpha$ and

$$\nu_i = (\,1\,,\, -\omega_a\,,\, \sigma^\alpha)\,, \tag{A.17}$$

and where $\mathcal{V}$ denotes the volume of the Calabi-Yau three-fold in string frame. The inverse Kähler metric has not been given in [17], but can be determined as follows. Let us make the following ansatz

$$G^{i\bar{j}} = \frac{4\mathcal{V}}{e^{2\phi}} \int \big[\rho^i \wedge e^{-B}\big] \wedge \star \big[\rho^j \wedge e^{-B}\big]\,, \tag{A.18}$$

with the dual forms

$$\rho^i = (\,\omega_0\,,\, -\sigma^a\,,\, \omega_\alpha)\,. \tag{A.19}$$

We now verify that (A.18) is indeed the inverse of (A.16). For that purpose, we note that

$$\int \big[\nu_i \wedge e^{+B}\big] \wedge \big[\rho^j \wedge e^{-B}\big] = \int \nu_i \wedge \rho^j = \delta_i^j\,. \tag{A.20}$$

This implies that we can expand the Hodge duals as

$$\star\big(\nu_i \wedge e^{+B}\big) = \mathcal{M}_{ij}\,\big(\rho^j \wedge e^{-B}\big)\,, \qquad \star\big(\rho^i \wedge e^{-B}\big) = \mathcal{N}^{ij}\,\big(\nu_j \wedge e^{+B}\big)\,, \tag{A.21}$$

with $\mathcal{M}$ and $\mathcal{N}$ some matrices. Applying the Hodge star to the second relation and noting that for even forms on six-dimensional manifold $\star^2 = 1$, gives $\mathcal{N}^{ij}\mathcal{M}_{jk} = \delta_k^i$. This allows us to compute

$$G_{i\bar{j}}\,G^{\bar{j}k} = \mathcal{M}_{ji}\mathcal{N}^{kj} = \mathcal{N}^{kj}\mathcal{M}_{ji} = \delta_i^k\,. \tag{A.22}$$

We have therefore shown that the metric (A.18) is indeed the inverse of (A.16).

## B Proof of general results

In this appendix we show that for vanishing $B$-field, $\star\mathcal{L}_{\rm NS\,NS}$ can indeed be cast as proposed in (3.34). We have already proved that (3.6), (3.18), (3.21), (3.25) and (3.26), comply with (3.34) when only one kind of flux is switched on at a time. When fluxes are turned on simultaneously we have to care about mixed terms. In the original NS-NS Lagrangian there are $FR$ and $HQ$ mixed terms in (2.22). On the other hand, given its structure, in (3.34) the only mixed terms are precisely of type $FR$ and $HQ$. Concretely, the relevant terms in (3.34) are $T_{HQ} + T_{FR}$, where

$$T_{HQ} = -H\wedge\star\left(Q\bullet\frac{1}{2}J^2\right) + \mathrm{Re}\,(\Omega\wedge H)\wedge\star\left(\overline{\Omega}\wedge Q\bullet\frac{1}{2}J^2\right)\,, \tag{B.1}$$

and similarly

$$T_{FR} = -F\circ J\wedge\star\left(R\llcorner\frac{1}{3!}J^3\right) + \mathrm{Re}(\Omega\wedge F\circ J)\wedge\star\left(\overline{\Omega}\wedge R\llcorner\frac{1}{3!}J^3\right)\,. \tag{B.2}$$

We will proceed by evaluating separately each term in the above relations.

Let us begin with (B.1). Using (3.19) and the property $J_{ij} = g_{im}\,I^m{}_j$ we find

$$-H\wedge\star\left(Q\bullet\frac{1}{2}J^2\right) = -\frac{1}{2}H_{i'j'k'}Q_i{}^{jk}I^{j'}{}_j\,I^{k'}{}_k\,g^{ii'}\,\star 1\,. \tag{B.3}$$

It is convenient to express the right hand side in a complex basis and then simplify it applying an appropriate Bianchi identity. With $F$, $H$ and $Q$ different from zero, the second identity in (3.4) yields

$$g^{a\bar a}\big(H_{abc}Q_{\bar a}{}^{bc} + H_{\bar a\bar b\bar c}Q_a{}^{\bar b\bar c}\big) - g^{a\bar a}\big(H_{\bar a bc}Q_a{}^{bc} + H_{a\bar b\bar c}Q_{\bar a}{}^{\bar b\bar c}\big) + 2g^{a\bar a}\big(F^{\bar c}{}_{ab}\,F^b{}_{\bar a\bar c} - F^c{}_{a\bar b}\,F^{\bar b}{}_{\bar a c}\big) = 0\,. \tag{B.4}$$

Notice that when only $F\neq 0$ this identity reduces to (3.11). Going to a complex basis and substituting (B.4) we arrive at

$$\begin{aligned}-H\wedge\star\left(Q\bullet\frac{1}{2}J^2\right) = {}& g^{a\bar a}\big(H_{abc}Q_{\bar a}{}^{bc} + H_{\bar a\bar b\bar c}Q_a{}^{\bar b\bar c} - H_{\bar a b\bar c}Q_a{}^{b\bar c} - H_{ab\bar c}Q_{\bar a}{}^{b\bar c}\big)\star 1\\ &-g^{a\bar a}\big(F^{\bar c}{}_{ab}\,F^b{}_{\bar a\bar c} - F^c{}_{a\bar b}\,F^{\bar b}{}_{\bar a c}\big)\star 1\,.\end{aligned} \tag{B.5}$$

The $F$ depending piece will cancel against an analogous contribution in $\frac{1}{2}\,\Xi_3\wedge\star\Xi_3$, $\Xi_3 = F\circ J$. In fact, from (3.10) we see that before using (3.11), the right hand side of (3.12) has an extra term that offsets the second line in (B.5). In the complex basis we also obtain

$$\mathrm{Re}(\Omega\wedge H)\wedge\star\left(\overline{\Omega}\wedge Q\bullet\frac{1}{2}J^2\right) = -2g^{a\bar a}\big(H_{abc}Q_{\bar a}{}^{bc} + H_{\bar a\bar b\bar c}Q_a{}^{\bar b\bar c}\big)\star 1\,. \tag{B.6}$$

Finally, for the $HQ$ term in (2.22) the Bianchi identity (B.4) further implies that

$$\begin{aligned}-\frac{1}{2}H_{mni}Q_j{}^{mn}g^{ij}\,\star 1 = {}& -g^{a\bar a}\big(H_{abc}Q_{\bar a}{}^{bc} + H_{\bar a\bar b\bar c}Q_a{}^{\bar b\bar c} + H_{\bar a b\bar c}Q_a{}^{b\bar c} + H_{ab\bar c}Q_{\bar a}{}^{b\bar c}\big)\star 1\\ &+g^{a\bar a}\big(F^{\bar c}{}_{ab}\,F^b{}_{\bar a\bar c} - F^c{}_{a\bar b}\,F^{\bar b}{}_{\bar a c}\big)\star 1\,.\end{aligned} \tag{B.7}$$



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The term involving $F$ is cancelled by a similar contribution in $\frac{1}{2}F^m{}_{ni}F^n{}_{mj}g^{ij}\star 1$ that also appears in (2.22). In the analysis of pure $F$ flux this extra contribution was absent by virtue of (3.11). Observe that adding the first line in (B.5) and (B.6) precisely matches the first line in (B.7). Hence, we have shown that the mixed terms in $T_{HQ}$ indeed lead to the $HQ$ term in the NS-NS Lagrangian.

To evaluate the mixed $FR$ terms we basically take the same steps as in the preceding calculation. A crucial ingredient is the Bianchi identity that follows from the fourth line in (3.4)

$$g_{a\bar{a}}\big(R^{\bar{a}bc}F^a{}_{bc}+R^{a\bar{b}\bar{c}}F^{\bar{a}}{}_{\bar{b}\bar{c}}\big)-g_{a\bar{a}}\big(R^{abc}F^{\bar{a}}{}_{bc}+R^{\bar{a}\bar{b}\bar{c}}F^a{}_{\bar{b}\bar{c}}\big)+2g_{a\bar{a}}\big(Q_{\bar{b}}{}^{ac}\,Q_c{}^{\bar{a}\bar{b}}-Q_b{}^{a\bar{c}}\,Q_{\bar{c}}{}^{\bar{a}b}\big)=0\,, \quad \text{(B.8)}$$

which clearly shortens to (3.20) when only $Q\neq 0$. Inserting this identity in the $FR$ term in (2.22) gives

$$\begin{aligned}-\frac{1}{2}R^{mni}F^j{}_{mn}\,g_{ij}\star 1 = {} & -g_{a\bar{a}}\big(R^{abc}F^{\bar{a}}{}_{bc}+R^{\bar{a}\bar{b}\bar{c}}F^a{}_{\bar{b}\bar{c}}+R^{\bar{a}b\bar{c}}F^a{}_{b\bar{c}}+R^{a\bar{b}c}F^{\bar{a}}{}_{\bar{b}c}\big)\star 1\\ & +g_{a\bar{a}}\big(Q_{\bar{c}}^{ab}\,Q_b{}^{\bar{a}\bar{c}}-Q_c^{a\bar{b}}\,Q_{\bar{b}}{}^{\bar{a}c}\big)\star 1\,. \qquad \text{(B.9)}\end{aligned}$$

The $Q$ part is nullified by an identical term with opposite sign in $\frac{1}{2}Q_m{}^{ni}Q_n{}^{mj}g_{ij}\star 1$. Using the identity (B.8) we also find that adding the pieces in $T_{FR}$ reproduces the first line in (B.9) up to an additional contribution that is cancelled by a similar one in $Q\bullet\frac{1}{2}J^2\wedge\star(Q\bullet\frac{1}{2}J^2)$.



## References


[1] M. Graña, *Flux compactifications in string theory: A Comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003] [INSPIRE].

[2] M.R. Douglas and S. Kachru, *Flux compactification*, *Rev. Mod. Phys.* **79** (2007) 733 [hep-th/0610102] [INSPIRE].

[3] F. Denef, M.R. Douglas and S. Kachru, *Physics of String Flux Compactifications*, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 119 [hep-th/0701050] [INSPIRE].

[4] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152] [INSPIRE].

[5] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097] [INSPIRE].

[6] S. Kachru, R. Kallosh, A.D. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240] [INSPIRE].

[7] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [hep-th/0502058] [INSPIRE].

[8] J. Shelton, W. Taylor and B. Wecht, *Nongeometric flux compactifications*, *JHEP* **10** (2005) 085 [hep-th/0508133] [INSPIRE].




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[9] B. Wecht, *Lectures on Nongeometric Flux Compactifications*, *Class. Quant. Grav.* **24** (2007) S773 [arXiv:0708.3984] [INSPIRE].

[10] G. Aldazabal, P.G. Camara, A. Font and L.E. Ibáñez, *More dual fluxes and moduli fixing*, *JHEP* **05** (2006) 070 [hep-th/0602089] [INSPIRE].

[11] G. Villadoro and F. Zwirner, *D terms from D-branes, gauge invariance and moduli stabilization in flux compactifications*, *JHEP* **03** (2006) 087 [hep-th/0602120] [INSPIRE].

[12] J. Shelton, W. Taylor and B. Wecht, *Generalized Flux Vacua*, *JHEP* **02** (2007) 095 [hep-th/0607015] [INSPIRE].

[13] B. de Carlos, A. Guarino and J.M. Moreno, *Flux moduli stabilisation, Supergravity algebras and no-go theorems*, *JHEP* **01** (2010) 012 [arXiv:0907.5580] [INSPIRE].

[14] G. Aldazabal, D. Marques, C. Núñez and J.A. Rosabal, *On Type IIB moduli stabilization and $N = 4, 8$ supergravities*, *Nucl. Phys.* **B 849** (2011) 80 [arXiv:1101.5954] [INSPIRE].

[15] G. Dibitetto, A. Guarino and D. Roest, *Charting the landscape of $N = 4$ flux compactifications*, *JHEP* **03** (2011) 137 [arXiv:1102.0239] [INSPIRE].

[16] M. Graña, J. Louis and D. Waldram, *Hitchin functionals in $N = 2$ supergravity*, *JHEP* **01** (2006) 008 [hep-th/0505264] [INSPIRE].

[17] I. Benmachiche and T.W. Grimm, *Generalized $N = 1$ orientifold compactifications and the Hitchin functionals*, *Nucl. Phys.* **B 748** (2006) 200 [hep-th/0602241] [INSPIRE].

[18] M. Graña, J. Louis and D. Waldram, $\mathrm{SU}(3) \times \mathrm{SU}(3)$ *compactification and mirror duals of magnetic fluxes*, *JHEP* **04** (2007) 101 [hep-th/0612237] [INSPIRE].

[19] A. Micu, E. Palti and G. Tasinato, *Towards Minkowski Vacua in Type II String Compactifications*, *JHEP* **03** (2007) 104 [hep-th/0701173] [INSPIRE].

[20] E. Palti, *Low Energy Supersymmetry from Non-Geometry*, *JHEP* **10** (2007) 011 [arXiv:0707.1595] [INSPIRE].

[21] D. Cassani and A. Bilal, *Effective actions and $N = 1$ vacuum conditions from* $\mathrm{SU}(3) \times \mathrm{SU}(3)$ *compactifications*, *JHEP* **09** (2007) 076 [arXiv:0707.3125] [INSPIRE].

[22] D. Robbins and T. Wrase, *D-terms from generalized NS-NS fluxes in type-II*, *JHEP* **12** (2007) 058 [arXiv:0709.2186] [INSPIRE].

[23] R. D'Auria, S. Ferrara and M. Trigiante, *On the supergravity formulation of mirror symmetry in generalized Calabi-Yau manifolds*, *Nucl. Phys.* **B 780** (2007) 28 [hep-th/0701247] [INSPIRE].

[24] R. Blumenhagen et al., *A Flux-Scaling Scenario for High-Scale Moduli Stabilization in String Theory*, *Nucl. Phys.* **B 897** (2015) 500 [arXiv:1503.07634] [INSPIRE].

[25] W. Siegel, *Two-vierbein formalism for string-inspired axionic gravity*, *Phys. Rev.* **D 47** (1993) 5453 [hep-th/9302036] [INSPIRE].

[26] W. Siegel, *Superspace duality in low-energy superstrings*, *Phys. Rev.* **D 48** (1993) 2826 [hep-th/9305073] [INSPIRE].

[27] C. Hull and B. Zwiebach, *Double Field Theory*, *JHEP* **09** (2009) 099 [arXiv:0904.4664] [INSPIRE].

[28] O. Hohm, C. Hull and B. Zwiebach, *Generalized metric formulation of double field theory*, *JHEP* **08** (2010) 008 [arXiv:1006.4823] [INSPIRE].

JHEP12(2015)122

[29] O. Hohm, C. Hull and B. Zwiebach, *Background independent action for double field theory*, *JHEP* **07** (2010) 016 [arXiv:1003.5027] [INSPIRE].

[30] G. Aldazabal, D. Marques and C. Núñez, *Double Field Theory: A Pedagogical Review*, *Class. Quant. Grav.* **30** (2013) 163001 [arXiv:1305.1907] [INSPIRE].

[31] D.S. Berman and D.C. Thompson, *Duality Symmetric String and M-theory*, *Phys. Rept.* **566** (2014) 1 [arXiv:1306.2643] [INSPIRE].

[32] O. Hohm, D. Lüst and B. Zwiebach, *The Spacetime of Double Field Theory: Review, Remarks and Outlook*, *Fortsch. Phys.* **61** (2013) 926 [arXiv:1309.2977] [INSPIRE].

[33] R. Blumenhagen, F. Hassler and D. Lüst, *Double Field Theory on Group Manifolds*, *JHEP* **02** (2015) 001 [arXiv:1410.6374] [INSPIRE].

[34] R. Blumenhagen, P. du Bosque, F. Hassler and D. Lüst, *Generalized Metric Formulation of Double Field Theory on Group Manifolds*, *JHEP* **08** (2015) 056 [arXiv:1502.02428] [INSPIRE].

[35] D. Geissbuhler, D. Marques, C. Núñez and V. Penas, *Exploring Double Field Theory*, *JHEP* **06** (2013) 101 [arXiv:1304.1472] [INSPIRE].

[36] O. Hohm and S.K. Kwak, *Frame-like Geometry of Double Field Theory*, *J. Phys.* **A 44** (2011) 085404 [arXiv:1011.4101] [INSPIRE].

[37] G. Aldazabal, W. Baron, D. Marques and C. Núñez, *The effective action of Double Field Theory*, *JHEP* **11** (2011) 052 [*Erratum ibid.* **11** (2011) 109] [arXiv:1109.0290] [INSPIRE].

[38] D. Geissbuhler, *Double Field Theory and $N = 4$ Gauged Supergravity*, *JHEP* **11** (2011) 116 [arXiv:1109.4280] [INSPIRE].

[39] M. Graña and D. Marques, *Gauged Double Field Theory*, *JHEP* **04** (2012) 020 [arXiv:1201.2924] [INSPIRE].

[40] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four folds*, *Nucl. Phys.* **B 584** (2000) 69 [*Erratum ibid.* **B 608** (2001) 477] [hep-th/9906070] [INSPIRE].

[41] R. Blumenhagen, X. Gao, D. Herschmann and P. Shukla, *Dimensional Oxidation of Non-geometric Fluxes in Type II Orientifolds*, *JHEP* **10** (2013) 201 [arXiv:1306.2761] [INSPIRE].

[42] P. Shukla, *Implementing odd-axions in dimensional oxidation of non-geometric type IIB action*, arXiv:1507.01612 [INSPIRE].

[43] A. Rocen and P. West, *E11, generalised space-time and IIA string theory: the R-R sector*, in *Strings, gauge fields, and the geometry behind: The legacy of Maximilian Kreuzer*, A. Rebhan, L. Katzarkov, J. Knapp, R. Rashkov and E. Scheidegger eds., World Scientific (2010), pp. 403–412 [arXiv:1012.2744] [INSPIRE].

[44] O. Hohm, S.K. Kwak and B. Zwiebach, *Unification of Type II Strings and T-duality*, *Phys. Rev. Lett.* **107** (2011) 171603 [arXiv:1106.5452] [INSPIRE].

[45] O. Hohm, S.K. Kwak and B. Zwiebach, *Double Field Theory of Type II Strings*, *JHEP* **09** (2011) 013 [arXiv:1107.0008] [INSPIRE].

[46] O. Hohm and S.K. Kwak, *Massive Type II in Double Field Theory*, *JHEP* **11** (2011) 086 [arXiv:1108.4937] [INSPIRE].

[47] I. Jeon, K. Lee and J.-H. Park, *Ramond-Ramond Cohomology and $O(D, D)$ T-duality*, *JHEP* **09** (2012) 079 [arXiv:1206.3478] [INSPIRE].

[48] R. Blumenhagen, D. Lüst and S. Theisen, *Basic concepts of string theory*, Springer (2013).

[49] C. Jeschek and F. Witt, *Generalised $G_2$-structures and type IIB superstrings*, *JHEP* **03** (2005) 053 [hep-th/0412280] [INSPIRE].

[50] T.W. Grimm and J. Louis, *The Effective action of $N = 1$ Calabi-Yau orientifolds*, *Nucl. Phys.* **B 699** (2004) 387 [hep-th/0403067] [INSPIRE].

[51] P. Berglund and P. Mayr, *Non-perturbative superpotentials in F-theory and string duality*, *JHEP* **01** (2013) 114 [hep-th/0504058] [INSPIRE].

[52] D. Cassani, *Reducing democratic type-II supergravity on* $\mathrm{SU}(3) \times \mathrm{SU}(3)$ *structures*, *JHEP* **06** (2008) 027 [arXiv:0804.0595] [INSPIRE].