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Relating Technology To Acquisition Costs: Aircraft Turbine Engines

J. R. Nelson and F. S. Timson

A Report prepared for

UNITED STATES AIR FORCE PROJECT RAND

Rand
SANTA MONICA, CA. 90406

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PREFACE

During the past several years, Rand studies concerning weapon system R&D and acquisition strategies have indicated that considerable improvement might be achieved in the cost estimation of aerospace equipment if it were possible to incorporate explicitly in the cost-estimating relationship a quantitative measure of the technical advance sought in a new program.(1-3) Analysis of past programs suggests that a strong correlation exists between the degree of technical advancement sought and subsequent performance, cost, and schedule variances. A more recent Rand report describes a technique for quantitatively assessing the technology sought in aircraft turbine engines in terms of the time of arrival of a demonstrated level of performance.(4) To the extent that these demonstrated levels of performance represent the *best efforts* of government and industry, they reflect the technological state of the art. The present report describes how that technique has been refined and related to development and production costs of military aircraft turbine engines. The technique has also been applied to product improvement for aircraft turbine engines.(5)

The technology and cost-estimation models developed in this study reflect mainstream trends. They are intended to provide planners with a broader understanding of the relationship between the technical requirements for a new or improved weapon system and the important variables associated with the technology, cost, and schedule of the aircraft turbine engine for that weapon system. The resulting models are not able to distinguish fine differences among particular aircraft turbine engines of comparable vintage, nor are they able to predict marginal costs of producing additional quantities of an existing engine. The primary goal is to provide a systems planner with information for making decisions concerning development and procurement strategies for new or improved weapon systems.

This Project RAND study is one in a continuing research program on the R&D and acquisition processes and the estimation of military equipment costs. Plans for further research include application of the findings of this study to aircraft turbine engine operational experience to determine if this measure of technical advance sought in a new engine can be related to improving the ability to anticipate the operations and maintenance costs of engines in the field and at the depot. Combining all of these cost elements would, for this particular military component, lead to a life-cycle cost analysis that is more detailed and is related to the technological content of the engine.

This report should be of use to various Air Force and other agencies that are engaged in R&D and system acquisition decisions and that must obtain the cost information on which such decisions rely. Air Force staff personnel in DCS/Research and Development, DCS/Systems and Logistics, Office of the Comptroller, ACS/Studies and Analysis, and the Air Force Systems Command should find this research particularly useful.

SUMMARY

Cost-estimating relationships for aircraft turbine engines can be significantly improved by incorporating within the estimating model a quantitative measure of an engine's technology content. The quantitative measure presented in this report is derived from a recent Rand study in which a technique was developed for assessing the date at which an aircraft turbine engine with a specified set of technical parameters should pass its 150-hr Model Qualification Test (MQT).(4)

The present study refines the earlier work and applies the results to the estimation of engine development and procurement costs. The resulting cost models, which incorporate the predicted Time of Arrival (TOA) of an engine with a specific set of technical characteristics at MQT along with other specific "conventional" variables known to affect engine cost, are compared to models that exclude the TOA measure. Models incorporating technical parameters by themselves are also investigated.

The refined aircraft turbine engine TOA model is based on 26 U.S. military turbojet and turbofan engines developed and produced during the past 30 years. The model predicts the man-rated 150-hr MQT date as a function of certain of the engine's performance and design parameters. The parameters include maximum thrust of the engine at sea-level static conditions, weight, specific fuel consumption at military thrust at sea-level static, turbine inlet temperature, and a pressure term (the product of flight envelope maximum dynamic pressure and the overall pressure ratio of the engine). The coefficients of the TOA equation are consistent with intuitive notions of what constitutes more technologically advanced achievement -- positive coefficients on variables for which larger values are more difficult to achieve and negative coefficients on variables for which smaller values are more difficult to achieve. To the extent that demonstrated levels of aircraft turbine engine performance represent the *best efforts* of government and industry, they accurately reflect the technological state of the art. A model is also presented that includes 11 U.S. commercial turbojet and turbofan engines. The commercial engines are shown to

lag the military engines technologically by about two and one-half years. In addition, 13 growth engines (achieved through product improvement) are shown not to be able to incorporate technological advance as readily as can new engine designs.

The TOA appears in the cost models in two forms: directly, as calculated by the TOA equation, and as ΔTOA, the difference between the calculated TOA and the actual or planned MQT. Because TOA and ΔTOA are functions of performance characteristics and are measures of time, the cost equations containing these variables can be used to analyze performance schedule/cost tradeoffs and their associated risks. The standard and technology parameters models do not have this capability.

All costs represent selling price to the government measured in 1973 dollars. The models for estimating costs include:

- development to MQT
- total development including MQT and all subsequent product improvement
- 1000th unit production cost
- cumulative average production unit cost progress curve slope
- production quantity cumulative cost

In all cases examined, incorporation of TOA and/or ΔTOA yields a cost-estimating relationship that is superior to the equations based only on the conventional variables used in past studies. (Only in the case of the progress slope was an equation incorporating additional technology parameters separately significantly better than the TOA equation.) The cumulative production cost equation also yields an average progress curve slope. Both the progress curve equations and the cumulative production cost equation indicate that the progress curve slope is somewhat shallower when some technology measure is included. In addition, in the slope models, as production rate increases, there is a higher rate of progress. Care must be exercised in using these models since small differences in progress curve slopes have a pronounced effect on cost estimates associated with the procurement of large numbers of engines.

Additional analyses in the report provide a basis for evaluating, at least in an approximate way, the evolution of development and production costs over two decades, and risks associated with time in developing and producing new aircraft turbine engines.

These TOA and cost models are intended for use by long-range military planners attempting to determine costs for new systems -- especially those of a technically advanced nature -- so that better estimates can be made, not only of the costs involved, but also of the associated development time and performance characteristics required. All parameters needed to make such TOA and cost estimates are readily available at an early stage of planning for a new system. Care must be exercised in using these models to ensure that inputs are consistent with the data base in this study. For example, cost estimates will reflect military technology and the manner in which programs were conducted during the 1950s and 1960s. To the extent that a new program differs from these conditions, the input data or cost estimates will require some adjustment.

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CONTENTS

FIGURES

TABLES

SYMBOLS

DEVMQTCOST = development cost to MQT, millions of 1973 dollars

DEVTIME = development time from start to MQT, calendar quarters

KTEMP = maximum turbine inlet temperature, °R/1000

KPRATE = average production rate, 1000 engines/quarter[*]

MACH = maximum flight envelope Mach number

MCDUM = military-commercial dummy (1 = commercial, 0 = military)

MFRDUM = manufacturer dummy (1 = Pratt & Whitney, 0 = others)

MQTQTR = man-rated 150-hr Model Qualification Test date, calendar quarters

MQTY = total quantity produced, millions of units

MVOLUME = engine volume (max. dia. and length, cu. in./10^6)

PROP = dummy variable when turboprop/turboshaft engines are included in technology model

PRQCOST = production quantity cumulative cost at quantity purchased, millions of 1973 dollars

PRUCOST = production unit cost at 1000th unit, millions of 1973 dollars

QMAX = maximum dynamic pressure in flight envelope, lb/ft^2

QTR = quarter of year

QTY = quantity of production engines procured

SFCMIL = specific fuel consumption at military thrust, sea-level static (SLS), lb/hr/lb thrust

SLOPE = cumulative average unit production cost progress slope

TEMP = maximum turbine inlet temperature, °R

THRMAX = maximum thrust (with afterburner if afterburning configuration), SLS, lb

THRMIL = military thrust (even if afterburner configuration), SLS, lb

TOA26 = time-of-arrival index of demonstrated performance for 26 military turbojet and turbofan engines, calendar quarters

TOA26F = time-of-arrival index of last growth engine, calendar quarters

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[*] Several variables are expressed in what appear to be unusual units in order to obtain significant figures in the computer output for various equations.

TOA37 = time-of-arrival index of demonstrated performance for 26 military and 11 commercial turbojet and turbofan engines, calendar quarters

ΔTOA26 = TOA26-MQT date, calendar quarters

ΔTOA26F = TOA26F-final production date, calendar quarters

TOTDEVCOST = total development cost including MQT and product improvement, millions of 1973 dollars

TOTDEVTIME = total development including MQT and product improvement, calendar quarters

TOTPRS = pressure term (product of QMAX x pressure ratio), lb/ft^2

WGT = weight of engine at configuration of interest, lb

I. INTRODUCTION

In the past several years, Rand has examined weapon system acquisition experience of the 1950s and 1960s and compared performance, cost, and schedule predictions with the actual outcomes of selected major programs for all three services.(1-3) During the course of the study it became apparent that considerable improvement could be achieved in cost and schedule estimates made at the beginning of a development program if a measure of the degree of technical advance sought in that program could be obtained. To that end, technological advance factors were generated.(2) They were subjective in the sense that individuals who had experience in various aspects of systems acquisition or who were knowledgeable in fields related to the systems under study rated the technical difficulty of each development. The ratings were on a scale from 1 to 20, with low numbers signifying systems which essentially were technologically straightforward, and higher numbers indicating increasing technological advance. Thus, technical difficulty was ordered, but the differences between the ratings for two systems did not necessarily imply a quantitatively measured difference.* These subjective ratings were related to performance, schedule, and cost predictions and had interesting and promising implications.

It seemed desirable to have a more objective technological advance measure. Toward this end a study was conducted to measure technological change in aircraft turbine engines.(4) This study developed a technology assessment equation using the date of achieving the Model Qualification Test (MQT) as a function of a particular bundle of performance-oriented characteristics, tested some of its implications, and briefly touched on the application of this technique to cost estimation.

The present study refines the earlier work on technology assessment in order to clarify a basic issue concerning the interpretation

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*That is, a rating of 16 did not necessarily indicate a degree of difficulty twice that indicated by a rating of 8. In mathematical terms, the technological advance factors were ordinal, not cardinal.

and application of the measure obtained and then relates the resulting measure to aircraft turbine engine acquisition costs.[*]

Refinements to the analyses involve a redefinition of this measure, changes to the data base, and modification of certain variables. The resulting implications are discussed in detail in Sec. II. The technique is then applied to cost estimation for new aircraft turbine engines. Section III presents the resulting cost-estimation equations. They include cost of development to MQT, cost of total development (where total development consists of development to MQT and further product improvement costs),[**] 1000th unit production cost, progress curve slope, and cumulative production cost. The data used in this study are summarized in Table 1.

Table 1

TURBOJET AND TURBOFAN DATA SUMMARY

	Engine Programs/Data Points
Military technology assessment.........	26/26
Military and commercial technology assessment..........................	37/37
Product improvement technology assessment..........................	13/13
Development (MQT) cost.................	14/14
Total development cost.................	8/25
Production unit cost and slope.........	18/18
Production quantity cost...............	18/88

Section IV first uses the cost equations to analyze factors contributing to increasing acquisition cost trends during the past twenty years, and then employs the equations to predict the time of arrival and related acquisition costs for new aircraft turbine engines.

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[*] The equations developed in this report supersede those given in Ref. 4.

[**] For a more detailed investigation of product improvement of aircraft turbine engines, see Ref. 5.

II. AIRCRAFT TURBINE ENGINE TECHNOLOGY ASSESSMENT REVISITED

The technology trending analysis used in this study is based on Ref. 4.* As was noted in that study, there are three major reasons why turbine engines were selected for the initial application of this approach: (1) turbine engines are an important subsystem, accounting for very large portions of the defense budget (on the order of several billions of dollars per year); (2) on a qualitative basis there has been a strong and continuous improvement in the level of technology of turbine engines over the past 30 years (as shown by the historical synopsis of technological developments displayed in Table 2); and (3) an adequate data base was available.

Since publication of Ref. 4, considerable interest has arisen concerning the use, interpretation, and implications of the trending technique. To provide a consistent set of equations in the cost-estimation work that follows, the technology assessment measure has been refined. Before describing the refinements, the nature of the measure will be discussed in some detail to assist in understanding its meaning, uses, and limitations.

TECHNOLOGY ASSESSMENT VERSUS TIME OF ARRIVAL

Technology is not a directly measurable quantity, so a substitute measure has been sought. Technology trends are tracked by recording the values of the substitute measure(s) at different times. The original Rand study on turbine engine technology(4) used multiple regression to relate the date of an engine's successful completion of its Model Qualification Test to certain of its technical characteristics. The present study uses the same approach.

The value given by the multiple regression equations is the date that an engine with a specified set of technical characteristics is expected to pass its MQT. In this study, calculated dates obtained from the regression equations are designated TOA (Time of Arrival) and

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*See Sec. II of that report for a detailed discussion.

Table 2

SYNOPSIS OF AIRCRAFT TURBINE ENGINE DEVELOPMENTS

Early 1940s (WW II)	Late 1940s	Early 1950s (Korean War)	Late 1950s	Early 1960s	Late 1960s (Vietnam)	Early 1970s
Turbojet	Turbojet	Turbojet, Turboprop/ Turboshaft	Turbojet, Turboprop/ Turboshaft	Turbojet, Turboprop/ Turboshaft, Turbofan	Turbojet, Turboprop/ Turboshaft, Turbofan	Turbojet, Turboprop/ Turboshaft, Turbofan
Trends in engineering development						
Increased thrust Centrifugal to axial compressor Single-design point mission Limited use of high-temperature steels; primarily conventional steels	Augmentation Two-position nozzle Stainless steel, aluminum, conventional steel Higher pressure ratio, dual rotor	High-pressure ratio, variable stators Titanium begins to replace aluminum Sustained supersonic flight Small helicopter engines Reliability/ durability Moderately higher turbine temperature	Cooled turbine Mach 3 Small lightweight engines Commercial turbojet Subsonic turbofan Titanium and superalloy material improvements Transonic compressor	Supersonic turbofan Multi-design point mission Superalloy materials Lightweight design Component improvements Commercial turbofan	High-bypass turbofan (military and commercial) High-temperature turbine Cooling techniques 3-spool rotor Compatibility/ integration Increasing sophistication of development Commercial technology and requirements becoming as advanced as military	High thrust/weight High component performance High-temperature materials Cooling techniques Composite materials
Companies						
General Electric Westinghouse	Allison Boeing Curtiss Wright Fairchild General Electric Pratt & Whitney Westinghouse	Allison Boeing Continental Curtiss Wright Fairchild General Electric Lycoming Pratt & Whitney Westinghouse	Allison Boeing Continental Curtiss Wright Fairchild General Electric Lycoming Pratt & Whitney	Allison Boeing Continental Curtiss Wright Garrett General Electric Lycoming Pratt & Whitney	Allison Continental Garrett General Electric Lycoming Pratt & Whitney	Allison Continental Garrett General Electric Lycoming Pratt & Whitney

the input dates for particular engines having specific bundles of characteristics at the 150-hr man-rated MQT are designated by MQTQTR. The difference, TOA - MQTQTR = ΔTOA, is the interval between the time when an engine is predicted to pass its MQT and the time it actually passes. Because all of the inputs to the regression analyses are cardinally scaled measures of time, the predictions obtained from the resulting equations are also cardinally scaled measures of time.*

While TOA is a measure of time, it is a function of the technological characteristics of turbine engines. As will be noted later in this section, the signs of the coefficients in the TOA equations are consistent with intuitive notions of what constitutes more technologically advanced achievement -- positive coefficients on variables for which larger values are more difficult to achieve and negative coefficients on variables for which smaller values are more difficult to achieve. The authors believe that there has been continual pressure to advance technology during the 30-year period covered by the data used in this study, and that continuous progress has been achieved. These observations and others** have led the authors to believe that the

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*The rules applying to the mathematical uses of TOA and MQTQTR are the same as those applying to any measure of time. Thus, in comparing two engines with different TOAs and MQTQTRs, the differences between the times are comparable but the ratios of the times are not. This is because all measures of time involve arbitrary choices of origins. For example, consider Engine A with TOA(A) = 50 and MQTQTR(A) = 40 and Engine B with TOA(B) = 40 and MQTQTR(B) = 30. The ΔTOAs are the same, 50 - 40 = 40 - 30 = 10, but the ratios of the TOAs and MQTQTRs are not, 50/40 ≠ 40/30.

**Among the other observations are: (1) analyses of the trend at different times indicate steady advancement; (2) analyses of military versus commercial engines show the commercial engines "behind" the military engines but slightly converging; (3) analyses of primary versus improved engines indicate a slower rate of technological growth for improved models; (4) analyses of advanced technology demonstrator engines show them to be significantly ahead of the military engine trend; (5) analyses of selected foreign versus U.S. engines show that at the present time U.S. industry should achieve a given operational version earlier than foreign industry, whereas 25 to 30 years ago the opposite was true; and (6) specific U.S. engines that are significantly ahead of or behind the trend equation are indeed thought of as being advanced or conservative by the aircraft turbine engine industry and the military.

MQT date generally orders the level of technology of engines, and that TOA provides an *ordinal* prediction of an engine's level of technology. In this technological sense the measure is not cardinal because the differences cannot be compared.

The technology and cost models given in this report are of interest to planners and cost estimators whether TOA is a cardinal or an ordinal measure. Interest has been focused on this issue because a cardinal measure of technology is intrinsically desirable: Such a measure would provide a sounder theoretical basis for cost estimation than do the "conventional" cost models which are concerned with variables that best explain the data regardless of their theoretical appeal. The TOA measure seems to be a more objective measure of technological advance than has been previously obtained and used in cost-estimating relationships, but it is not a cardinal measure of technology. It is a proxy for technology, it does relate technology to time, and time is used in a cardinal sense in the cost-estimating models.

CHANGES TO THE DATA BASE AND MODIFICATIONS OF VARIABLES

The data base used in Ref. 4 has been extensively revised. The engines included in the present analysis consist of 37 turbojet and turbofan engines -- 26 military and 11 commercial.

The 12 turboprop/turboshaft engines used in the original study were removed because detailed development and production cost data were not available on enough of the models and because there were inconsistencies in definitions of certain technological parameters in some of the data. The J44, J93, J97, and J100 were removed because they did not pass a 150-hr man-rated Model Qualification Test. The TF35, TF37, JT8D, JT9D, and CF6 were removed from the military sample and included in an 11-engine commercial sample which is shown to be significantly different from the military sample. Also, the MQT dates and performance characteristics for the J52 and J85 corresponding to the more stringent 150-hr man-rated MQT were substituted for non-man-rated MQT values used previously. These changes result in a data base of 26 military turbojets and turbofans which have all passed a 150-hr man-rated MQT, and 11 commercial engines which passed the FAA Certification. The specific models and manufacturers are listed in Table 3. The 37 military and commercial engines are "primary" engines in the sense that each is used only once according to its first MQT or FAA Certification -- no product improvement models are included in the technology analysis. The 13 product improvement models listed in Table 3 are used to analyze the relative rates of technological advance for new versus improved engines.

In addition to the data changes, two of the parameters in the TOA model of the previous study have been modified. These are the substitution of maximum thrust for military thrust and a pressure term for dynamic pressure. Maximum thrust is the thrust that an engine can generate at afterburner or military power (depending upon its configuration) at sea-level static conditions. The pressure term is the product of maximum dynamic pressure in the flight envelope and the pressure ratio of the engine.

Many engines have both afterburning and non-afterburning models. The model that first passed a Model Qualification Test or FAA Certification is the criterion for selecting which model to use in the time-of-arrival equation. If the MQT engine had no afterburner, but subsequent models did, as in the case of the J57, the non-afterburning sea-level static thrust was used. Similarly, if the MQT engine had an afterburner and subsequent models did not, for example, the TF30, the afterburning sea-level static thrust was used. The performance data for each engine in the data base are listed in Table 4.

STATISTICAL RESULTS

Statistical analyses of the turbine engine data were performed with several objectives in mind. The primary objective was to determine the shape and movement of the tradeoff surface of performance parameters. Statistical tests were made of the assumptions that the shape of the surface did not change over time and that movement was uniform.

Table 3

TURBINE ENGINE DATA BASE

Early 1940s	Late 1940s	Early 1950s	Late 1950s	Early 1960s	Late 1960s
			a. Military		
J30 W[a] J31 GE J33 GE/A J34 W J35 GE/A	J40 W J42 PW J46 W J47 GE J48 PW J57 J71 A J73 GE	J52 PW J65 CW J69 C J75 PW J79 GE	J58 PW J60 PW J85 GE TF30 PW TF33 PW		TF34 GE TF39 GE TF41 A
5	8	5	5		3
			b. Commercial		
			JT3C PW JT4A PW JT12 PW JT3D PW JT8D PW CJ805-3 GE CJ805-23 GE	CJ610 GE CF700 GE	JT9D PW CF6 GE
			7	2	2
			c. Growth (military)		
		J33 A J35 A J47 GE J57 PW	J71 A J75 PW J69 C	TF33 PW	J52 PW J60 PW J79 GE J85 GE TF30 PW
		4	3	1	5

[a] Engine manufacturer is indicated as follows: W = Westinghouse; GE = General Electric; A = Allison; PW = Pratt & Whitney; CW = Curtiss Wright; C = Continental.

Table 4

U.S. AIRCRAFT TURBINE ENGINE TECHNOLOGY DATA

Designation	Turbine Inlet Temp. (°R)	Thrust Max. (lb)	Weight (lb)	Pressure Term (lb/ft^2)	SFC (lb/hr/lb)	Mach No.	Max. Dia. (in.^{2})	Length (in.)	MQT (qtr)
Military									
J30	1830	1560	686	1575	1.17	0.9	19.0	94	17
J31	1930	1600	850	1710	1.25	0.9	41.5	72	11
J33	1960	3825	1875	3400	1.22	1.0	50.5	103	19
J34	1895	3250	1200	3400	1.06	1.0	27.0	120	27
J35	2010	4000	2300	3400	1.08	1.0	40.0	168	21
J40	1985	10900	3580	5750	1.08	1.8	41.0	287	45
J42	1825	5000	1729	3640	1.25	1.0	49.5	103	25
J46	1985	6100	1863	6625	1.01	1.8	29.0	192	44
J47	2060	4850	2475	5375	1.10	1.0	37.0	144	26
J48	2030	6250	2040	4880	1.14	1.0	50.0	107	33
J52	2060	8500	2050	12840	0.82	1.8	31.5	150	74
J57	2060	10000	4160	11400	0.80	1.4	41.0	158	41
J58	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)	87
J60	2060	3000	460	10360	0.96	1.0	24.0	80	71
J65	2030	7220	2815	8500	0.92	1.8	38.0	127	46
J69	1985	920	333	3400	1.12	1.0	22.0	44	56
J71	2160	9570	4090	11000	0.88	1.5	40.0	195	47
J73	2060	8920	3825	8750	0.92	1.9	37.0	147	49
J75	2060	23500	5950	16724	0.80	2.0	43.0	259	59
J79	2160	15000	3225	18056	0.87	2.0	37.5	208	57
J85	2100	3850	570	10360	1.03	2.0	20.0	109	74
TF30	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)	92
TF33	2060	17000	3900	19240	0.52	1.0	53.0	136	71
TF34	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)	120
TF39	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)	109
TF41	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)	107
Commercial									
JT3C	1995	13500	4234	11050	0.78	(b)	(b)	(b)	59
JT4A	1995	15800	5020	10200	0.80	(b)	(b)	(b)	59
JT3D	1995	17000	4150	11050	0.52	(b)	(b)	(b)	71
JT12	2000	2700	465	5525	0.96	(b)	(b)	(b)	71
CJ805-3	2100	11200	2800	11050	0.83	(b)	(b)	(b)	71
CJ805-23	2100	16100	3800	11050	0.56	(b)	(b)	(b)	77
CJ610	2060	2850	399	5780	0.99	(b)	(b)	(b)	82
CF700	2100	4125	725	5525	0.65	(b)	(b)	(b)	87
JT8D	2180	14000	3160	13600	0.59	(b)	(b)	(b)	81
JT9D	(a)	(a)	(a)	(a)	(a)	(b)	(b)	(b)	107
CF6	(a)	(a)	(a)	(a)	(a)	(b)	(b)	(b)	112
Growth									
J33	2065	5900	1954	3613	1.10	(b)	(b)	(b)	51
J35	2140	7200	2830	6250	1.11	(b)	(b)	(b)	42
J47	2060	5970	2707	6500	1.06	(b)	(b)	(b)	41
J52	(a)	(a)	(a)	(a)	(a)	(b)	(b)	(b)	111
J57	2060	16000	5045	11400	0.84	(b)	(b)	(b)	51
J60	2060	3300	460	10360	0.96	(b)	(b)	(b)	96
J69	1985	1025	364	3400	1.14	(b)	(b)	(b)	67
J71	2160	10200	4090	11000	0.92	(b)	(b)	(b)	55
J75	2070	24500	5875	17612	0.82	(b)	(b)	(b)	63
J79	(a)	(a)	(a)	(a)	(a)	(b)	(b)	(b)	98
J85	2200	4300	600	10360	1.04	(b)	(b)	(b)	95
TF30	(a)	(a)	(a)	(a)	(a)	(b)	(b)	(b)	115
TF33	2210	21000	4605	23680	0.61	(b)	(b)	(b)	82

[a] Deleted due to security or proprietary considerations.

[b] Not used in this study.

Equations were investigated that incorporated input variables (characteristics that are associated with the technological state of the art such as turbine inlet temperature and pressure ratio), output variables (characteristics that are desired in the final product such as thrust and specific fuel consumption), and a mixture of both. In selecting variables for inclusion in the time-of-arrival equations (and in the cost equations to follow), several criteria were used. All regressions were obtained using the stepwise-least-squares procedure.*

Table 5 shows the results obtained using the military and commercial engine data. Equations are presented for an all-military case (26 engines) and a combined military and commercial case (37 engines). For comparison, an equation using the 26 military engines with the variables from the original study(4) is shown, as well as the 47 mixed-engine equation from the original study. The statistical improvement is good.

The equation that best represents the military trend of the technological tradeoff surface contains three output variables (maximum thrust, weight, specific fuel consumption (SFC), one input variable (turbine inlet temperature), and one mixed variable (the product of maximum dynamic pressure and overall pressure ratio, an output variable and an input variable, respectively).**

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*The variables in the equations in this report are listed in the order that the stepwise procedure includes them.

In determining the validity of the models, the usual statistics of coefficient of determination (R^2), standard error of the estimate (SE), and significance levels of the coefficients were employed, as well as the following criteria: (1) whether the variables were meaningful from engineering and cost points of view; (2) whether the signs of the coefficients were consistent with conventional wisdom regarding the variables' impact on technology and cost; (3) statistical improvement of equation due to added variables; (4) degree of correlation between independent variables; and (5) the distribution of the residuals. (Not all of these criteria were satisfied in all cases.)

**This set of variables evidences some problems of multicollinearity, particularly between weight and thrust. However, attempts at leaving thrust or weight out of the TOA equation had mixed effects on the cost equations and it was decided to leave in both. The only adequate solution for multicollinearity is more observations, which are not feasible

Table 5

AIRCRAFT TURBINE ENGINE TIME-OF-ARRIVAL (TOA) EQUATIONS

Final Models/Current Data (26 Military Turbojets & Turbofans, 11 Commercial Turbojets & Turbofans)

$$\text{TOA26} = -856.38 + 110.10 \underset{(<0.0001)}{\ln \text{TEMP}} + 11.407 \underset{(0.0057)}{\ln \text{TOTPRS}} - 26.077 \underset{(<0.0001)}{\ln \text{WGT}} - 16.024 \underset{(0.0105)}{\ln \text{SFCMIL}} + 18.369 \underset{(0.0111)}{\ln \text{THRMAX}}$$

$R^2 = 0.956$

SE = 6.9

F = 92

$$\text{TOA37} = -772.85 + 98.151 \underset{(<0.0001)}{\ln \text{TEMP}} + 9.860 \underset{(0.0016)}{\text{MCDUM}} + 11.970 \underset{(0.0004)}{\ln \text{TOTPRS}} - 26.465 \underset{(<0.0001)}{\ln \text{WGT}} - 15.668 \underset{(0.0013)}{\ln \text{SFCMIL}} + 19.038 \underset{(0.0016)}{\ln \text{THRMAX}}$$

$R^2 = 0.964$

SE = 6.13

F = 134

Previous Model/Current Data (26 Military Turbojets & Turbofans)

$$\text{TOA} = -1074.0 + 132.83 \underset{(<0.0001)}{\ln \text{TEMP}} - 22.778 \underset{(0.0121)}{\ln \text{SFCMIL}} + 19.070 \underset{(<0.0008)}{\ln \text{QMAX}} - 27.820 \underset{(0.0003)}{\ln \text{WGT}} + 21.525 \underset{(0.0130)}{\ln \text{THRMIL}}$$

$R^2 = 0.944$

SE = 8.0

F = 67

Previous Model/Previous Data (47 Military and Commercial Turbojets, Turboprops, & Turbofans)

$$\text{TOA47} = -1187.5 + 156.20 \underset{(<0.0001)}{\ln \text{TEMP}} - 20.647 \underset{(0.0042)}{\ln \text{SFCMIL}} - 26.533 \underset{(<0.0001)}{\ln \text{WGT}} + 18.767 \underset{(0.0012)}{\ln \text{THRMIL}} + 11.742 \underset{(0.0112)}{\ln \text{QMAX}} + 13.011 \underset{(0.0405)}{\text{PROP}}$$

$R^2 = 0.903$

SE = 9.6

F = 62

NOTES: Variables are listed in order of introduction using stepwise-least-squares procedure. Probability that coefficient is zero--from the *t*-test of significance--is shown in parentheses below each variable.

Turbine inlet temperature was the most important variable in the analysis. One obvious explanation for the statistical power of turbine inlet temperature, even after the major performance parameters have been included in the equation, is that temperature plays a dominant role in the thermodynamics of engines, and consequently a major development goal has been ever higher temperatures. These higher temperatures have been one of the chief sources of improved performance as measured by the major performance parameters. In addition to the major parameters, many less important engine characteristics have not been included in the equation, and turbine inlet temperature may be considered a proxy for these parameters.*

The equation best representing the military/commercial tradeoff surface includes a military/commercial dummy as well as variables present in the pure military equation. For both equations, parameter coefficients have the correct signs with respect to technological advance. Weight and SFC, being more highly valued as they are reduced, have negative coefficients; i.e., holding other variables constant, SFC and weight have fallen over time. Thrust, temperature, and pressure have positive coefficients, indicating growth over time. The military/commercial dummy variable takes on the value zero if the engine is a military turbojet or turbofan, and one if it is a commercial product. This dummy variable automatically incorporates certain adjustments required for differences in the military/commercial engines. For example, commercial engines enter service designed for longer life and higher reliability. The value of the dummy suggests that on the average they enter service about 10 quarters (2.5 years) later than their military counterpart.

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in the present case. Furthermore, multicollinearity is only a problem when making predictions for new systems when the collinearity pattern for the new system is different. It is not expected that this should be a problem in the near future, and as new engines are qualified, they can be added to the data base and the equations updated.

*In this context, temperature can be considered a technical budget from which expenditures are made. The major expenditures are accounted for by the major parameters, but they do not exhaust the budget. The residual effect of temperature measures its contribution to the excluded variables. In addition, the advanced materials and production techniques used to achieve high temperatures were available for secondary uses throughout the engine.

It should be stressed that *these models represent the time of arrival* (150-hr MQT or FAA Certification) *of a demonstrated level of performance*. This demonstrated performance is assumed to represent the best efforts of the aircraft turbine engine industry; it thus is considered to be the technological state of the art of U.S. aircraft turbine engines.

A graphical representation of the military technological trend equation is plotted in Fig. 1. The combined military and commercial trend is presented in Fig. 2. The calculated TOA, which is determined by inserting an engine's characteristics into the equations, is plotted on the vertical axis.[*] The horizontal axis shows the actual date the engine passed its MQT. Note that the scatter of the residuals about the 45-degree line does not appear to violate any assumptions usually made about the distribution of errors.

The 45-degree line represents the average trend or expected date of MQT over the period. Points plotted above the 45-degree line represent engines "ahead of their time;" that is, engines with characteristics yielding TOAs greater than their actual MQT dates appeared earlier than predicted. Likewise, points below the line are "late" or "conservative" developments. The average deviation from the line (the standard error of the equation) is 6.1 quarters for the military and commercial model and 6.9 quarters for the pure military model.[**] This means that approximately two-thirds of the observations could be expected to fall within ± 6.1 (or 6.9) quarters of the 45-degree line. This spread of three years around the trend line illustrates one of the problems of interpreting the results. There are many possible reasons for deviations about the trend, and an individual engine can be identified as advanced or conservative only if it is significantly removed from the trend. The equation therefore cannot be used for making fine distinctions, but if certain points or trends deviate

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[*] The dates on both axes are measured in quarters of a year beginning with zero as the third quarter of 1942.

[**] The standard error is actually the square root of the average squared deviation.

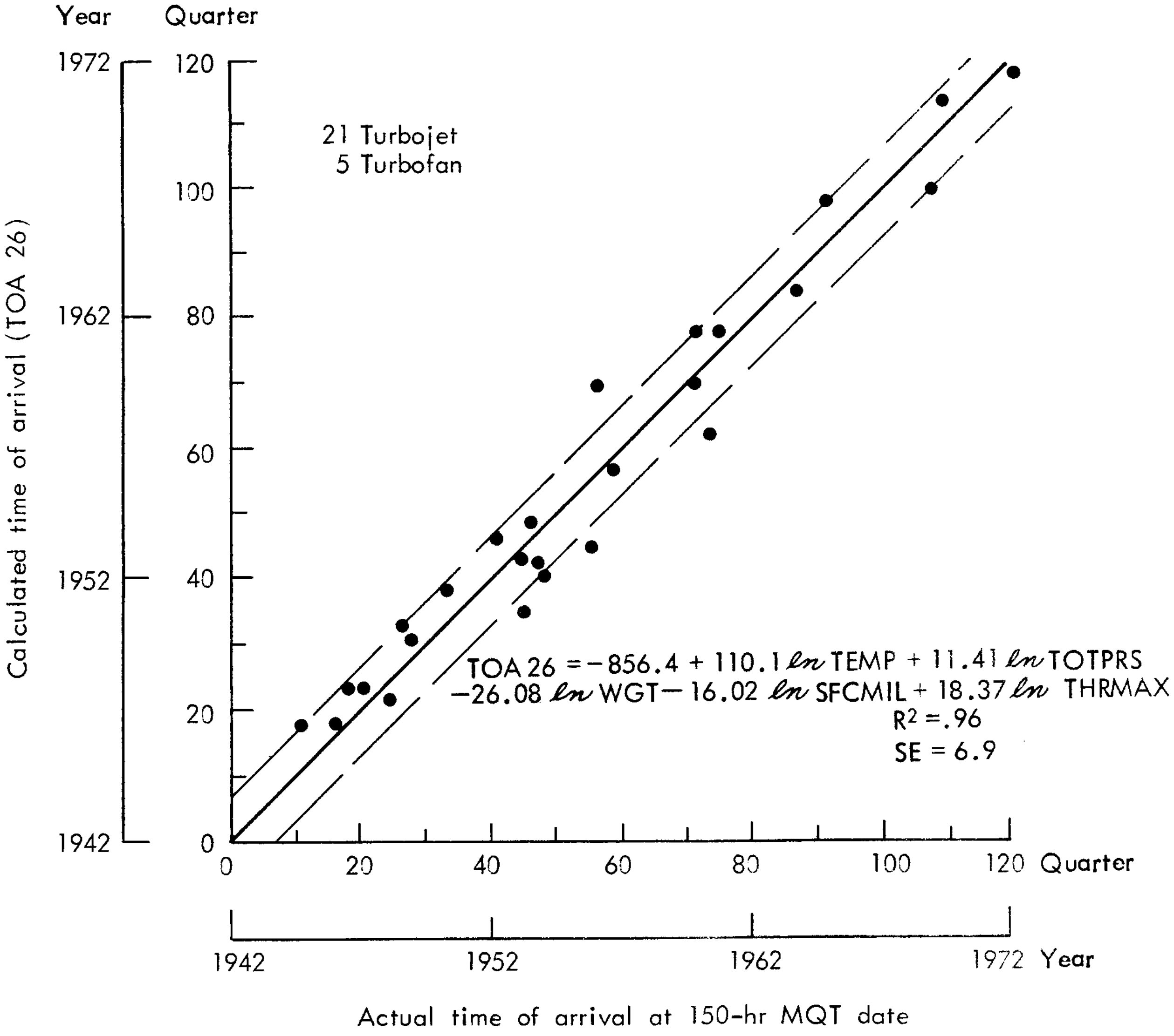
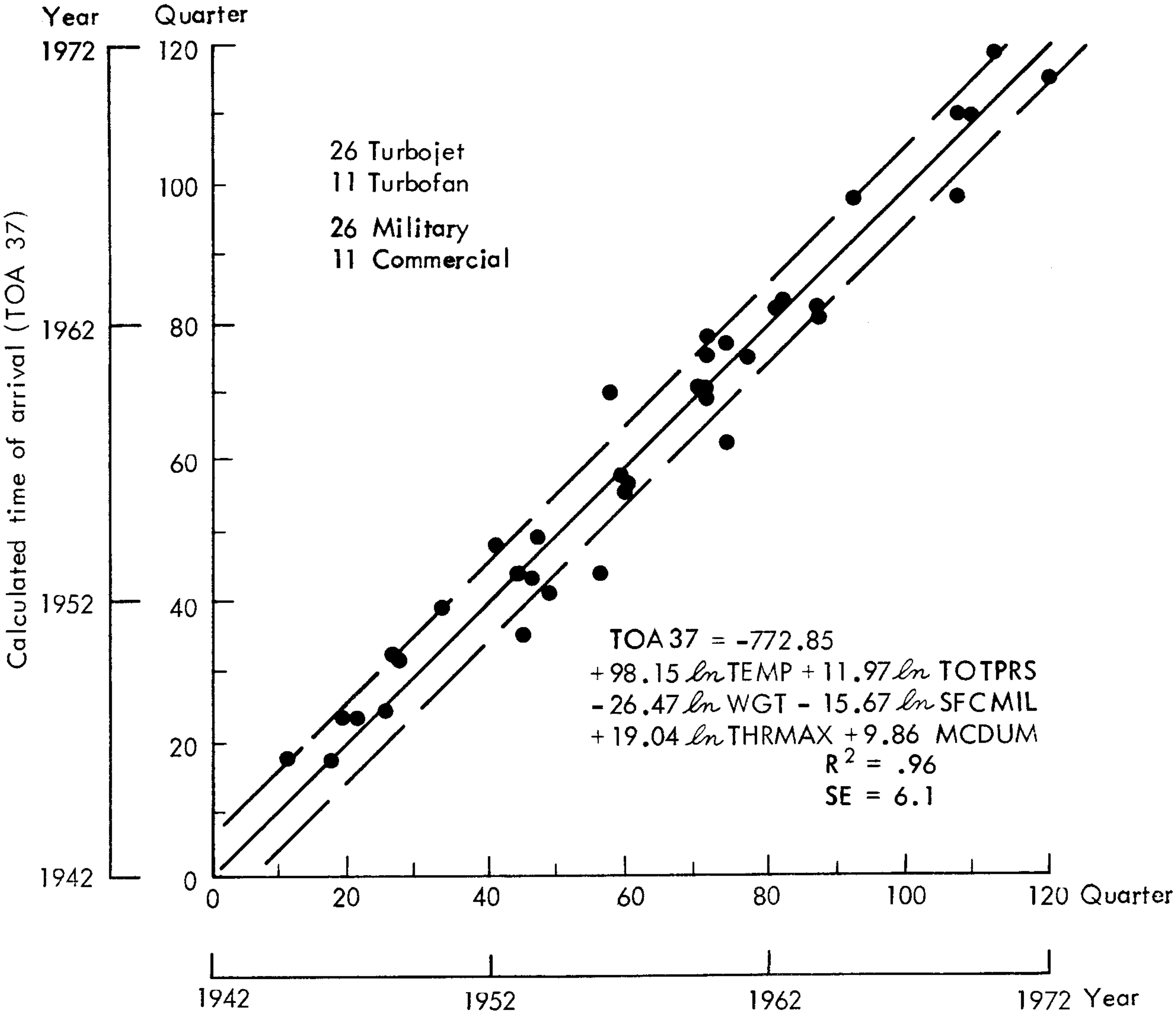


Fig. 1 — Military turbine engine time of arrival



Actual time of arrival at 150-hr MQT or FAA Certification

Fig. 2—Military and commercial turbine engine time of arrival

sharply from the average, it should be possible to distinguish them from the "ambient noise." As noted earlier, this study considers deviations greater than one standard error as being significant.

In an attempt to determine whether the equation changed over time, the sample was split into subsamples covering various time periods. Dividing the sample into equal halves yielded the surprising result that there was little statistically discernible difference between the trends for earlier and later periods.*

Figure 3 shows the results of putting the commercial engine data into the military model (TOA26) and fitting a line to the points. The commercial trend line lies below and is approaching the 45-degree line military model. This figure corroborates the combined military/commercial model which showed commercial engines to be behind military engines on the average. However, from this analysis it appears that commercial technology is approaching military technology. Indeed, some engine designers feel that commercial technology could surpass military technology in the future, especially (for example) if noise abatement requirements are explicitly considered in the technology index. Another possible factor is the absence of any new military programs started in the early 1960s.** Until development of the Pratt & Whitney JT9D, all commercial engines were direct derivatives of military programs. The JT9D is the first example of a major new U.S. aircraft turbine engine entering commercial service with no prior military experience.

The development of engines beyond the MQT is often more costly than the entire development program up to the MQT. An analysis was made of the additional technological growth of 13 engines after their original MQT. It is expected intuitively that design flexibility of a growth version of an engine already in production is limited because many of its features are constrained by the existing hardware and production capabilities. Hence, technology improvement for updated engines should be less than that demonstrated by new engines. This

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*The Chow test for significance was used.(6)

**See Table 3.

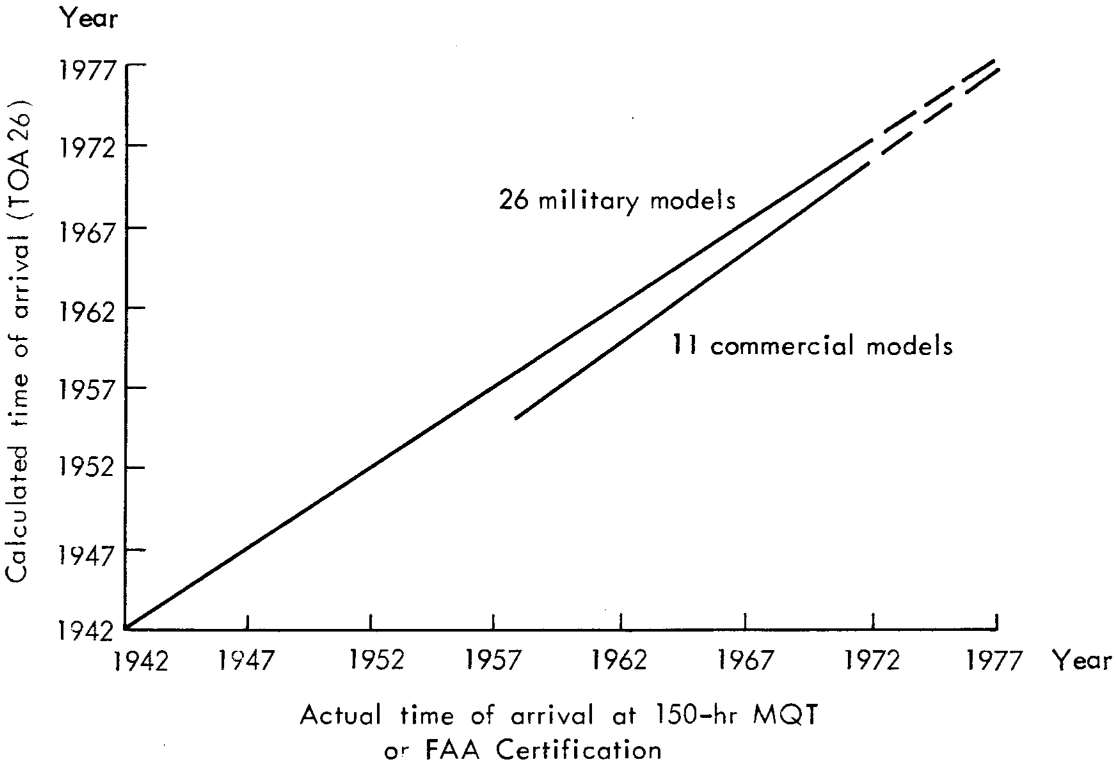

Fig. 3 — Comparison of military and commercial aircraft turbine engine technology

expectation is borne out by the plot in Fig. 4 of post-MQT technology growth for the 13 engines. The left-hand point of each pair of points is the TOA of the original MQT engine, and the right-hand point is the TOA of the most improved version. The connecting line indicates the rate of technological growth for each engine relative to the state-of-the-art trend. All engines showed growth curves of less than 45 degrees. A recent Rand study(5) analyzes the type, amount, and cost of technological change through growth models of turbine engines. Those findings are generally consistent with what is shown here.*

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*There are, however, some slight differences because the engine data in that study are not exactly the same as those used here.

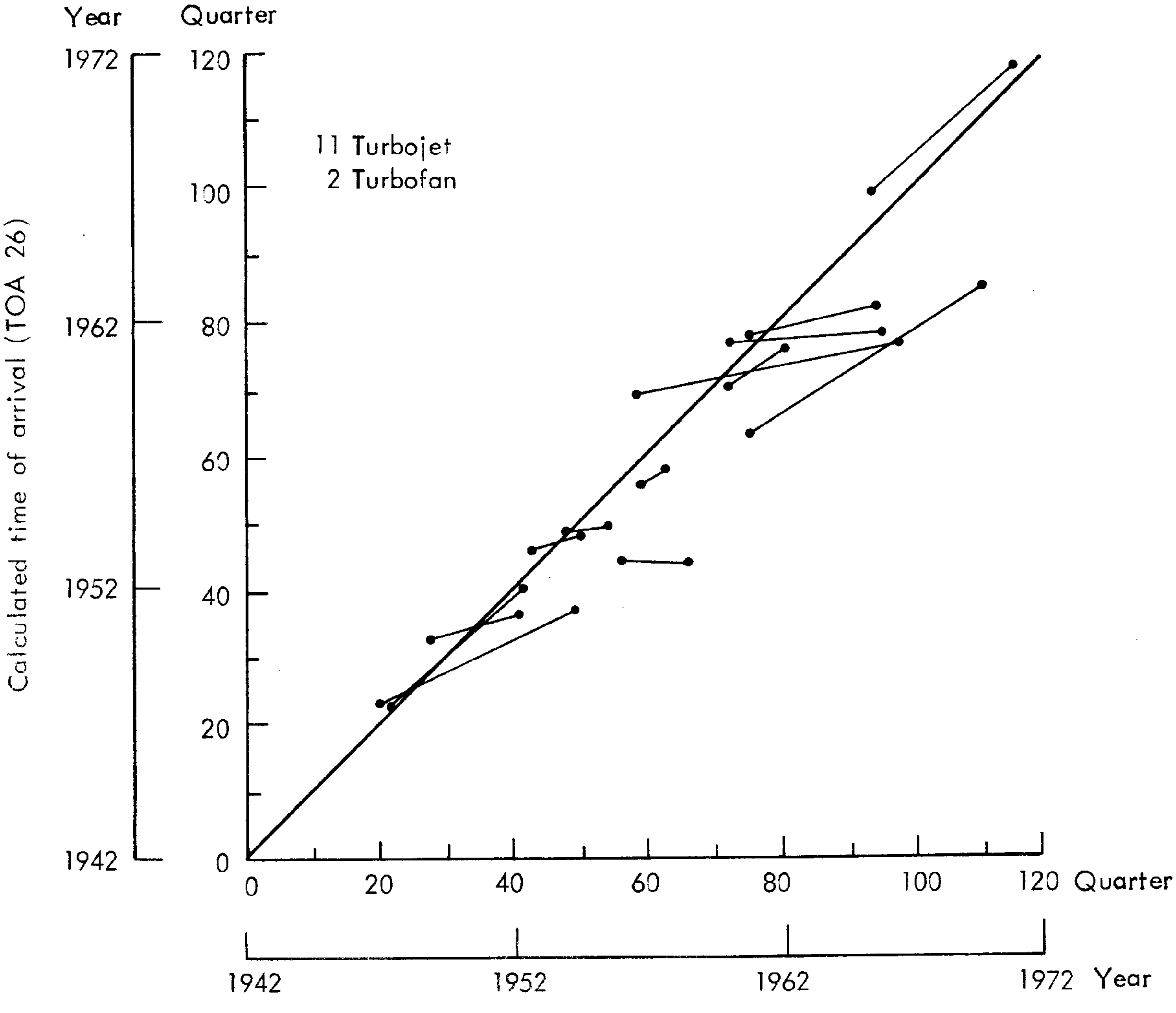

Fig. 4 — Military growth engine time of arrival

III. AIRCRAFT TURBINE ENGINE COST ESTIMATING

This section gives the results of estimating aircraft turbine engine costs using the time-of-arrival model discussed in the preceding section. The purpose is to improve upon the available techniques for predicting costs associated with aircraft turbine engine development and production processes. It is hoped that ultimately this relationship can be investigated for the operational portion of turbine engine costs, so that an overall life-cycle analysis can be accomplished.

Considerable effort has been devoted to estimating costs of aircraft turbine engines over the past decade. Rand work during this period included that of Watts,(7) which related the costs of development and production to thrust and quantity for turbojets and to horsepower and quantity for turboprop/turboshaft engines (turbofans were not considered); and Large,(8) which related development and production costs to thrust, Mach number, and quantity for turbojet and turbofan engines.* Large also included a qualitative assessment of the impact of technology on cost by dividing the engine data base into technology classes. The present study verifies the variables that Large found to be important for turbojets and turbofans and adds the TOA measure, which is a function of technological parameters. This measure eliminates the need to estimate the technology class for a new engine.**

Other notable work in estimating turbine engine costs includes that of Brennan and Taylor,(9) of the Naval Air Development Center, which related production costs of turbojet and turbofan engines to the materials content and the difficulty of manufacturing parts from these materials. The Brennan and Taylor approach involves very specific knowledge of the design of an engine, because all of the materials and parts of the various components must be known. Thus, that method is

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*The Large study omitted turboprop/turboshaft engines, and as previously mentioned, turboprop/turboshaft engines will not be discussed further in this report.

**Large's technology classes correspond to time periods by decades.

more appropriately used to obtain estimates later in the system planning cycle when the design is much closer to manufacture.

Equations are presented for (1) the cost of development of the engine to the 150-hr man-rated MQT; (2) total development cost, which includes MQT and all subsequent product improvement during the life of the engine; (3) production unit cost; (4) progress curve slope; and (5) cumulative production quantity cost. Each equation is discussed in detail, including definitions, background information, data base, modifications to the data, and results.

DATA ADJUSTMENTS

Two major adjustments were made to the data in this study -- one for price level changes and the other for engine configuration and quantity -- so comparisons could be made on the basis of constant dollars and at production quantities for homogeneous engine models.

The price levels were adjusted using the indexes shown in Table 6.

Because cost information at the individual cost element levels (such as engineering, tooling, materials, etc.) was not available in a consistent form, total contract costs were used. Thus, the term *cost*, as used in this report, refers to the selling price to the government. The price index adjusts the cost to 1973 dollars.

Occasionally, during the course of an engine production program, new models are introduced that differ substantially from the original version. To provide a homogeneous data base for equations that include production quantity as a variable, the data were divided into four categories. Category 1 contains engines that were produced entirely in one configuration, either afterburning or non-afterburning, for a man-rated application. This category presents no problem. Category 2 includes engines in which a non-afterburning version was produced before an afterburning version started production, or vice versa. There is no problem with respect to homogeneous data up to the point where the different configuration is introduced. It is then handled similarly to Category 3. Category 3 contains engines for which both afterburning and

Table 6

PRICE LEVEL ADJUSTMENT INDEX FOR AIRCRAFT ENGINES[a]

Year	Index	Year	Index
1946	3.824	1960	1.832
1947	3.623	1961	1.779
1948	3.289	1962	1.718
1949	3.185	1963	1.672
1950	3.012	1964	1.618
1951	2.703	1965	1.577
1952	2.577	1966	1.506
1953	2.513	1967	1.462
1954	2.439	1968	1.370
1955	2.348	1969	1.292
1956	2.232	1970	1.220
1957	2.128	1971	1.147
1958	1.992	1972	1.064
1959	1.894	1973	1.000

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[a] Based on the average hourly earnings of production workers in the aircraft engine industry as given by Bureau of Labor Statistics data on employment and earnings to 1969 (Ref. 10) and adjusted to 1973 by E. Ojdana of The Rand Corporation. (Note that no productivity adjustment has been made to this index.)

non-afterburning versions were being produced at the same time. To ensure that the progress curve for the basic engine is captured, the costs of the non-afterburning engines are adjusted (set equal) to the afterburning version costs to obtain a homogeneous set of data for the basic engine. For cost-estimating purposes, the engine performance data correspond to the afterburning version. Category 4 is a special situation involving the J52 and J85 only, where non-man-rated missile engines came first and a man-rated MQT and production version followed. In the case of the J85 there was the additional problem of afterburning and non-afterburning configurations for the man-rated applications. Costs and quantities in these cases are associated with the time period after the manned MQT has been achieved, but the missile engine quantities have been factored into the cost data to adjust the learning curve for those quantities produced.[*] This is perhaps the most suspect adjustment to the data in this study. However, since it occurs for only these two engines, the effects should be minimal.

Another data adjustment was made for the production cost analyses. Because several of the engines went through a number of models during their production history, it was possible that the progress curves might exhibit discontinuities at the model changes, producing a sawtoothed progress curve.[**] To determine whether such changes were present, the "significant" model variations were identified,[***] and progress curves were determined for the various models in addition to the overall production runs. Seven engines were identified as having growth models that differed significantly from the original MQT models. For these seven engines, progress curves were determined for the first and second models in addition to the overall progress curve that ignored the model variations. The progress curves did not exhibit the expected sawtoothed pattern, and in four cases the slope decreased while for the other three it increased. Chow tests(6) were performed for all of the model splits and only one case failed to reveal a significant difference.[****] Because of these differences, all production data beyond the significant model changes were omitted from the analyses.

As Large(8) notes, there are comparability problems with respect to differing production rates, commonality among engine models, differences in efficiency, and pricing policy anomalies concerned with overhead costs and specific contract add-ons such as Independent Research and Development, Component Improvement Programs, and recently,

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[*] For the J52, a substantial number of missile engines were produced and included in the production totals. For the J85, only a few missile engines were produced before the man-rated MQT and they were omitted.

[**] For an investigation of variations in progress curves related to deliberate cost-reducing product improvement, see Ref. 5.

[***] "Significance" was determined by the various growth models' TOAs. A difference of approximately four quarters was considered to indicate a new model.

[****] The 0.10 level of significance was used. The progress curves may differ for two reasons: A shift due to a change in unit costs or a rotation due to a change in slope.

Integrated Logistics Systems. These considerations cannot be adjusted for in any systematic, justifiable way and are mentioned in Ref. 8 only as a reminder that the cost data are somewhat less than perfect -- a valid statement for the present study as well.

ALTERNATIVE MODELS FOR COST ESTIMATING

The following subsections investigate alternative models for estimating the costs of development to MQT, costs of total development, costs of unit production and cumulative production, and the progress curve slope. Three alternatives for the cost models are presented:*

- The "standard variables" equation, which includes thrust, Mach number, development time and/or quantity, and a dummy for Pratt & Whitney engines in the cumulative cost equation. These equations are called standard because they include only variables that have been found to be important in past studies.(7,8)
- The time-of-arrival cost estimating equation, which includes the standard variables plus either TOA, ΔTOA, or both.** TOA and ΔTOA are illustrated in Fig. 5.
- The "technology parameters" equation, which includes a mixture of standard variables and technology parameters present in the TOA equation.***

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*There is a fourth equation in the progress curve slope and cumulative production cost tables due to the inclusion of a dummy to adjust for a difference in the accounting practice at Pratt & Whitney, prior to 1971, described later in this section.

**Maximum thrust appears in the TOA cost equations directly and indirectly. Because thrust is the last variable to enter the TOA equation and because it is highly collinear with weight, a complete set of TOA and cost equations was determined excluding thrust from TOA. The results were inconclusive. The predictive ability of TOA was not improved, the development cost equations worsened, and the production cost equations improved. Because the results were mixed and the authors believe thrust should be in the TOA equation, it was left that way.

***These equations were determined by applying the stepwise-least-squares procedure to the standard variables plus the technology parameters. The number of variables in the resulting equations were limited

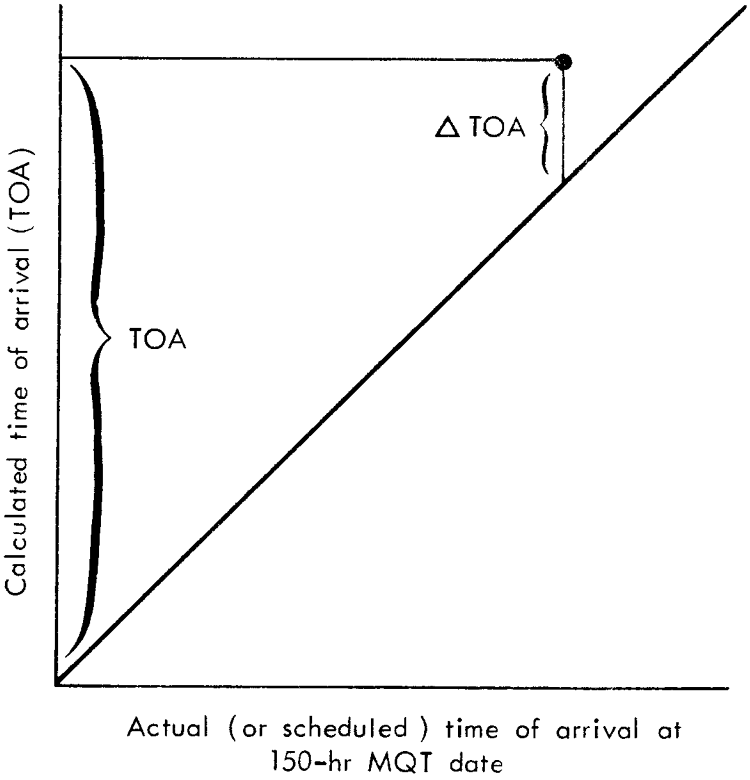

Fig. 5 — Relationship between predicted time of arrival and deviation from the trend

The progress curve slope equations differ from the cost equations in that there are no previous studies to indicate what would be "standard" variables. Consequently, an extensive search was made to find the best regression equations possible.

All cost equations are in logarithmic form, for two major reasons. First and most important, this form produces a distribution of residuals that appears to be normal, of constant variance, and not serially correlated; hence, all of the statistics are meaningful.[*] Second, the resulting distribution of errors on dollar costs is log normal with constant percentage error, which is intuitively more appealing than a normal distribution with constant absolute error.[**]

DEVELOPMENT COSTS

Two distinct development costs (defined as cost to the government adjusted to 1973 dollars) are analyzed in this study. The *development*

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to the same number as in the TOA equation (with the exception of the total development, which has one less variable because no additional variable was statistically significant). For example, the TOA-development-to-MQT cost equation has four variables, so the corresponding technology parameters equation is limited to four variables. (It was not always possible to satisfy all criteria for variable selection in every equation.)

In general, the technology parameters equations are *not* recommended for predicting the costs of new engines by themselves because they include only one or two technology parameters. New engines that are markedly advanced or conservative with respect to characteristics in the technology parameters equations (and not so relative to the other parameters in TOA) may be over or underestimated relative to the corresponding TOA equation's estimate. TOA measures an engine's relative advancement on the basis of five variables, not one or two. Specific instances are discussed in Sec. IV.

[*] In the cumulative production cost models, subsets of the data are serially correlated. Consequently, the statistics are not meaningful for that particular case.

[**] The log normal distribution ranges from zero to positive infinity and is skewed toward higher values, while the normal distribution ranges from negative infinity to positive infinity and is symmetric. Because sums of logarithms translate into products when antilogs are taken, standard errors for the logarithmic forms translate into percentages of the corresponding dollar values. Thus a standard error of ± 0.200 yields multiplicative factors of $\exp(+0.2) = 1.22$ and $\exp(-0.2) = 0.82$, which correspond to additive factors of +22 percent and -18 percent. Both values will be shown in the tables to follow.

cost to MQT is associated with the 150-hr man-rated endurance test, after which the engine is considered to be sufficiently developed for installation in a production military aircraft and is suitable for operational use in the field. The development cost to MQT includes initial design, engineering, prototype tooling, materials and fabrication, and assembly and testing of components and complete engines. Not included are any costs associated with demonstrator programs that may have provided the basic technology required to develop the new engine or any costs of production tooling associated with the procurement phase. No flight test engines are included.

Total development cost includes the expenses involved in developing a new engine to MQT, as outlined above, costs to correct service-related deficiencies, and costs for continued performance and reliability improvement over time. A performance improved model must pass an additional 150-hr endurance test at the higher performance level. The cost of continued development beyond MQT can exceed the cost of development up to MQT. This is illustrated in Fig. 6, which plots percentage of cost to MQT versus percentage of time to MQT for eight engine programs.

The three equations for development costs to MQT are shown in Table 7. The 14 data points represent 14 separate development programs. The results for the standard equation and the time-of-arrival equation are displayed graphically in Figs. 7a and 7b, respectively. The points represent the calculated (predicted) versus actual costs for the engines in the data base. The superior statistical quality of the time-of-arrival equation is indicated by the narrower scatter of the points. Similar results are presented in Table 8 and Figs. 8a and 8b for eight total development programs. The 25 data points represent 8 engines that passed several MQTs during their lifespan. The improvement achieved by the time-of-arrival equation in this case is not significant. All three models exhibit the same level of statistical accuracy. However, the SFC term in the technology parameters model has a counter-intuitive sign.

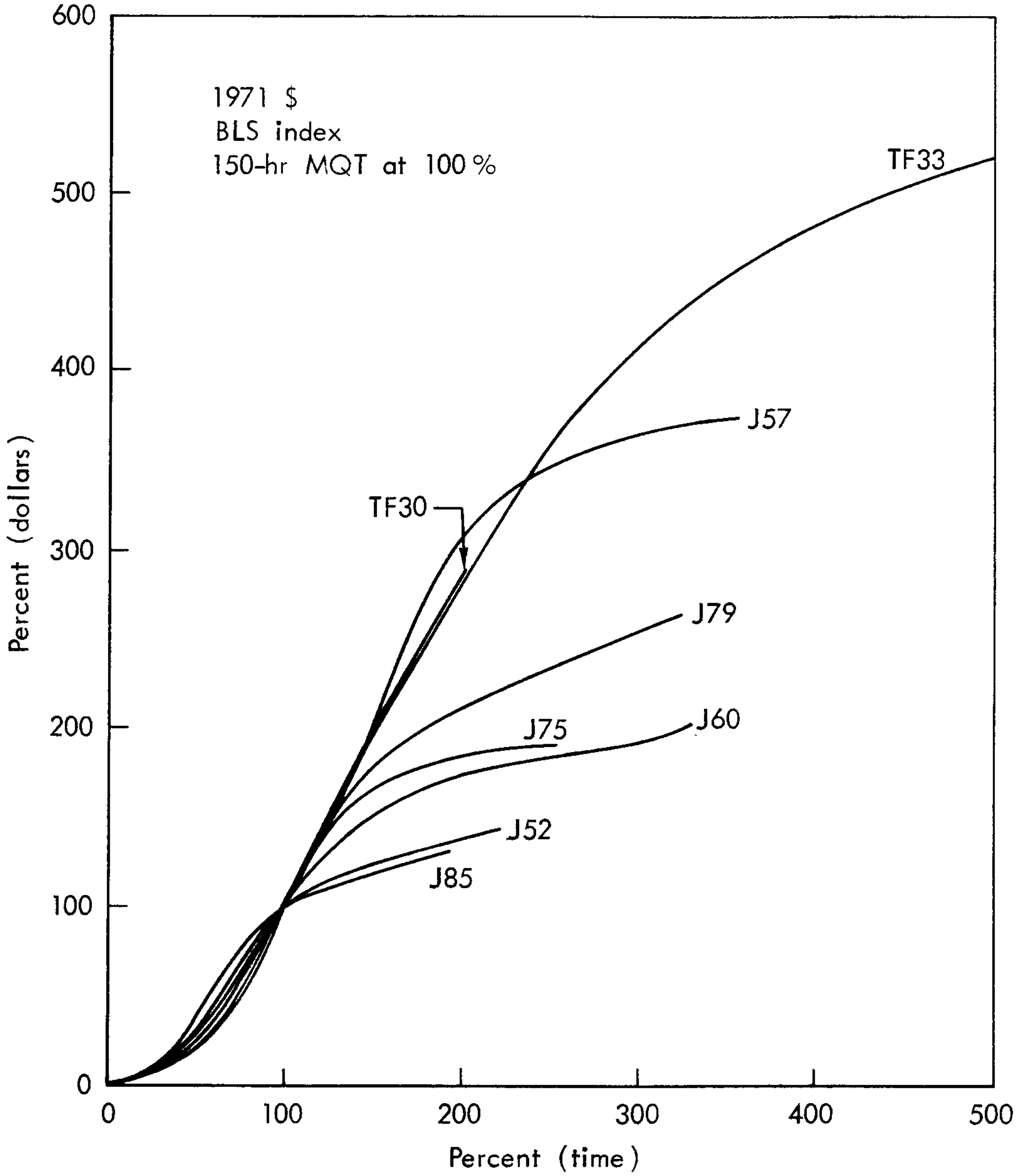

Fig. 6 — Total development cost versus time for selected aircraft turbine engines

Table 7

AIRCRAFT TURBINE ENGINE DEVELOPMENT COST TO MQT

(14 Turbojet and Turbofan Engines/14 Data Points)
(BLS Index--1973 $ millions)

Time-of-Arrival Model

$R^2 = 0.961$

$SE = 0.182\ (+20\%,\ -17\%)$

$F = 55.7$

$$\ell n\ \text{DEVMQTCOST} = -1.4461 + 0.08538\ \underset{(<0.0001)}{\text{DEVTIME}} + 0.49630\ \underset{(<0.0001)}{\ell n\ \text{THRMAX}} + 0.04099\ \underset{(0.0009)}{\Delta\text{TOA26}} + 0.41368\ \underset{(0.0493)}{\ell n\ \text{MACH}}$$

Technology Parameters Model

$R^2 = 0.882$

$SE = 0.317\ (+37\%,\ -27\%)$

$F = 16.8$

$$\ell n\ \text{DEVMQTCOST} = -1.5723 + 0.07184\ \underset{(0.0049)}{\text{DEVTIME}} + 0.81292\ \underset{(0.0178)}{\ell n\ \text{THRMAX}} + 0.58532\ \underset{(0.1016)}{\ell n\ \text{MACH}} - 0.33430\ \underset{(0.2165)}{\ell n\ \text{WGT}}$$

Standard Model

$R^2 = 0.858$

$SE = 0.329\ (+39\%,\ -28\%)$

$F = 20.2$

$$\ell n\ \text{DEVMQTCOST} = -1.0779 + 0.07463\ \underset{(0.0039)}{\text{DEVTIME}} + 0.47611\ \underset{(0.0036)}{\ell n\ \text{THRMAX}} + 0.5112\ \underset{(0.1502)}{\ell n\ \text{MACH}}$$

NOTES: Variables are listed in order of introduction using stepwise-least-squares procedure. Probability that coefficient is zero--from the *t*-test of significance--is shown in parentheses below each variable.

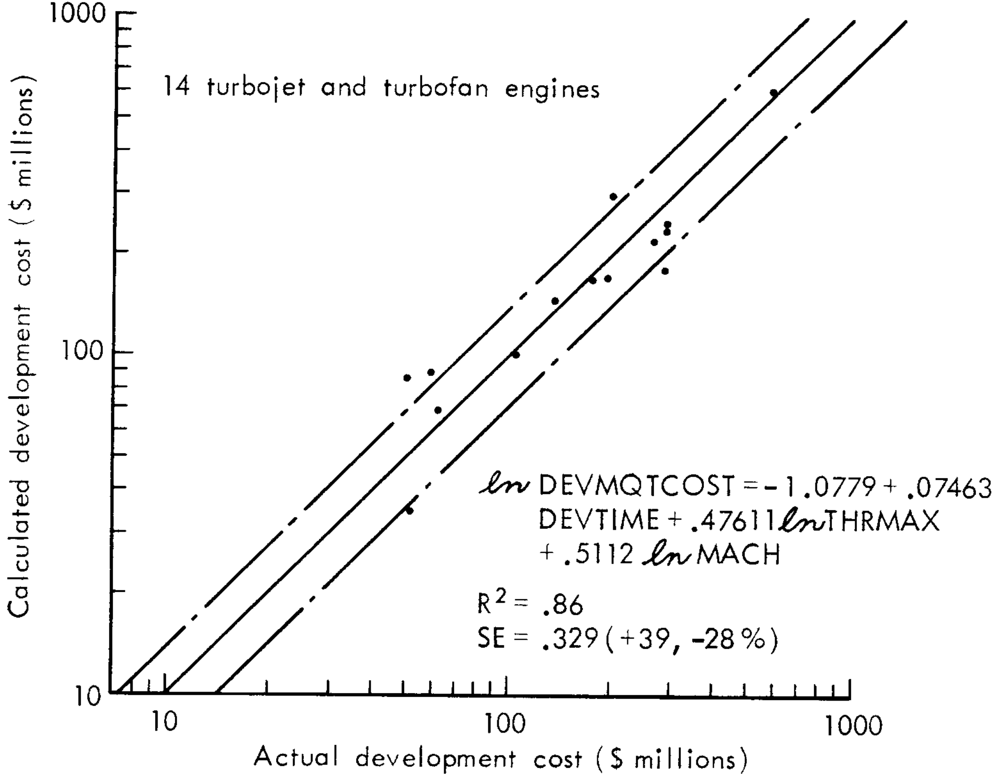

Fig. 7a — Development cost (MQT), standard equation
BLS index - 1973 $

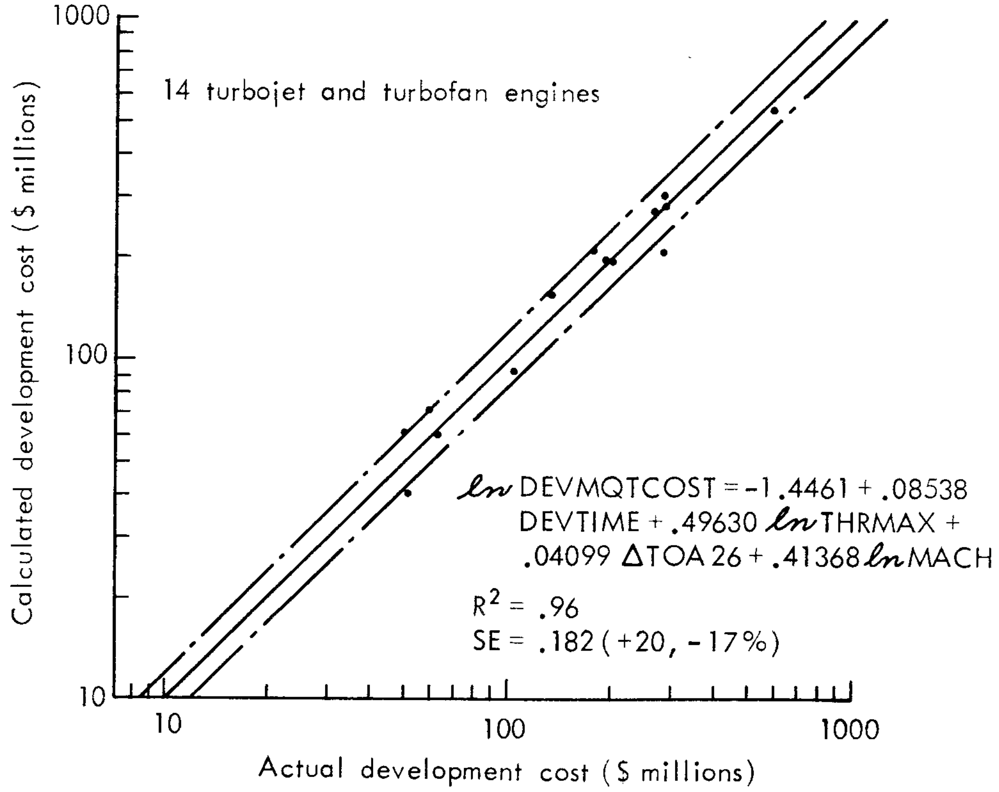

Fig. 7b — Development cost (MQT), time-of-arrival equation
BLS index - 1973 $

Table 8

AIRCRAFT TURBINE ENGINE TOTAL DEVELOPMENT COST

(8 Turbojet and Turbofan Engines/25 Data Points)
(BLS Index--1973 $ millions)

Time-of-Arrival Model

$R^2 = 0.927$

$$\ln \text{TOTDEVCOST} = 0.23198 + \underset{(<0.0001)}{1.0193 \ln \text{MACH}} + \underset{(0.0048)}{0.06228 \ln \text{QTY}} + \underset{(<0.0001)}{0.44251 \ln \text{THRMAX}} + \underset{(0.0862)}{0.01418\ \text{TOTDEVTIME}} + \underset{(0.1097)}{0.01302\ \Delta\text{TOA26F}}$$

SE = 0.209 (+23%, -19%)

F = 48.0

Technology Parameters Model

$R^2 = 0.930$

$$\ln \text{TOTDEVCOST} = -10.485 + \underset{(0.0004)}{1.0098 \ln \text{MACH}} + \underset{(<0.0001)}{0.07119 \ln \text{QTY}} + \underset{(0.0001)}{0.43019 \ln \text{WGT}} + \underset{(0.1231)}{1.5642 \ln \text{TEMP}} + \underset{(0.1279)}{0.84530 \ln \text{SFCMIL}}$$

SE = 0.204 (+23%, -19%)

F = 50.7

Standard Model

$R^2 = 0.914$

$$\ln \text{TOTDEVCOST} = 0.79747 + \underset{(<0.0001)}{1.2867 \ln \text{MACH}} + \underset{(<0.0001)}{0.08146 \ln \text{QTY}} + \underset{(<0.0001)}{0.39884 \ln \text{THRMAX}}$$

SE = 0.216 (+24%, -19%)

F = 74.1

NOTES: Variables are listed in order of introduction using stepwise-least-squares procedure. Probability that coefficient is zero--from the t-test of significance--is shown in parentheses below each variable.

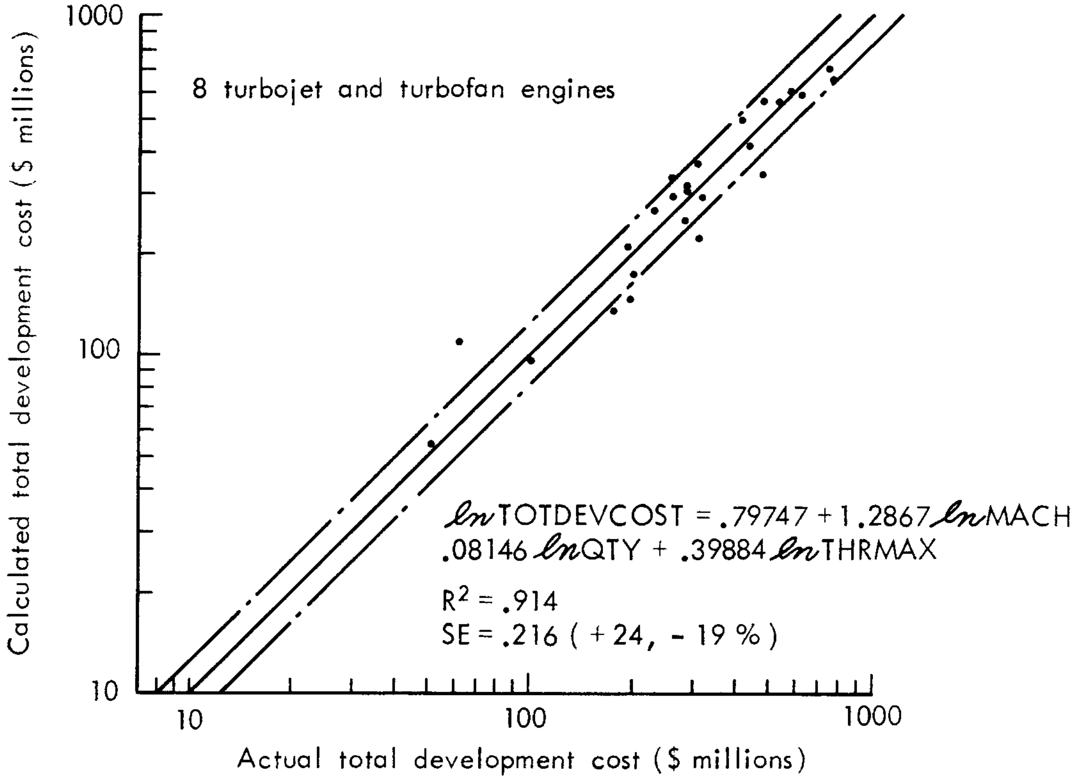

Fig. 8a — Total development cost, standard equation
BLS index - 1973 $

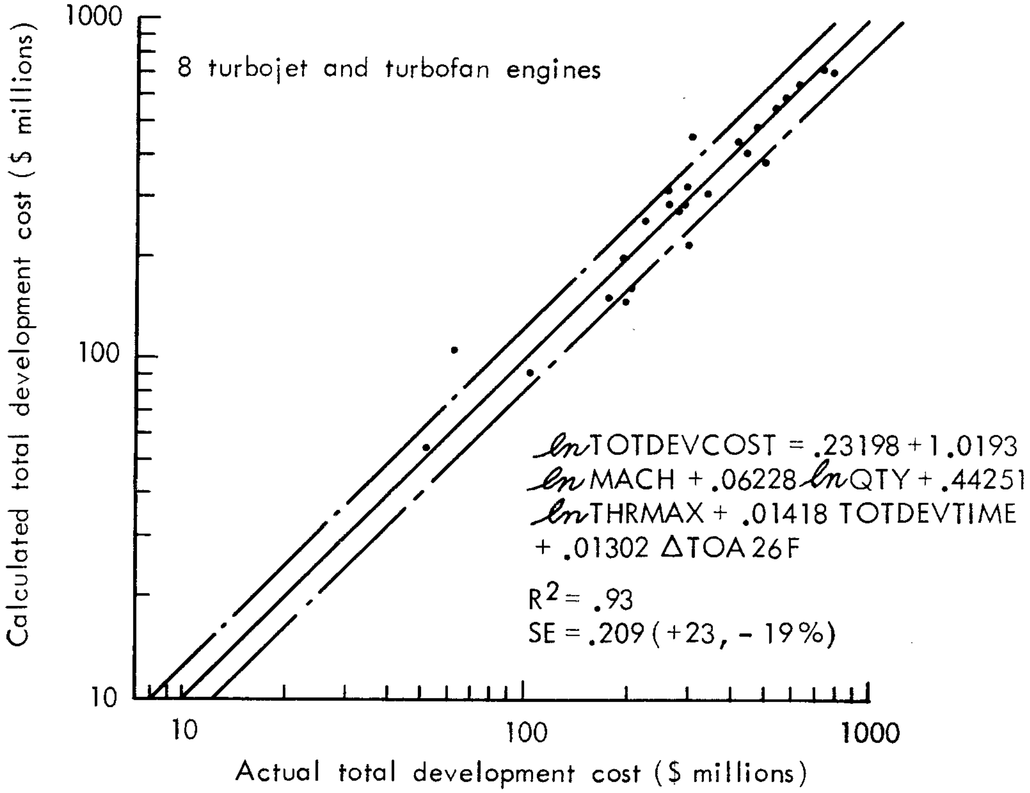

Fig. 8b — Total development cost, time-of-arrival equation
BLS index - 1973 $

The significant variables in the development-to-MQT equation are development time, maximum thrust, Mach number, and ΔTOA. Quantity takes the place of development time in the total development equations. All these variables have intuitive appeal as well as statistical significance.

Maximum thrust can be considered a measure of the physical size of the engine. More than half of the cost of developing an engine is for test hardware, and as an index of engine size, thrust reflects the cost of hardware. Engine development programs will use as many as 50 test engines and equivalent spares.

The Mach number can be considered an indicator of the environment in which the engine must operate, and the operational environment is a strong determinant of the amount of testing required.* More than one-quarter of the cost of the development program is associated with testing.

The length of the development program is an important determinant of cost which enters linearly in the development cost equations.** When estimating the cost of a new engine, an appropriate development time must be selected with care. For new engine development that is at or below the technology trend (planned MQT $\geq$ TOA), the minimum development should be about 20 quarters (five years), which is approximately the average development time in the data base. A technologically advanced engine should probably take longer. A minimum development time of about 24 quarters (six years) in this case appears reasonable. Selection of a development time to use in cost predictions is discussed extensively in Sec. IV.

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*In the cost models, one is the lowest value of Mach number used for subsonic engines. (Design considerations for subsonic engines are basically similar.) Because Mach number enters in logarithmic form, there is no incremental contribution to the cost for a subsonic engine. This must be kept in mind when using the models for predicting new engine costs.

**It is perhaps more reasonable to expect that a quadratic form would give better results. There is some length of time that yields the minimum cost, and either speeding up or stretching out the program raises costs. The present data base is not sufficiently large to capture this form of variation. The development time term can be viewed as the upward sloping, right-hand portion of a quadratic form.

The technology proxy enters the development cost equations in the form of the deviation from the trend, ΔTOA. The other variables in the equation can be viewed as capturing the resources required to keep the trend going, whereas the TOA deviation term can be viewed as that amount of additional resources required to provide the additional technology. The development time coefficient is more than twice as large as the ΔTOA coefficient. Schedule slippages for technologically advanced engines will result in increased costs. However, the risk associated with large ΔTOA must also be recognized. This risk assessment feature of these cost equations is illustrated in Sec. IV.

The total development regression also includes a production quantity term. This variable is intuitively appealing, since the amount of resources devoted to improving a particular engine model through retrofitting should be related to the quantity of engines that are produced and operating in the field. In the total development cost model, it should be noted that the ΔTOA26F term is calculated at the end of the program (the "F" is for final). The total development time and ΔTOA26F coefficients are nearly equal. For a given engine, the increase in predicted cost due to lengthening the development program after MQT will be almost completely offset by the associated changes in the ΔTOA term. Thus, in the absence of any change in the technology level of a given engine, the total development cost reflects improved reliability, which is related to the quantity of engines produced. However, when performance is upgraded, the equation captures both the costs of reliability and performance improvement.*

This TOA cost-estimating model sheds light on important variables, but the standard model still provides as good a statistical estimate. Both should be used and compared when estimating total development costs for new engine programs.

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*This reflects the rationale behind spending money in product improvement programs. Post-MQT development is funded primarily on the basis of the quantity of engines already in the field and in production, and on the upgrading of technology in the engine. In the development cost-to-MQT equation, development time is more important and enters into the relationship more strongly than ΔTOA.

It should be noted that the definition of total development cost as the cumulative cost associated with cumulative values of time, quantity, and technology might violate a basic assumption of regression theory -- that errors of the individual observations are serially independent. In the present case, the Durbin-Watson test indicates that serial correlation is not a problem;* however, it is a problem for cumulative production quantity cost, which is discussed later in this section.

PRODUCTION COSTS

As stated previously, engine production costs used in this study reflect the selling price to the government adjusted to 1973 dollars. This price includes all items listed in the representative example in Table 9. Because of the changing nature of costs in the aircraft turbine engine industry, there is a question whether an equation based on experience of the 1950s and 1960s will be able to predict costs for the 1970s. Because the intent is to use these cost equations to predict aircraft turbine engine experience in the 1970s and even 1980s, any anticipated changes that are radically different from the past, such as plant production capacity, production rate, differences in overhead rate, and changes in contract add-ons, must be reflected in any cost assessment. The left column in Table 9 gives a production cost breakdown typical of previous experience, and the right column projects how costs resulting from rising overhead and add-on factors might be broken down in the future. If these overhead and add-on factors are changing sporadically, such that the left column is representative of the past 20 to 30 years and the right column is 1975, then an explicit variable must be introduced into the equation to capture their effects in predicting new costs. On the other hand, if the right-hand column represents the continuation of an evolutionary process taking place over the past 30 years, then the equation already

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*See Refs. 11 and 12 for a discussion of this test. For the first equation in Table 8, d = 1.93, which indicates no serial correlation.

Table 9

TYPICAL AIRCRAFT ENGINE PRODUCTION ACCOUNTS

Cost Element	Approximate Percentage of Selling Price (Cumulative) Current Experience	A Future Possibility
Direct material		
Direct assembly labor		
Indirect manufacturing expense		
Shop cost	75	70
Tooling		
Technical publications		
Field service		
Manufacturing cost	79	75
General and administrative expense		
Total cost	85	83
Indirect component improvement		
Contributing engineering & IR&D (ILS in 1970s)		
Total cost plus add-ons	90	92
Profit		
Selling price	100	100

captures that trend. Cost predictions in this study are based on the assumption that such a trend is continuing. There is insufficient detailed data available to attempt to measure the effect of an assumption of a stepwise function in the present study. This study, therefore, assumes that these changes are evolving over a reasonably long time period, and that this trend is inherently captured in the existing models.

It would be preferable to deal in shop costs only, because they represent the actual cost of producing the product. The other cost elements can vary, obviously, for reasons other than changes in manufacturing activities. Because the purpose of the present study is to construct a model for estimating the selling price to the government, however, it must include all the costs associated with field service, component improvement, profit, general and administrative expenses, and so forth. Hence, the data used in this study are considered adequate, as they include all currently allowable costs (as outlined in Table 9).

Unit Production Cost

The unit cost of an engine is the additional cost (to the government in 1973 dollars) of one engine. For example, the cost of the 1000th engine is the difference between the cumulative cost of 1000 engines and the cumulative cost of 999 engines. Because of learning or progress effects, the unit cost decreases as the number of engines produced increases.

The data base for this portion of the study consists of the production contract costs and quantities for 18 turbojet and turbofan engines, giving a total of 88 data points.

These data were used to determine total-cost progress curves[*] for each of the 18 engines. Progress curves have the form

$$C = aQ^b ,$$

where C is some measure of cost, Q is cumulative quantity, and a and b are parameters that are fit to the data. The cost measure can be cumulative cost, cumulative average unit cost, or unit cost. The cumulative average unit cost is equal to the cumulative cost divided by the cumulative quantity, and thus the two are equivalent except that the b parameters differ by -1. For the present study, the first measure was used because the available data (annual total costs and quantities) were not sufficiently precise for calculation of the unit cost. The form used for the individual engine progress curves was

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[*] The term progress curve is used here because this study analyzes total costs, including overhead allocations, etc. The term learning curve is usually reserved for cases where only labor hours are involved. For total costs the effect of labor learning is mixed with other effects, such as changing allocation of indirect charges with production rate variations and other items.

$$\ell n\ \text{CUMCOST} = \ell n\ a + b\ \ell n\ \text{CUMQTY}.$$

The 1000th unit production cost and the cumulative average unit production cost progress curve slope for the 18 engines were used as dependent variables in a series of regression analyses.* The 1000th unit production cost equations are displayed in Table 10 and the cumulative average slope equations are in Table 11. The time-of-arrival equation and the standard equation for the 1000th unit production cost are compared graphically in Figs. 9a and 9b. The technology parameters equation and the standard equation for the progress curve slope are compared in Figs. 10a and 10b because in this case the technology parameters equation is significantly better. (Note that the 1000th unit and cumulative average slope are obtained. Proper adjustment must be made to obtain the cumulative cost.)

The 1000th unit cost (rather than the first unit cost or the progress curve parameter a) was chosen for two reasons. First, the cost of the 1000th unit has been used so frequently that it has become almost a standard. Second, the 1000th unit cost gives much better statistical results than the first unit cost or the progress curve parameter a. This is probably due to the fact that the 1000th unit cost lies nearer the middle of the actual data than the first unit cost. Furthermore, production progress will be stabilized by the time 1000 units have been produced, whereas there can be more variations at first unit.

The best 1000th unit cost equation contains thrust (size), Mach number (environment), TOA (time), and ΔTOA (deviation from the trend). Again, cost estimates will change with schedule changes due to variations in ΔTOA, as discussed further in Sec. IV. The equation that contains the

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*An earlier paper on learning curves by Baloff(13) discusses parameter prediction and variability of slope among various firms in a given industry. Those results and conclusions are not strictly comparable to the present study because they deal with labor hours and the learning curve whereas this study deals with total cost and a progress curve.

Table 10

AIRCRAFT TURBINE ENGINE 1000TH UNIT PRODUCTION COST

(18 Turbojet and Turbofan Engines/18 Data Points)
(BLS Index--1973 $ millions)

Time-of-Arrival Model

$R^2 = 0.951$ $$\ell n\ \text{PRUCOST} = -8.3433 + \underset{(<0.0001)}{0.70532\ \ell n\ \text{THRMAX}} + \underset{(0.0158)}{0.00674\ \text{TOA26}} + \underset{(0.0215)}{0.45710\ \ell n\ \text{MACH}} + \underset{(0.0327)}{0.01804\ \Delta\text{TOA26}}$$

SE = 0.215 (+24%, -19%)

F = 63.0

Technology Parameters Model

$R^2 = 0.953$ $$\ell n\ \text{PRUCOST} = -24.94 + \underset{(<0.0001)}{0.72338\ \ell n\ \text{THRMAX}} + \underset{(0.0085)}{0.38018\ \ell n\ \text{TOTPRS}} + \underset{(0.0641)}{1.7696\ \ell n\ \text{TEMP}} + \underset{(0.0699)}{0.63575\ \ell n\ \text{SFCMIL}}$$

SE = 0.212 (+24%, -19%)

F = 65.5

Standard Model

$R^2 = 0.905$ $$\ell n\ \text{PRUCOST} = -9.2122 + \underset{(<0.0001)}{0.85176\ \ell n\ \text{THRMAX}} + \underset{(0.1262)}{0.35944\ \ell n\ \text{MACH}}$$

SE = 0.279 (+32%, -24%)

F = 71.6

NOTES: Variables are listed in order of introduction using stepwise-least-squares procedure. Probability that coefficient is zero--from the t-test of significance--is shown in parentheses below each variable.

Table 11

CUMULATIVE AVERAGE PRODUCTION UNIT PROGRESS SLOPE

(18 Turbojet and Turbofan Engines/18 Data Points)
(BLS Index--1973 $ millions)

Time-of-Arrival Model With Manufacturer Dummy

$R^2 = 0.789$ $\quad \text{SLOPE} = 0.85735 + \underset{(0.0344)}{\text{MFRDUM}^a} - \underset{(0.0202)}{0.53243\ \text{KPRATE}} + \underset{(0.0411)}{33.743\ \text{MQTY}} - \underset{(0.0576)}{0.07170\ \text{MVOLUME}} + \underset{(0.1007)}{0.00106\ \text{TOA26}}$

$SE = 0.044$

$F = 9.0$

Technology Parameters Model With Manufacturer Dummy

$R^2 = 0.839$ $\quad \text{SLOPE} = 0.52913 + \underset{(0.0037)}{\text{MFRDUM}^a} - \underset{(0.0076)}{0.54334\ \text{KPRATE}} + \underset{(0.0251)}{32.993\ \text{MQTY}} - \underset{(0.0087)}{0.11854\ \text{MVOLUME}} + \underset{(0.0160)}{0.18489\ \text{KTEMP}}$

$SE = 0.039$

$F = 12.5$

Standard Model With Manufacturer Dummy

$R^2 = 0.734$ $\quad \text{SLOPE} = 0.92100 + \underset{(0.0282)}{\text{MFRDUM}^a} - \underset{(0.0088)}{0.63427\ \text{KPRATE}} + \underset{(0.0447)}{35.318\ \text{MQTY}} - \underset{(0.2265)}{0.03991\ \text{MVOLUME}}$

$SE = 0.048$

$F = 9.0$

Standard Model Without Manufacturer Dummy

$R^2 = 0.609$ $\quad \text{SLOPE} = 0.94552 - \underset{(0.0005)}{0.90984\ \text{KPRATE}} + \underset{(0.0027)}{56.702\ \text{MQTY}} - \underset{(0.2391)}{0.04505\ \text{MVOLUME}}$

$SE = 0.056$

$F = 7.3$

NOTES: Variables are listed in order of introduction using stepwise-least-squares procedure. Probability that coefficient is zero--from the t-test of significance--is shown in parentheses below each variable.

[a] Coefficients for MFRDUM are not shown because they do not apply to new engines. Significance levels are shown to indicate the importance of distinguishing between one particular manufacturer and the other companies historically. Also note that the dummy term enters the equation first in stepwise procedure.

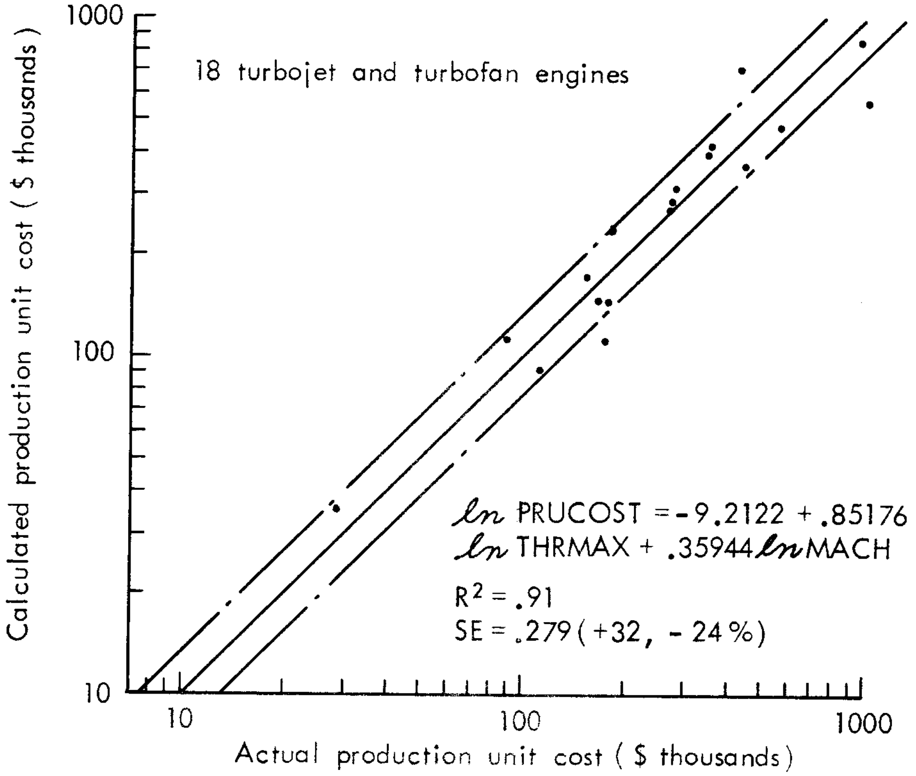

Fig. 9a — Production unit 1000 cost, standard equation
BLS index - 1973 $

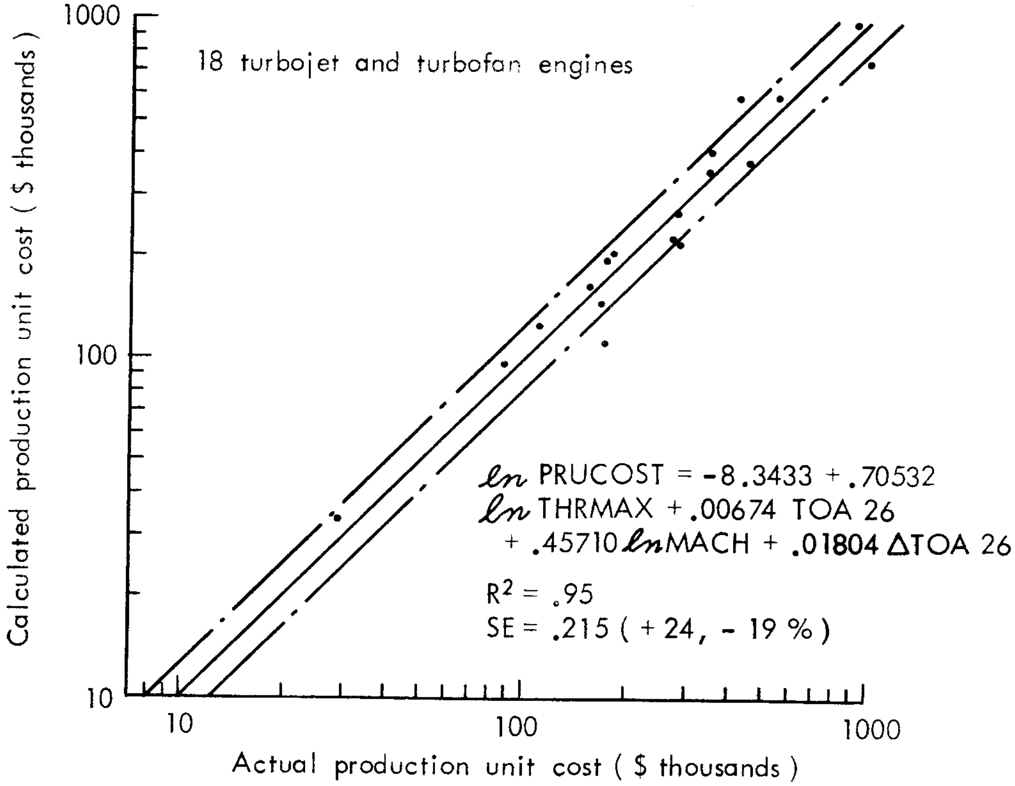

Fig. 9b — Production unit 1000 cost, time-of-arrival equation
BLS index - 1973 $

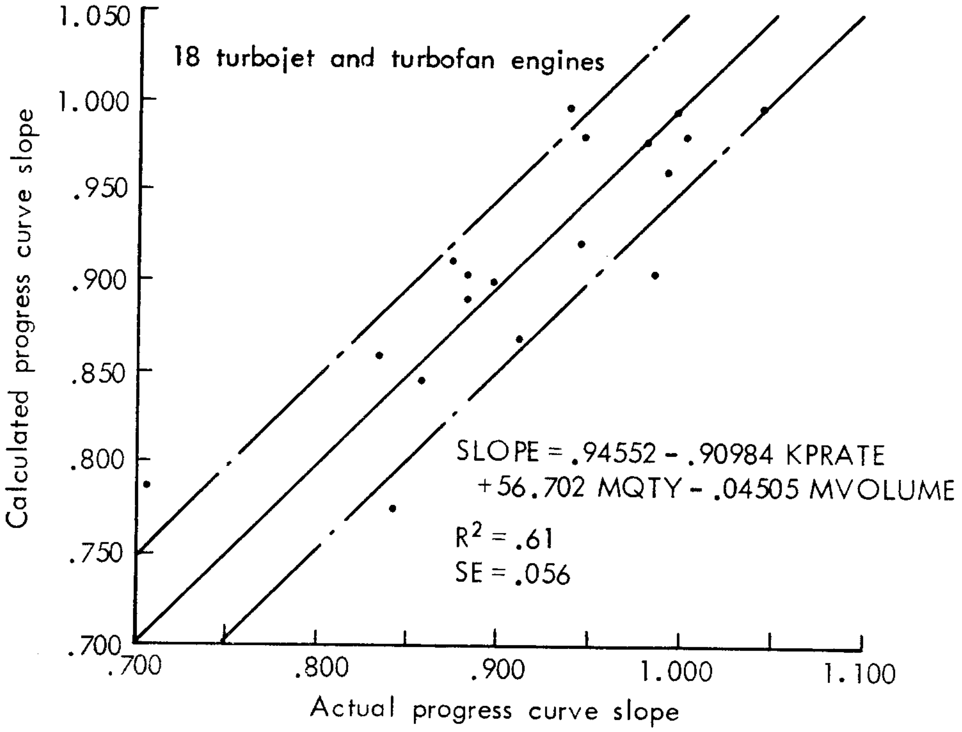

Fig. 10a — Cumulative average progress curve slope, standard equation

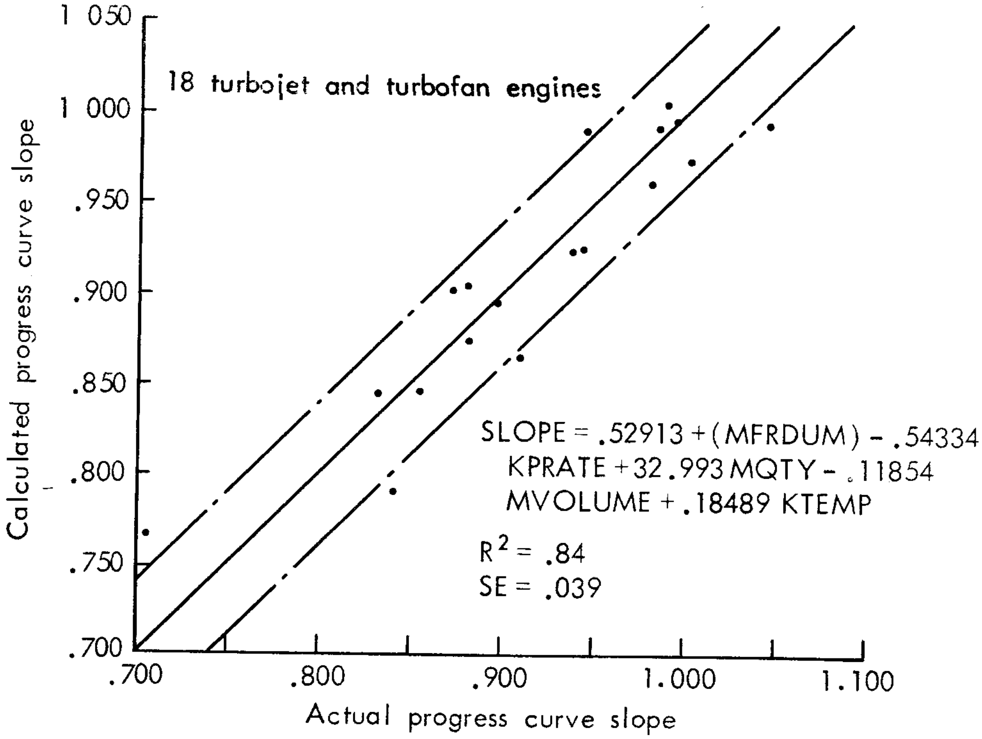

Fig. 10b — Cumulative average progress curve slope, technology equation technology parameters is not statistically different from the TOA equation.*

Both of the technology equations are statistically superior to the standard equation. The technology parameters used in these regressions correspond to the performance capabilities of the 1000th engine, which may be different from the original MQT engine. In the present case, nine of the engines had the same performance at the 1000th unit as at MQT. For the purpose of forecasting costs of a new engine, it is best to use the MQT performance.

The best "standard" variables for the slope regressions are the average production rate, the total quantity to be produced, and the volume of the engine as given by its length and maximum projected diameter. The relationship is very significantly improved by including a dummy term for Pratt & Whitney engines.** The slope for Pratt & Whitney engines has been strongly affected by accounting practices which have recently been changed to conform more closely to the rest of the industry.*** (There are additional potential sources for

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*The stepwise procedure yields an equation containing weight instead of SFC. That equation is inferior to the equation presented here wherein weight was removed from the list of candidate variables. This was the only instance where stepwise regression results were modified to obtain a "better" model. (Note that the coefficient of SFC has the "wrong" sign.)

**The dummy was one for Pratt & Whitney and zero for the rest of the industry. The dummy was also tried in the unit 1000 regressions, but it was not significant. The dummy was significant in regressions obtained for the cumulative average cost at 1000 units. It was thus concluded that the progress curves for Pratt & Whitney and other manufacturers cross over in the neighborhood of 1000 units.

***Reference 14. In Congressional testimony before the Senate Armed Services, Mr. David J. Hines of Pratt & Whitney, a division of United Aircraft Corporation, stated, "The impact of the method followed prior to 1971, Mr. Chairman, was really to distribute some of the starting costs on the new engines over the older engines. This had the effect of leveling the price of an engine in its lifetime so that in the beginning of its lifetime and in the middle and toward the end of it, it didn't have much deviation in price.

The change in the system was to a method of direct cost identity by part number of material and labor, with the ultimate intent that the material and labor costs would be specifically identified with a given product and would not be contaminated by any other product's different slopes being obtained by Pratt & Whitney.) Regression equations that include the dummy term are recommended for estimating new engines' slopes. However, because the dummy has zero value for all non-Pratt & Whitney engines and also for all new Pratt & Whitney engines (due to the change in accounting practices), the coefficients for the dummies have been omitted from Table 11. Inclusion of a technology related variable (TOA26 or TEMP) improves the model, with the best results for TEMP.*

Logical explanations of the signs of the coefficients in the slope regressions can be constructed; however, they should be treated as hypotheses until confirmed by some other methods. Positive signs indicate a lower rate of progress and negative signs a higher rate. The coefficients of MVOLUME indicate that as engine physical size increases, all other things being equal, there is a higher rate of progress. Larger engines have a larger labor component than smaller ones, thus there is more opportunity for learning.

The MQTY coefficients indicate that as the total quantity produced increases, there is a lower rate of progress. This is consistent with the argument that the progress curve levels off after a start up with a higher rate of learning.**

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material and labor costs. The impact of this was to show the cost progression curve from the beginning to the end of the life of an engine, the so-called learning curve depiction, if you will."

*It is important to note that the consequences of a small error in estimating a progress or learning curve slope are quite large when large quantities are involved. For example, if the true slope is 1.04 times the estimated slope, then the total cost factor is 1.30 at a quantity of 100, 1.42 at 500, 1.48 at 1000, and 1.62 at 5000.

**This argument is discussed by Baloff(13) in the context of a labor hours learning curve. Baloff indicates a horizontal, steady-state portion, which may hold for a labor learning curve but is not as likely to be true for a total cost progress curve because of the additional factors at work. In addition, there may be model changes that displace the progress curve upward at the same time the rotation due to lower learning rate occurs. This would result in a sawtooth effect. Additional discussion of the learning curve for aircraft turbine engines can be found in Ref. 5.

The KPRATE coefficients indicate that as the production rate increases, there is a higher rate of progress. Several factors can contribute to this: labor learning should be higher with higher production rates; there may be more favorable prices on materials because of larger quantity purchases; and there may be a more favorable allocation of indirect costs.

The TOA26 and TEMP coefficients indicate that more technologically advanced engines exhibit a lower rate of progress. This may be explained by the fact that higher technology has generally involved more advanced designs and/or materials, which require more difficult manufacturing processes. As experience with new processes is gained, there is an improvement in learning. At the same time newer materials and processes are used to advance technology further. Thus the forward area of technology is always characterized by advanced materials and processes, resulting in lower progress rates at the frontier.*

To predict costs and slopes for new engines, the first equation in Table 10 and the second equation in Table 11 are recommended, as discussed further in Sec. IV.

Cumulative Production Cost

The cumulative production cost at a given quantity, say 5000, is the total production cost of manufacturing the first 5000 engines. This cost does not include any development monies, either pre- or post-MQT. The data base for cumulative production consists of the same set of 88 observations used in the unit production and slope analysis. The individual observations were constructed from contract data by summing these dollars after deflating by the appropriate price index. The corresponding quantities of engines were also accumulated.

A significant problem in using a series of observations for each engine that represent the cumulative costs and quantities at the end of successive years is that one of the basic assumptions of regression

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*New high-technology engine programs may also exhibit low production rates, at least initially. This also will result in a lower rate of learning because of the effect of the KPRATE term.

theory is violated -- the assumption that the errors in the successive data points are independent of each other. The failure of this condition is called serial (or auto) correlation, and its effect is to invalidate the standard error, student t, and F statistics relating to confidence measures for the equation and its coefficients.[*] The approach of dealing with unit production cost and progress curve slope separately as described in the previous section circumvents this difficulty.[**]

Another problem inherent in this procedure is that individual engines with more observations than others will have a stronger influence on the outcome. The least-squares procedure minimizes the sum of the squares of all residuals counted equally.

The equations for cumulative production cost were obtained by regressing the cumulative production cost against a variety of independent variables. The results are shown in Table 12. The first equation shows the best results using the technology variables TOA26 and ΔTOA26F. The technology parameters equation is not quite as good, and the standard equation is slightly worse. The fourth equation was determined without correcting for the slope difference for Pratt & Whitney engines and it is the worst of the four. Figures 11a and 11b present a comparison of data for the TOA and "standard" models.

The Pratt & Whitney slope difference was accounted for by adding a dummy to the ℓn QTY term for Pratt & Whitney engines. Thus the ℓn QTY coefficients for the first three equations in Table 12 apply to all non-Pratt & Whitney engines and all post-1971 Pratt & Whitney engines.

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[*] The resulting coefficients are still maximum likelihood estimates, but the variances are understated. That is, the standard error, t, and F statistics are not as good as indicated.

[**] The present case is further complicated by the pooling of several sets of serially correlated data. The cumulative data for each engine are serially correlated and there are 18 engines. Thus, the problem is one of serial correlation among the subsets of the data. This is referred to as pooled time series and cross-section data in the econometric literature. One method of analyzing such data sets is to include a dummy variable for each engine (cross section). However, this approach is infeasible from a planner's point of view, for he must forecast a dummy for each new engine.

Table 12

AIRCRAFT TURBINE ENGINE CUMULATIVE PRODUCTION COST

(18 Turbojet and Turbofan Engines/88 Data Points)
(BLS Index--1973 $ millions)

Time-of-Arrival Model With Manufacturer Dummy

$R^2 = 0.978$

$$\ln \text{PRQCOST} = -7.9854 + \underset{(<0.0001)}{0.92753 \ln \text{QTY}} + \underset{(<0.0001)}{0.77831 \ln \text{THRMAX}} + \underset{(<0.0001)}{0.01542\ \Delta\text{TOA26F}} + \underset{(<0.0001)}{\text{MFRDUM}} + \underset{(<0.0001)}{0.37779 \ln \text{MACH}} + \underset{(0.0002)}{0.00447\ \text{TOA26F}}$$

SE = 0.198 (+22%, -18%)

F = 591.2

Slope = 0.951

d = 1.08

Technology Parameters Model With Manufacturer Dummy

$R^2 = 0.968$

$$\ln \text{PRQCOST} = -25.130 + \underset{(<0.0001)}{0.86943 \ln \text{QTY}} + \underset{(<0.0001)}{0.86883 \ln \text{THRMAX}} + \underset{(0.0015)}{\text{MFRDUM}} + \underset{(0.0027)}{0.30170 \ln \text{MACH}} + \underset{(0.0005)}{2.2107 \ln \text{TEMP}} + \underset{(0.0310)}{0.47005 \ln \text{SFCMIL}}$$

SE = 0.235 (+27%, -21%)

F = 414.9

Slope = 0.913

d = 0.69

Standard Model With Manufacturer Dummy

$R^2 = 0.963$

$$\ln \text{PRQCOST} = -8.5461 + \underset{(<0.0001)}{0.87079 \ln \text{QTY}} + \underset{(<0.0001)}{0.90865 \ln \text{THRMAX}} + \underset{(<0.0001)}{\text{MFRDUM}} + \underset{(0.0140)}{0.24242 \ln \text{MACH}}$$

SE = 0.251 (+29%, -22%)

F = 541.5

Slope = 0.914

d = 0.79

Standard Model Without Manufacturer Dummy

$R^2 = 0.948$

$$\ln \text{PRQCOST} = -7.9417 + \underset{(<0.0001)}{0.84172 \ln \text{QTY}} + \underset{(<0.0001)}{0.84755 \ln \text{THRMAX}} + \underset{(0.0630)}{0.21462 \ln \text{MACH}}$$

SE = 0.295 (+34%, -26%)

F = 513.4

Slope = 0.896

d = 0.64

NOTES: Variables are listed in order of introduction using stepwise-least-squares procedure. Probability that coefficient is zero--from the t-test of significance--is shown in parentheses below each variable. All statistics except R^2 are uncertain due to serial correlation; they are shown only to indicate relative comparisons between models. Coefficients for MFRDUM are not shown because they do not apply to new engines. Significance levels are shown to indicate the importance of distinguishing between one particular manufacturer and the other companies historically. Also note that the dummy term enters the equation first in stepwise procedure.

d = Durbin-Watson statistic.

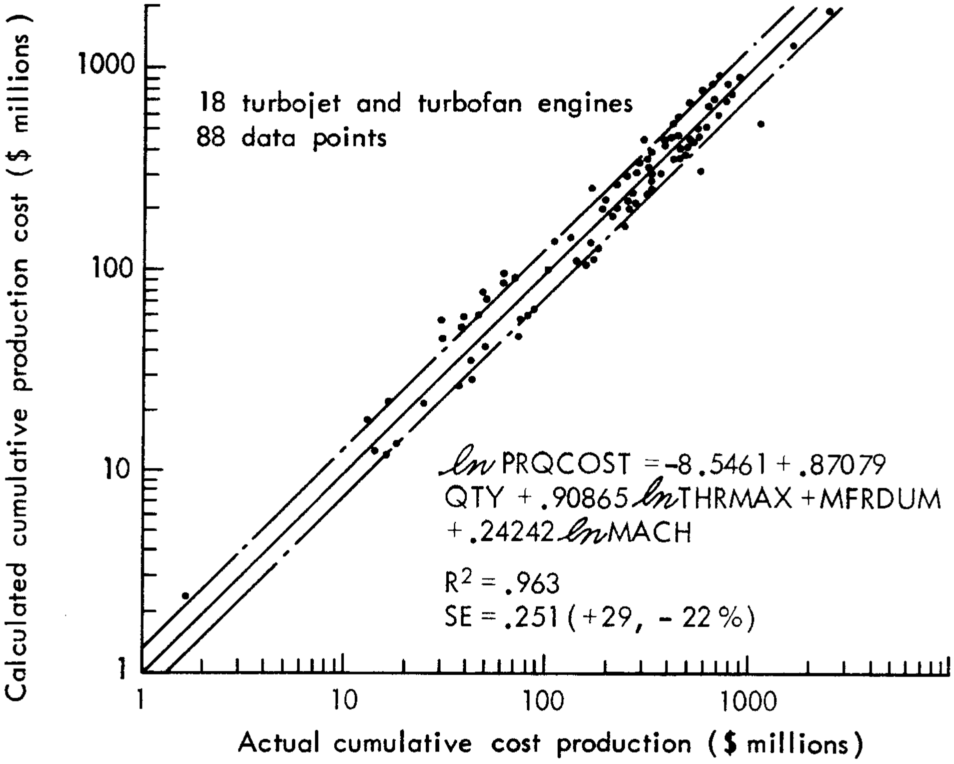

Fig. 11a — Cumulative production cost, standard model
BLS index - 1973 $

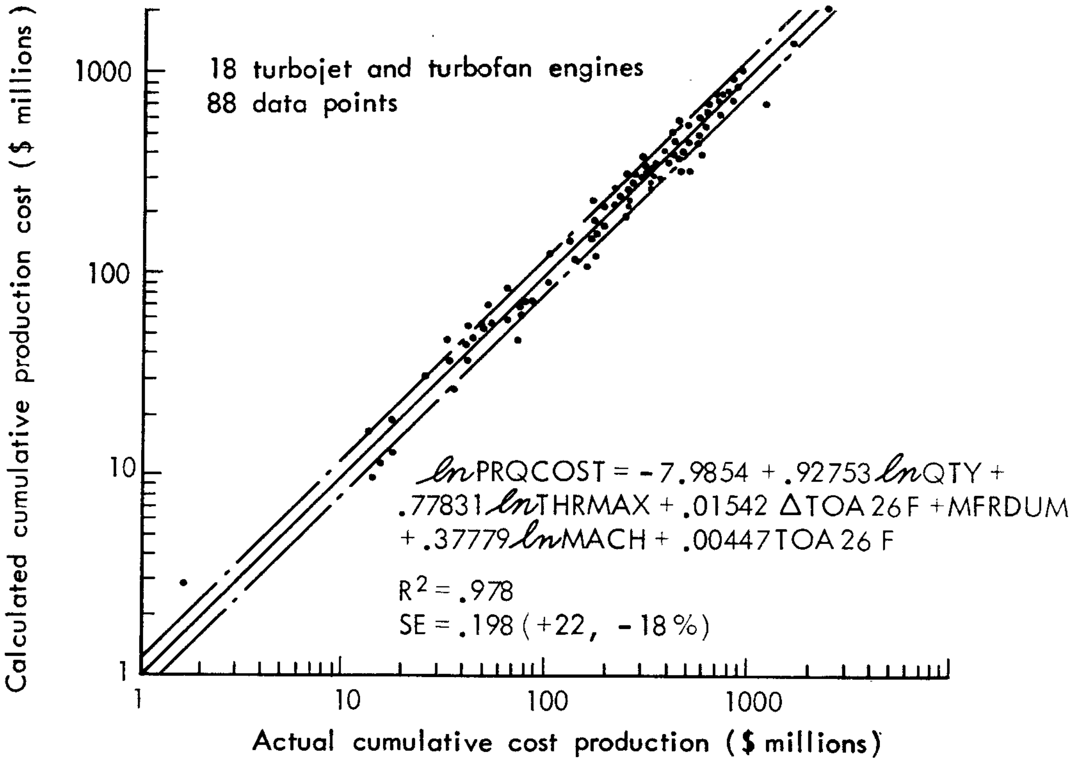

Fig. 11b — Cumulative production cost, time-of-arrival model
BLS index - 1973 $

The dummy adjusts the ℓn QTY coefficients for pre-1971 Pratt & Whitney engines.*

Predicted cumulative production cost can be obtained by using the equations in Table 12 or the unit 1000 production cost and slope equations in Tables 10 and 11. As an overall measure of comparability between these two approaches, the TOA equation for unit 1000 cost and the technology parameters equation for slope were used to predict the costs of the 88 input observations for the cumulative cost regressions. The calculated coefficient of determination is 0.968, which compares favorably with the R^2 for the TOA cumulative production cost equation (0.978).

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*The dummy values are excluded from the table as they are not needed for application of the equations to future programs.

IV. APPLICATIONS AND PREDICTIONS: COST AND RISK ANALYSIS

Over the 30-year time period covered by the aircraft turbine engine technology and cost data base, there has been a significant increase in the costs of developing and producing new engines. It is useful, when predicting the costs of new engines, to keep in mind the underlying factors. This section is divided into two parts. The first part analyzes the factors leading to higher costs as embodied in the cost regressions (including the time-of-arrival terms). The second part is structured as a working guide for MQT date and cost prediction, and risk assessment for new engine programs.

EVOLUTION OF COSTS OVER THE PAST TWO DECADES

Analyses were conducted to obtain breakdowns of the contributing factors to the evolution of aircraft turbine engine development and production costs during the past two decades. For this type of analysis it is useful to have equations that contain TOA or development time terms so as to capture technological effects not present in the other terms. The development-to-MQT cost equations in Table 7 all contain development time. The first equation will be used because of its statistical superiority. The first equation for the 1000th unit production cost in Table 10 contains TOA, so it will be used.

Analysis of cost growth requires specific values for the variables over the time period of interest -- 1953 to 1973 in the present case. The average engine in 1953 had a development time of approximately 17 quarters, a maximum thrust of 8500 lb, a TOA of 45, a ΔTOA of zero (by definition the average engine is on the trend), and a Mach number of 1.4. These values were obtained by determining single-variable regressions for development time, thrust, and Mach number as functions of time. These and similarly obtained values for the typical engine in 1973 are shown in Table 13.

The development and unit production cost equations are used to estimate the costs of the typical 1973 engine in 1973. This is shown in the first line of figures in Table 14. This cost is deflated to

Table 13

TYPICAL ENGINE CHARACTERISTICS (ON TREND)

Year	DEVTIME (qtr)	THRMAX (lb)	TOA26	ΔTOA26	Mach No.
1953	17	8,500	45	0.0	1.4
1973	21	24,500	125	0.0	1.8

Table 14

SUMMARY OF ENGINE COST GROWTH ELEMENTS

Item	Development		Production	
	Cost, $M	Factor	Cost, $K	Factor
1973 engine size 1973 development 1973 dollars	273.2	6.7	901.4	10.2
1973 engine size 1973 development 1953 dollars	108.7	2.7	358.7	4.1
1953 engine size 1973 development 1953 dollars	57.7	1.4	151.5	1.7
1953 engine size 1953 development 1953 dollars	41.0	1.0	88.4	1.0

1953 dollars to remove the effect of inflation (the second line of figures in Table 14). Next, the cost of an engine having the 1953 values for thrust and Mach number, but developed in 1973 (with that typical development time), is estimated and deflated to 1953 dollars (the third line in Table 14). The difference between this and the previous estimate represents cost differences due to changes in the engine's thrust and Mach number, with the major portion due to the large increase in thrust. Then the 1953 cost of the average 1953 engine developed in 1953 is estimated (the fourth line in Table 14). The difference between this and the preceding estimate represents cost growth due to factors captured by development time and TOA26. As indicated by the table, today's engine development program costs 1.4 times more due to technological improvements (TOA and development time), a factor of 1.9 due to the larger average thrust and speed capability of today's engine, and a final factor of 2.5 due to inflation, for an overall increase of 6.7. The increased development time reflects the need for more elaborate testing of today's more complex engine. The higher thrust and Mach number values do not capture all of this complexity. The unit production costs show similar results with an overall cost factor increase of 10. In both cases inflation is a major contributor.*

PREDICTING MQT DATE AND COSTS, AND ASSESSING RISKS FOR NEW ENGINES

This subsection summarizes the points made throughout this report on the underlying assumptions and restrictions on the technology and cost equations. The equations apply to military development and production styles and pricing practices similar to those of the 1950s and 1960s. They do not apply to engines that can be expected to fall

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*The cost index used in these calculations does not correct for productivity changes. The importance of TOA and DEVTIME is increased and the importance of inflation is decreased if some measure of productivity could be accounted for. This was the case when development and production cost growth was analyzed using a national wholesale price index corrected for overall manufacturing productivity. Unfortunately, productivity data are not available specifically for aircraft turbine engine manufacture.

outside the historical distribution because of special features, such as "quiet technology" or commercial pricing. The results of the equations can be adjusted to technologies other than military but should not be used directly. It is also important to remember that the equations cannot be used for marginal analysis of existing engines, whether they fit the assumptions or not, and they cannot be used to make fine distinctions among various engines. They are intended for planning estimates for new engine programs.

There are several cautionary points regarding the input data. First, the data must reflect the maximum capability of the engine, not the aircraft in which it is to be installed. A specific example is the J52, which first appeared as a supersonic missile engine. Because it subsequently has been used in subsonic aircraft, a casual observer might use a Mach number of 1.0. However, the engines for the subsonic aircraft were not downrated. The engine is capable of Mach numbers substantially in excess of Mach 1.0, and the user is paying for that capability whether or not he actually uses it.*

Second, the engine performance data must be consistent in terms of the thermodynamic cycle.

Third, data at growth points require some sort of forecast, a fact that has not been analyzed in the present study. The problem is basically to determine an improved technology level after some quantity of engines has been produced. If only a few engines are produced, technology may not be uprated. If many engines are produced, it is highly unlikely that technology will not be uprated. Some estimate of "few," "many," and "how much improvement" should be made. Additionally, the length of time involved and the implied production rate should be considered.

Fourth, there are the limitations on the values used to ensure consistency with the historical data, as previously discussed. No turbojet and turbofan Mach numbers are less than one. The minimum

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*This is not a criticism of using the J52 in subsonic applications. The cost of developing a new engine to meet the requirement would have more than offset the additional cost of buying the J52.

development time for a new engine is at least 20 quarters for a state-of-the-art design and about 24 quarters for an advanced design by today's standards.

Lastly, predictions can be made with greater confidence when the parameter values for a new engine fall within the range of the sample data. This may occur frequently for quantities of engines and for development and production program lengths. However, it will frequently not occur for at least one technology parameter. As a guide to the estimator, Table 15 shows the ranges of the input data for all equations presented in this report.

The remaining subsections will carry through the calculations for qualification date and costs of a hypothetical military turbofan engine with characteristics as shown in Table 16. (Note that the engine is beyond the range of the input data for turbine inlet temperature and pressure and that the estimated development time is short and production rate high relative to the historical sample.)

All cost equations from Sec. III will be used and the results compared. The equations will be numbered according to the table numbers in Sec. II and III. Thus, Eq. (5-1) is the first equation in Table 5.

Predicting Model Qualification Test Date

Either Eq. (5-1) or Eq. (5-2) is used for predicting the expected MQT or FAA Certification date of a new engine. If the new engine is a military model, Eq. (5-1) is preferred. If it is a commercial model, Eq. (5-2) should be used (with MCDUM = 1). All cost equations in this report are based on TOA26 (Eq. 5-1), for military engines only.* The

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*Commercial development costs will differ considerably from military development costs. Preliminary analyses using development costs for two commercial engines indicate that the difference can be accounted for by the dummy in TOA37 as above and obtaining a cost model for military and commercial engines, or by obtaining a new TOA model excluding the dummy and then including it in the cost model. In either case, the military and commercial data must be distinguished from each other.

Commercial production selling price is an entirely different matter because the market environment will be a very important consideration.

Table 15

MEANS AND RANGES OF INPUTS TO FINAL REGRESSION ANALYSES

Parameter	TOA (26 observations)			DEVMQTCOST (14 observations)			TOTDEVCOST (25 observations)			Unit 1000 Cost (18 observations)			Progress Slope (18 observations)			Cumulative Cost (88 observations)		
	Low	Mean	High	Low	Mean	High	Low	Mean	High	Low	Mean	High	Low	Mean	High	Low	Mean	High
Cumulative cost[a]																11.4	423.0	2055.1
Development cost[a]				42.5	163.2	515.0	42.5	293.3	667.3									
DEVTIME (qtr)				9.0	18.8	29.0												
KPRATE (units/qtr/10^3)													0.029	0.152	0.566			
KTEMP (°R/10^3)													1.960	2.159	∿2.800			
MACH				1.0	1.7	∿3.0	1.0	1.8	2.4	1.0	1.5	2.2				1.0	1.5	2.2
MQTQTR (qtr)	11.0	54.9	120.0															
MQTY (units/10^6)													0.0004	0.0021	0.0078			
MVOLUME (cu in./10^6)													0.0167	0.2757	1.744			
PRODSPAN (qtr)													7.0	16.2	30.0			
Pressure ratio	3.5	9.1	26.0															
QMAX (lb/in.^{2})	450	1192	3020															
QTY							0	2132	16000							103	1858	7752
SFCMIL (lb/hr/lb thrust)	0.32	0.93	1.25				0.52	0.82	1.04	0.32	0.89	1.22				0.32	0.89	1.22
SLOPE													0.704	0.917	1.046			
TEMP (°R)	1825	2139	∿2800				2060	2191	2610	1960	2159	∿2800				1960	2155	∿2800
THRMAX (lb)	920	10457	∿41000	3000	15337	∿41000	3000	14803	25100	1025	11547	∿41000				920	11447	∿41000
TOA26 (qtr)										21.3	59.5	113.1	21.3	59.5	113.1	21.3	63.1	113.1
ΔTOA26 (qtr)				-11.8	0.4	11.9	-37.4	-7.9	12.0	-12.1	1.1	12.3				-45.1	-11.2	9.4
TOTDEVTIME (qtr)							9.0	35.0	77.0									
TOTPRS (lb/ft^2)	1575	11611	51340							3400	13929	51340						
Unit cost[a]										0.0245	0.2825	∿0.850						
Weight (lb)	333	2789	∿7000	460	3555	∿7000	460	3396	5950									

[a] All costs are adjusted to 1973 dollars and expressed in millions.

Table 16

EXAMPLE ENGINE CHARACTERISTICS

Symbol	Quantity	Value
MQTQTR	Planned Model Qualification Test date (qtr)	132
TEMP	Turbine inlet temperature (°R)	3100
THRMAX	Maximum thrust, SLS (lb)	30000
WGT	Weight (lb)	3500
SFCMIL	SFC, military power, SLS (lb/hr/lb thrust)	0.65
P/P	Pressure ratio	25.0
QMAX	Maximum dynamic pressure (lb/ft^2)	3000
TOTPRS	Pressure (P/P × QMAX) (lb/ft^2)	75000
MACH	Mach number	2.4
DIA	Maximum diameter (in.)	60
LNG	Length (in.)	250
VOL	Volume (cu in.)	706858
DEVTIME	Planned development program length (qtr)	16
QTY	Quantity to be produced	2000
PRODSPAN	Length of production run (qtr)	8

predicted qualification date for the hypothetical engine in Table 16 is thus:

$$\text{TOA26} = 140.2 \quad \text{qtrs}$$

Predicting Development Costs and Assessing Risk

Equations (7-1) through (7-3) may be used to predict the cost of developing a new engine up to and including its Model Qualification Test. Comparing the statistical qualities of the three equations indicate that Eq. (7-1) is the best. The other two equations can be used with less confidence.

Before substituting the hypothetical characteristics into these equations, observe that ΔTOA26 is 8.2 quarters, which is larger than the standard error of the TOA26 equation (6.9 quarters). This indicates that the hypothetical engine is significantly advanced for the time at which it is being sought.[*] Also, note that the planned development program in this example is 16 quarters, which is short for an advanced program according to the rule of thumb that state-of-the-art (on or below the trend) engines should have a minimum development time of at least 20 quarters and advanced engines should have a minimum development time of about 24 quarters. When an engine is predicted to be significantly advanced there is a good chance that its schedule will slip and costs will increase.

Using the values in Table 16, the predicted costs of development to MQT are:

"on time" DEVMQTCOST (7-1) = \$309.4 million
"on time" DEVMQTCOST (7-2) = \$311.6 million
"on time" DEVMQTCOST (7-3) = \$237.9 million

The predicted costs from Eqs. (7-1) and (7-2) are not significantly different. The cost predicted by Eq. (7-3) is outside the standard error of Eq. (7-1). The value obtained using Eq. (7-1) is preferred because the statistics are better.

Assessing Risks in Predicted Development Cost

Next, determine the predicted cost for the more likely outcome that the engine will pass its qualification test on the trend line. Both the development time and the planned qualification date must be changed.

"on trend" DEVTIME = "on time" DEVTIME + "on time" ΔTOA26
= 16 + 8.2 = 24.2

By definition

"on trend" ΔTOA26 = 0

The predicted costs now are

"on trend" DEVMQTCOST (7-1) = \$445.2 million
"on trend" DEVMQTCOST (7-2) = \$561.7 million
"on trend" DEVMQTCOST (7-3) = \$438.7 million

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[*] This is an approximate criterion for degree of technological advance. Prediction intervals for the new engine would be more appropriate but also more complicated.

The cost predicted by Eq. (7-2) is outside the standard error of Eq. (7-1). It should be kept in mind that both Eqs. (7-2) and (7-3) have only one term sensitive to schedule slips (DEVTIME), whereas Eq. (7-1) has two which trade off. As the schedule slips (all other things remaining constant), the engine becomes less advanced relative to the average trend at its qualification date. This is another reason for preferring Eq. (7-1) over the other two.

The "on trend" cost is the much more likely outcome. The analysis reveals that the planners are asking for an engine two years ahead of its time, with a good chance of a $136 million overrun.* *It is recommended that this type of analysis be carried out whenever* ΔTOA26 *for a new engine exceeds the standard error of the* TOA26 *equation (6.9 quarters).*

Equations (8-1) through (8-3) may be used to predict the total development cost of a new engine, which includes development to MQT and subsequent product improvement.** The statistics in Table 8 indicate so little statistical difference between these equations that all three should be used for estimating purposes and the results compared.

Because the TOA equation is sensitive to schedule slips, "on time" and "on trend" calculations can be made. If the engine qualifies "on time" as planned, the development time is 16 quarters and the total development time is

$$\text{TOTDEVTIME} = \text{DEVTIME} + \text{PRODSPAN} = 16 + 8 = 24 \text{ quarters}$$

and MQTQTR "on time" is 132, so

$$\Delta\text{TOA26F} = \text{TOA26} - \text{MQTQTR} - \text{PRODSPAN} = 140.2 - 132 - 8 = 0.2$$

The predicted total development costs are

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*A crude estimate of the expected value of the "game" being played by the planners is to assign a probability of 1/6 to the engine qualifying "on time" when it is considered an advanced engine (outside the standard error) and a probability of 5/6 to its qualifying "on trend" when it is not an advanced engine. The expected value is then $309.4/6 + 5 x $445.2/6 million = $422.6 million, or a $113 million expected overrun.

**Note that by inserting a quantity of one, and DEVTIME to MQT, these equations yield an additional estimate of development cost to MQT.

"on time" TOTDEVCOST (8-1) = \$666.8 million
TOTDEVCOST (8-2) = \$781.7 million
TOTDEVCOST (8-3) = \$776.4 million

By inserting the time for development to MQT, the ΔTOA26 at MQT, and a quantity of one, these equations give the cost of development to MQT.

"on time" DEVMQTCOST (8-1) = \$411.5 million
DEVMQTCOST (8-2) = \$455.0 million
DEVMQTCOST (8-3) = \$418.0 million

The inputs for "on trend" qualification are

"on trend" DEVTIME = 24.2
"on trend" TOTDEVTIME = 32.2
"on trend" MQTQTR = 140.2
"on trend" ΔTOA26 = 0
"on trend" ΔTOA26F = -8

The predicted costs are

"on trend" TOTDEVCOST (8-1) = \$673.2 million
"on trend" DEVMQTCOST (8-1) = \$415.4 million

Equations (8-2) and (8-3) do not contain any variables that change with schedule slips, so they cannot distinguish between "on time" and "on trend" qualifications.

There are several observations regarding these estimates. First, for the hypothetical engine considered here, Eq. (8-1) produces lower estimates than Eq. (8-2) or Eq. (8-3). However, the total development cost estimated by Eq. (8-2) is just outside the standard error of Eq. (8-1), and the development to MQT cost estimated by Eq. (8-2) is within the standard error of Eq. (8-1).* Hence, the differences are not very significant.

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*The standard error of Eq. (8-1) is +23%, -19%. The total development cost of Eq. (8-2) is 27% greater than that of Eq. (8-1), and the development-to-MQT cost of Eq. (8-2) is only 11% greater than that of Eq. (8-1).

It is not surprising that Eqs. (8-2) and (8-3) give higher estimates than Eq. (8-1) considering how advanced the hypothetical engine is relative to the inputs to the total development cost regression equations (see columns 7-9, Table 15). The hypothetical engine's SFC is near the lower limit of the range of inputs, its Mach number is at the upper limit, and its maximum thrust and turbine inlet temperature are considerably outside the range of inputs for this particular model.

The "on time" and "on trend" costs of development to MQT estimated by Eqs. (8-1) through (8-3) are greater than the "on time" costs and less than the "on trend" costs given by Eqs. (7-1) through (7-3). The "on time" and "on trend" costs predicted by Eq. (8-1) differ by a very small amount because the coefficients of TOTDEVTIME and ΔTOA26F are nearly equal. This is not necessarily inconsistent with what might happen in the real world. If an advanced engine is qualified early ("on time") and at a low cost for its initial MQT, it is not unlikely that a substantial amount of post-MQT development expense will be required to correct performance and reliability deficiencies. However, if the engine passes its MQT on the trend, there will have been more time to discover and correct technical difficulties, and it should require less post-MQT development expense. (It is hoped that future data will provide better information concerning the cost of reliability versus the cost of performance.)

The small difference does not make sense in the case of development to MQT where estimated cost overrun according to Eq. (8-1) is only $6.4 million. According to Eq. (7-1), it is $135.8 million, a much more likely figure for a two-year schedule slip. This is one reason for not using the total development cost equations to predict development-to-MQT costs.

Predicting Production Costs and Assessing Risks

Production costs can be predicted in two ways: using the cumulative production cost equations, or using the 1000th unit production cost equations and a progress curve slope determined by the regression equation or some other method.

Equations (10-1) through (10-3) may be used to predict the unit production cost of the 1000th engine. The statistics in Table 10 show that the first two equations are superior to the third equation. Three out of four of the coefficients in the first equation are more significant than those in the second equation. The coefficient of the SFC term in the second equation appears intuitively to have the wrong sign. Hence, the first equation is preferred.

The first equation contains ΔTOA26, which is sensitive to schedule slips, so two calculations are desired in the case of significantly advanced engines. The predicted unit 1000 production costs are

"on time" PRUCOST (10-1) = \$1.523 million
"on time" PRUCOST (10-2) = \$2.090 million
"on time" PRUCOST (10-3) = \$0.890 million
"on trend" PRUCOST (10-1) = \$1.314 million[*]

The differences between the "on time" predictions are larger than would be expected considering the statistics for the equations.[**] Part of the explanation for the large difference between the cost predicted by Eq. (10-3) and that predicted by the other equations is that the hypothetical engine is very advanced relative to the turbine inlet temperature and pressure values in the sample, and consequently the contributions of these two terms in Eq. (10-2) are quite large. These two terms are not present in Eq. (10-3), and the maximum thrust and

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[*] Equations (10-2) and (10-3) give the same predictions for "on trend" and "on time." The PRUCOST (10-1) "on trend" equation yields a lower cost than the "on time" equation. This should be considered valid only before commitment to go ahead with the program. Once the program is started, any schedule slip which puts the production engine back on the trend will not result in a lower cost engine.

[**] The standard error of Eq. (10-1) is +24%, -19%.

Mach number values for the hypothetical engine are not outside the range of sample values; hence the cost from Eq. (10-3) is lower.

To obtain a total cost for 2000 engines, it is necessary to use a progress curve slope. A value can be obtained using Eqs. (11-1) through (11-3).* However, a note of caution on using these equations is in order. Equations (11-1) and (11-2) contain technology-related variables that can, for extreme values, drive the predicted slope greater than one. Equation (11-3) contains no technology-related variable; hence, it cannot capture the effect of higher technology on the slope. Predicted slopes greater than one can only be supported after the fact,** and consequently some other source should be used to obtain a slope. On the basis of the statistics in Table 11, Eq. (11-2) is preferred.

Using the inputs for the hypothetical engine, the predicted slopes are

SLOPE (11-1) = 0.890
SLOPE (11-2) = 0.949
SLOPE (11-3) = 0.805

The differences between these slopes are larger than would be expected on the basis of the statistics, and all differences are greater than the standard errors of the equations. As was noted with regard to the technology-parameters unit-1000 production-cost equation, the turbine inlet temperature for the hypothetical engine is beyond the range of

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*Equation (11-4) is not recommended because it does not adjust for the differences due to accounting practices at Pratt & Whitney.

**Factors that can result in an overall progress curve slope greater than one for a given production program include: (1) increasing direct costs resulting from inflation, (2) increasing direct costs due to loss of favorable price breaks when production rate declines; (3) increasing allocation of indirect costs when production rate declines or when other business is lost; (4) introduction of technologically advanced models; and (5) engineering changes required to improve reliability/maintainability, which can increase both direct and indirect costs. (Inflation and the introduction of technologically advanced models have been accounted for in this study.) These factors usually cannot be predicted; consequently, it is not possible to justify a progress curve slope greater than one before the fact.

the sample data and the predicted values may be misestimated. This is also true of Eq. (11-2) here. In the present example, slopes of 0.90 and 0.95 will be used for the remainder of the calculations.

The cumulative cost of Q engines is given by

$$\text{CUMCOST} = \text{U1} \cdot Q^{F(s)}$$

where U1 is the cost of the first unit and F(s) is a factor related to the slope of the progress curve.

$$F(s) = 1 + \ln s/0.693$$

The cost of the first unit is given by

$$\text{U1} = \frac{\text{U1000}}{1000^{F(s)} - 999^{F(s)}}$$

For a unit 1000 cost of \$1.523 million ("on time" cost from Eq. (10-1)) and a slope of 0.90

$$\text{"on time," U1 } (0.90) = \frac{1.523}{1000^{F(0.90)} - 999^{F(0.90)}}$$

$$= \$5.133 \text{ million}$$

and

$$\text{"on time," CUMCOST } (2000,\ 0.90) = 5.133 \times 2000^{F(0.90)}$$

$$= \$3232 \text{ million}$$

For a slope of 0.95

$$\text{"on time," U1 } (0.95) = \$2.742 \text{ million}$$

and

$$\text{"on time," CUMCOST } (2000,\ 0.95) = \$3125 \text{ million}$$

The "on trend" unit 1000 cost is \$1.314 million ("on trend" cost from Eq. (10-1)) resulting in the following:

"on trend," U1 (0.90) = \$4.429 million

"on trend," CUMCOST (2000, 0.90) = \$2789 million

"on trend," U1 (0.95) = \$2.366 million

"on trend," CUMCOST (2000, 0.95) = \$2696 million

The two "on time" and two "on trend" results are within 5 percent, while the overall variation between these four cumulative cost estimates is 20 percent at the most (and the standard errors of the cost models are on the order of 20 percent). It is interesting to note that the procurement cost of 2000 units "on trend" is about \$500 million lower than the procurement of 2000 units "on time." The unit one costs vary greatly, which is to be expected because they are outside of the range of the data, and because small differences in progress curve slopes lead to large differences when extrapolated.

The cumulative production cost can also be predicted using the cumulative cost Eqs. (12-1) through (12-3). The statistics in Table 12 indicate that Eq. (12-1) is superior to Eq. (12-2), which is marginally superior to Eq. (12-3). Equation (12-2) depends on only two technology parameters and may yield over or underestimates depending on the input values. Note that the coefficient of ℓn TEMP is very large. These equations are adjusted for pre-1971 Pratt & Whitney accounting practices which influence the learning curve slope.[*]

The "on time" and "on trend" cost predictions for Eqs. (12-1) through (12-3) are:

"on time" PRQCOST (12-1) = \$3130 million

"on trend" PRQCOST (12-1) = \$2758 million

$$\text{Not time oriented}\ \left\{\begin{array}{l}\text{PRQCOST (12-2) = \$3901 million}\\ \text{PRQCOST (12-3) = \$2105 million}\end{array}\right.$$

The "on time" and "on trend" results of Eq. (12-1) are not very different from those obtained above. The prediction from Eq. (12-2) is just outside the standard error of the prediction from Eq. (12-1), but the prediction

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[*] Again, the fourth equation is omitted because it does not reflect the difference between one set of accounting procedures at Pratt & Whitney and a different set of procedures for the rest of the industry prior to 1971. Hence, it should not be used.

from Eq. (12-3) is nearly two standard errors removed from the Eq. (12-1) prediction. The cumulative costs obtained using the unit 1000 cost and slopes of 0.90 and 0.95 are very close to the estimates from Eq. (12-1).

The technology variables estimate (12-2) is probably high relative to the TOA estimate (12-1) because of the influence of turbine inlet temperature. The turbine inlet temperature of the hypothetical engine is advanced relative to the inputs to the regressions (see columns 16-18, Table 15), and the coefficient in Eq. (12-2) is large. The standard equation estimate (12-3) is probably low because it contains no variables that are technologically advanced in the hypothetical engine relative to the regression inputs. The hypothetical engine's Mach number is advanced but the coefficient in Eq. (12-3) is small.

As might be expected, the "on time" program resulted in higher costs than the "on trend" program in terms of 1973 dollars. However, "on trend" costs reflect a later time period. If inflation and/or discounting are important factors, then the comparisons may change.

In summary, it is recommended that all models and techniques at the disposal of the planner be studied in order to obtain a more complete picture of the planning information for the program under analysis. Only with a wide spectrum of information available can judgment be exercised by the planner to make a "best" assessment of a new program.

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