Relation of accelerations in two inertial frames in Special Relativity theory

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ABSTRACT

In special relativity theory, we discover the relation of inertial frames' accelerations. In this theory, we can understand general state of the relation of inertial frames' accelerations

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I.Introduction

In special relativity theory, we discover the relation of inertial frames' accelerations.

At first, 4-dimension-Lorentz differential transformation is

$$d\vec{x} = d\vec{x}' + \gamma\vec{v}_0 dt' - (1-\gamma)\frac{\vec{v}_0 \cdot d\vec{x}}{v_0^{\ 2}}\vec{v}_0 \tag{1}$$

$$dt = \gamma(dt' + \frac{\vec{v}_0}{c^2}\cdot d\vec{x}) \ , \ \gamma = 1/\sqrt{1-\frac{v_0^{\ 2}}{c^2}} \tag{2}$$

If we rewrite Eq(1)-(2) in the other way,

$$\frac{d\vec{x}}{d\tau} = \frac{d\vec{x}'}{d\tau} + \gamma\vec{v}_0\frac{dt'}{d\tau} - (1-\gamma)\frac{\vec{v}_0}{v_0^{\ 2}}\cdot\frac{d\vec{x}'}{d\tau}\vec{v}_0 \tag{3}$$

$$dt = \gamma(dt' + \frac{\vec{v}_0}{c^2}\cdot d\vec{x}) = \gamma dt'(1+\frac{\vec{v}_0}{c^2}\cdot\vec{u}), \ \vec{V} = \frac{d\vec{x}}{dt}, \vec{u} = \frac{d\vec{x}'}{dt'} \tag{4}$$

2. The relation of inertial frames' accelerations

For calculating the acceleration,

$$\begin{aligned}\vec{a} = \frac{d}{dt}(\frac{d\vec{x}}{d\tau}) &= \frac{1}{\gamma(1+\frac{\vec{v}_0\cdot\vec{u}}{c^2})}[\frac{d}{dt'}(\frac{d\vec{x}'}{d\tau}) + \gamma\vec{v}_0\frac{d}{dt'}(\frac{dt'}{d\tau}) - (1-\gamma)\frac{\vec{v}_0}{v_0^{\ 2}}\cdot\frac{d}{dt'}(\frac{d\vec{x}'}{d\tau})\vec{v}_0] \\ &= \frac{1}{\gamma(1+\frac{\vec{v}_0\cdot\vec{u}}{c^2})}[\vec{a}' + \gamma\vec{v}_0\frac{d}{dt'}(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}) - (1-\gamma)\frac{\vec{v}_0\cdot\vec{a}'}{v_0^{\ 2}}\vec{v}_0]\end{aligned} \tag{5}$$

In this time, the constant acceleration a_0' is

$$a_0' = \frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \rightarrow u = \frac{a_0't'}{\sqrt{1+\frac{a_0'^2t'^2}{c^2}}} \tag{6}$$

Hence,

$$\frac{d}{dt'}(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}) = \frac{d}{dt'}(\sqrt{1+\frac{a_0'^2t'^2}{c^2}}) = \frac{a_0'}{c^2}\frac{a_0't'}{\sqrt{1+\frac{a_0'^2t'^2}{c^2}}} = \frac{a_0'}{c^2}u \tag{7}$$

$$\vec{v}_0\cdot\vec{u} = v_0 u\cos\theta' \quad , \qquad \vec{v}_0\cdot\vec{a}_0' = v_0 a_0'\cos\theta' \tag{8}$$

Therefore, Eq(5) is

$$\vec{a} = \frac{1}{\gamma(1+\frac{\vec{v}_0\cdot\vec{u}}{c^2})}[\vec{a}_0{}' + \gamma\vec{v}_0\frac{a_0{}'}{c^2}u - (1-\gamma)\frac{\vec{v}_0\cdot\vec{a}_0{}'}{v_0{}^2}\vec{v}_0]$$

$$= \frac{1}{\gamma(1+\frac{v_0 u\cos\theta'}{c^2})}[\vec{a}_0{}' + \gamma\vec{v}_0\frac{a_0{}'}{c^2}u - (1-\gamma)\frac{v_0 a_0{}'\cos\theta'}{v_0{}^2}\vec{v}_0] \qquad (9)$$

If we calculate Eq(9),

$$a^2 = \vec{a}\cdot\vec{a} = \frac{1}{\gamma^2(1+\frac{v_0 u\cos\theta'}{c^2})^2}[a_0{}'^2 + \gamma^2 v_0^2\frac{a_0{}'^2}{c^4}u^2 + (1-\gamma)^2 a_0{}'^2\cos^2\theta'$$

$$+2a_0{}'^2\gamma v_0\cos\theta'\frac{u}{c^2} - 2\gamma v_0\frac{a_0{}'^2}{c^2}u(1-\gamma)\cos\theta' - 2(1-\gamma)a_0{}'^2\cos^2\theta']$$

$$= \frac{1}{(1+\frac{v_0 u}{c^2}\cos\theta')^2}a_0{}'^2[\frac{1}{\gamma^2} + (\frac{v_0{}^2 u^2}{c^4} + 2\frac{v_0 u}{c^2}\cos\theta' + \frac{v_0{}^2}{c^2}\cos^2\theta')] \qquad (10)$$

Finally, the relation of inertial frames' accelerations is

$$a = \frac{\sqrt{(1-v_0{}^2/c^2) + (\frac{v_0{}^2u^2}{c^4} + 2\frac{v_0 u}{c^2}\cos\theta' + \frac{v_0{}^2}{c^2}\cos^2\theta')}}{(1+\frac{v_0 u}{c^2}\cos\theta')}a_0{}' \qquad (11)$$

In special case, if $\theta' = 0$ in Eq(11), [1]

$$a = a_0{}' = \frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) = a_0 = \frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}), \quad \vec{V} = \frac{d\vec{x}}{dt}, \vec{u} = \frac{d\vec{x}'}{dt'} \qquad (12)$$

Eq(11) is the general state of Eq(12).

According to the relativity principle, if we use $-v_0, V, \theta$ instead of v_0, u, θ' in Eq(11), we can obtain the next relation of inertial frames' accelerations.

$$a' = \frac{\sqrt{(1-v_0{}^2/c^2) + (\frac{v_0{}^2V^2}{c^4} - 2\frac{v_0 V}{c^2}\cos\theta + \frac{v_0{}^2}{c^2}\cos^2\theta)}}{(1-\frac{v_0 V}{c^2}\cos\theta)}a_0 \qquad (13)$$

In this time, the constant acceleration a_0 is

$$a_0 = \frac{d}{dt}\left(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}\right) \ , \ \vec{v}_0 \cdot \vec{V} = v_0 V \cos\theta \quad , \quad \vec{v}_0 \cdot \vec{a}_0 = v_0 a_0 \cos\theta \qquad (14)$$

Additional special case, if $\theta = 0$ in Eq(13), [1]

$$a = a_0 = \frac{d}{dt}\left(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}\right) = a_0' = \frac{d}{dt'}\left(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\right), \quad \vec{V} = \frac{d\vec{x}}{dt}, \vec{u} = \frac{d\vec{x}'}{dt'} \qquad (15)$$

3.Conclusion

We can understand general state of the relation of inertial frames' accelerations in this theory.

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